Addressing Geothermal Applications in the BSc Course "Physical Transport Phenomena"

For over 30 years, at two universities, I have taught BSc and graduate courses in physical transport phenomena. TU Delft has a distinguished history in teaching this subject. In the US the standard textbook is by professors Bird, Stewart and Lightfoot, first published in 1960. In 1958 Prof. Bird spend a sabbatical year at TU Delft, and was impressed by how the subject was taught here. He acknowledges his debt to that experience in the preface to the 1960 edition of their book.

In the Applied Earth Sciences BSc, Physical Transport Phenomena covers elementary fluid mechanics, heat transfer and chemical diffusion processes. I love teaching this course because it has applications in all aspects of our professional work and also in the kitchen and garage. In some versions of the course students compose, and then solve, their own problems using the tools of the course. This stimulates students to see the varied connections of the course to many aspects of their experience. The submissions have been a joy to see. For instance:

- How wide was the crack in the hull that sank the *Titanic*? (I incorporate that into our weekly homework now. Just last year I realized our solution left out important factors and needed to be modified.)
- Why does the ice on the top of a glacier move faster than ice on the bottom?
- Consider the art exhibit "Peanut Butter Floor" then on display in Rotterdam. If one tipped the floor, would the peanut butter slide down, or not?
- If one cooks goose eggs, that are some given factor larger than chicken eggs, in boiling water, how much longer should you cook them?

The methods we learn are elementary, and in most cases we have to make simplifying assumptions that do not apply in the real world. We learn ways to represent individual steps in heat transfer: for instance, convective heat transfer between water flowing through a well and the pipe wall, conduction through the pipe wall, and unsteady heating or cooling of the rock surrounding the well.

The most basic concept of the course is the "shell balance:" we define a small region of space, and then consider conservation of mass (or momentum, or energy, or charge) within the region: what comes in across the boundaries, minus what goes out, plus what is created inside, minus what is destroyed inside, equals the change in the property. We then let the region shrink to zero size and get a differential equation. I took my first course in transport phenomena was in my third year of college, and this exercise was a revelation to me: "Oh, so *that's* what Newton was thinking when he invented calculus!"

In the absence of a complete solution, we learn how to pick the best approximate answer among those that focus on the individual steps. The principle is similar to what we learn in secondary-school physics about electrical resistors in series or in parallel. If the processes are "in series" as in the example just given, focus on the biggest resistance. Solve the simplified solution for each step in the process. The best approximate answer is the one with the slowest heat transfer. If it is *much* slower than the other answers, your solution (which assumes it is the *only* process that matters) is probably a very good approximation.

If the processes are "in parallel," where heat can move by several independent means, then focus on the path that offers the least resistance. Cooling of a cup of coffee can be by evaporation or heat conduction through the cup wall. If evaporation is the fastest step, focus on that process.

In recent years the GSE Department has greatly expanded its research and education in geothermal engineering. I have tried to incorporate applications of our tools to those problems into the class. Here are some examples:

- How fast does water in the pores of rock come to the same temperature as the grains of rock? The answer is striking: within about 0.02 sec., while the fluid travels about 0.7 microns. Figure 1 shows the lecture notes where this is covered. As the flow path gets wider and velocity increases, distances and times increase. Water traveling at 1 m/s through a fracture 2 cm wide would flow about 1.7 m (taking less than 2 sec.) before reaching 2/3 of the way to the temperature of the rock wall.
- In lecture we model the advance of a cold-water front through a homogeneous geothermal formation. There are two lessons: the temperature front advances more slowly than the water itself, and most of the heat we harvest was originally in the rock, not in the water.

We study unsteady heat conduction in solids using dimensionless variables and charts from Carslaw and Jaegar's classic 1949 text *Conduction of Heat in Solids*. The same charts apply to unsteady conduction, unsteady diffusion, unsteady electrical current, and Darcy flow through geological formations. (At Chevron, the company where I first worked when I came out of graduate school, their first reservoir simulator had been a large "analog computer" mounted on a lab wall. Charged capacitors represented pressure in the reservoir and electrical connections the flow paths in the reservoir. I have speculated that the whole thing must have discharged in a matter of seconds, representing depressurization of the reservoir over years.)

