

# Green-wave analysis in a tandem of traffic-light intersections

A. Oblakova, A. Al Hanbali, R.J. Boucherie,  
J.C.W. van Ommeren, W.H.M. Zijm





# Overview

---

- ▶ Network of intersections
- ▶ Stochastic model
- ▶ Numerical results

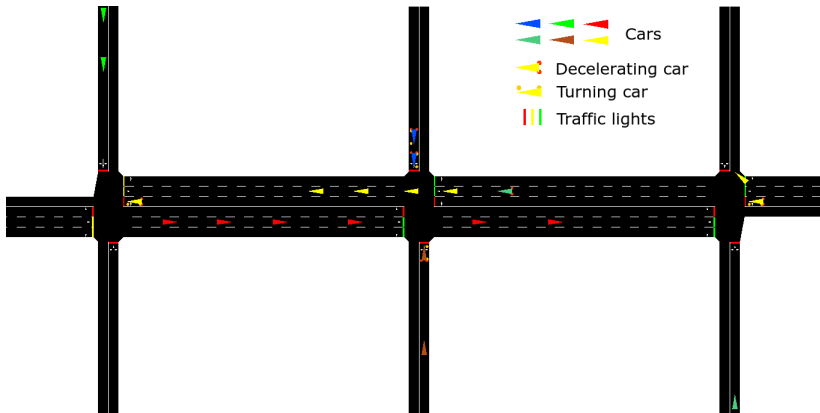


# Overview

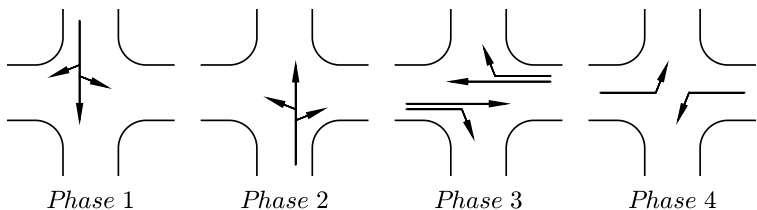
---

- ▶ Network of intersections
- ▶ Stochastic model
- ▶ Numerical results

# Network of Intersections



# Network of intersections: phases



- Fixed length of each phase.



## Network of intersections: fixed control

---


- ▶ Each lane has fixed green and red times.  
no real-time data
- ▶ Fixed common cycle length,  $c$ , in the network.  
coordination between intersections
- ▶ Control parameters: green times and offsets.  
offset is time between coordinated phases  
of two intersections



# Overview

---

- ▶ Network of intersections
- ▶ Stochastic model
- ▶ Numerical results




# Stochastic model: problems

---

- ▶ Service process is time-dependent.  
discrete-time model






## Stochastic model: problems

---

- ▶ Service process is time-dependent.  
discrete-time model
- ▶ High dimension of the system.  
network decomposition into separate lanes

