# Application of Cross Entropy Method to solving an Optimal Road Network Design problem for Improving Intersections

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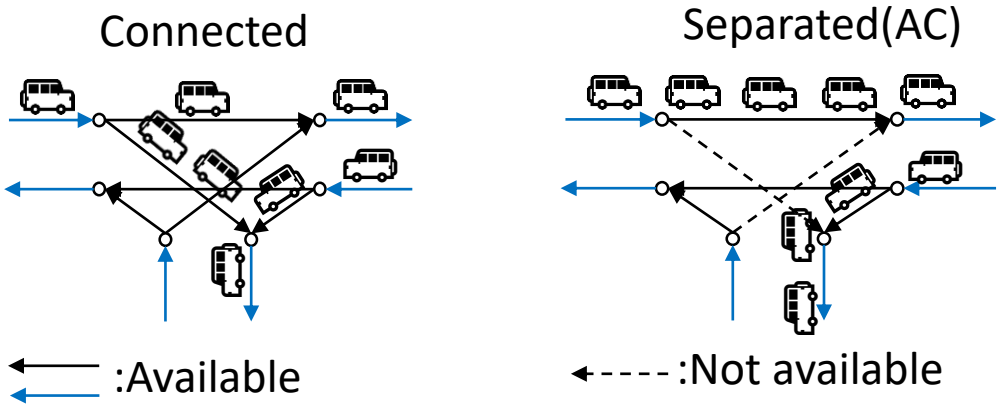
# Background



- Access control(AC) on some intersections contributes to improvement of congestion in roadway network.

Example of AC

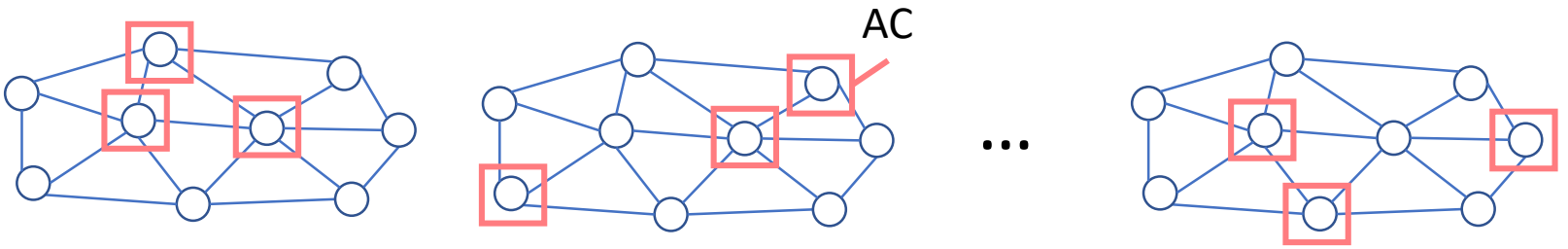



- Connection of trunk road and secondary road may cause traffic congestion and increase total travel time.
- Access control(AC) on some intersections may also cause traffic congestion and increase total travel time.
- Finding optimal location of AC is important to reduce total travel time.

# Objective



- Formulation
  A model to find the optimal location of AC intersections that minimizes the total travel time in roadway network




Alternatives of location pattern

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Finding the optimal location of AC

- Solution method
  Cross Entropy method to solve the formulated model

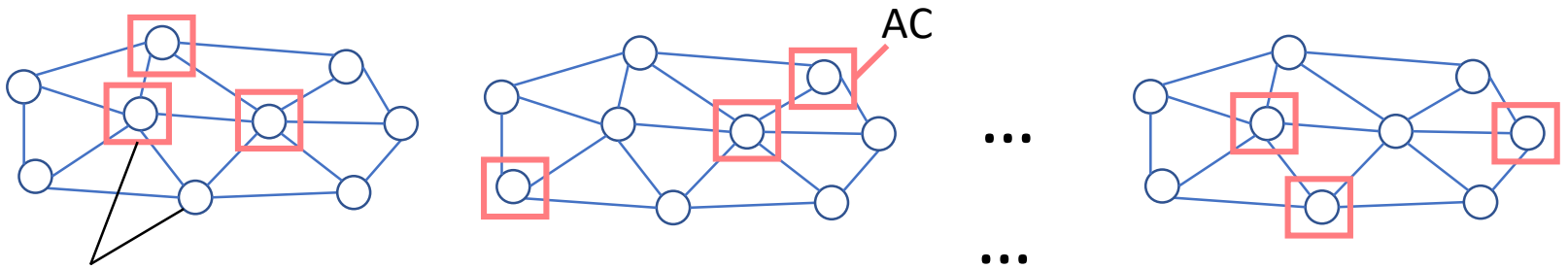
- Numerical analysis
  Sensitivity analysis of parameters

# Formulation

# Formulation



- Input：Capacity of link, OD demand
- Objective function：Total travel time(TT)
- Decision variable：location of AC intersection



