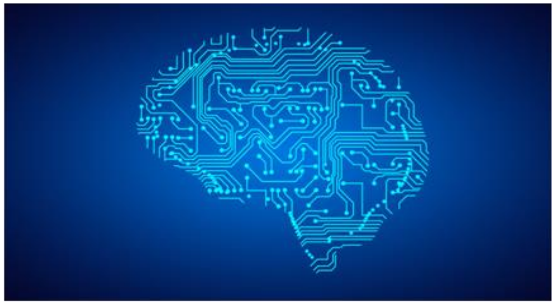
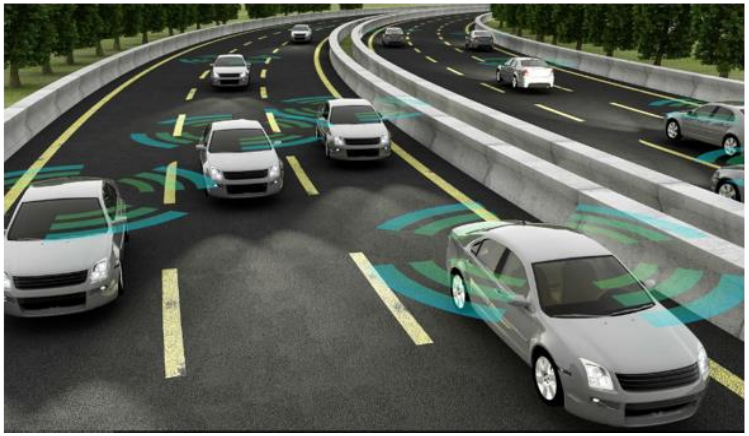
Calibrating Car Following Models





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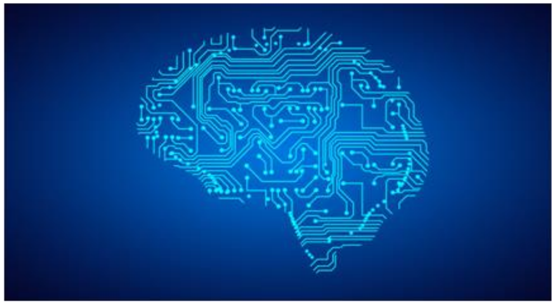
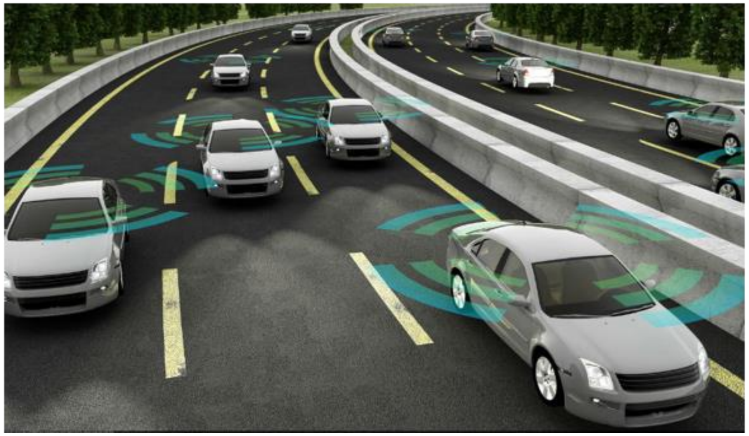
Calibration:

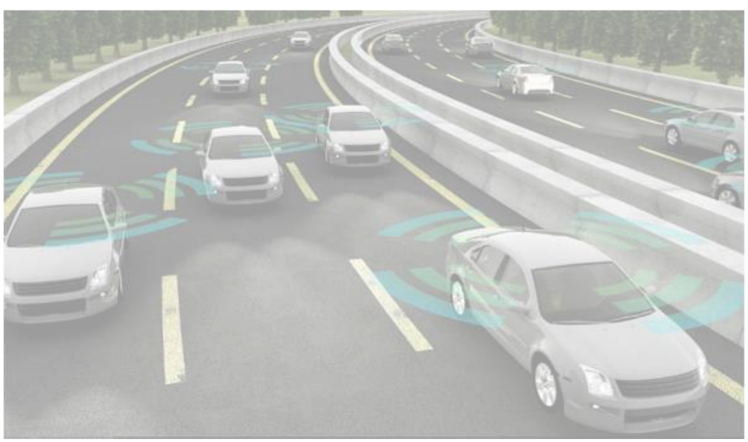




Calibrating Car Following Models

Calibration:

