Stationary Flow Approximation In Floor-Field Models Using Markov-Chain Theory

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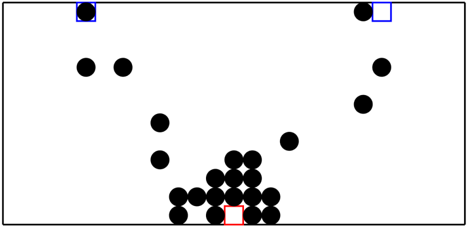
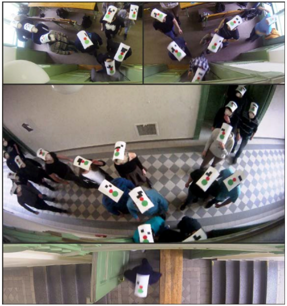
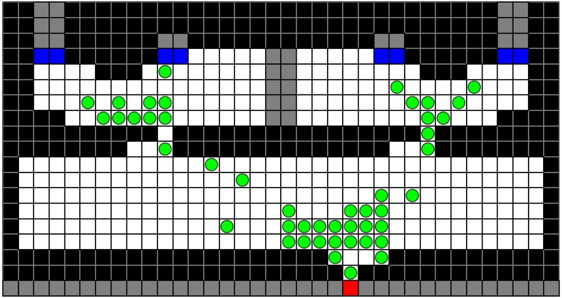
What is it about?



Outline

Cellular evacuation model

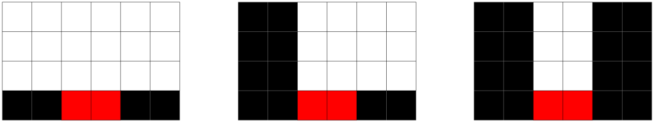




- Lattice consisting of cells 40 $\times$ 40 cm.
- How to model exit of the width 75 cm?
- Two cells representing 80 cm are “too much”.

Not the width, but maximal flow

- In *Seyfried et al. 2009* linear dependence $J = 1.9 \cdot \text{width}$ suggested.
- Same bottleneck width, different flow value.



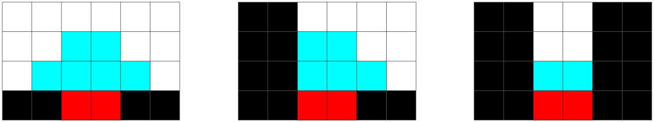
- Key feature is the “maximal flow through the bottleneck”
- Measured experimentally or estimated by some linear model

$$J = 1.9 \cdot \text{width} + \beta_2 \cdot \text{property}_2 + \beta_3 \cdot \text{property}_3 + \dots$$

- How to adjust the cellular model to produce wanted bottleneck flow?

Locally dependent friction

- Locally dependent “friction parameter”.



- Wanted equation saying:

  “For this flow value choose this value of the friction parameter”
- Finding implicit equations of the form

$$J = J(\text{parameters})$$

- How?

Outline

Floor-field models

Floor-field conception

- Static floor-field

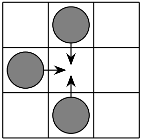
$$S_b = \mathrm{dist}(b, b_{\mathrm{exit}})$$

- Probabilistic choice of next cell

$$\Pr(b_{\mathrm{act}} \to b) \propto \exp\{-k_S \cdot S_b\}$$

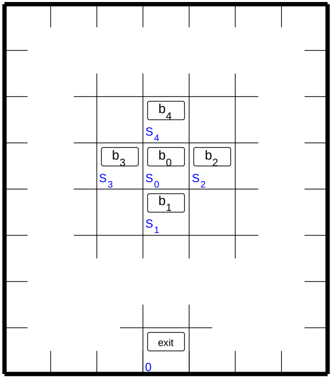
Conflicts and friction

- Conflict: k agents to one cell.



- No one wins with prob. $\phi(\zeta, k)$.