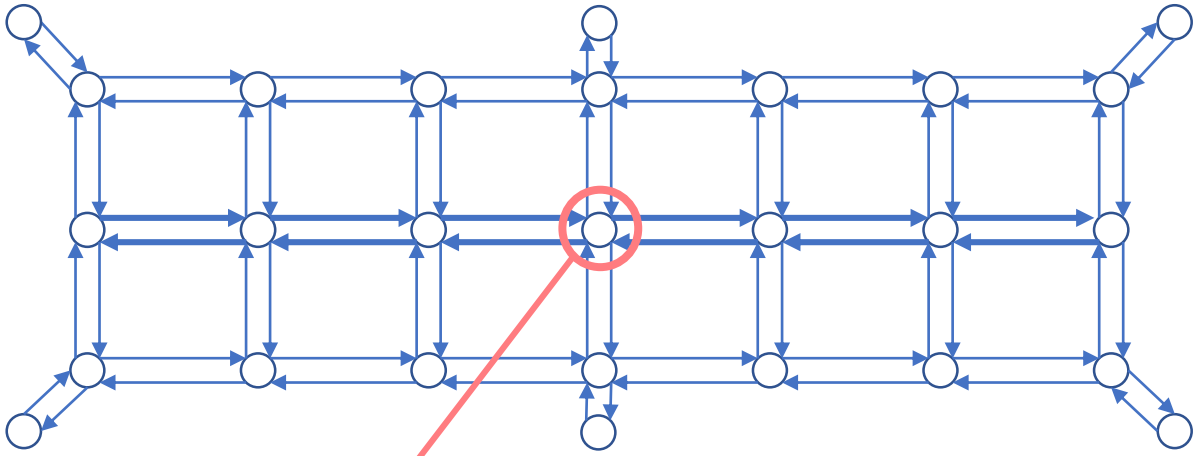

$ac_i$ = 1 : if an intersection “i” is AC
0 : Otherwise

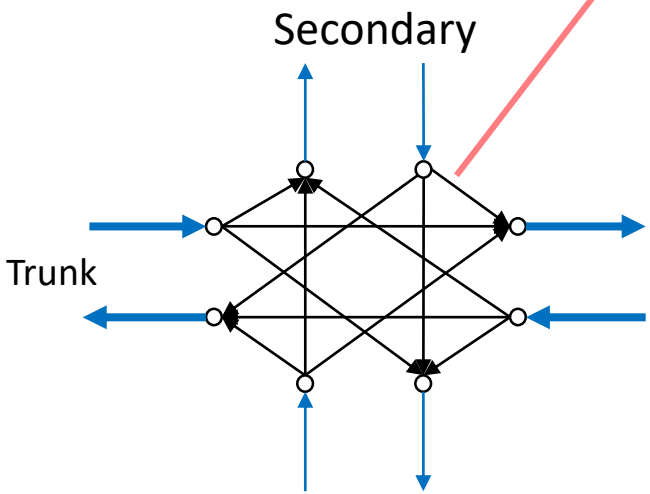
- Flow：Static User Equilibrium
- Target ：Optimal location of AC intersections which minimizes TT