Given some data and a model, find the model parameters that best fit the data



Outline

- Macro and Micro refresher

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- Problem and Motivation

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- Calibration as Optimization Problem

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- Calibration as Optimization Problem
- Adjoint method
- Preliminary Results and Discussion

Macroscopic Models

- Cell Transmission Model (CTM)/Lighthill-Whitham-Richard (LWR)

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- Fundamental Diagram

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- Hyperbolic conservation law; Link to hydrodynamics

Macroscopic Models

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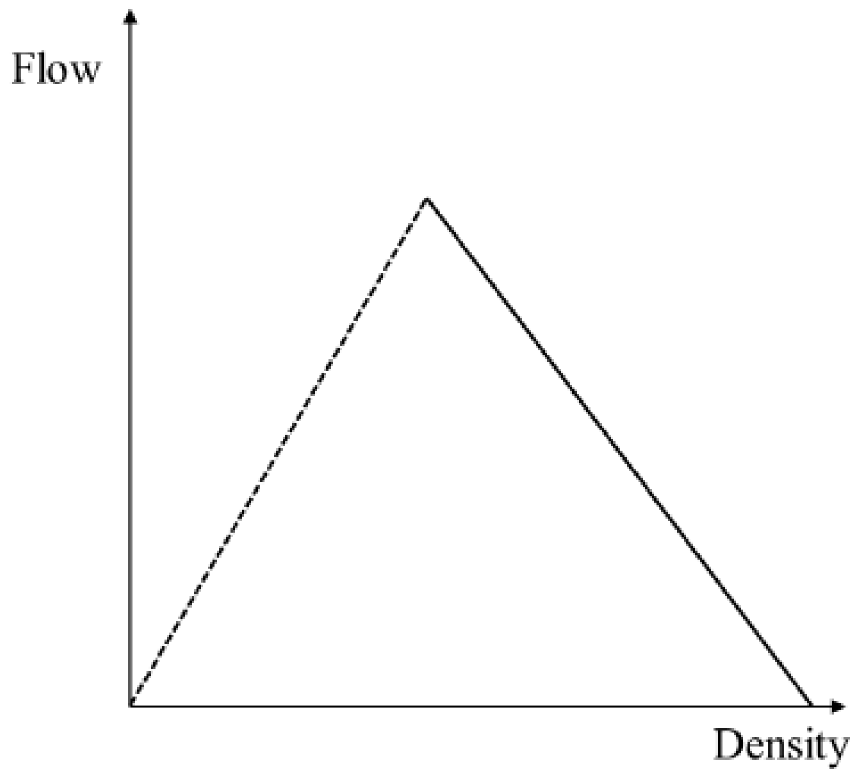
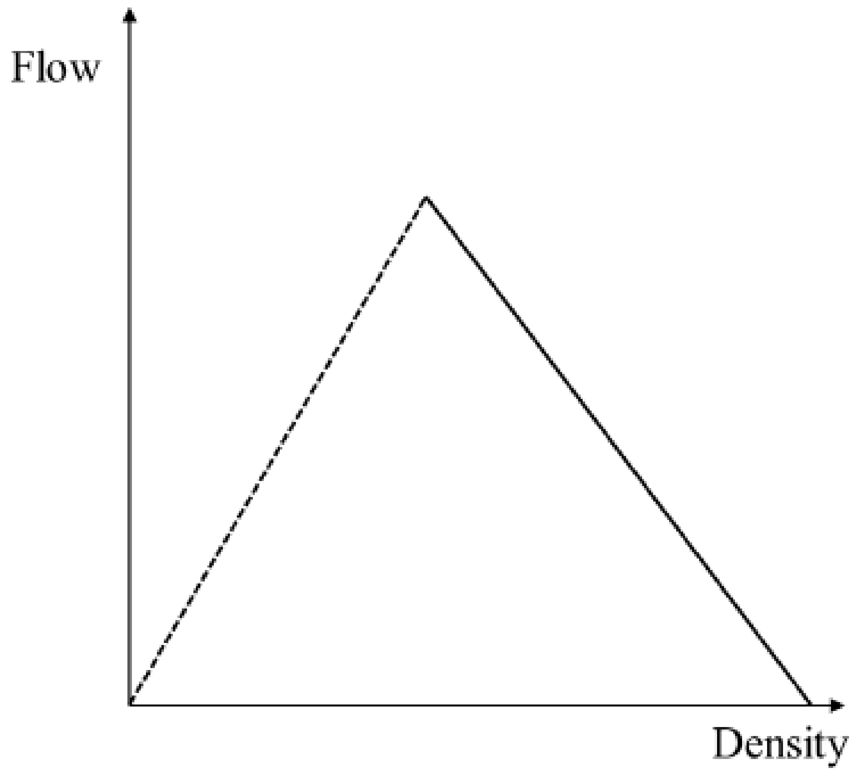
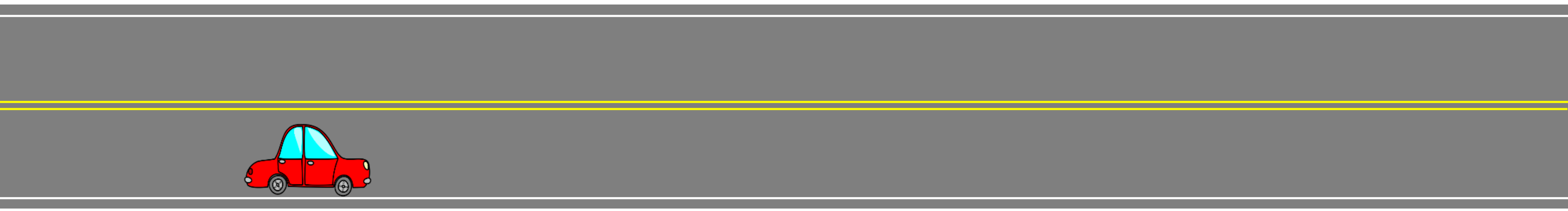


