

The Modeling of Velocity Enhancement in Polymer Flooding

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(Received: 27 February 1996; in final form: 23 July 1996)

Abstract. In single-phase polymer flooding experiments it has repeatedly been observed that the average velocity of the polymer molecules is higher than the average velocity of the water molecules. This effect is incorporated in many conventional Enhanced Oil Recovery (EOR) simulators by the introduction of a constant velocity enhancement factor. In this paper we show that, in absence of dispersion, a constant enhancement factor in the mathematical model for two-phase polymer flow (Buckley–Leverett displacement) leads to ill-posedness of the model equations. We propose a saturation dependent enhancement factor, derived from a model based on percolation concepts, for which this problem does not occur.

Key words: polymer flooding, velocity enhancement, inaccessible pore volume, excluded pore volume, loss of hyperbolicity, ill-posed model equations, polymer accumulation.

1. Introduction

Polymer flooding is an Enhanced Oil Recovery (EOR) process. Water-soluble polymer is added to the flood water to improve the mobility ratio in the flooding process. When a mixture of polymer and tracered brine is injected into a core the breakthrough of polymer molecules at the production end of the core is significantly earlier than the breakthrough of the tracer molecules (see Sorbie *et al.*, 1987). This effect is called velocity enhancement. In colloid chemistry this phenomenon is known as hydrodynamic chromatography (see Small, 1974). A common explanation for this effect is that not all of the pore space is available to the polymer molecules. In the porous medium there exists inaccessible pore volume (IPV) and excluded pore volume (EPV). Inaccessible pores are pores into which the large polymer molecules cannot enter. Excluded pore volume is that part of the pore volume, near the pore wall, that cannot contain the centers of mass of the polymer molecules due to their size. Polymer molecules tend to flow in the center of the pores where the velocity is highest according to the parabolic Poiseuille velocity profile. A similar effect occurs for non-Poiseuille, i.e., non-Newtonian flow conditions. Teeuw and Hesseling (1980) consider EPV as the important cause of the velocity enhancement effect. Also Sorbie (1991) mentions the EPV effect as the

dominating effect for porous materials with large representative pore sizes. However, the IPV effect is not negligible. In this paper we restrict ourselves to the IPV effect, because it is easier to model.

The velocity enhancement factor is defined as the ratio α of the interstitial velocity v_p of the polymer component and the interstitial velocity v_w of the water phase (see also Gilham and MacMillan, 1987). However, the concept of velocity enhancement is essentially based on single-phase experiments. In two-phase flow models it is usually assumed that the velocity enhancement is constant, independent of the water saturation s_w , or the Darcy velocity u_w of the water phase. In this conventional model (see Pope, 1980) the velocity enhancement factor α is thus given by the ratio of the porosity ϕ over the effective porosity ϕ_p for the polymer. This constant factor α can be measured in single-phase experiments. However, we show that the mathematical model for two-phase flow with a constant velocity enhancement factor α is ill-posed, when dispersive terms are ignored. In frontal displacement problems there is an unbounded peak in the polymer concentration. Dispersive terms keep the concentration peak finite. This peak has not been noticed before due to the large numerical dispersion in simulators, unless an unpractically large number of grid cells has been used.

For the IPV effect we introduce a percolation type model, that does not lead to ill-posed equations. The basic idea is that the smallest pores form isolated water-filled clusters that do not contain the large polymer molecules. In water-wet media water prefers to enter the pores in the sequence of increasing radius. This means that it is possible to define a minimum radius r^* that must be exceeded before polymer can enter the porous medium. The water saturation needed to fulfill this condition is called the threshold saturation s^* . The enhancement factor α is no longer a constant but depends on the water saturation s_w and this threshold saturation s^* . If IPV is the only cause of velocity enhancement, the threshold saturation s^* can be determined from single-phase experiments.

2. Conventional Description

2.1. PHYSICAL MODEL

Major assumptions necessary to model the polymer flooding process are

- (1) The flow is one-dimensional.
- (2) Both phases are incompressible, and Darcy's Law is valid for both phases.
- (3) The polymer solution shows Newtonian behavior, i.e., the velocities are low.
- (4) Polymer dispersion, gravity and capillary forces are disregarded.
- (5) The reservoir is homogeneous and water-wet.
- (6) Adsorption to the porous medium is an increasing function of the polymer concentration.