# Network and Intersection



- Network of trunk and secondary road
- A unit with 8 nodes as an intersection






：Trunk road link

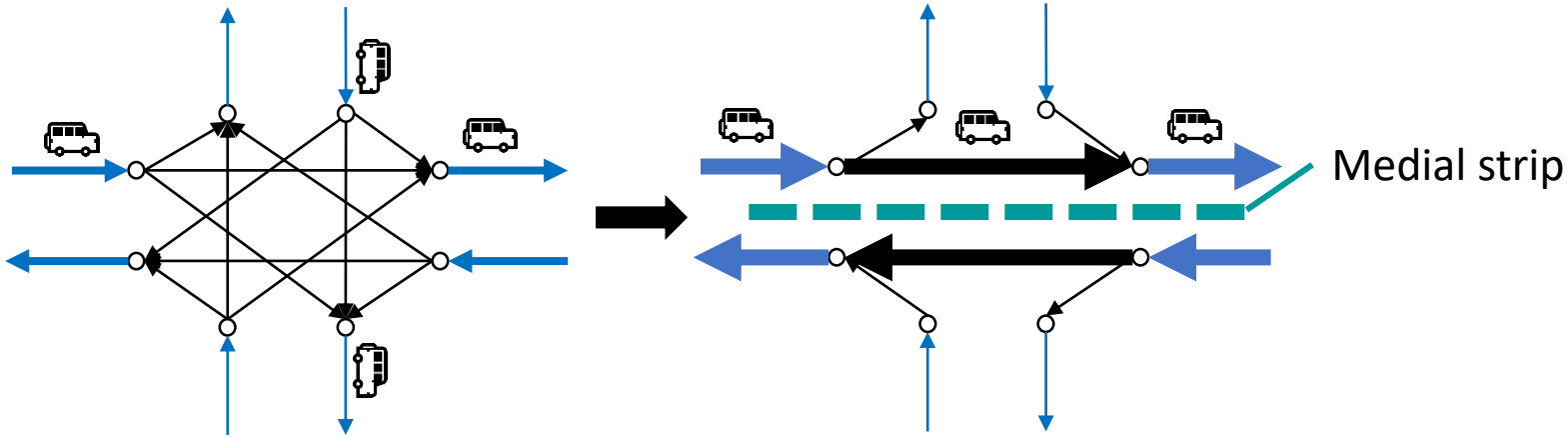
：Secondary road link

：Link in an intersection

# Access Control on intersections



- Prohibition of right turn and going straight ahead from secondary road link by introducing medial strip (I call this operation “Access control” , “AC”)
- By access control, performance of trunk road will be improved.




：Trunk road link adjacent with an AC intersection

：Trunk road link in an AC intersection

：Other trunk road link

：Other secondary road link

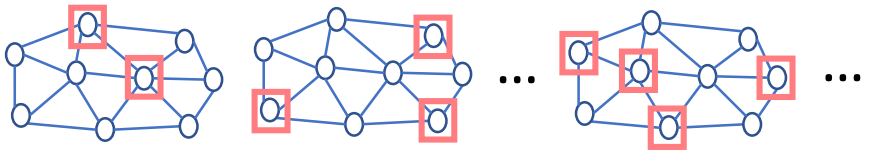
：Secondary road link in an intersection

# Solution Method

# Cross Entropy Method(CEM)

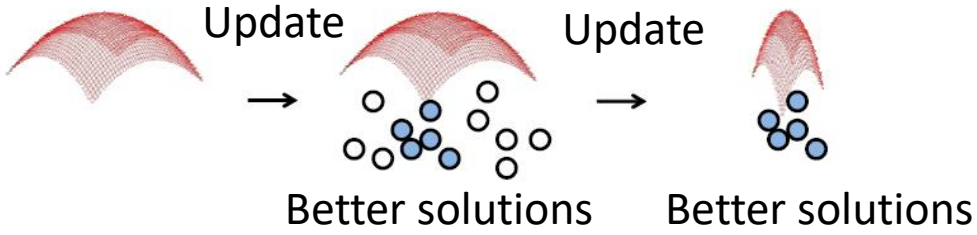


- When there are M intersections, the number of possible combinations is $2^M$



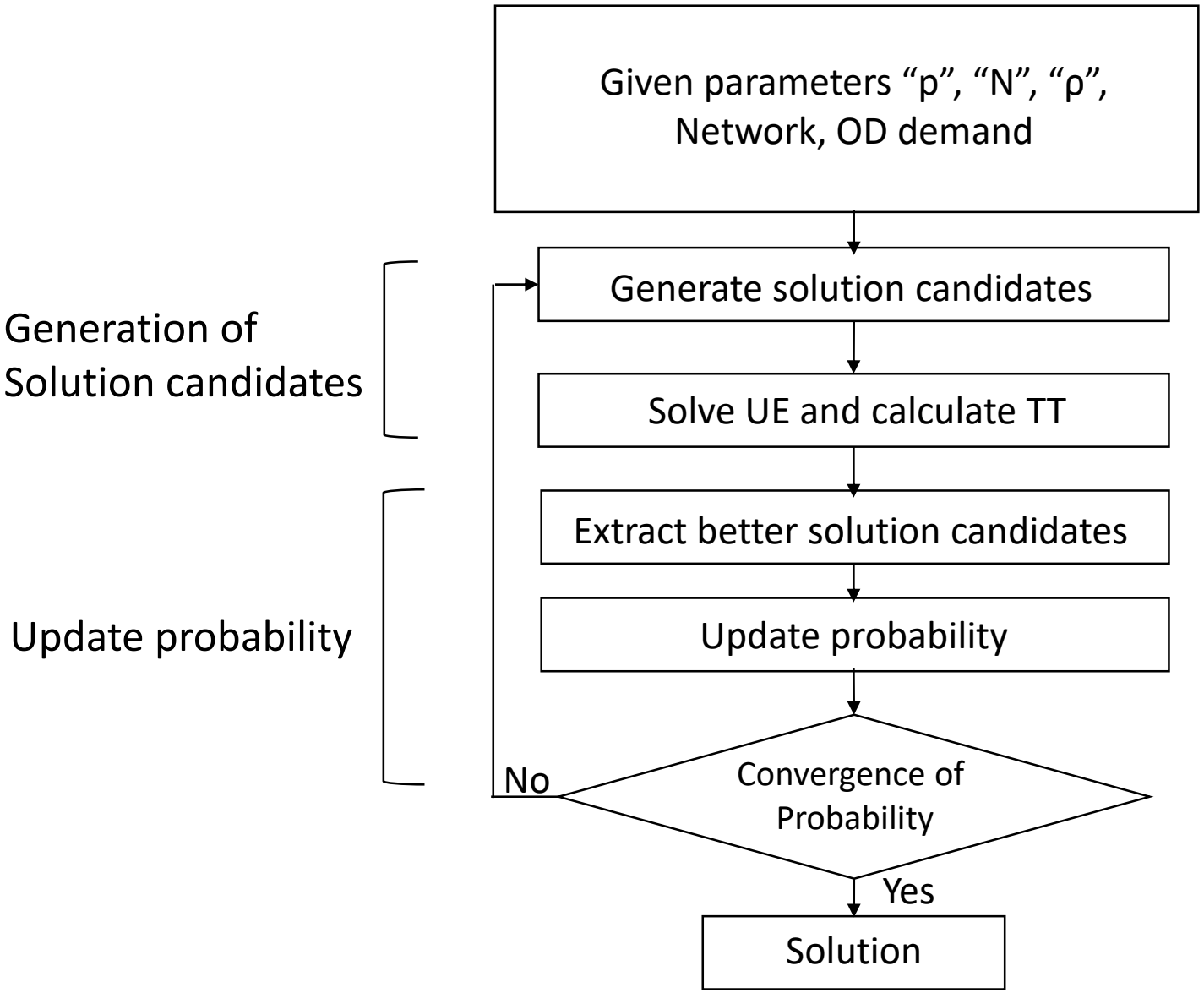
If M equals 9,
there are $2^9$solution candidates.

