UNIVERSITEIT VAN AMSTERDAM

Centrum Wiskunde & Informatica

User equilibrium analysis with travel uncertainties and hypercongestion

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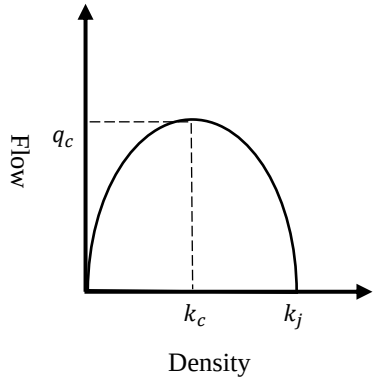
Outline

- Hypercongestion
- Vickrey model
- Bathtub model
- Our model
- Bottleneck models with arrival time uncertainty
- Stochastic bottleneck model

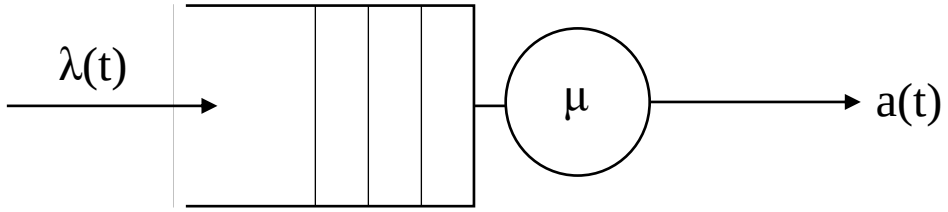
Bottleneck models and hypercongestion

- The bottleneck models of congestion with endogenous scheduling are standard tools of transportation economics.
- Not all of the bottlenecks model captures the empirical relation between flow and density as in the fundamental diagram.