Most of these assumptions are weakened easily. Here we focus on concepts and

understanding, and therefore disregard factors which only lead to unnecessary complexity.

2.2. MODEL EQUATIONS

According to Pope (1980) the conservation equations for water and the dissolved polymer are

$$\phi \frac{\partial s_w}{\partial t} + u \frac{\partial f_w}{\partial x} = 0, \quad (1)$$

$$\phi \frac{\partial c s_w}{\partial t} + \phi \frac{\partial a}{\partial t} + \alpha u \frac{\partial c f_w}{\partial x} = 0, \quad (2)$$

where s_w is the water saturation, c the polymer concentration in the water phase, u the total velocity, $f_w(s_w, c)$ the fractional flow function for water, α the velocity enhancement factor,

$$\alpha = \frac{v_p}{v_w} = \frac{u_w/\phi_p}{u_w/\phi} = \frac{\phi}{\phi_p} \quad (3)$$

and $a(c)$ the concentration of polymer adsorbed to the porous medium. Here we ignore the influence of ϕ_p on the adsorbed concentration (see, however, Pope (1980)), and assume that the adsorption term is a function of the polymer concentration only, such that

$$\frac{da}{dc} \geq 0. \quad (4)$$

The fractional flow f_w for water is a function of both the water saturation and the polymer concentration, such that

$$\frac{\partial f_w}{\partial c} < 0 \quad \text{and} \quad \frac{\partial f_w}{\partial s_w} \geq 0. \quad (5)$$

The conservation Equation (2) for the polymer is only valid under the condition that α is constant.

2.3. ANALYSIS OF THE EQUATIONS

To analyze this system of equations, we write it in semi-linear form. That is, after expanding the derivatives, we multiply Equation (1) by c and then subtract this from Equation (2). After some rearrangements this results in the following system of equations

$$\frac{\phi}{u} \frac{\partial s_w}{\partial t} + \frac{\partial f_w}{\partial s_w} \frac{\partial s_w}{\partial x} + \frac{\partial f_w}{\partial c} \frac{\partial c}{\partial x} = 0, \quad (6)$$

$$\frac{\phi}{u} \frac{\partial t}{\partial t} + \frac{(\alpha - 1)c \frac{\partial f_w}{\partial s_w}}{s_w + \frac{da}{dc}} \frac{\partial s_w}{\partial x} + \frac{\alpha f_w + (\alpha - 1)c \frac{\partial f_w}{\partial c}}{s_w + \frac{da}{dc}} \frac{\partial c}{\partial x} = 0. \quad (7)$$

This can be written in matrix notation as

$$\frac{\partial}{\partial t} \begin{pmatrix} s_w \\ c \end{pmatrix} + \mathcal{A} \frac{\partial}{\partial x} \begin{pmatrix} s_w \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad (8)$$

where the matrix \mathcal{A} is defined by

$$\mathcal{A} = \frac{u}{\phi} \begin{pmatrix} \frac{\partial f_w}{\partial s_w} & \frac{\partial f_w}{\partial c} \\ \frac{(\alpha - 1)c \frac{\partial f_w}{\partial s_w}}{s_w + \frac{da}{dc}} & \frac{\alpha f_w + (\alpha - 1)c \frac{\partial f_w}{\partial c}}{s_w + \frac{da}{dc}} \end{pmatrix}. \quad (9)$$

The system of conservation laws is called hyperbolic if the eigenvalues of the matrix \mathcal{A} are real for all values of s_w and c . If for some values of s_w and c the eigenvalues are complex, the system is said to have elliptic regions. A simple Fourier analysis (see, e.g., Bell *et al.* (1986)) shows that the system is linearly unstable in these elliptic regions. This means that the system of equations is ill-posed.