- Cross Entropy Method(CEM) is a simulation method to solve combinational optimization problem.
- CEM is useful for problems which have many local solutions.
- CEM generates solution candidates stochastically.
- CEM updates probability $[p_1, p_2, \ldots p_9]$.



# Structure of network optimization model with CEM





# Case of introducing CEM to network optimization problem



- Mather, Liu, Nogduy
  Signal optimization using the cross entropy method
  (2013)
- Takei, Nagae
  The release of earthquake resistance problem of road network by road selection by random selection algorithm selection by random selection
  (2015)
- Wada, Usui, Yaginuma
  Optimization of traffic signal group considering queue extension based on Cross Entropy Method
  (2015)

- There is little case that CEM is introduced to optimization of direction control on intersections.

- In order to introduce the model with CEM to general network optimization problems, sensitivity analysis on CEM parameters is necessary.

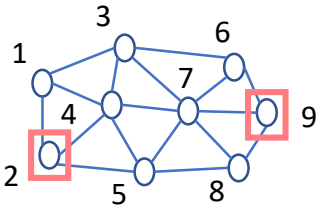
# Structure of network optimization model with CEM

## Generate solution candidates

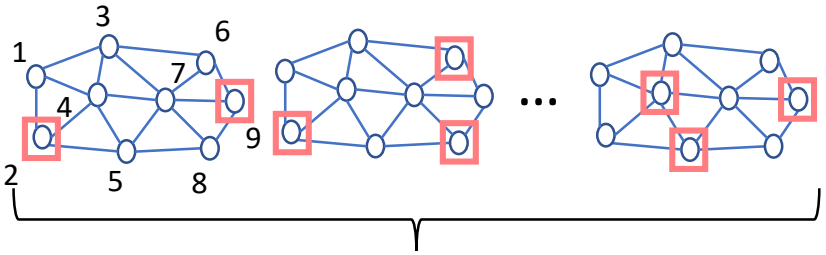
- Initial probability to each intersections : $[p_1, p_2, \ldots p_M]$ = [0.5, 0.5, …0.5].
- CEM generates N solution candidates stochastically by possibility $[p_1, p_2, \ldots p_M]$ .
- "N" is sample size , it means the number of solution candidate this model generates.

Ex) $[p_1, p_2, p_3, \ldots p_9]$ = [ 0.3, 0.9, 0.1 …0.8]