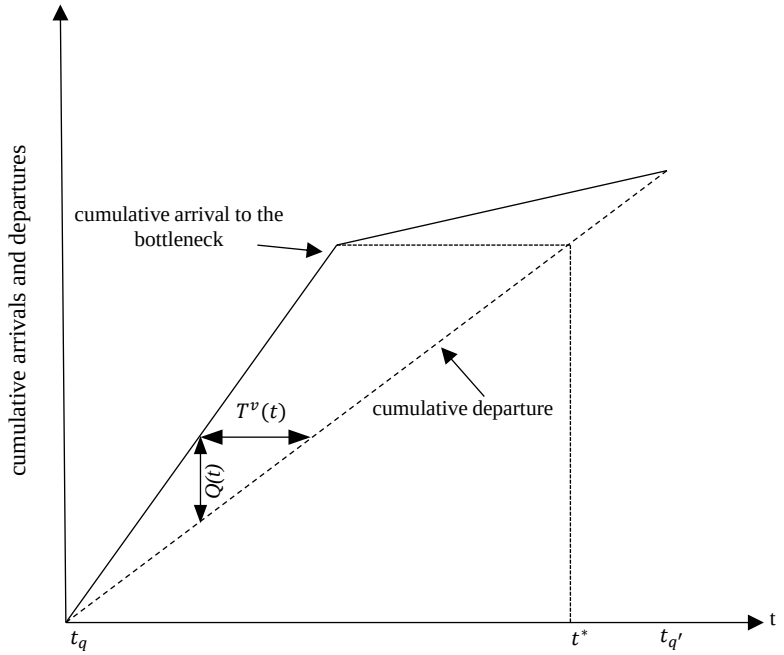


Greenshields' fundamental diagram

Vickrey model



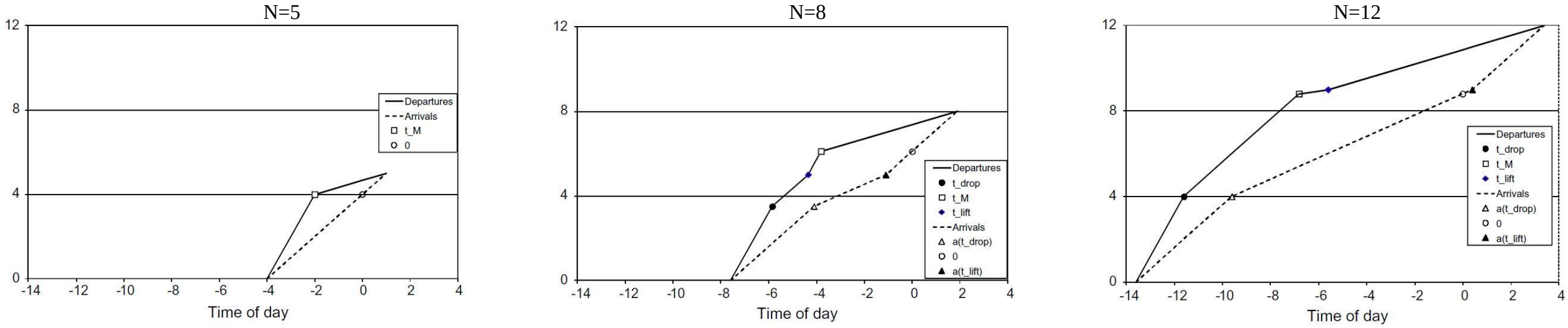
$$C = waiting\ time + \ early\ arrival\ cost + lateness\ cost$$



Vickrey model
Arnott et al. (1990)

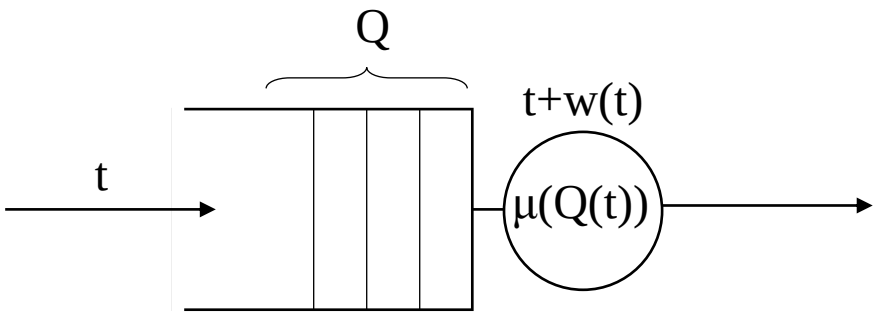
Bathtub model

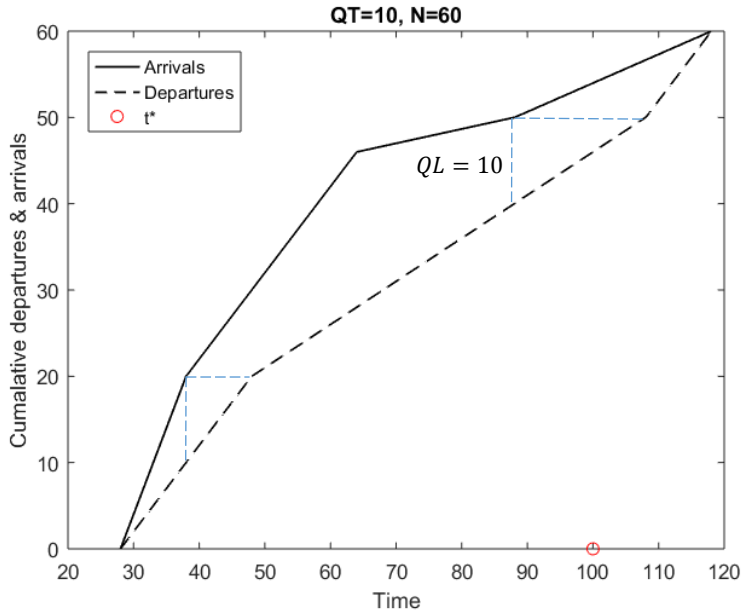
- The service rate of the bottleneck declines with occupancy.
- A system where later entrants can influence earlier ones.