The eigenvalues λ of the matrix \mathcal{A} are given by the quadratic equation

$$\det(\mathcal{A} - \lambda \mathcal{I}) = 0, \quad (10)$$

where \mathcal{I} denotes the identity matrix. They are real if the discriminant D of that quadratic equation is nonnegative. The discriminant is given by

$$D = \left(\frac{u}{\phi}\right)^2 \left[\left(\frac{\partial f_w}{\partial s_w} - \frac{\alpha f_w + (\alpha - 1)c \frac{\partial f_w}{\partial c}}{s_w + \frac{da}{dc}} \right)^2 + 4 \left(\frac{\partial f_w}{\partial c} \frac{(\alpha - 1)c \frac{\partial f_w}{\partial s_w}}{s_w + \frac{da}{dc}} \right) \right]. \quad (11)$$

Let us first consider the special case that $\alpha = 1$, so no velocity enhancement. From fractional flow theory (see Pope (1980)) it is known that the velocity and the

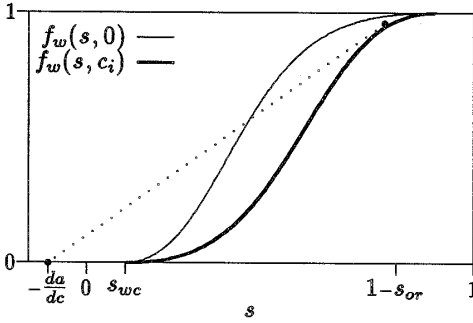


Figure 1. Fractional flow curves for water ($c = 0$) and polymer-water ($c = c_i$). The dotted line shows the construction for the polymer-oil shock.

saturations at the polymer-oil front are found by equating the shock velocity to the Buckley–Leverett velocity for polymer-solution/oil flow

$$\frac{\partial f_w}{\partial s_w} = \frac{f_w}{s_w + \frac{da}{dc}}. \tag{12}$$

This is graphically shown in Figure 1. This means that at the polymer-oil front the discriminant D is zero for $\alpha = 1$ (see (11)), and thus that the two eigenvalues are equal. Let H be the set of all pairs (s_w, c) such that the two eigenvalues of the quadratic Equation (10) are equal for $\alpha = 1$, so

$$H = \left\{ (s_w, c) \left| \frac{\partial f_w}{\partial s_w} = \frac{f_w}{s_w + \frac{da}{dc}} \right. \right\}. \tag{13}$$

We now show that for $\alpha > 1$ the system of conservation laws is, in general, no longer hyperbolic. Let $\alpha = 1 + \varepsilon$, where $\varepsilon > 0$ and sufficiently small. The discriminant D then becomes

$$D = \left(\frac{u}{\phi}\right)^2 \left[\left(\frac{\partial f_w}{\partial s_w} - \frac{f_w + \varepsilon f_w + \varepsilon c \frac{\partial f_w}{\partial c}}{s_w + \frac{da}{dc}} \right)^2 + 4 \frac{\partial f_w}{\partial c} \frac{\varepsilon c \frac{\partial f_w}{\partial s_w}}{s_w + \frac{da}{dc}} \right]. \tag{14}$$

For $(s_w, c) \in H$ this simplifies to

$$D = \left(\frac{u}{\phi}\right)^2 \left[\frac{\varepsilon}{s_w + \frac{da}{dc}} \left(\frac{\varepsilon}{s_w + \frac{da}{dc}} \left(c \frac{\partial f_w}{\partial c} + f_w \right)^2 + 4c \frac{\partial f_w}{\partial c} \frac{\partial f_w}{\partial s_w} \right) \right]. \tag{15}$$

Table I. Values for parameters in numerical examples.

Parameter	Value	Unit	Meaning
L	1	m	Length of core
ϕ	0.3	–	Porosity
u	10^{-6}	m/s	Injection rate
s_{wc}	0.1	–	Connate water saturation
s_{or}	0.1	–	Residual oil saturation
$\mu_w(0)$	1	cP	Water viscosity
μ_o	30	cP	Oil viscosity
c_i	1000	ppm	Polymer concentration of stock solution

Because ε was chosen sufficiently small, the second term on the right-hand side dominates. It follows from (5) that this dominating term is negative, so the discriminant itself is negative. This implies that both eigenvalues are complex, and thus that there exists an elliptic region centered around the set H . The model is ill-posed.