→ $[ac_1, ac_2, ac_3, \ldots ac_9]$ = [ 0, 1, 0…1] →




→As such, N solutions are generated stochastically

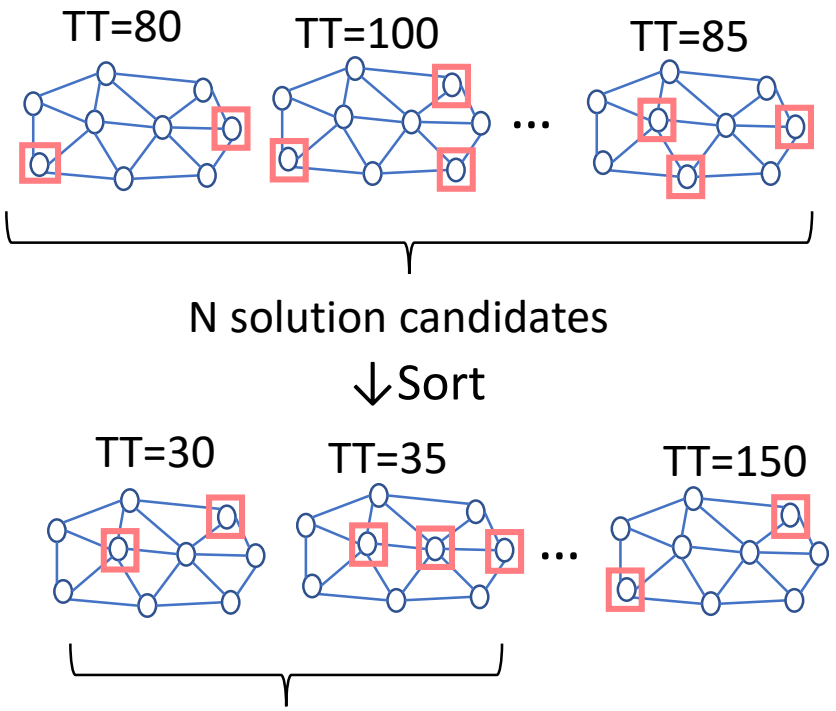



N solution candidates

# Structure of network optimization model with CEM

## Solve UE & calculate TT

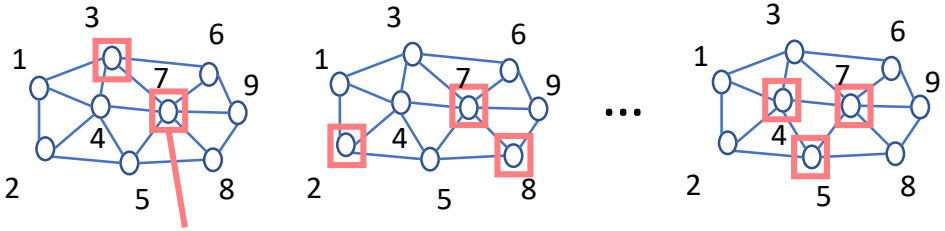
- Solve UE and calculate TT of each solution candidate
- Sort solution candidates in ascending order of TT
- Extract up to ρN th solution candidates
- “ρ” is a parameter of CEM “Extract ratio” , $0 < \rho < 1$



## Update probability

$$p_i^{Updated} = \frac{\sum_{n \in \{extracted\ solution\ candidates\}} ac_i}{\rho N}$$

- $p_i^{Updated}$ is updated probability of AC on the intersection "i".
- $ac_i$ = 1 : if an intersection "i" is AC
  0 : Otherwise




- If No.7 intersection is located AC in ten samples, $\sum_{\rho N} ac_7$=10.
- When CEM parameter(N, ρ) are (100, 0.3),

$$p_7^{updated} = \frac{\sum_{30} ac_7}{0.3 \times 100} = \frac{10}{0.3 \times 100} = \frac{1}{3} \cong 0.33$$

…do the same calculation to each intersection

- Repeat this process until all $p_i$ converges to either 1 or 0.

[0.5,0.5,…0.5] Update → [0.33,0.8, …0.66] Update… → Converge → [0,1,…1]

# Sensitivity analysis

# Input & Output



## Input

- Link Capacity
- OD demand
- CEM parameters
  - The number of solution candidates of AC location (I call it as “Sample size N”)
  - Extract ratio “ρ”

