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Analysis in Kantrovich geometric space for quasi-stable patterns in 2D-OV model

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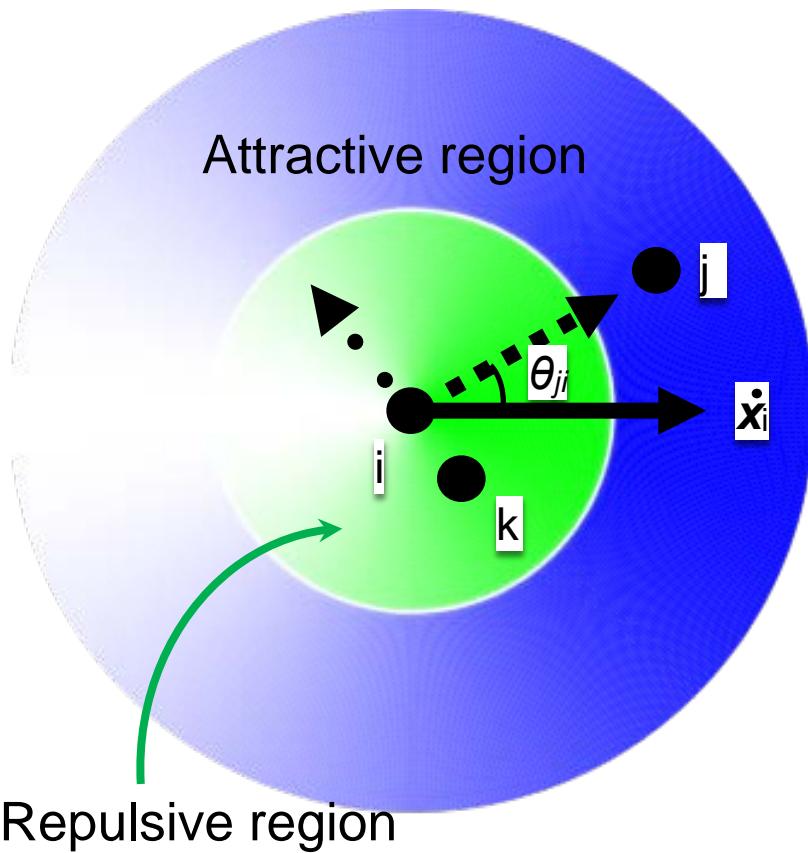
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1-1. Two-dimensional Optimal Velocity Model

Basic Idea

An organism moves to maintain an optimal velocity which depends on distances to others.

Equation of motion



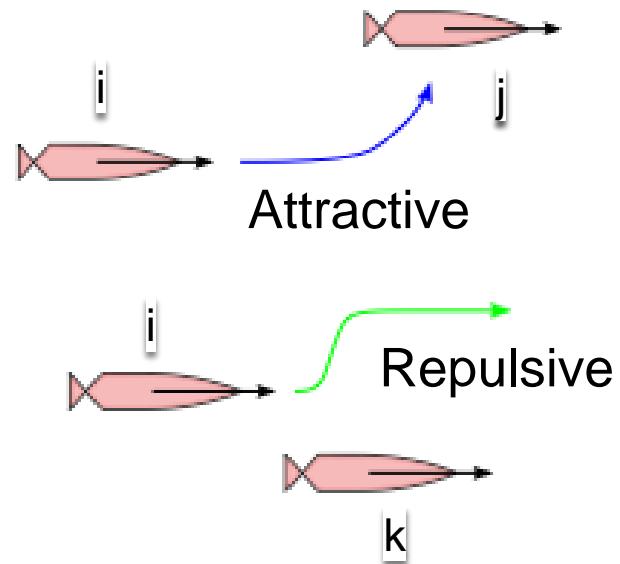
$$\frac{d^2}{dt^2} \mathbf{x}_i = a \left[\sum_j \mathbf{V}(\mathbf{r}_{ji}) - \frac{d}{dt} \mathbf{x}_i \right]$$

$$\mathbf{V}(\mathbf{r}_{ji}) := (1 + \cos \theta_{ji}) f(|\mathbf{r}_{ji}|, c) \frac{\mathbf{r}_{ji}}{|\mathbf{r}_{ji}|}$$

$$\mathbf{r}_{ji} : \mathbf{x}_j - \mathbf{x}_i$$

a : sensitivity

c : control parameter of the threshold of **attractive** and **repulsive** interactions