2.4. SIMULATION EXAMPLES

To illustrate the consequences of the elliptic region for the two-phase polymer flow problem, we consider the simple example of the polymer flooding of a linear core. Initially the core is filled with oil and connate water. Polymer-water is then injected into the core. The injected water has a polymer concentration c_i outside the reservoir. Continuity of the water and polymer flux at the injection point implies that the polymer concentration just inside the porous medium is c_i/α (see Ligthelm (1996)). Values for relevant parameter are listed in Table I, moreover we take

$$k_{rw} = \left(\frac{s_w - s_{wc}}{1 - s_{wc} - s_{or}} \right)^2, \quad (16)$$

$$k_{ro} = \left(\frac{1 - s_w - s_{or}}{1 - s_{wc} - s_{or}} \right)^2, \quad (17)$$

$$\mu_w(c) = \mu_w(0)(1 + 5c + (5c)^2). \quad (18)$$

This yields the fractional flow function

$$f_w(s_w, c) = \frac{(s_w - s_{wc})^2}{(s_w - s_{wc})^2 + (1 - s_w - s_{or})^2 \mu_w(c)/\mu_o}. \quad (19)$$

In the first simulation example we consider the case without velocity enhancement, so $\alpha = 1$. The solution of the polymer model Equations (1), (2) is approximated numerically by a second order upwind scheme on a grid with 1000 grid points.

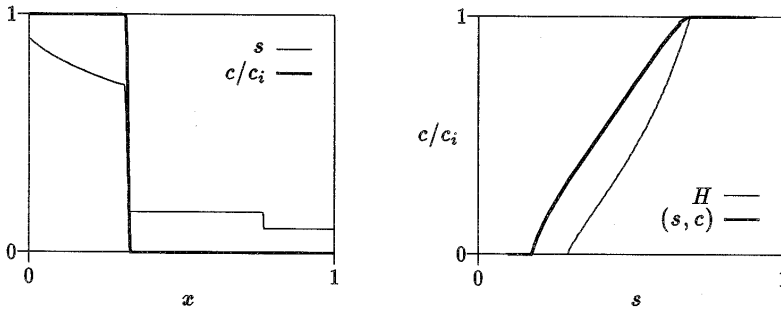


Figure 2. Numerical solutions in x -space (left) and phase-space (right) at $t = 0.25$ PV injected for $\alpha = 1$.

The time step is chosen such that the CFL-number equals 0.1. Figure 2 shows the numerical approximation of the solution both in x -space and in (s_w, c) -space at 0.25 Pore Volume injected. The solution consists of two shocks (see Pope, 1980). The upstream shock separates the polymer-water region from a region without polymer, the so-called water bank, and the downstream shock separates a region with movable water from a region with connate water only. The upstream shock occurs at the eigenvalue degeneracy $(s_w, c) \in H$ (see Allen *et al.*, 1988).

Next we consider the solution of the same problem with a constant velocity enhancement factor $\alpha = 1.1$. Figure 3 shows the numerical solution. There is a peak in the polymer concentration just before it drops to its initial value zero. The phase-space solution plot shows that this rise in the polymer concentration occurs in the neighborhood of the elliptic region. The solution tends to avoid the elliptic region. The numerical solution is not stable with respect to the number of grid points used. If finer meshes are used, the peak in the polymer concentration gets higher. This is another indication of the instability of the system of equations. Notice that for the standard flooding situation the initial condition $(s_w, c) = (s_{wc}, 0)$, and injection values $(s_w, c) = (1 - s_{or}, c_i)$ are separated in the phase plane by the elliptic region (see Figure 3). The solution must always cross the elliptic region, and cannot avoid this region of instability.

3. Percolation Description

3.1. PHYSICAL MODEL

In order to describe the velocity enhancement with a percolation-type approach, we introduce a three-phase concept of polymer flow. These three phases are

- (1) Water which cannot contain polymer (water phase 1, with water saturation s_{w1}).
- (2) Water which may potentially contain polymer (water phase 2, with water saturation s_{w2}).