## Output

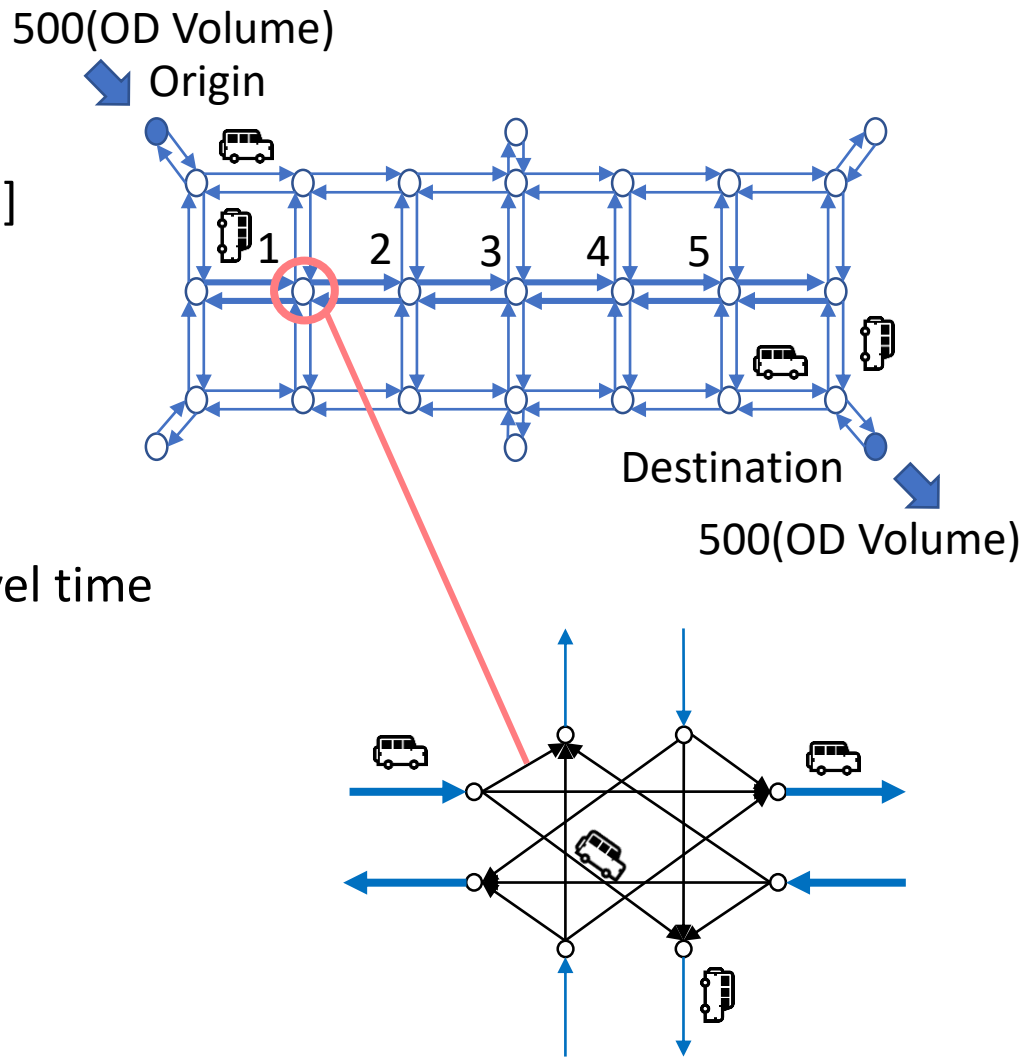
- Confidence ratio,  it means difference of accuracy
  under different CEM parameter(N,  ρ) and OD patterns

- Link flow from different OD cases.

# Network



- network with 3 × 7 nodes
- Target intersections are [1,2,3,4,5] which are connected with trunk road and secondary road.
- Input :
  - Single OD with 500 vehicles
  - Link capacity and free flow travel time



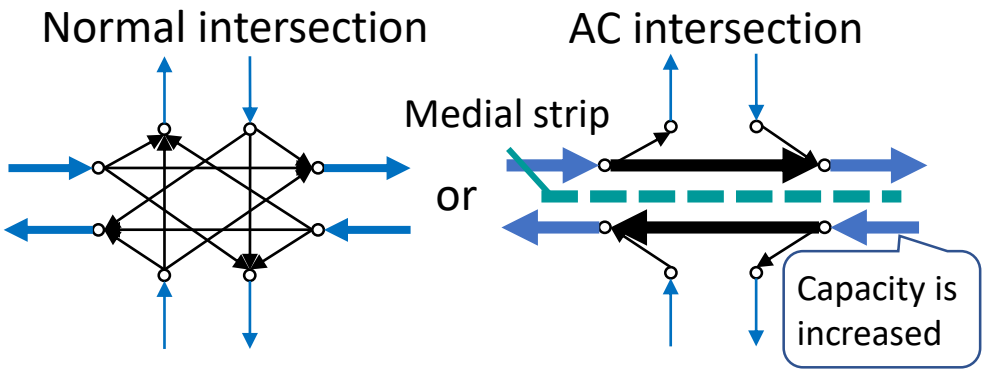

: Secondary road link
$(t_0, Cap) = (10,100)$

: Trunk road link$(5,100)$

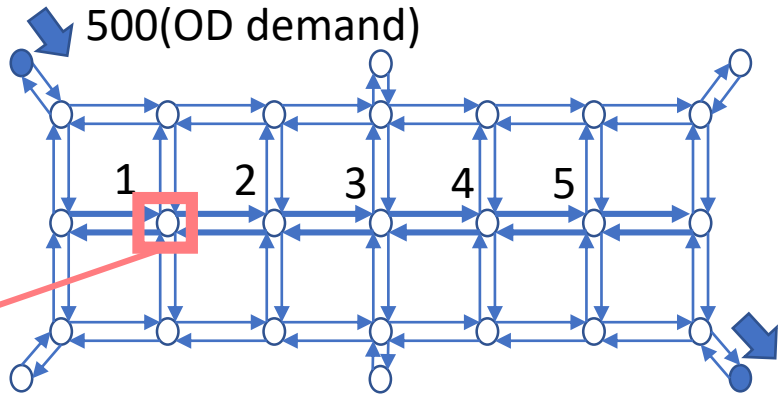
: All secondary link
in Intersections$(1,100)$

# Network & Operation as AC






If AC intersection is located, capacity of trunk road link in the intersection and adjacent with the intersection is increased.

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The number of solution candidates are $2^5 (= 32)$.