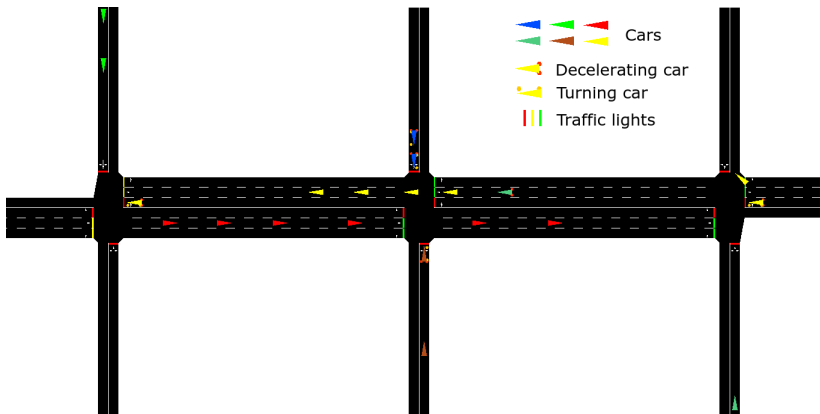


# Stochastic model: problems

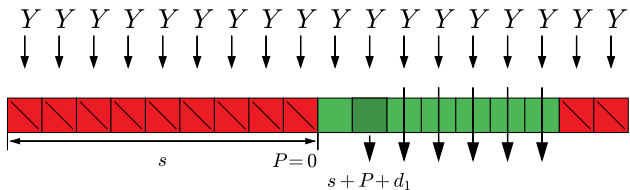
---

- ▶ Service process is time-dependent.  
discrete-time model
- ▶ High dimension of the system.  
network decomposition into separate lanes
- ▶ Dependency between lanes.  
arrival process

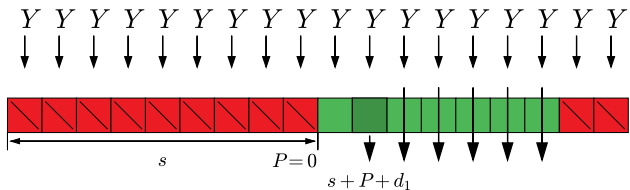
# Stochastic model: network



# Stochastic model: external lane

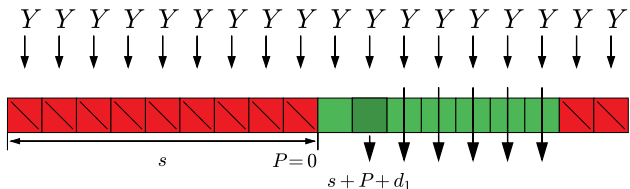


## Stochastic model: external lane



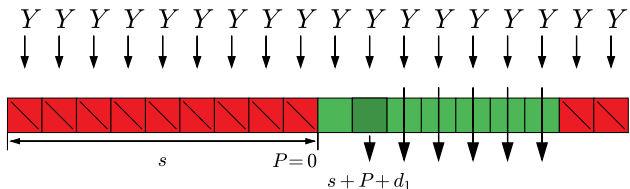
- Bernoulli arrivals: i.i.d.  $Y$ .

## Stochastic model: external lane



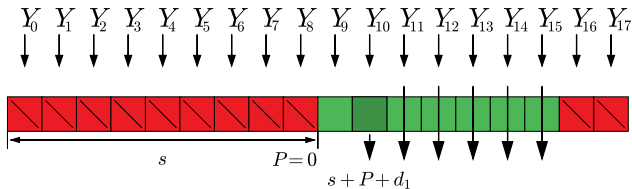
- ▶ Bernoulli arrivals: i.i.d.  $Y$ .
- ▶ Delayed departure at second  $s + P + d_k$ , where  $s$  — beginning of the green time,  $P$  — distraction variable,  $d_k$  — deterministic second of the  $k^{\text{th}}$  delayed vehicle.

## Stochastic model: external lane



- ▶ Bernoulli arrivals: i.i.d.  $Y$ .
- ▶ Delayed departure at second  $s + P + d_k$ , where  $s$  — beginning of the green time,  $P$  — distraction variable,  $d_k$  — deterministic second of the  $k^{\text{th}}$  delayed vehicle.
- ▶ If the queue becomes empty, all the arrivals proceed without stopping.

## Stochastic model: internal lane

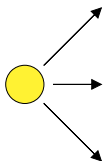


- ▶ Correlated arrivals.
- ▶ Acceleration of the delayed departures.



