Modeling Traffic Accidents Caused by Random Misperception

Marcel Kleiber

Institute of Probability and Statistics & House of Insurance

Leibniz Universität Hannover

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The talk is based on joint work with Volker Berkhahn, Chris Schiermeyer, & Stefan Weber.

Outline

1. Introduction
2. The Traffic Model
3. Case Studies
4. Conclusion

Outline

1 Introduction

2 The Traffic Model

3 Case Studies

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Introduction

- **Classical Approach in Motor Insurance**
  - Determine probability of traffic accidents on the basis of empirical data.
- **Disruption of Mobility**
  - Automotive industry is undergoing a massive disruption with the appearance of autonomously driving vehicles (cf., e.g., Bertoncello and Wee (2015)).
  - Existing studies (e.g., Blanco et al. (2016)) indicate that the number of accidents will significantly be reduced when vehicles are controlled by computers.
- **Overcome Lack of Data**
  - Get insight into the occurrence of accidents and their effects on traffic flow by a simulation based approach (cf. Berkhahn et al. (2018)).

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The Traffic Model

Idea

- **Notation**
  - $x^i(t)$: Position of the i-th vehicle at time t, $v^i(t)$: Momentary speed
  - $\Delta x^i(t)$: Distance to preceding vehicle, $\Delta v^i(t)$: Approaching rate
- **Classical Car-Following Model in Stylized Form**

$$\begin{cases} \frac{\mathrm{d}x^i(t)}{\mathrm{d}t} = v^i(t), \\ \frac{\mathrm{d}v^i(t)}{\mathrm{d}t} = f(v^i(t), \Delta x^i(t), \Delta v^i(t), \dots). \end{cases} \tag{1}$$

- **Car-Following Model with Random Misperception**
  - Let $(\varepsilon_t^{i,1})$, $(\varepsilon_t^{i,2})$ and $(\varepsilon_t^{i,3})$ be three stochastic processes, fluctuating around 1.

$$\begin{cases} \frac{\mathrm{d}x^i(t)}{\mathrm{d}t} = v^i(t), \\ \frac{\mathrm{d}v^i(t)}{\mathrm{d}t} = f(\varepsilon_t^{i,1} v^i(t), \varepsilon_t^{i,2} \Delta x^i(t), \varepsilon_t^{i,3} \Delta v^i(t), \dots). \end{cases} \tag{2}$$

The Traffic Model

Mathematical Foundation

Definition (Random Ordinary Differential Equation (RODE))

Let $(\varepsilon_t)_{t\geq 0}$ be a stochastic process on some probability space $(\Omega, \mathcal{F}, P)$ with values in $\mathbb{R}^m$ and continuous paths. Suppose that $f : \mathbb{R}^d \times \mathbb{R}^m \to \mathbb{R}^d$ is continuous. A *random ordinary differential equation* in $\mathbb{R}^d$ for some function $y : [0, \infty) \to \mathbb{R}^d$ is given by

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y, \varepsilon_t).$$

- For each scenario $\omega \in \Omega$, a RODE defines a non-autonomous ordinary differential equation via

$$\frac{\mathrm{d}y}{\mathrm{d}t} = F_\omega(t, y) := f(y, \varepsilon_t(\omega)).$$

- Pathwise RODEs are ODEs $\Rightarrow$ Standard numerical methods for ODEs can be used to solve RODEs.

The Traffic Model

Movement of Vehicles

Let $\mathcal{M} := \{1, 2, \dots\}$ denote the collection of vehicles. Let $(\varepsilon_t^{i,1})_{t\geq 0},\ (\varepsilon_t^{i,2})_{t\geq 0},\ (\varepsilon_t^{i,3})_{t\geq 0}$, $i \in \mathcal{M}$, be stochastic processes with continuous paths.

Intelligent Driver Model with Random Misperception (IDMrm):

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}t} x^i(t) = \max\{v^i(t), 0\}, \\ \frac{\mathrm{d}}{\mathrm{d}t} v^i(t) = \max\Bigg\{ a_{\max}^i \cdot \Bigg(1 - \left(\frac{\varepsilon_t^{i,1} v^i(t)}{v_{\mathrm{d}}^i} \right)^\delta \\ \qquad - \left(\frac{s^*(\varepsilon_t^{i,1} v^i(t), \varepsilon_t^{i,1} v^i(t) - \varepsilon_t^{i,3} v^{i-1}(t))}{\varepsilon_t^{i,2} \Delta x^i(t)} \right)^2 \Bigg), a_{\min}^i \Bigg\}, \\ x^i(t_0^i) = 0,\ v^i(t_0^i) = v_0^i,\ t \geq t_0^i,\ i \in \mathcal{M} \end{cases}$$

The Traffic Model

Accidents

- **The Event of an Accident**
  - For $i \in \mathcal{M}$, let $A^i(t)$ denote the area of the road which is occupied by vehicle i at time $t > 0$.
  - An accident occurs, if

$$\exists\ i, j \in \mathcal{M},\ i \neq j,\ \exists\ t > 0 \colon A^i(t) \cap A^j(t) \neq \emptyset.$$

- **Effects of an Accident in the Model**
  1. If two vehicles collide, their velocities are set to 0.
  2. At the time of the first collision, a waiting time

$$t_{\text{removal}} \sim \text{Exp}(\gamma), \quad \gamma > 0,$$

is triggered. After this time, all collided vehicles (in this accident) are removed from the model.