: Trunk road link adjacent with an AC intersection $(t_0, \mathrm{Cap}) = (10,100)$

: Trunk road link in an AC intersection $(1,120)$

: Other trunk road link $(5,100)$

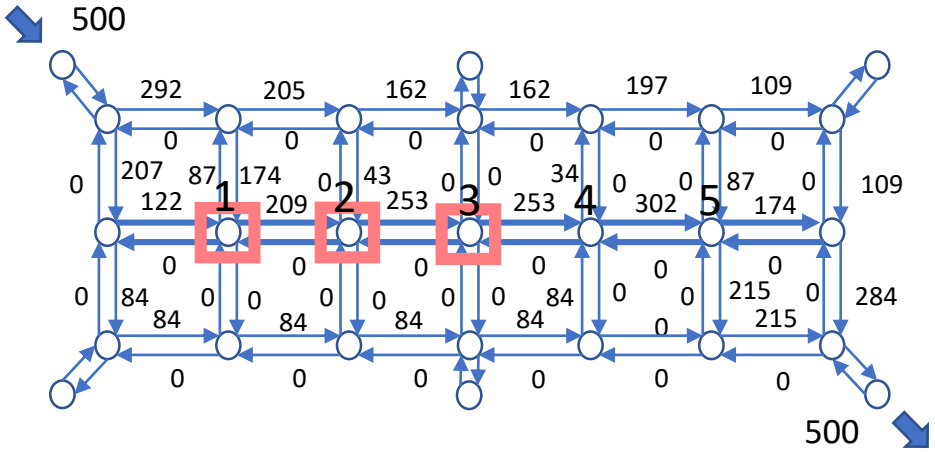
: Other secondary road link $(10,100)$

: Secondary road link in an intersection $(1,100)$

# Sensitivity Analysis -OD_1-



OD_1




Optimal solution and link flow

- If there is single OD in network, trunk road with multiple AC intersections can transport more vehicles than with normal intersections.
- Intersection No.4 and 5 are not to be AC in order to let vehicles escape from trunk road and avoid congestion of links near the destination.

| AC location | TT |
|---|---|
| No access controlled | 868.11 |
| All access controlled | 872.89 |
| [1,2,3] access controlled | 845.98 |
| [1,2,3,4] access controlled | 848.79 |

# Sensitivity Analysis -Confidence ratio for CEM parameters(N,ρ)-

・Calculate “confidence ratio” in difference of N and ρ

・ Confidence ratio = $\frac{The\ number\ of\ trials\ with\ optimal\ solution}{The\ number\ of\ total\ trials(10trials)}$

Confidence ratio(Sample size N, extraction rate ρ)

| N＼ρ | 0.4 | 0.6 |
|---|---|---|
| 40 | 1.0 | 1.0 |
| 30 | 1.0 | 1.0 |
| 20 | 1.0 | 0.8 |
| 10 | 0.8 | 0.7 |
| 5 | 0.5 | 0.3 |

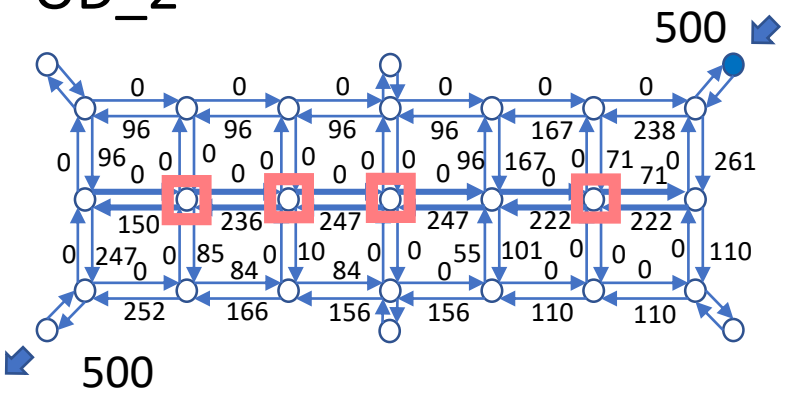
$N$：The larger N is, the higher confidence ratio is derived.

ρ：The bigger ρ is, lower confidence ratio is derived because not good solutions are also extracted as good solutions and also used in update of possibility.

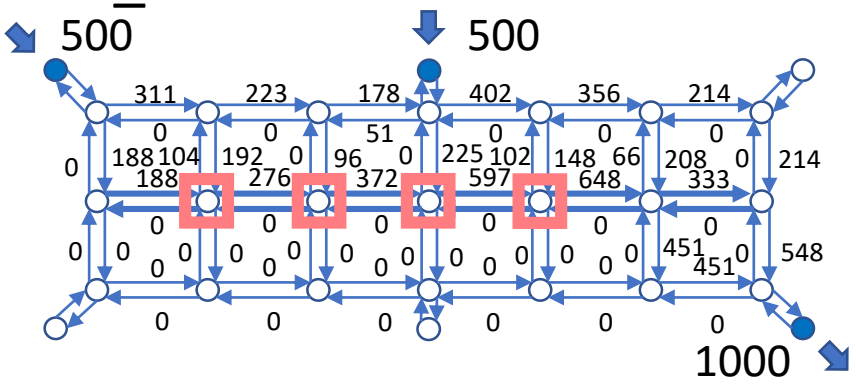
# Sensitivity Analysis -Link flow for OD patterns-



OD_2




OD_3




- Vehicles are prohibited to enter trunk road by right turn at AC intersections. The reason why there also must be normal intersections.