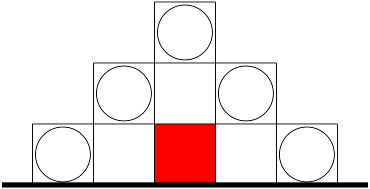
Cellular lattice



Flow approximation by Ezaki et. al 2012

T. Ezaki, D. Yanagisawa, and K. Nishinari, 2012

- They investigated 25×25 cells lattice regarding the stationary flow
- Maximal bottleneck flow approximated by means of reduced state space

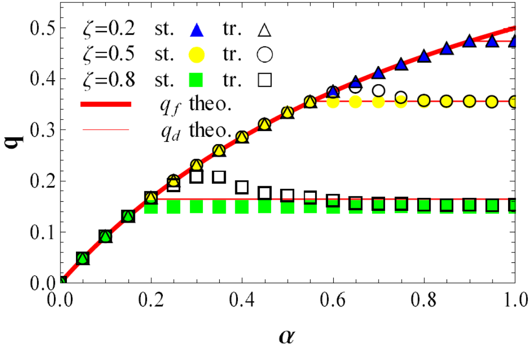


- Markov chain with 2^4 states having stationary distribution π, thus

$$J \approx \sum_s j_s \pi_s$$

Assumptions of the approximation

- $k_S \approx +\infty$, thus almost deterministic motion
- Surrounding cells always occupied in congestion
- Result: “it works well”



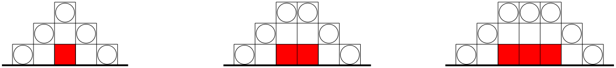
[1]

[1] Taken from T. Ezaki, D. Yanagisawa, and K. Nishinari, 2012

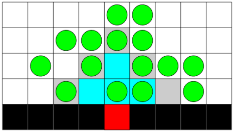
Outline

Questions to be answered by our research

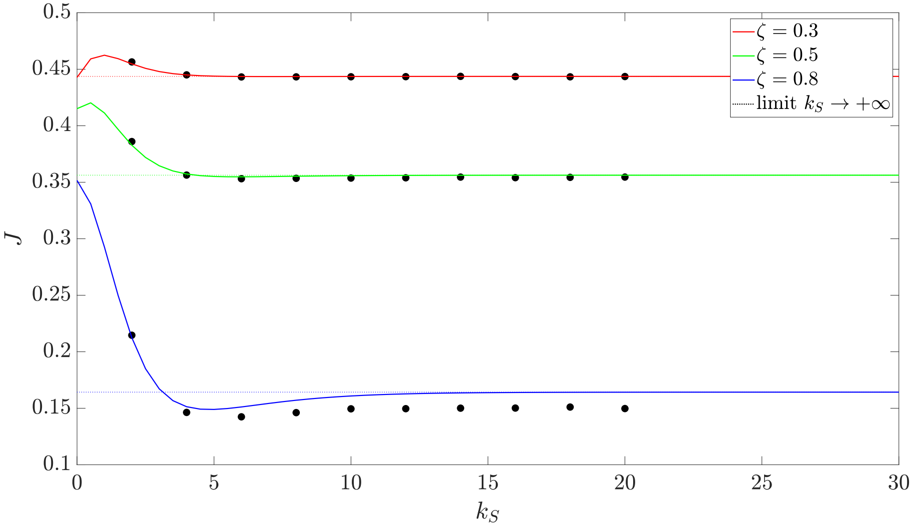
- Does this work for general $k_S \in (0, +\infty)$?
- Does this work for multiple-cell wide bottlenecks?



- Is the assumption of always occupied surrounding cells valid?