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Case Studies

Measures of Efficiency and Safety

1. **Measure of Efficiency**
   Traffic flow per time unit, measured at position d:

$$Q = \frac{\operatorname{card}\{j \in \mathcal{M} :\ \exists\ t \le T_{\text{sim}} : x^j(t) = d\}}{T_{\text{sim}}}.$$

2. **Measure of Safety**
   Number of accidents per time unit: For $M \subseteq \mathcal{M}$ we define $A^M(t) := \bigcup_{i \in M} A^i(t)$.

$$f_{\text{acc}} = \frac{1}{T_{\text{sim}}} \cdot \operatorname{card}\{\emptyset \ne M \subseteq \mathcal{M} : \exists\ t \le T_{\text{sim}}\ \forall\ i \in M : A^i(t) \cap A^{M \setminus \{i\}}(t) \ne \emptyset \text{ and } \forall\ t \le T_{\text{sim}} : A^M(t) \cap A^{M^c}(t) = \emptyset\}.$$

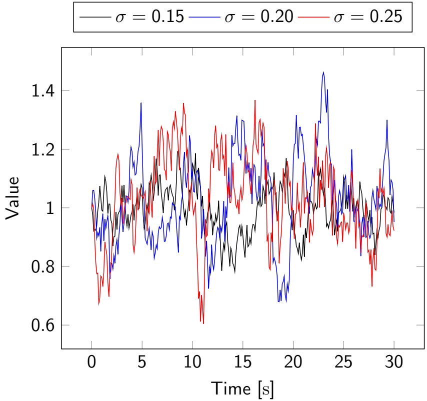
Case Studies

Errors: Ornstein-Uhlenbeck Process

Let $\beta \in \mathbb{R}$ and $\alpha, \sigma > 0$. A stochastic process $(\varepsilon_t)_{t \geq 0}$ is called an Ornstein-Uhlenbeck process, if $\varepsilon_0 = a \in \mathbb{R}$ and $(\varepsilon_t)_{t \geq 0}$ solves the following stochastic differential equation:

$$d\varepsilon_t = \alpha(\beta - \varepsilon_t)dt + \sigma dW_t,$$

where $(W_t)_{t \geq 0}$ denotes a one-dimensional standard Brownian motion.



Case Studies

Scenario A: One-Lane Road Segment (1/2)

1. **Scenario**
   Segment of a one-lane road with length $L = 2,000\,\mathrm{m}$.

2. **Vehicle Generation**
   Vehicles are generated at the origin and removed at the end of the road.
   - Vehicles are created deterministically with a constant demand (here, $1,500\,^{\mathrm{veh}}/_{\mathrm{h}}$), if there is enough space available - otherwise the generation is delayed.
   - The initial velocity of any new vehicle matches the velocity of the preceding vehicle.

Case Studies

Scenario A: One-Lane Road Segment (2/2)

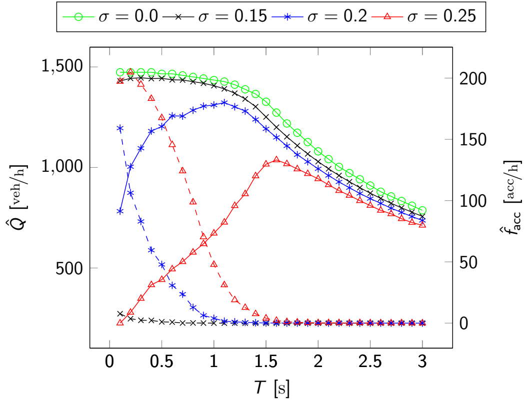


Figure: Averaged flow (solid lines) and number of accidents (dashed lines) for varying T and fixed σ.

Case Studies

Scenario B: Left-Turning on T-Junction (1/5)

We use IDMrm on one-lane road segments as building blocks to explain traffic in more complex scenarios.