1-2. Pattern formations of 2D-OV model

Attractive interaction

$c=1.0$

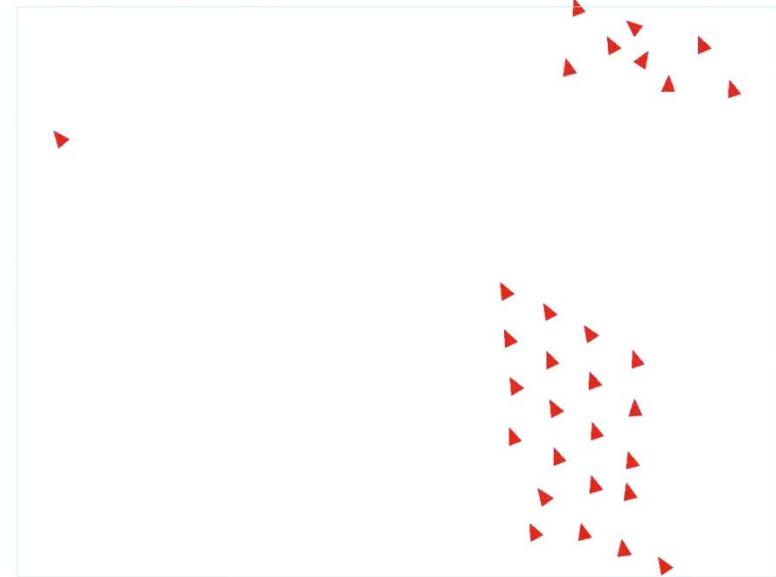
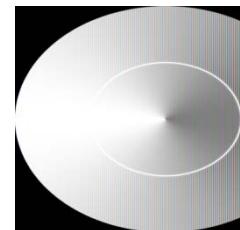


(a) $a=5.0, c=1.0, N=50$

String-like flow

Attractive + Repulsive mixed Interaction

$c=0.0$



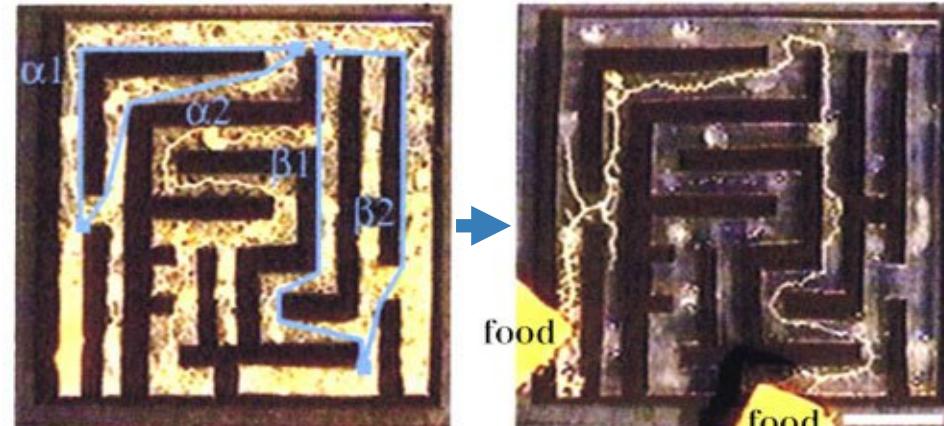
(a) $a=5.0, c=0.0, N=30$

Cluster flow

2-1. Optimal patterns of Collective bio-motion(Amoeboid) and Flow of self-driven particles(2D-OV)