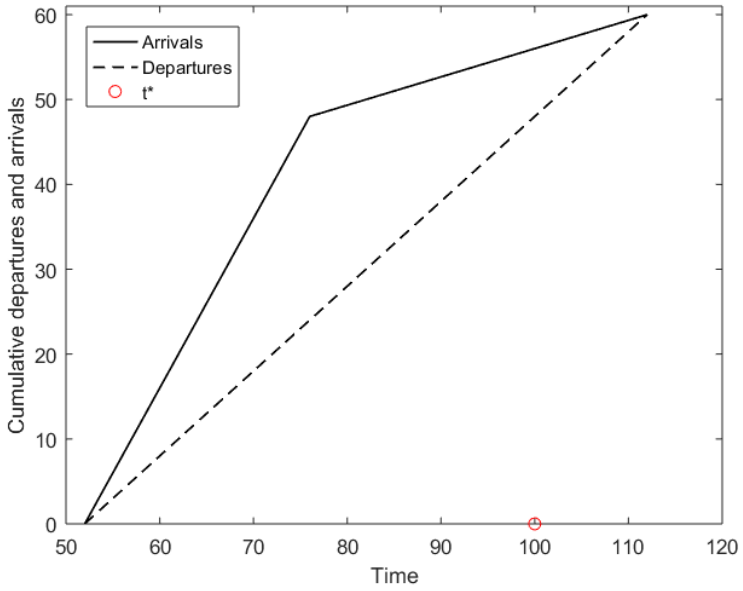
Cumulative departure and arrival patterns for various N.
Fosgerau &Small (2013)

Our model

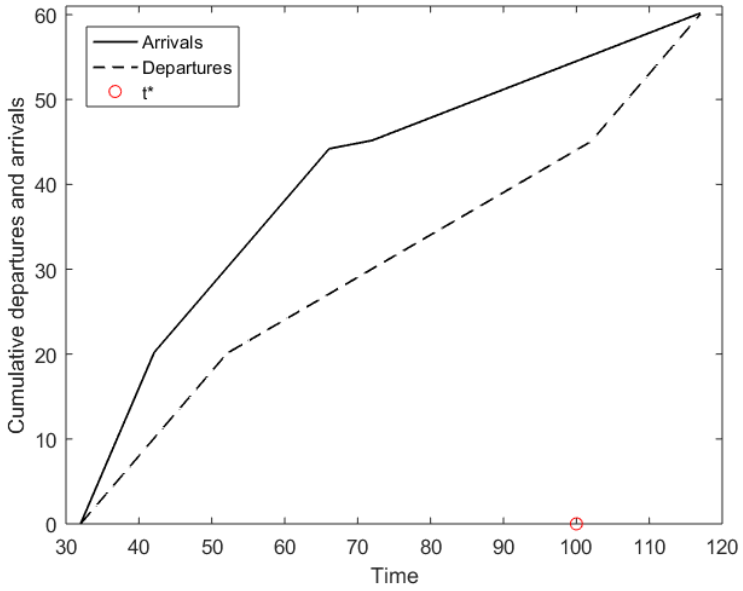

Q
t+w(t)
t
μ(Q(t))


QT=10, N=60
Arrivals
Departures
t*
$QL = 10$
Cumalative departures & arrivals
Time
0
10
20
30
40
50
60
20
30
40
50
60
70
80
90
100
110
120

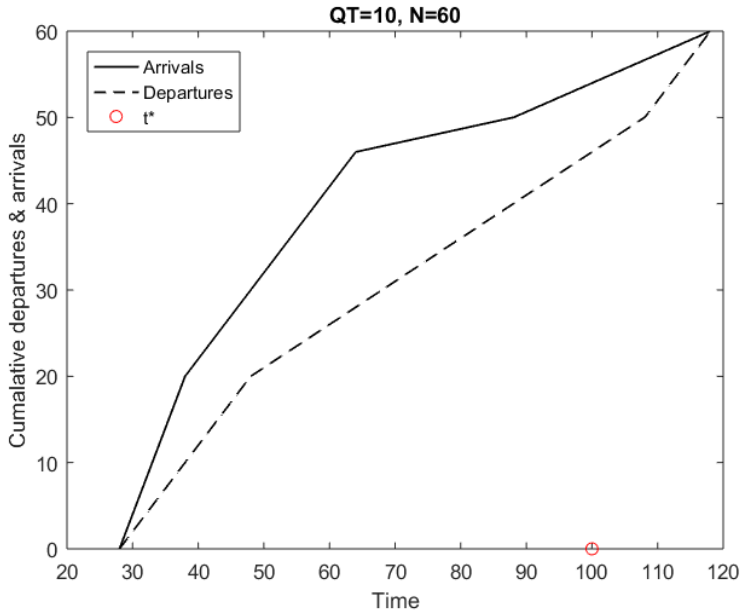
Dynamic user equilibrium of bottleneck models