Macroscopic Models



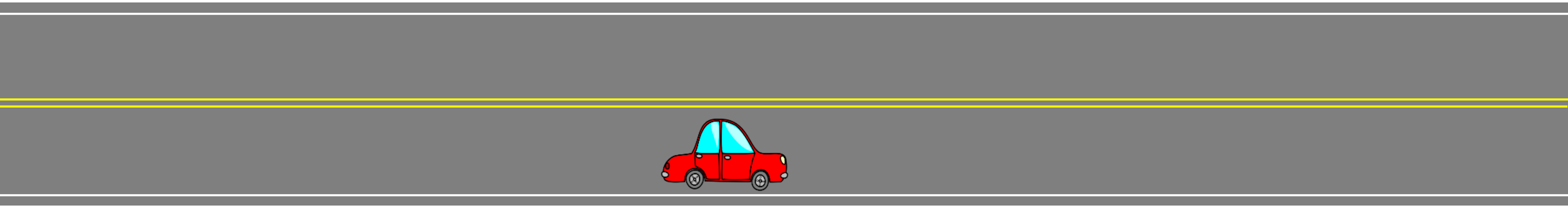
MFD and conservation laws agreed upon framework for macro- modeling.

Car-following = Microsimulation



Suppose we know the trajectory of a “lead vehicle”

Car-following = Microsimulation



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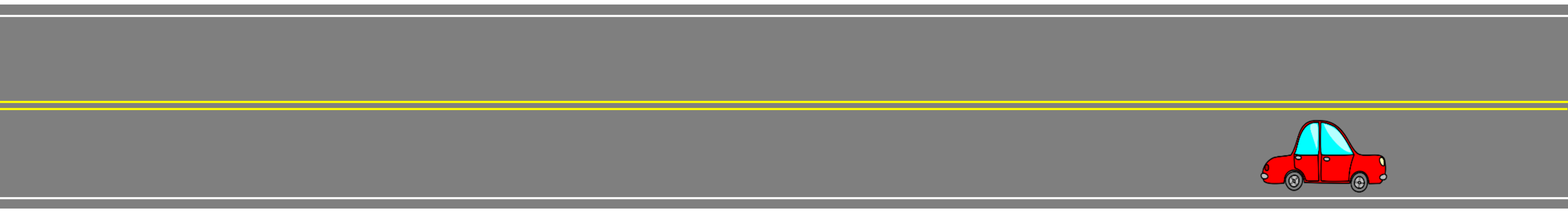

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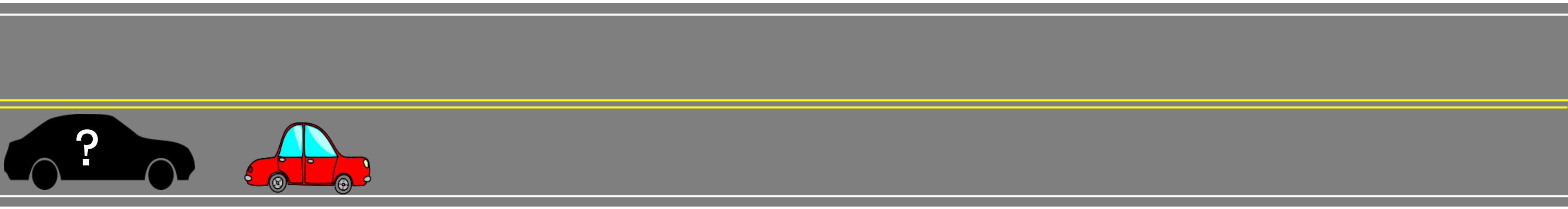

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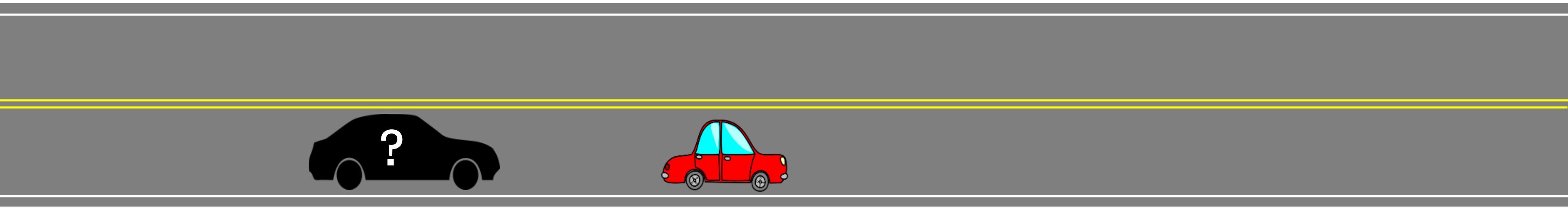

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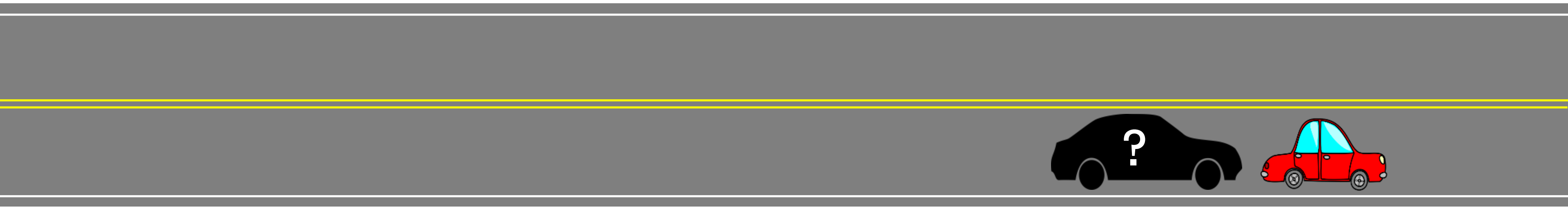