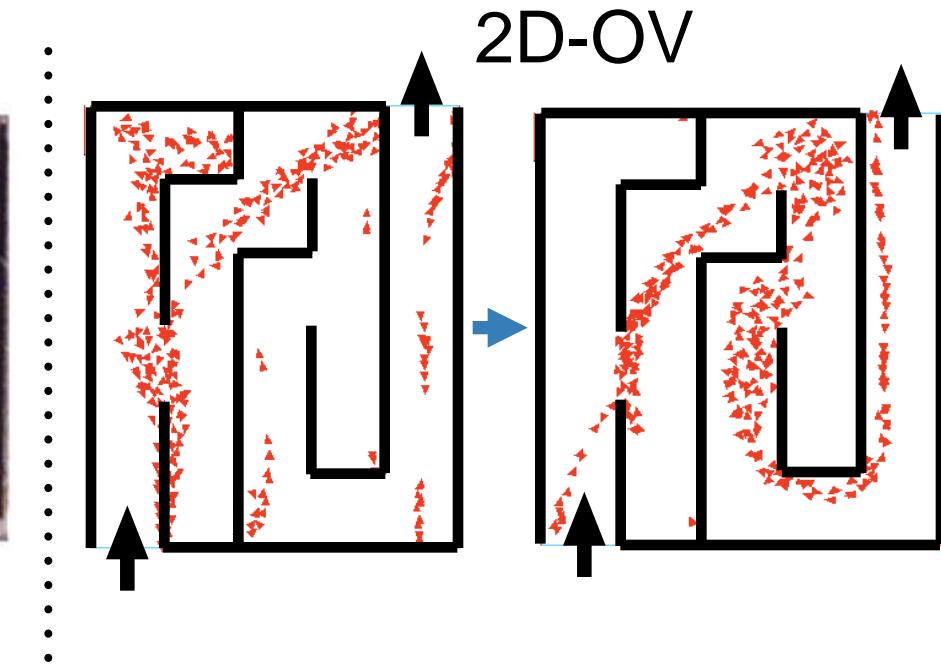
- e.g. Maze solving -

Amoeboid



Nakagaki, T., Yamada, H., & Tóth, A.
(2000). Maze-solving by an amoeboid
organism. *Nature*, 407(6803), 470.

- Amoeboid organism organize the optimal path between the two locations of foods in the maze.
- 2D-OV particles organize optimal flow pattern between two gates connected with the periodic boundary condition in the maze.



Is there any quantity to estimate the stability
of the optimal flow pattern?

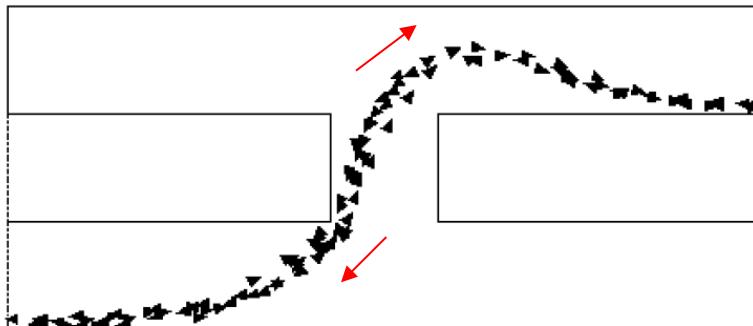
Formation of solutions for a simple maze