Vickrey model
Arnott et al. (1990)

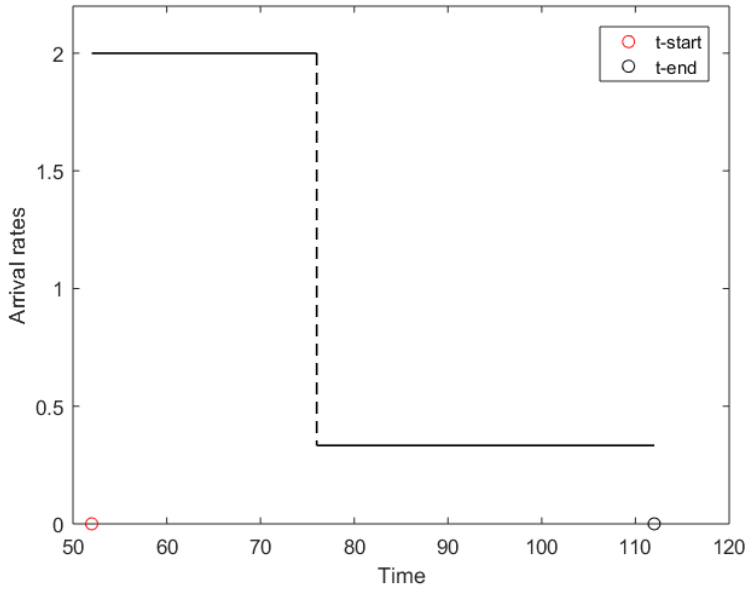


Bathtub model
Fosgerau and Small (2013)



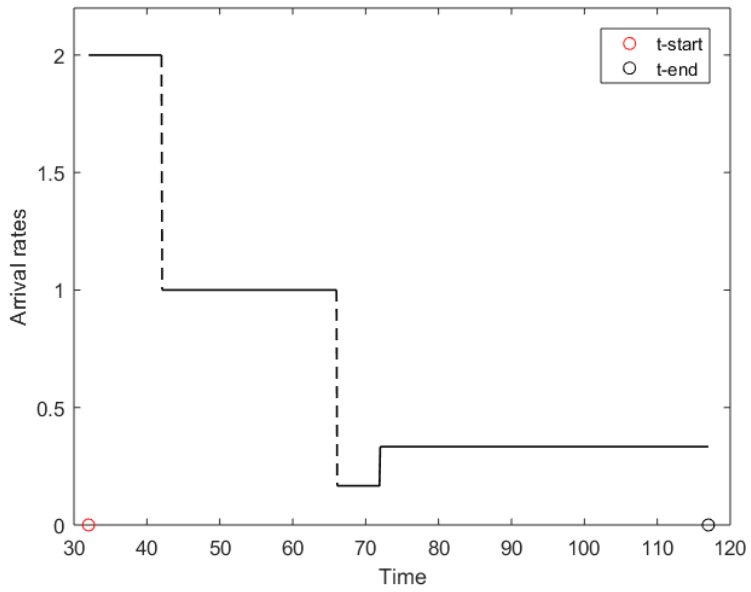
Our model

Arrival rates and cost values



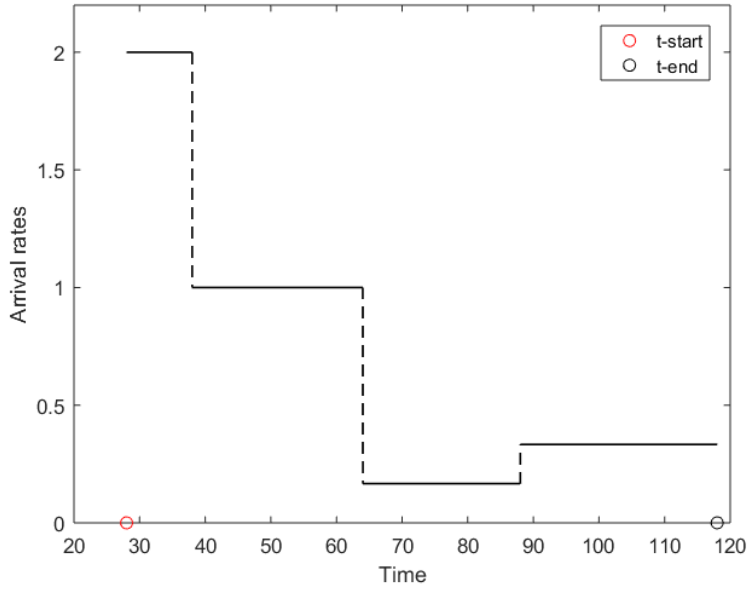
Vickrey model
Arnott et al. (1990)