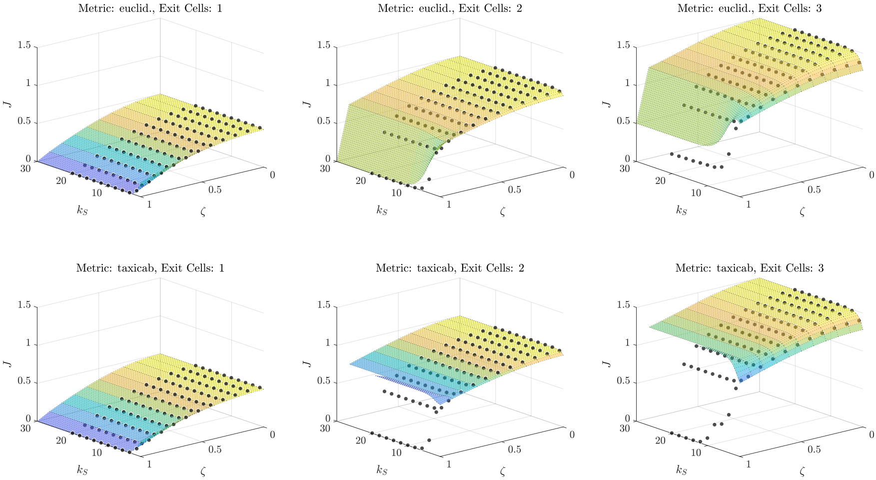


General k_S



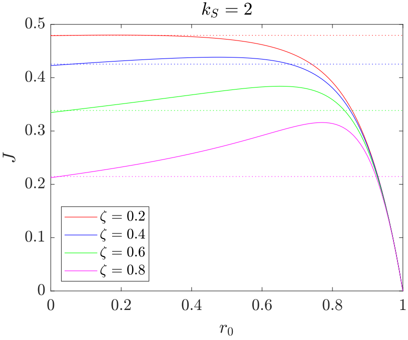
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
J
0
5
10
15
20
25
30
k_S
ζ = 0.3
ζ = 0.5
ζ = 0.8
limit k_S → +∞

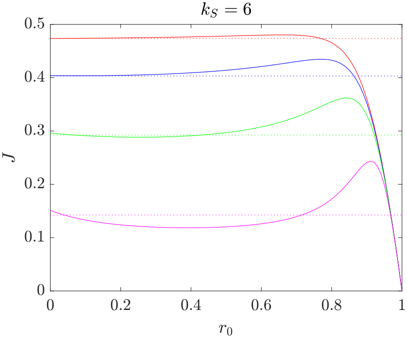
Multiple-cell exits



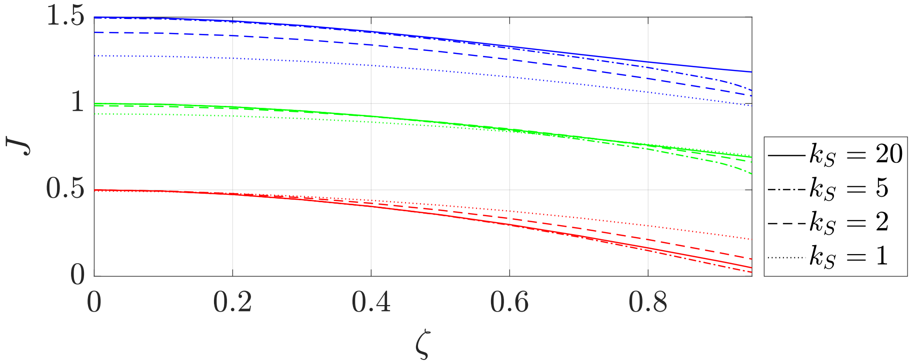
Assumption of always occupied surrounding cells

- Simulation of the 25 × 25 cells lattice
- Theoretical result with $r_0 \in [0, 1]$, probability of the nearest cell being empty.