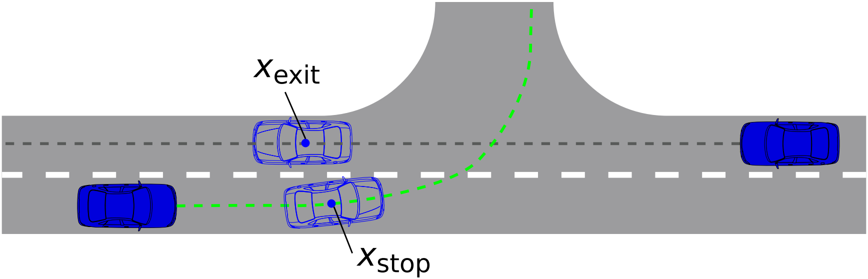


Figure: Simplified left-turning scenario on t-junction.

Case Studies

Scenario B: Left-Turning on T-Junction (2/5)

- **Vehicle Generation**
  - Generate vehicles with an exponentially distributed headway.
- **Movement of Vehicles**
  - Vehicles accelerate (positively) according IDMrm (in this case study with perfect perception).
  - Additional feature on lower lane: Conflict detection and reaction.
- **Random Misperception**
  - Random misperception in conflict detection and reaction creates accidents in the context of left-turning maneuvers.

Case Studies

Scenario B: Left-Turning on T-Junction (3/5)

1 Conflict Detection

- Turning vehicle i estimates the distance $\hat{d}^{ij}(t)$ to the approaching vehicle j (based on an extrapolation of trajectories).
- The situation is classified as a conflict, if $\varepsilon_t^{i,4}\hat{d}^{ij}(t) \leq d_s$ for a safety threshold $d_s > 0$.

2 Conflict Reaction

- Compute a_{naive} (using $\varepsilon_t^{i,5}v^i(t)$) s.t. vehicle i stops at x_{stop} with a constant (negative) acceleration. This needs a time of t_{naive}.
- Let $\hat{t}^j$ be the time vehicle j needs to reach x_{exit}.
- Set the acceleration

$$a^{i,j}_{\mathrm{conflict}}(t) = \begin{cases} a^i_{\mathrm{naive}}, & \text{if } \hat{t}^j > t_{\mathrm{naive}}, \\ a^{i,j}_{\mathrm{smooth}}, & \text{if } \hat{t}_j \leq t_{\mathrm{naive}} \end{cases}$$

where $a^{i,j}_{smooth}$ is determined (using $\varepsilon_t^{i,5}v^i(t)$) s.t. vehicle i reaches x_{stop} with constant acceleration when vehicle j arrives at x_{exit}.

Case Studies

Scenario B: Left-Turning on T-Junction (4/5)

In summary, vehicles move according to

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}t} x^i(t) &= \max\{v^i(t), 0\}, \\ \frac{\mathrm{d}}{\mathrm{d}t} v^i(t) &= \max\{\min\{a^i_{\mathrm{IDMrm}}(t), a^{i,j_1}_{\mathrm{conflict}}(t), a^{i,j_2}_{\mathrm{conflict}}(t), \dots\}, a^i_{\min}\} \end{cases}$$

Case Studies

Scenario B: Left-Turning on T-Junction (5/5)

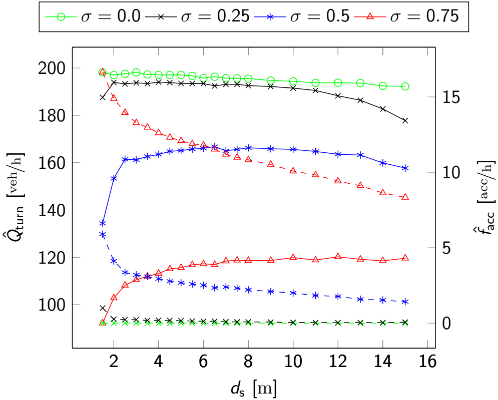


Figure: Averaged turn flow (lower lane, solid lines) and number of accidents (dashed lines) for varying d_s and fixed σ.

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Conclusion

1. **Microscopic Model of Traffic Accidents**
   - We introduced a traffic model that admits accidents.
   - Accidents are caused by random misperception.
2. **Tradeoff between Risk and Efficiency**
   - The simulation model admits a characterization of the tradeoff between safety and efficiency of traffic systems.
3. **Generation of Data**
   - The causal stochastic model produces simulated data that provide guidance to the design and risk management of future traffic systems.

Thank your for your attention!

Literature

Berkhahn, V., M. Kleiber, C. Schiermeyer, and S. Weber (2018). "Modeling Traffic Accidents Caused by Random Misperception". *Accepted: The 21st IEEE International Conference on Intelligent Transportation Systems*.

Bertoncello, M. and D. Wee (2015). *Ten ways autonomous driving could redefine the automotive world*. https://www.mckinsey.com/industries/automotive-and-assembly/our-insights/ten-ways-autonomous-driving-could-redefine-the-automotive-world.

Blanco, M., J. Atwood, S. Russell, T. Trimble, J. McClafferty, and M. Perez (2016). *Automated vehicle crash rate comparison using naturalistic data*. Tech. rep. Virginia Tech Transportation Institute.