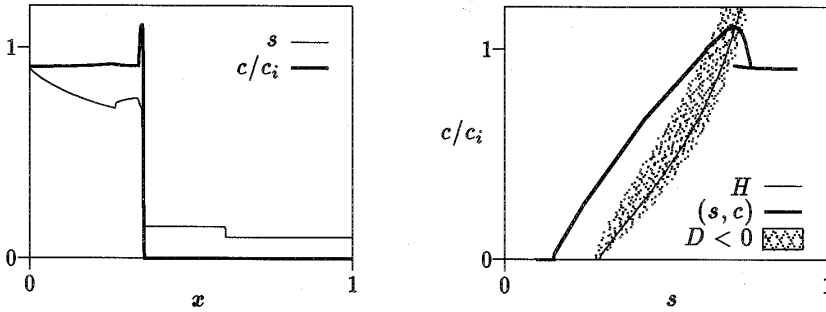


Figure 3. Numerical solutions in x -space (left) and phase-space (right) at $t = 0.25$ PV injected for $\alpha = 1.1$.

(3) Oil without polymer (oil phase, with oil saturation s_o).

Exchange of water molecules between water phase 1 and 2 is possible. Polymer is, however, restricted to remain in water phase 2. The distinction between the two water phases comprises both the EPV and IPV models, because it does not specify whether phase 1 and 2 belong to the same pore or to different pores. There is local equilibrium in the polymer concentration. This means that there is instantaneous diffusion of polymer molecules between the part of the connate water phase that is accessible for polymer molecules and the movable water phase.

In the percolation model only the inaccessible pore volume is taken into account. The basic idea is that the smallest pores, with a radius less than a critical radius r^* , do not contain the large polymer molecules. The underlying assumption is that small pores also have small pore throats giving access to those pores. The small pore throats actually prevent that polymer enters the small pores. We disregard the presence of isolated clusters of small pores (see Bedrikovetsky and Bruining, 1995).

In water-wet media water prefers to enter the pores in a sequence of increasing radius. This means that all pores with radius $r < r^*$ must be filled with water before the polymer can enter the porous medium. The water saturation needed to fulfill this condition is called the threshold saturation s^* . Here we confine our interest to the situation that $s^* < s_{wc}$. The probability density function (see Davis, 1986) shown in Figure 4 illustrates this model. The horizontal axis denotes the pore radius r , and $P(r) dr$ is the fractional volume occupied by these pores. The light grey area ($r < r^*$) denotes the pore space filled with water phase 1, and corresponds to a water saturation $s_{w1} = s^*$. The dark grey area ($r^* < r < r_w$) indicates the pore space saturated with water phase 2, and corresponds to the water saturation $s_{w2} = s_w - s^*$. Finally the black area ($r_w < r$) denotes the pore space filled with oil. These are the biggest pores, where in a water-wet reservoir the oil tends assemble.

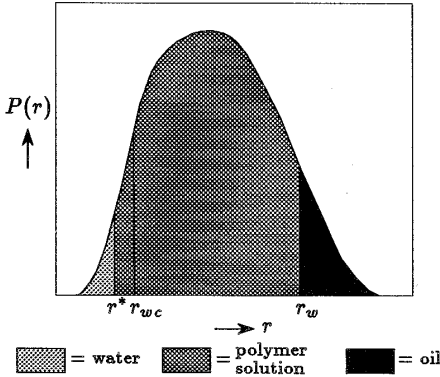


Figure 4. Example of a probability-density function showing the critical radius r^* and the three different phases.

3.2. MODEL EQUATIONS

The conservation equations for water phase 1 (without polymer), water phase 2 (with polymer), and for the dissolved polymer are

$$\phi \frac{\partial s_{w1}}{\partial t} + \frac{\partial u_{w1}}{\partial x} = +R, \tag{20}$$

$$\phi \frac{\partial s_{w2}}{\partial t} + \frac{\partial u_{w2}}{\partial x} = -R, \tag{21}$$

$$\phi \frac{\partial c_{w2} s_{w2}}{\partial t} + \phi \frac{\partial a}{\partial t} + \frac{\partial c_{w2} u_{w2}}{\partial x} = 0, \tag{22}$$

where R denotes the net transfer rate of water from phase 1 to phase 2, and c_{w2} the concentration of polymer dissolved in water phase 2. It is convenient to introduce the concentration c ,

$$c = \frac{c_{w2} s_{w2}}{s_{w1} + s_{w2}}, \tag{23}$$

which is the average polymer concentration in the total water phase. For simplicity we assume that the adsorption a is a function of this average concentration only.