Periodic boundary

Elastic boundary

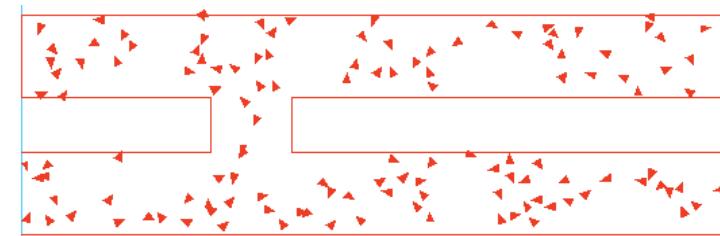
Solutions of optimal path



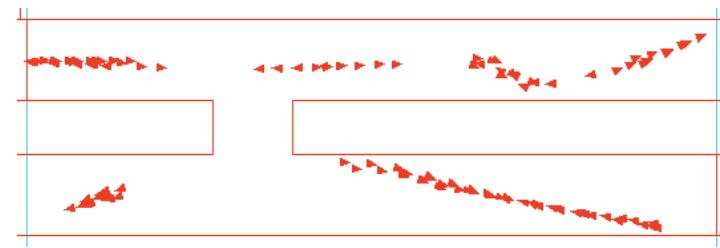
i) Solution of periodic boundary

($a=20.0, N=128$)

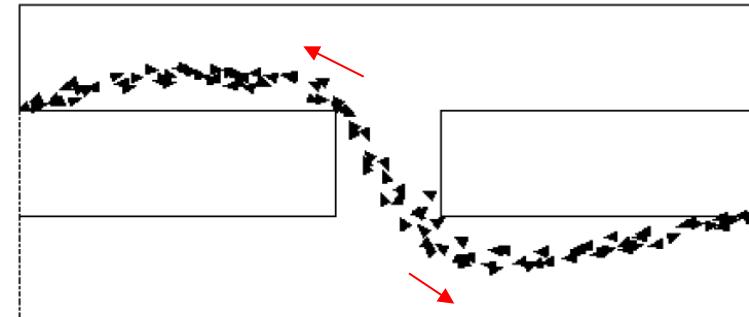
Initial state (random motions)



Intermediate state (creating strings)