For heat conduction within finite-size bodies, equilibration time scales like the square of the dimension of the solid. The "product method" for unsteady conduction allows us to extend the solution for a finite-width slab to rectangular-column matrix blocks in fractured reservoirs. (My geology colleagues have finally convinced me not to model cubic blocks of matrix: horizontal fractures are relatively rare.) Some specific applications:

- Heat loss from a geothermal well into the overburden rock surrounding the well. The result was a surprise to me. I assumed that insulation would be needed in the well, but the slow, controlling step is radial heat conduction into the surrounding formation. A lot of heat is lost in for the first day or so, but soon the region immediately around the well is warmed and the rate of subsequent heat conduction drops greatly. Within about 7 hr of hot water flowing up the well this radial conduction process becomes the rate-limiting step. Figure 2 shows the exam where I and the students worked out this result.
- Heat conduction into a geothermal reservoir from overburden and underburden rock. Once a particular location reaches a colder temperature, the heat-transfer rate at the boundary scales like the square root of time starting at that moment.
- Heating from impermeable layers interspersed with permeable layers in a geothermal reservoir. Their contribution is significant, and it both delays and spreads out the cold-temperature front. The time to come to equilibrium with the adjacent permeable layers scales like the square of the layer thickness.
- Using superposition we can represent the temperature of formation surrounding a heat-storage well as cold water is pumped down in the winter and warmer water in the summer. we can also estimate the amount of heat expelled or harvested from the well in the respective seasons. Figure 3 shows an exam problem where this process is addressed.

Teaching this course has been a joy and a learning experience. I hope to communicate to students both the usefulness and fascination of the subject and its applications in may aspects of daily and professional life.

Figure 1.

Examples of Heat Transfer in Tubes

1) Heat transfer to pore walls

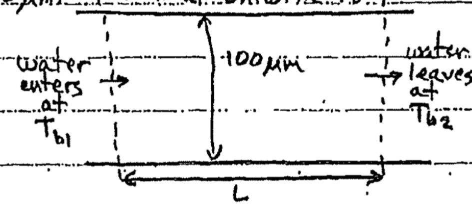
Assume pore is circular tube, $D = 100\,\mu m$. Wall at uniform T_0.

Assume vel. = 10 ft/d = $3.5 \cdot 10^{-5}$ m/s

Assume $\rho = 1000$ kg/m³

$\mu = 0.001$ Pa·s $\hat{C}_p = 4190$ J/kg

$K = 0.67$ W/m K.



What is L for $\frac{T_{02} - T_{b2}}{T_{01} - T_{b1}} = 0.01$? (99% of way to equilibrium)

Solution: $Re = \frac{D v \rho}{\mu} = \frac{(10^{-4})(3.5 \cdot 10^{-5})(1000)}{(0.001)} = 0.0035 \ll 2100$

$\frac{T_{02} - T_{b2}}{T_{01} - T_{b1}} < 0.2$; ∴ cannot use Fig. 14.3-2 or Eq. 14.3-17

Use Fig 14.2-1 "const. wall T (tube)" and Eq. "III".

Eq. III : $Nu_{ln} = \frac{h_{ln} D}{K} = \frac{T_{b2} - T_{b1}}{(T_0 - T_b)_{ln}} Re\,Pr \frac{D}{4L}$ **OR Eq. 3.122**

Since T_0 is uniform, $\frac{T_{b2} - T_{b1}}{(T_0 - T_b)_{ln}} = \ln\left(\frac{T_0 - T_{b1}}{T_0 - T_{b2}}\right) = \ln(100)$

$Re = 3.5 \cdot 10^{-5}$; $Pr = \frac{\hat{C}_p \mu}{K} = (4190)(0.001)/(0.67) = 6.25$

Eq. III → $Nu_{ln} = \ln(100)(0.0035)(6.25) \frac{10^{-4}}{4L} = 2.52 \cdot 10^{-6}/L$

Fig 14.2-1 without knowing L, we need some sort of trial + error here. Note if $\frac{L}{D\,Re\,Pr} > 0.3$, $Nu_{ln} \approx 3.657$. First assume $\frac{L}{D\,Re\,Pr} > 0.3$, then check at end.

If $Nu_{ln} = 3.657 = 2.52 \cdot 10^{-6}/L$, $L = 6.9 \cdot 10^{-7}$ m

Check: $\frac{L}{D\,Re\,Pr} = \frac{6.9 \cdot 10^{-7}}{(10^{-4})(3.5 \cdot 10^{-3})(6.25)} = 0.31 > 0.3$ Assumption verified.

Note that time for equilibration is $\frac{L}{\langle v \rangle} = \frac{6.9 \cdot 10^{-7}}{3.5 \cdot 10^{-5}} = 0.02$ s.