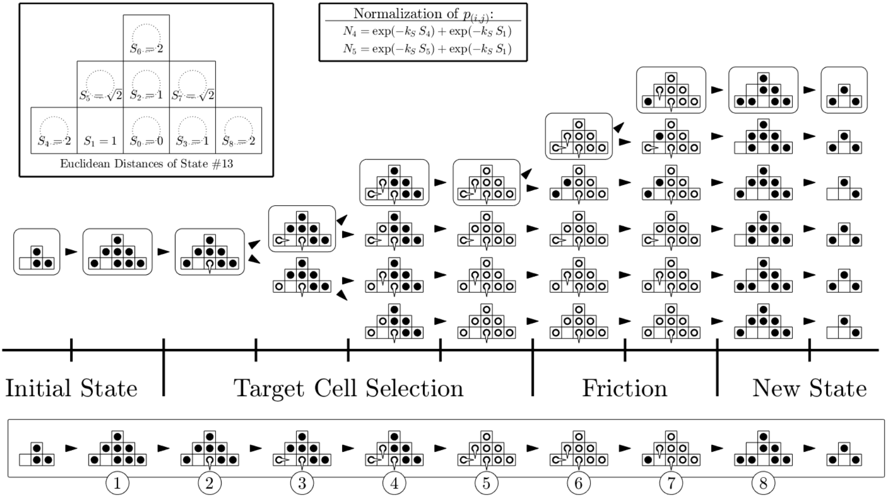


Flow depending on friction – main result



Outline

Building the transition matrix **P** algorithmically



Stationary flow calculation

- Stationary distribution satisfying

$$\boldsymbol{\pi} = \boldsymbol{\pi} \cdot \mathbf{P}$$

- Due to numerical issues

$$\boldsymbol{\pi} = \boldsymbol{p}(0) \cdot \lim_{n \to +\infty} \mathbf{P}^{n} \approx \boldsymbol{p}(0) \cdot \mathbf{P}^{n_0}$$

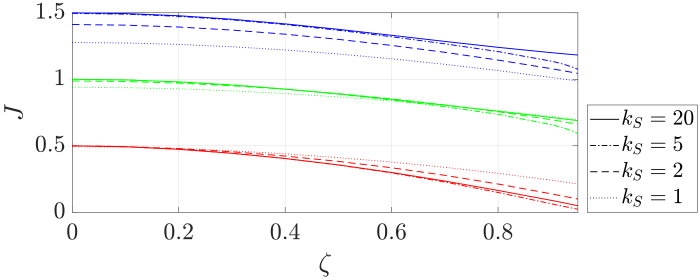
- The flow calculation

$$J \approx \sum_{s} j_s \cdot \pi_s \,, \qquad j_s = \mathrm{E}(\text{outflowing agents} \mid s)$$

Outline

Results

- Algorithm for building the transition matrix (MATLAB)
- Symbolic matrices
- Flow depending on frinction ζ



- Valid rather for lower values of ζ

Outline

Heterogeneity

- $H \geq 1$ types of agents
- Various attraction strength to the exit

$$\boldsymbol{k_S} = (k_{S;1}, \ldots, k_{S;H})$$

- Various friction ability

$$\boldsymbol{\zeta} = (\zeta_1, \ldots, \zeta_H)$$

- Generalized friction function

$$\phi(\bar{\zeta}, k)$$

- Aggressiveness

$$\boldsymbol{A} = (A_1, \ldots, A_H)$$

$$\Pr(j \in \{1, \ldots, k\} \text{wins}) \propto A_{i_j}$$

- Input distribution

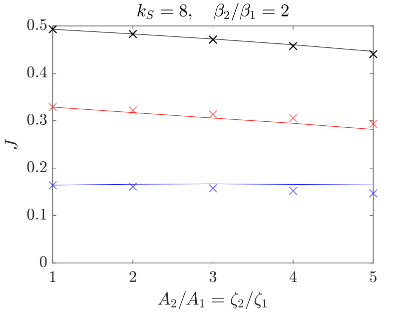
$$\boldsymbol{\beta} = (\beta_1, \ldots, \beta_H)$$

Heterogeneity

- Problem: how to determine the distribution of nearest cell occupation

$$\boldsymbol{r} = (r_1, \dots, r_H)\,.$$

- Substituted from the simulation $\implies$ approximation works well



Thank you for your attention!