Optimal path



ii) Solution of elastic boundary

2-2. Search for the quantity of stability of a pattern

- e.g. Simple maze -

Measurement of the similarity
between patterns in real space



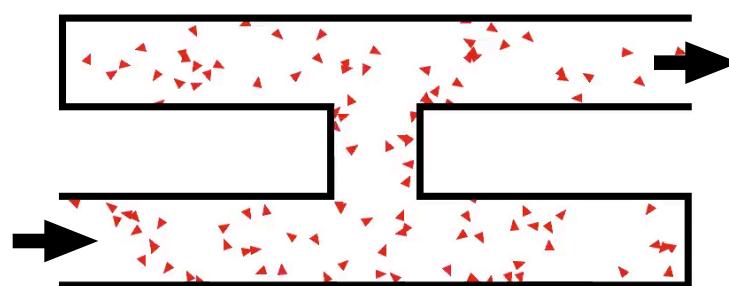
Kantorovich metric space

One pattern of particles
in real space

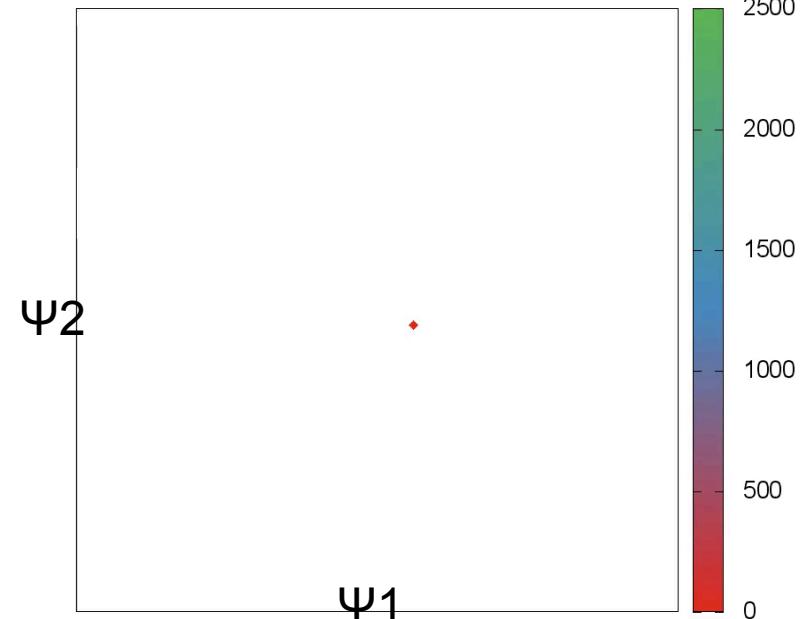


One point
in the Kantorovich metric space

Time step : 0001



Motion of 2D-OV particles
in real space



Trajectory for the changing
of flow patterns
in Kantorovich metric space

Kantorovich Metric

(measurement of affinity, similarity, resemblance)

Distance of
kr (P_i, P_j)

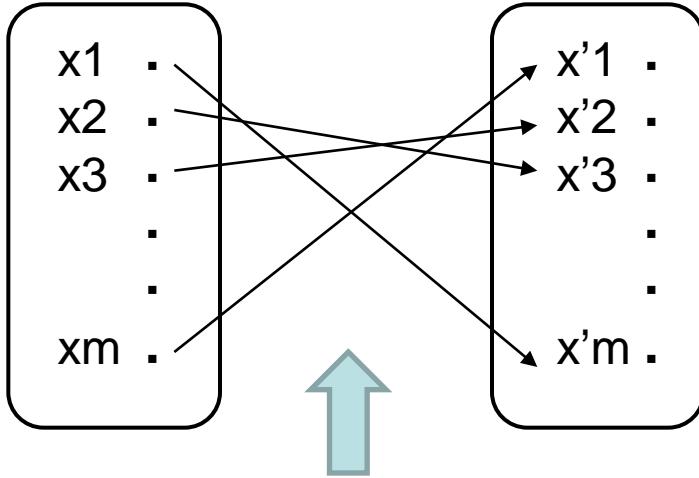
Pattern P_i

x₁ .
x₂ .
x₃ .
. .
. .
x_m .

Pattern P_j

x'₁ .
x'₂ .
x'₃ .
. .
. .
x'_m .

$x \cdot$: point



Transportation of points from P_i to P_j
: several plans

$$kr(P_i, P_j) := \min \{ \text{Cost of transportation } (P_i, P_j) \}$$

\Leftrightarrow “Optimal Transportation problem”

Time sequence of patterns : $P(t_1) \rightarrow P(t_2) \rightarrow P(t_3) \rightarrow \dots \rightarrow P(t_n)$

“Affinity matrix”

$$B := \begin{bmatrix} Kr(P(t_1), P(t_1)) & Kr(P(t_1), P(t_2)) & \dots & Kr(P(t_1), P(t_n)) \\ Kr(P(t_2), P(t_1)) & Kr(P(t_2), P(t_2)) & \dots & Kr(P(t_2), P(t_n)) \\ \vdots & \vdots & \ddots & \vdots \\ Kr(P(t_n), P(t_1)) & Kr(P(t_n), P(t_2)) & \dots & Kr(P(t_n), P(t_n)) \end{bmatrix}$$

↓

“Kantorovich space” : eigen vector : { $\Psi_1, \Psi_2, \dots, \Psi_r$ } $r = \text{rank } B$
 eigen value : $\lambda_1 > \lambda_2 > \dots > \lambda_r$

a pattern $P(t_i) \Rightarrow c_1\Psi_1 + c_2\Psi_2 + \dots$: a vector in Kantorovich space