Cost = 24



Bathtub model
Fosgerau and Small (2013)

Cost = 34



Our model

Cost = 36

Bottleneck models with arrival time uncertainty

The actual arrival time to the bottleneck deviates from the planned time.

- The actual arrival rate function:

$$\tilde{a}(t) = \int_{u=-\infty}^{t} f(u)a(t-u)du$$

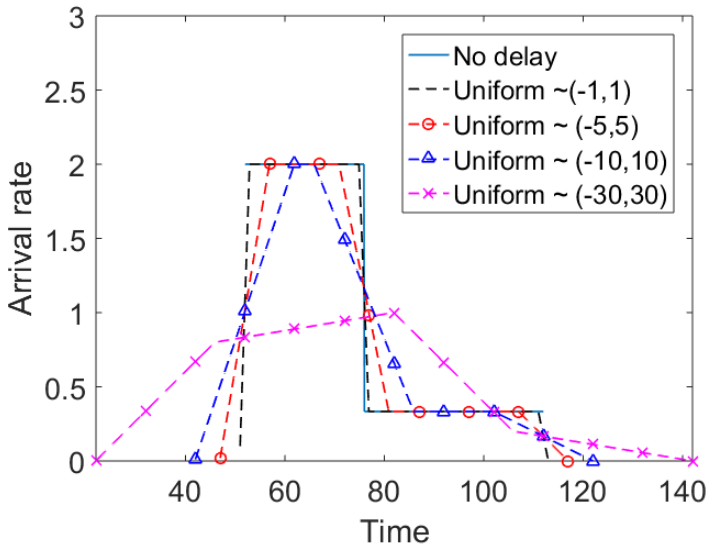
- The expected cost of an arrival at time t:

$$\tilde{C}(t) = \alpha W(t) + \beta\left(t^* - (t + W(t))\right)^+ + \gamma\left(t + W(t) - t^*\right)^+$$

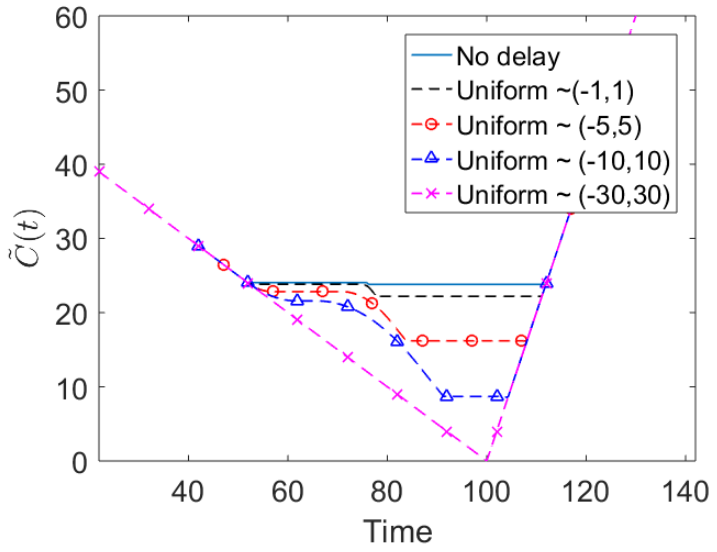
- The expected cost for a traveller who intends to arrive at t:

$$\mathbb{E}C(t) = \int_{u=-\infty}^{\infty} \tilde{C}(t+u)f(u)du$$

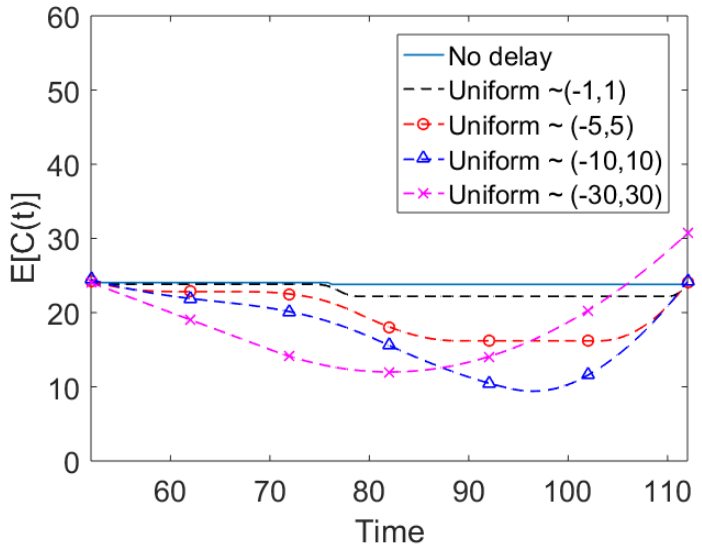
Vickrey model with uniform delay



Arrival rate

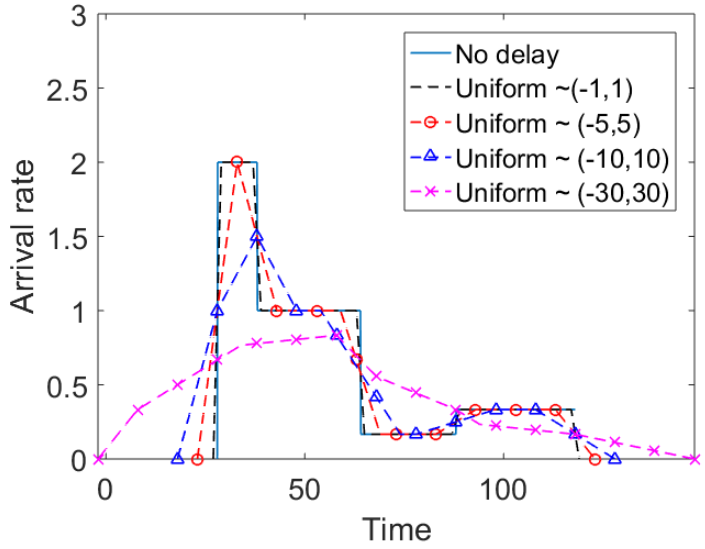


Cost per time unit

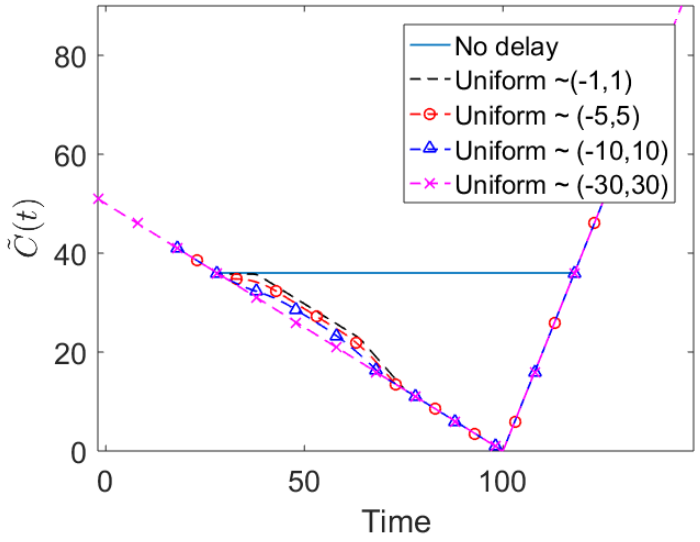


Expected cost for a traveller with intended time t

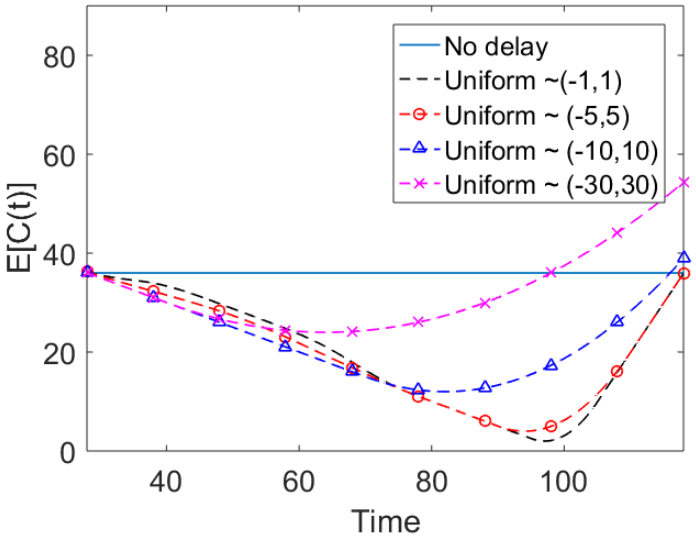
Our model with uniform delay



Arrival rate



Cost per time unit



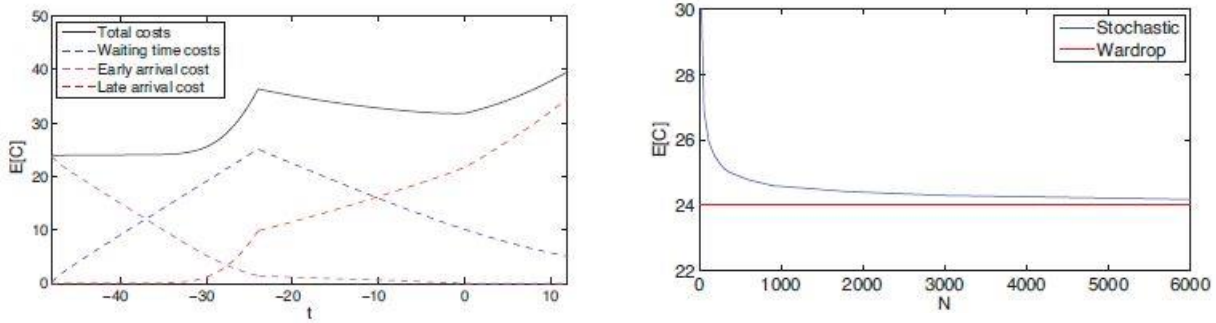
Expected cost for a traveller with intended time t

User equilibrium for bottleneck models with arrival time uncertainty

- For the Vickrey model we showed that neither a pure equilibrium nor a continuous mixed equilibrium exist when there are uniformly distributed delay functions on arrivals.