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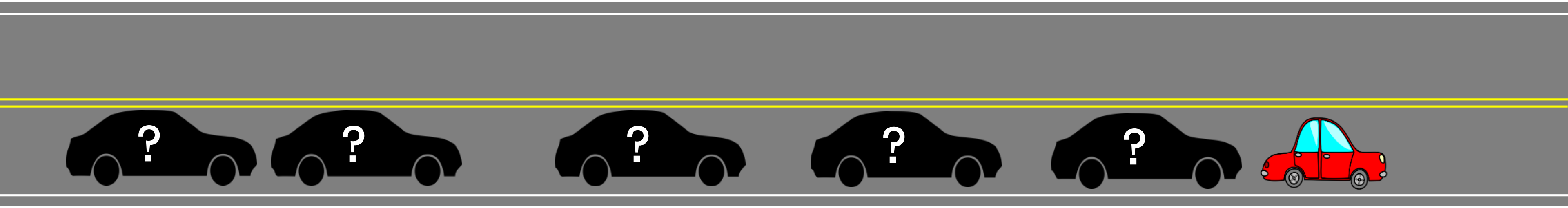

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- Nonlinear Nonconvex program. Formalisms later

Why Study Car-following?

- Reason 1

- Reason 2

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  Micro-models have different strengths compared to macro-

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- Micro- can be better for short links; low densities
- Micro- gives more flexibility for inhomogeneous traffic

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- Reason 2 Many new sources for trajectory data

- GPS (smart phones)

Photo credit: CNBC

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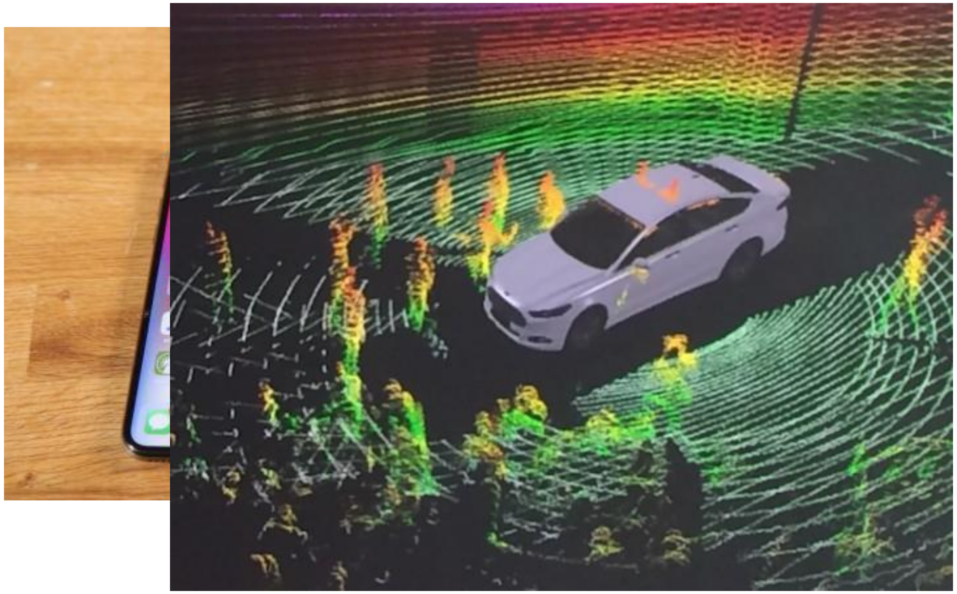