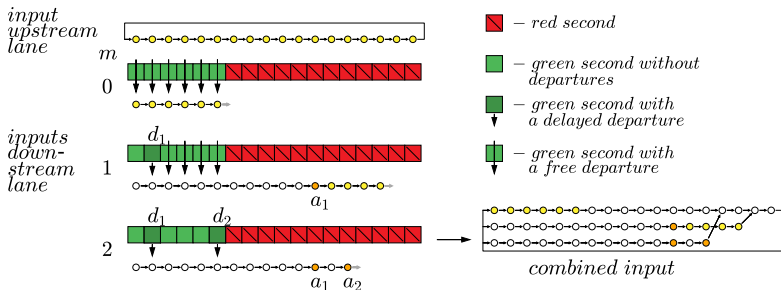
# Markovian arrival process


---



- ▶ Underlying Markov chain  $L_i, i = 0, \dots, c - 1$ .
- ▶ States represent information that determines arrivals, e.g., the number of delayed departures at the upstream lane.
- ▶  $\mathbb{P}(Y_i = 1 | L_i = l, Y_0, \dots, Y_{i-1}) = \lambda_l^i$ .

# Markovian arrival process






# Independence assumption

---

- ▶ The arrivals during different cycles are independent.



## Independence assumption

---

- ▶ The arrivals during different cycles are independent.

Under this assumption, we prove that the pgf of the queue length at a lane at the beginning of the cycle has form:

$$X(z) = \frac{\sum_{j=0}^{n-1} x_j f_j(z)}{z^n - A(z)C(z)},$$

where  $n$  is the maximum capacity,  $x_j = \mathbb{P}(X_0 = j)$ ,  $A(z)$  — the pgf of arrivals,  $C(z)$  — the pgf of the lost capacity due to randomness of  $P$ ,  $f_j(z)$  — polynomials.



# Overview

---

- ▶ Network of intersections
- ▶ Stochastic model
- ▶ Numerical results



# Green-wave efficiency

---

What is a **good green wave**?



# Green-wave efficiency

---

What is a **good green wave**?

**Definition** The *green-wave efficiency* is the expected number of intersections passed without stopping for an arbitrary vehicle.



## Green-wave efficiency

---

What is a **good green wave**?

**Definition** The *green-wave efficiency* is the expected number of intersections passed without stopping for an arbitrary vehicle.

- ▶ In an ideal green wave, the green-wave efficiency is equal to the expected number of intersections for a vehicle.





## Green-wave efficiency

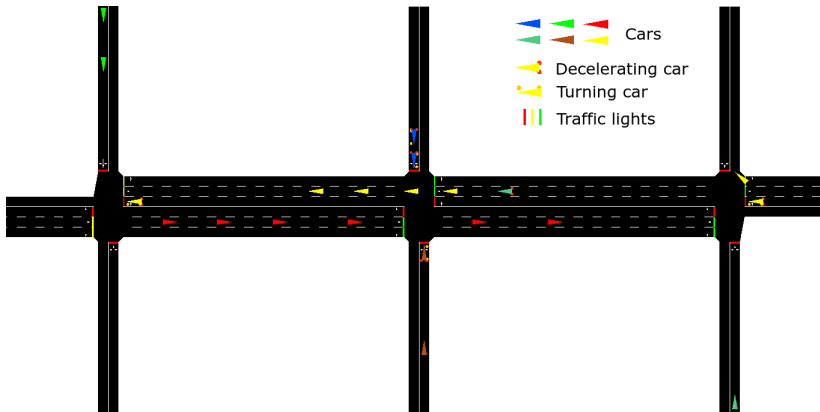
---


What is a **good green wave**?

**Definition** The *green-wave efficiency* is the expected number of intersections passed without stopping for an arbitrary vehicle.

- ▶ In an ideal green wave, the green-wave efficiency is equal to the expected number of intersections for a vehicle.
- ▶ In the worst case scenario, all of the vehicles need to stop, and our measure is equal to 0.

# Optimisation: network of intersections





## Optimisation: parameters

---

We consider a tandem of 3 intersections (100 meters apart):

- ▶ the arrival rate from west is  $\lambda$ ,
- ▶ the arrival rate from east is  $0.5\lambda$ ,
- ▶ the arrival rate from north and south is  $0.2\lambda$ ,
- ▶ 16% of the major traffic turns south or north,
- ▶ 40% (20%) of the minor traffic turns east (west).