- Since OD demand is increased compared with OD_1, more intersections are considered to be AC intersections than OD_1 to transport more vehicles.

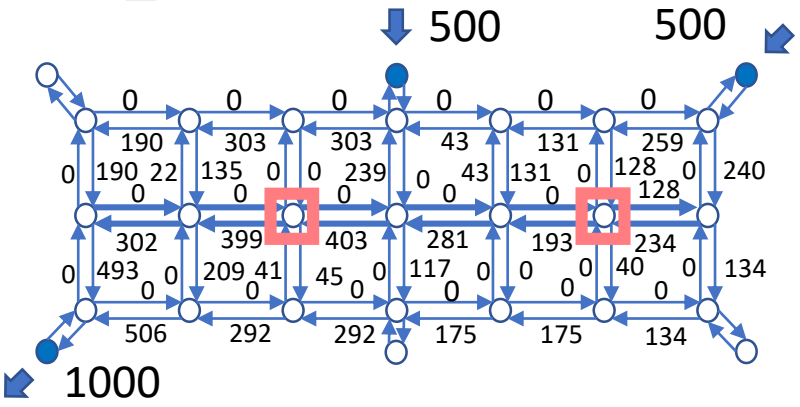
| AC location | TT |
|---|---|
| No access controlled | 864.7 |
| All access controlled | 883.99 |
| [1,2,3,5] access controlled | 841 |
| [1,2,3] access controlled | 842.17 |

| AC location | TT |
|---|---|
| No access controlled | 1403.49 |
| All access controlled | 1513.15 |
| [1,2,3,4] access controlled | 1348 |
| [1,3,4] access controlled | 1364 |

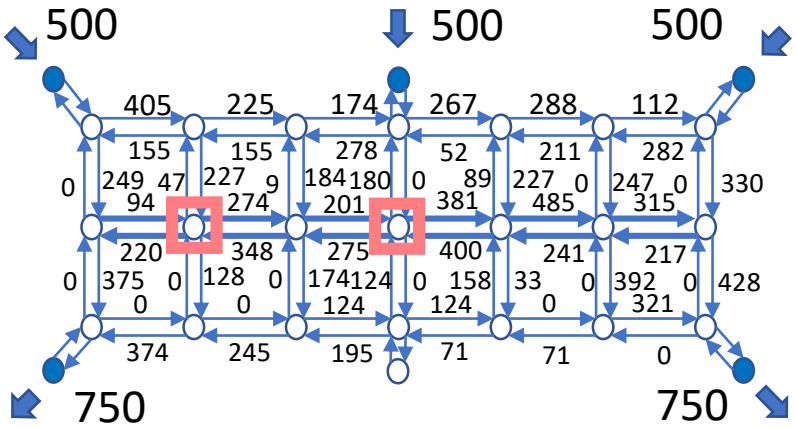
# Sensitivity Analysis -Link flow for OD patterns-



OD_4




OD_5




- OD demand is increased compared with OD_2, but there are less AC intersections than OD_2 .
- In order to let vehicles enter trunk road, AC intersections may be reduced.

- The same reason as OD_4

| AC location | TT |
|---|---|
| No access controlled | 1234.51 |
| All access controlled | 1385.29 |
| [2,5] access controlled | 1221.19 |
| [2] access controlled | 1223.64 |

| AC location | TT |
|---|---|
| No access controlled | 1453.79 |
| All access controlled | 1731 |
| [1,3] access controlled | 1432 |
| [1] access controlled | 1442.71 |

# Sensitivity Analysis - Confidence ratio for OD patterns-



Confidence ratio($N = 40, \rho = 0.4$)

| OD | Confidence ratio |
|---|---|
| OD_2 | 1.0 |
| OD_3 | 1.0 |
| OD_4 | 0.9 |
| OD_5 | 0.9 |

- With enough sample size N,
  a stable confidence ratio can be brought regardless of OD volume.

- With enough samples size N,
  higher score of confidence ratio can be expected
  even with small ρ.

# Summary



## Summary

- Formulation of a model to derive the optimal placement of AC intersections that minimizes the total travel time in roadway network

- Sensitivity analysis
  - Difference of accuracy of this model under different CEM parameter(N, ρ) and some OD cases
  - Validity of link flow for different OD patterns

## Future tasks

- Considering the combination of other granting data (Give data)
  (ex. Volume of link capacity , other types of operation on intersections)