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Photo credit: CNBC, cleantechnica

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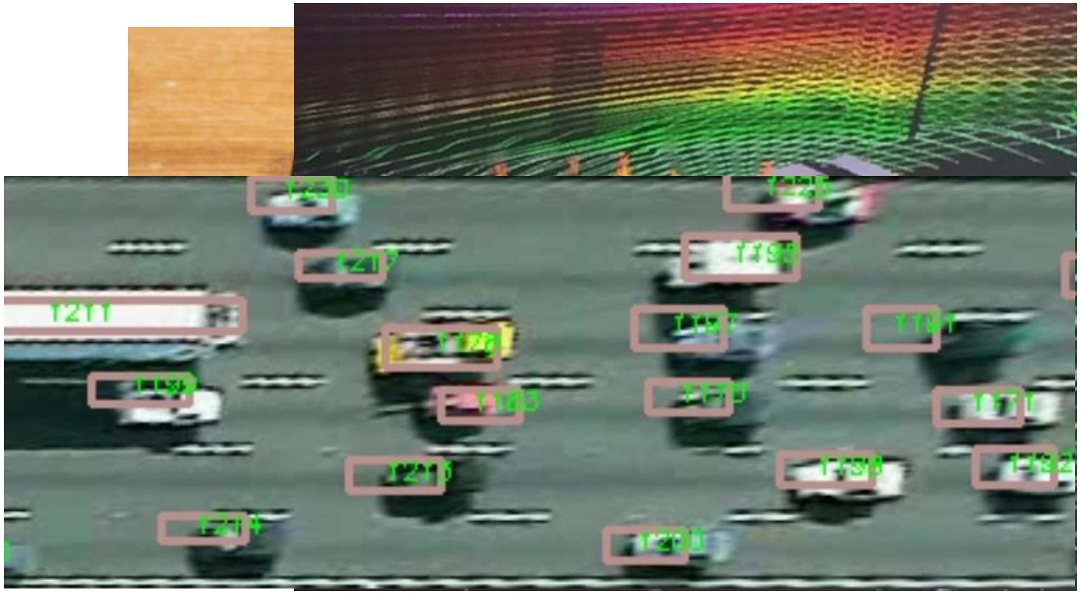

- GPS (smart phones)
- Lidar/Radar (A.V.)
- Computer Vision (traffic cameras)

Photo credit: CNBC, cleantechnica, FHWA

Formulation

Some notation

- $x_n(t)$: vector consisting of position and speed of vehicle n, at time t, calculated from the car-following model.
- $h_n(x_n(t), x_n(t-\tau_n), x_{n-1}(t-\tau_n), p_n)$: generic car-following model such that $\dot{x}_n = h_n$.
- p_n: a vector consisting of model parameters describing vehicle n.
- $\hat{x}_n(t)$: "true" measurements of vehicle n, taken from data.

Formulation

Define Calibration as the following optimization problem:

$$
\begin{aligned}
\min_{p} \quad & F = \sum_{i=1}^{n} \int_{t_i}^{T_{i-1}} f(x_i, \hat{x}_i, t)dt \\
\text{s.t.} \quad & \dot{x}_i(t) - h_i(x_i(t), x_{i-1}(t), p_i) = 0, \quad t \in [t_i, T_{i-1}], \quad i = 1, \ldots, n \\
& x_i(t_i) - (\hat{x}_i(t_i) + a_i) = 0 \quad i = 1, \ldots, n \\
& b_{\text{low}} \leq p \leq b_{\text{high}}
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$$

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Define Calibration as the following optimization problem:

Maximize Goodness of fit (minimize loss)

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Initial conditions (for model)

Box constraints on parameters

Video Interlude

Next Generation Simulation (NGSim) dataset created by FHWA.

1 camera of 1 location (U.S. I-80) for 15 minutes

Adjoint Method

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Adjoint method is faster, especially when the model has a lot of parameters.