Nearly istantaneous equilibration. Why? Small D.

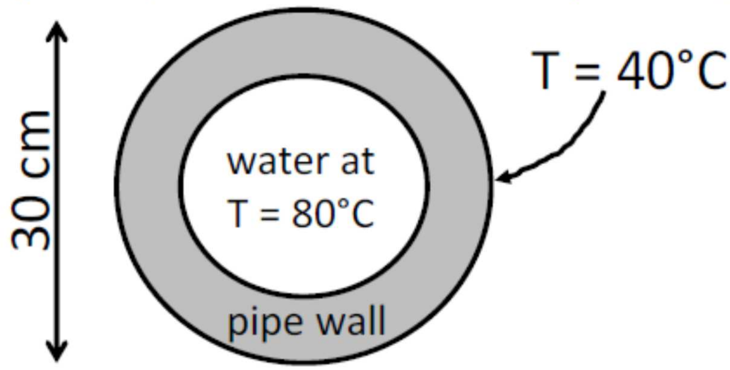
Figure 2

4. In a given geothermal project, engineers are concerned about losing heat from the hot water as it flows up through the well through the thick layer of rock above the geothermal formation. In one case, hot water flows through a 20-cm inner diameter pipe (30 cm outer diameter) cemented into the rock. Water flows at a rate 1 m^3/s. The properties of water are given in problem 2, and the properties of the pipe wall are given below. Suppose water is at 80°C and the rock on the other side of the plastic pipe is at a temperature of 40°C. What is the rate of heat loss from the water per m length of pipe?

(25 pts)

<u>properties of pipe wall</u>

$\rho = 1300\ kg/m^3$ $k = 0.30\ W/(m\ K)$ $C_p = 1950\ J/(kg\ K)$



5. Consider the same issue as in problem 4, but now from the point of view of heat conduction into the rock. Assume in this case that there is no resistance to heat transfer between the water and the pipe, or from conduction through the pipe. Thus the outer surface of the pipe is maintained at 80°C starting at t=0. How long would it take before the rate of heat transfer to the rock (per m length of pipe) would be less than that you calculated in part (a)? If you were unable to complete problem 4, assume a value of 500 W/m as the answer to problem 4.

(15 pts)

<u>properties of rock</u>

$k = 2.61\ W/(m\ K)$ $\rho = 2270\ kg/m^3$ $C_p = 999\ J/(kg\ K)$

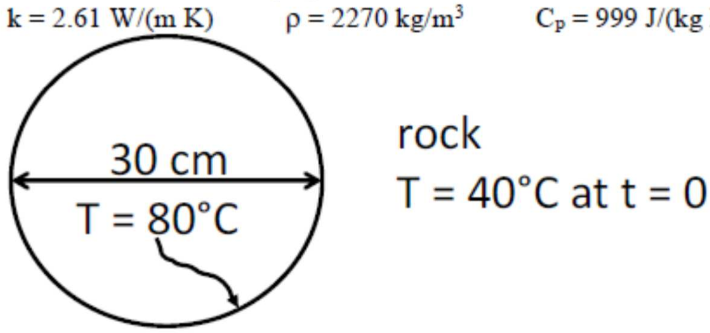


Figure 3

2. A scheme for heat storage into the subsurface involves pumping hot water through a loop of piping underground during the summer and then cold water in the winter, to extract the heat that was stored in the surrounding ground during the summer. For the purposes of this problem, simplify the description as follows:

   A solid (the subsurface) with properties below extends very far (infinitely) in the radial direction, and is 200 m thick. It is perfectly insulated on top and bottom surfaces. Assume a cylindrical, vertical pipe penetrates the subsurface; the pipe has a diameter of 30 cm. The subsurface is initially at a temperature of 10°C. Starting at time t = 0, we pump hot water through the pipe, such that the inner surface of the formation around the pipe is brought to and maintained at 40°C. Starting 180 days later, the surface of the pipe is brought to and maintained at 10°C.

   a. What is the temperature of the formation at a distance of 1 m from the center of the well at day 179, just before the temperature is reduced on day 180?
   b. What is the temperature of the formation at a distance of 1 m from the center of the well at day 210, 30 days after the temperature is reduced on day 180?

   (30 points)

Properties of formation

$\rho = 2270$ kg/m^3 $C_p = 1000$ J/(kg K) $k = 2.61$ W/(m K)

Cross-section view