- The question is if the same situation holds for our model.

Stochastic Vickrey bottleneck model

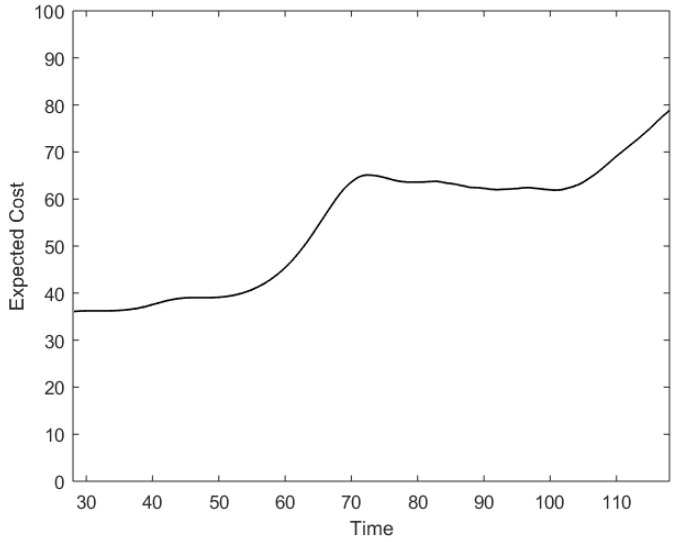
- Travellers arrive at the queue according to a time-dependent Poisson process
- The bottleneck can serve only a single traveller at a time, which takes an independent and identically distributed exponential time with rate μ



van Leeuwen & van de Ven (2017)

Stochastic bottleneck model

- Time-dependent Poisson arrivals
- Time-dependent Poisson service rates



❖ Future goal: to calculate the stochastic equilibrium

Conclusion

- Analysis of the bottleneck model that incorporates hypercongestion
  - Fits reality
  - Easy to simulate
  - Doesn’t underestimate the traveling cost
- The more freedom the travellers have, the less congested the system gets

Thank you!