Adjoint method can also be more accurate.

Adjoint Method

Adjoint method is for optimization problems constrained by differential equations

Adjoint: O(1)

Finite Differences: O(m)

m is number of parameters.

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Objective function:

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Augmented Objective function:

$$L = \int_{t_n}^{T_{n-1}} dt \Big[f(x_n, \hat{x}_n, t) + \lambda^T(t) \big(\dot{x}_n(t) - h_n(x_n(t), x_{n-1}(t), p_n) \big) \Big]$$

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Adjoint variables (think of as Lagrange multiplier)

$$\frac{dF}{dp} = \frac{dL}{dp}$$

1. Solve model for x
2. Solve adjoint system for Lambda
3. Calculate gradient

Numerical Experiment

Goal: To compare different ways of solving calibration problem.

1. Gradient-free

2. Gradient-based using finite differences

3. Gradient-based using adjoint method

Numerical Experiment

	Finite BFGS	Adjoint BFGS	NM Simplex	Differential Evolution
# Objective Evaluations	972	34	1965	3246
# Gradient Evaluations	0 (118)	6	0	0
Final Objective Value	1723	3509	1613	2914
Runtime (s)	9.64	1.23	24.69	32.74

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	Objective Function	Finite Differences	Adjoint method
Runtime (s)	0.011	0.047	0.032

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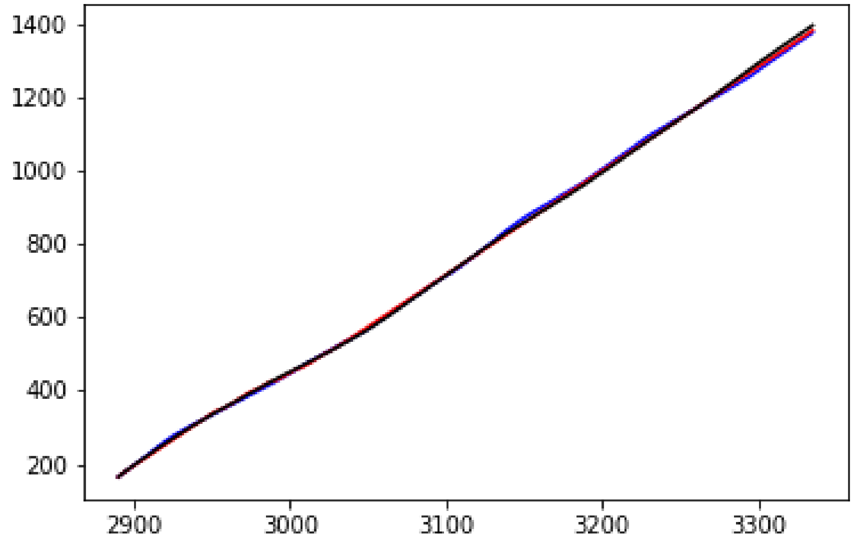
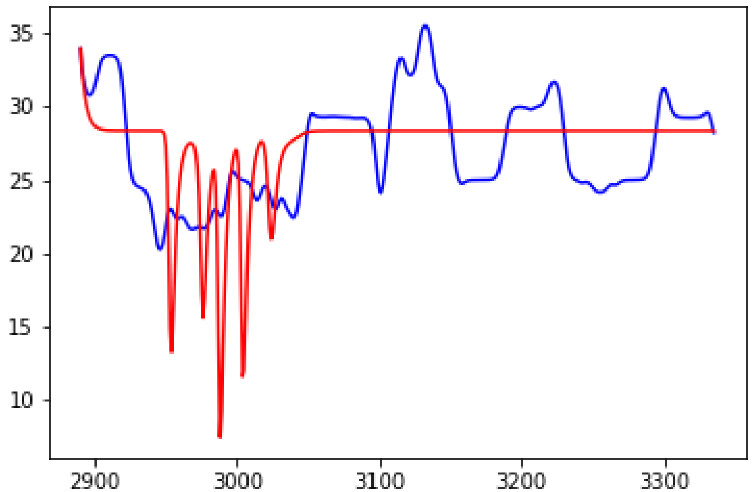