Using the definition (3) of the velocity enhancement factor α , we obtain

$$\alpha = \frac{v_p}{v_w} = \frac{v_{w2}}{v_w} = \frac{u_{w2} s_w}{s_{w2} u_w} = \frac{u_{w2}}{u_{w1} + u_{w2}} \frac{s_{w1} + s_{w2}}{s_{w2}}. \tag{24}$$

From the assumption $s_{w1} = s^* < s_{wc}$ it follows that $\partial s_{w1} / \partial t$ is zero, and that water phase 1 is unable to flow, so

$$u_{w1} = 0. \tag{25}$$

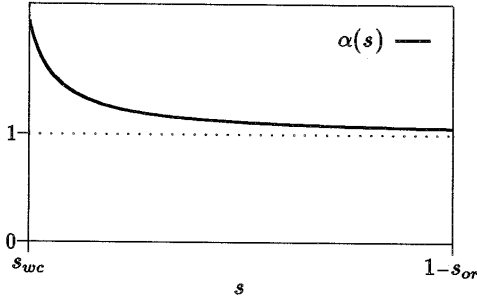


Figure 5. Typical plot of saturation dependent velocity enhancement factor $\alpha(s_w)$.

Hence the net accumulation term R in the conservation law (20) is zero: there is no net water transfer from phase 1 to phase 2. Substituting $s_{w1} = s^*$, $s_{w2} = s_w - s^*$, and (25) in (24), we obtain a saturation dependent enhancement factor,

$$\alpha(s_w) = \frac{s_w}{s_w - s^*}, \quad s^* < s_{wc}. \quad (26)$$

A typical plot of $\alpha(s_w)$ is shown in Figure 5. The velocity enhancement factor is a decreasing function of the water saturation, and $\alpha(s_w) > 1$.

Note that the threshold saturation s^* can be determined easily from one-phase flow experiments. When the elution time of both the polymer and the tracered brine is measured, the enhancement factor α follows directly from (26)

$$s^* = (1 - s_{or}) \frac{\alpha - 1}{\alpha}. \quad (27)$$

We remark that the restriction $s^* < s_{wc}$ is acceptable indeed for practical conditions in two-phase polymer flow. A commonly observed value for the enhancement factor is $\alpha = 1.1$. This results in a threshold saturation of at least 0.091. Because most reservoirs have a connate water saturation larger than 0.1, the restriction $s^* < s_{wc}$ is valid for practical flooding situations.

Using (23), (24) and the definition of the fractional flow function, $u_w = f_w u$, the conservation equations become

$$\phi \frac{\partial s_w}{\partial t} + u \frac{\partial f_w}{\partial x} = 0, \quad (28)$$

$$\phi \frac{\partial c s_w}{\partial t} + \phi \frac{\partial a}{\partial t} + u \frac{\partial \alpha c f_w}{\partial x} = 0. \quad (29)$$

These equations are the same as the conservation Equations (1), (2) stated by Pope, except that in (29) the enhancement factor α is a function of the water saturation.

3.3. ANALYSIS OF THE EQUATIONS

We analyze the equations in the same way as in Section 2.3. Again the linearized equations are written in matrix form

$$\frac{\partial}{\partial t} \begin{pmatrix} s_w \\ c \end{pmatrix} + \mathcal{A} \frac{\partial}{\partial x} \begin{pmatrix} s_w \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad (30)$$

where

$$\mathcal{A} = \frac{u}{\phi} \begin{pmatrix} \frac{\partial f_w}{\partial s_w} & \frac{\partial f_w}{\partial c} \\ \frac{\frac{d\alpha}{ds_w} c f_w + (\alpha - 1) c \frac{\partial f_w}{\partial s_w}}{s_w + \frac{da}{dc}} & \frac{\alpha f_w + (\alpha - 1) c \frac{\partial f_w}{\partial c}}{s_w + \frac{da}{dc}} \end{pmatrix}. \quad (31)$$