# Optimisation: objectives and constraints

---

Optimisation with multiple objectives:

- ▶ maximising the green-wave efficiency,
- ▶ minimising the average delay



# Optimisation: objectives and constraints


---

Optimisation with multiple objectives:

- ▶ maximising the green-wave efficiency,
- ▶ minimising the average delay

for

- ▶ fixed cycle length of 60 seconds,
- ▶ given phase schedule.

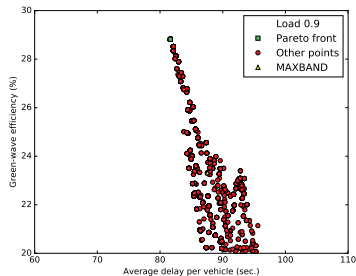
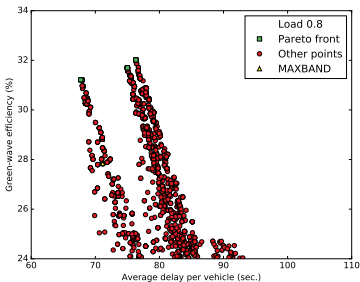
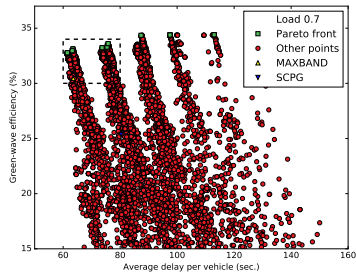
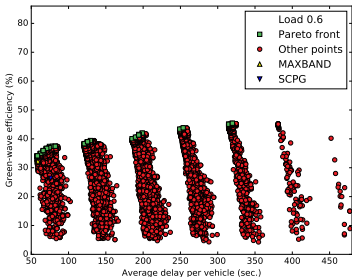


## Optimisation: approaches

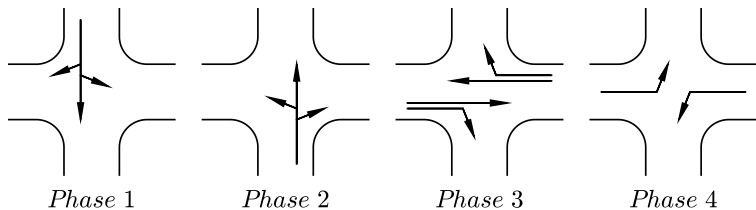
---

- ▶ Genetic algorithm coupled with our model, multiple objectives
- ▶ SUMO cycle program generator (SCPG), Webster (proportional) green time allocation
- ▶ MAXBAND.  
bandwidth maximisation

# Optimisation results: Pareto optimality

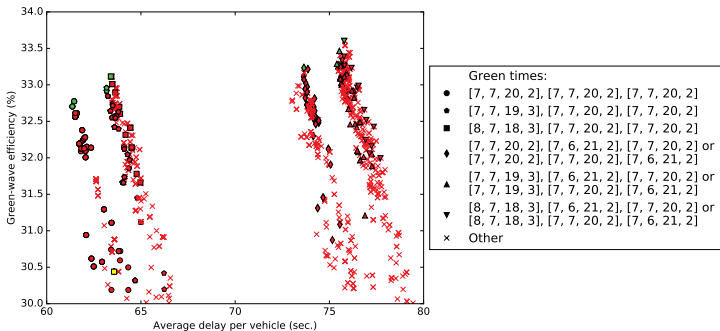


# Optimisation: phases





# Optimisation results: Pareto optimality load 0.7





## Conclusions

---

- ▶ It is important to take the real behaviour of traffic into account.
- ▶ Optimisation for the best green wave may be disadvantageous for the average delay.
- ▶ The average delay per vehicle is very sensitive to the changes in the green times.