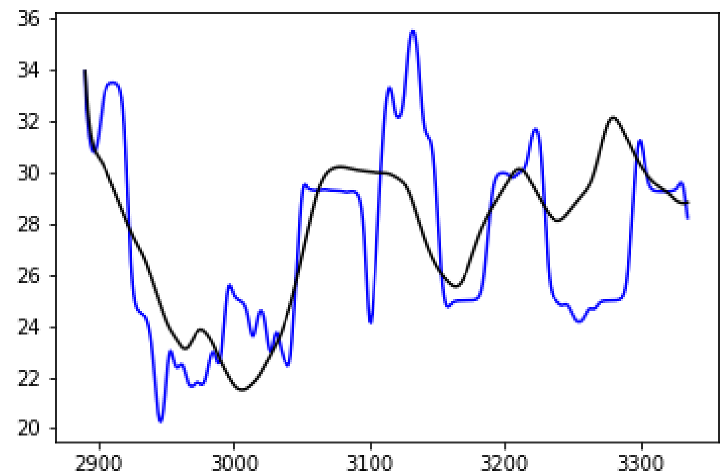
Numerical Experiment

Finite Dif. BFGS vs Adjoint BFGS



Speed-Time Planes

Blue is measurements (data)



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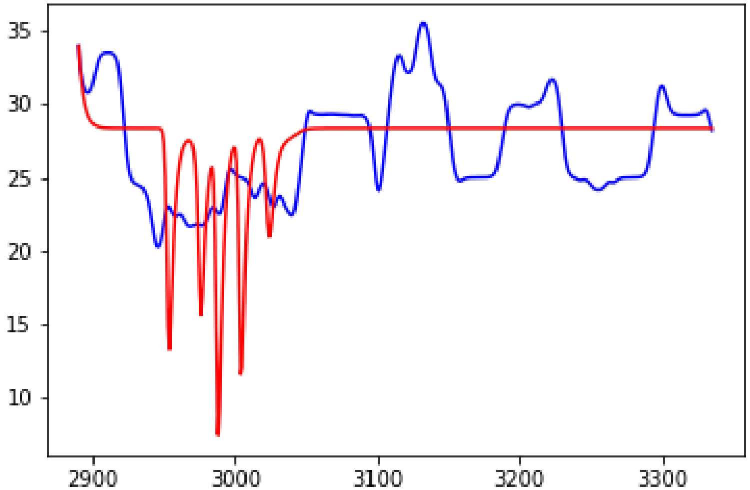

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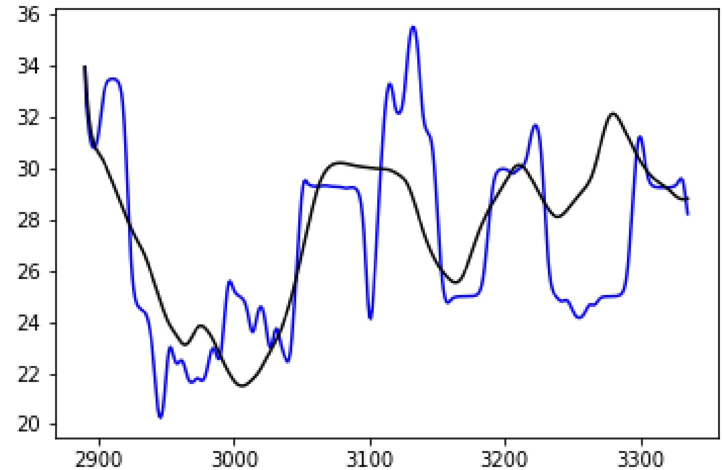
Finite Differences has given strange result because of inaccurate gradient



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- Can use gradient-based algorithms to solve calibration problem faster compared to grad-free
- Choice of optimization algorithm affects results; lower objective value not necessarily better!