The discriminant of the quadratic equation for the eigenvalues is

$$D = \left(\frac{u}{\phi}\right)^2 \left[\left(\frac{\partial f_w}{\partial s_w} - \frac{\alpha f_w + (\alpha - 1) c \frac{\partial f_w}{\partial c}}{s_w + \frac{da}{dc}} \right)^2 + 4 \left(\frac{\partial f_w}{\partial c} \frac{\frac{d\alpha}{ds_w} c f_w + (\alpha - 1) c \frac{\partial f_w}{\partial s_w}}{s_w + \frac{da}{dc}} \right) \right]. \quad (32)$$

We now prove that there is no elliptic region for the saturation dependent enhancement factor (26): the eigenvalues of the matrix in (31) are always real.

One easily verifies from Equation (26) that

$$\frac{d\alpha}{ds_w} = -\frac{\alpha(\alpha - 1)}{s_w}. \quad (33)$$

It is possible to rewrite (32) as

$$\begin{aligned} D &= \left(s_w + \frac{da}{dc}\right)^2 \left(\frac{\phi}{u}\right)^2 \\ &= \left(\frac{\partial f_w}{\partial s_w} \left(s_w + \frac{da}{dc}\right) - \alpha f_w - (\alpha - 1) c \frac{\partial f_w}{\partial c}\right)^2 \\ &\quad + 4 \frac{\partial f_w}{\partial c} \left(s_w + \frac{da}{dc}\right) \left(\frac{d\alpha}{ds_w} c f_w + (\alpha - 1) c \frac{\partial f_w}{\partial s_w}\right). \end{aligned}$$

We expand the quadratic term of this equation and use (33). This results in

$$\begin{aligned}
 D & \left(s_w + \frac{da}{dc} \right)^2 \left(\frac{\phi}{u} \right)^2 \\
 & = \left(\frac{\partial f_w}{\partial s_w} \left(s_w + \frac{da}{dc} \right) \right)^2 + 2 \frac{\partial f_w}{\partial s_w} \left(s_w + \frac{da}{dc} \right) (\alpha - 1)c \frac{\partial f_w}{\partial c} \\
 & \quad - 2 \frac{\partial f_w}{\partial s_w} \left(s_w + \frac{da}{dc} \right) \alpha f_w + \left((\alpha - 1)c \frac{\partial f_w}{\partial c} \right)^2 + 2(\alpha - 1)c \frac{\partial f_w}{\partial c} \alpha f_w \\
 & \quad + (\alpha f_w)^2 - 4 \frac{\partial f_w}{\partial c} \left(s_w + \frac{da}{dc} \right) \frac{\alpha(\alpha - 1)}{s_w} c f_w \\
 & = \left(\frac{\partial f_w}{\partial s_w} \left(s_w + \frac{da}{dc} \right) + (\alpha - 1)c \frac{\partial f_w}{\partial c} - \alpha f_w \right)^2 \\
 & \quad - 4\alpha(\alpha - 1)c f_w \frac{\partial f_w}{\partial c} \frac{da}{dc}.
 \end{aligned}$$

It follows from (4), (5) and $\alpha(s_w) > 1$ that the last term on the right-hand side is nonnegative, therefore the discriminant D is always nonnegative. Hence the eigenvalues of \mathcal{A} are always real. This means that the system of conservation laws is hyperbolic for every combination of s_w and c under the condition that $s^* < s_{wc}$.

3.4. SIMULATION EXAMPLE

We reconsider the test problem of Section 2.4, but now with the saturation dependent velocity enhancement term (26) according to the percolation model. We assume that the single-phase enhancement factor is 1.1, so $s^* \approx 0.082 < s_{wc}$ (see (27)). Figure 6 shows the numerical solution for this problem. The solution again consist of two shocks. The polymer concentration is monotone, and there is no accumulation of polymer at the upstream shock front.

4. Discussion

We have shown that the model equations are ill-posed for a constant enhancement factor. Therefore these equations are nonphysical. In spite of this, a constant enhancement factor is often used in computer simulations, and solutions are obtained. A numerical solution of the equations includes numerical dispersion, which depends on the grid size. The system of equations with a constant enhancement factor and a dispersion term is well-posed. In other words, if a constant enhancement factor is introduced in the simulator, the solution depends on the grid spacing and does not converge to the solution of the model equations in the limit that the grid spacing tends to zero. Essentially an important model feature is

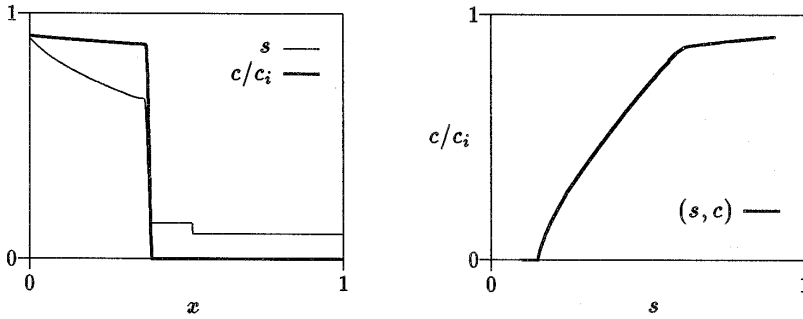


Figure 6. Numerical solutions in x -space (left) and phase-space (right) at $t = 0.25$ PV injected for $\alpha(s_w) = s_w / (s_w - s^*)$, with $\alpha(1 - s_{or}) = 1.1$.

discarded in the numerical simulation, being the unlimited pile-up of polymer at the polymer-oil shock. From the physical point of view the constant enhancement factor model is not based on any clear concept. It is just a convenient mathematical means to introduce the speed-up of the polymer molecules.

In the case that there is no polymer adsorption to the porous medium and zero connate water saturation, we can consider the capillary bundle model to elucidate the pitfall in the constant enhancement model from the physical point of view. The capillary model for two-phase flow is in some sense contradictory. Interfacial tension prevents the interface between oil and water from stretching out according to the parabolic velocity profile. Indeed near the interface we do not longer have a parabolic velocity profile (see Chun Huh and Scriven, 1971). If the idea of a parabolic velocity profile near the interface is maintained, polymer would move out of the water phase due to the velocity enhancement effect. However, polymer does not dissolve in the oil phase and consequently accumulation of polymer near the oil-water interface, the so-called pile-up effect, should occur. In the literature we do not find any experimental evidence for a pile-up effect. However, only a few core experiments of polymer solutions displacing oil have been reported (Jennings *et al.*, 1971; Patton *et al.*, 1971; Teeuw and Hesseling, 1980). None of these studies had the aim to investigate the existence of any accumulation effect.

The situation is more difficult to understand from the physical point of view, if retardation effects are taken into account. Diffusion of polymer molecules into the connate water and polymer adsorption are such retardation effects. Our analysis and the numerical simulations show that in that case also accumulation of polymer occurs, upstream of the water bank devoid of polymer (see Figure 3). Irrespective of any retardation effects, the equations used in classical EOR simulators are not adequate.

It was our aim to develop a velocity enhancement model based on a physical concept. Indeed the percolation model is based on such a clear concept. It leads to a new velocity enhancement factor, that describes the IPV effect. The model equations are well-posed provided that the threshold saturation s^* is smaller than

the connate water saturation s_{wc} . Our new enhancement factor is preferred for use in simulators, because it does not force the simulator to solve ill-posed equations. This is not to say that the EPV effect, or other mechanisms that may lead to a pile-up effect of polymer, should not be considered. However, this aspect needs to be left for future work.

5. Conclusions

The conventional treatment of polymer velocity enhancement in two-phase flow leads to ill-posed model equations when capillary dispersion is not present. A constant enhancement factor results in the accumulation of polymer near the polymer-oil shock, irrespective of any retardation effects. We have derived a percolation model for the IPV velocity enhancement effect. In this model the velocity enhancement factor is a function of the water saturation. The new model does not have the stability problems inherent to models with a constant enhancement factor.

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