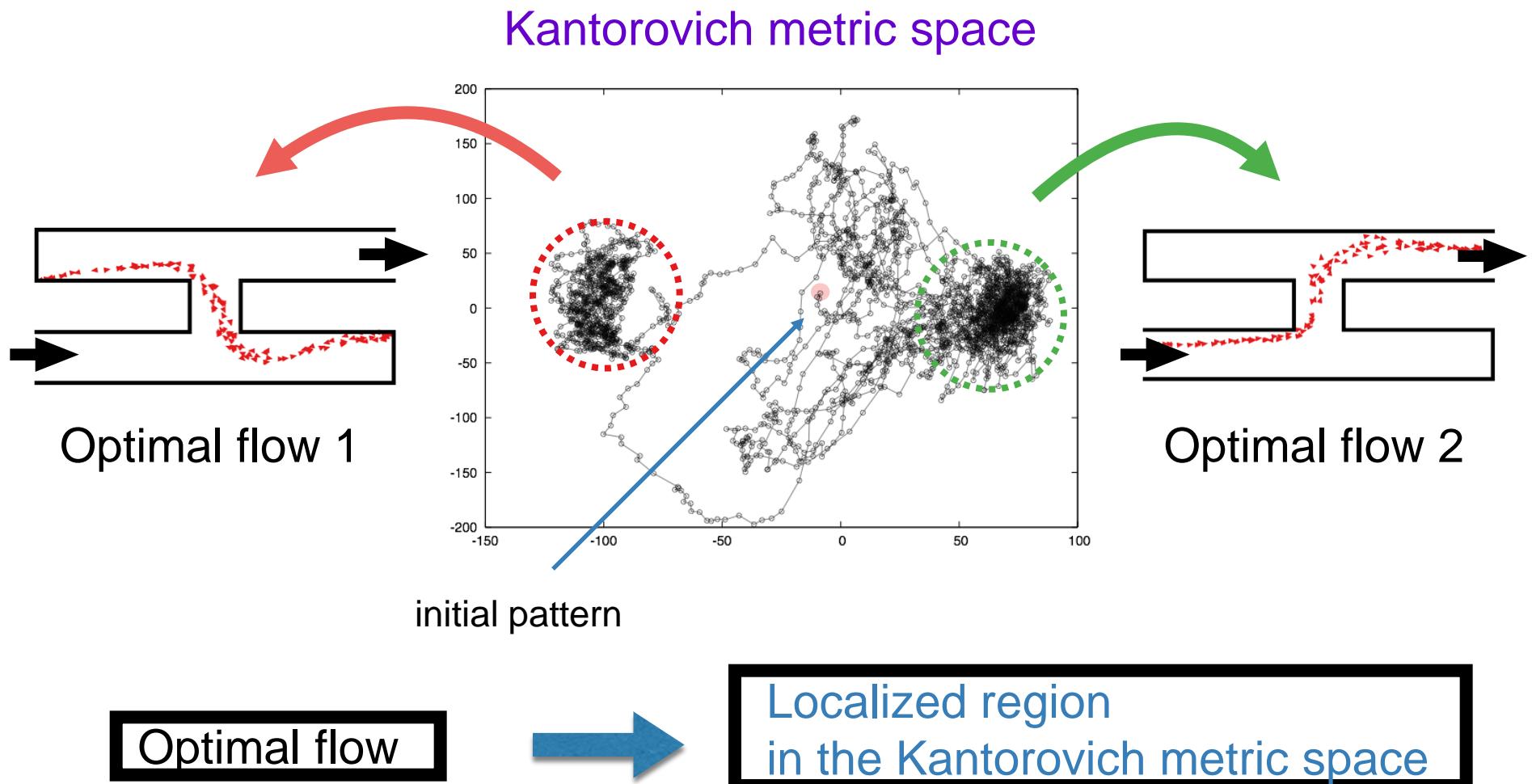
$p_i \in$ vector in Euclidean space $R^r \sim R^2$
dimensional reduction

Transition of patterns :

$P(t_1) \rightarrow P(t_2) \rightarrow P(t_3) \rightarrow \dots \rightarrow P(t_n)$

⇒ Motion of a point in Kantorovich space: $p_1 \rightarrow p_2 \rightarrow p_3 \rightarrow \dots \rightarrow p_n$

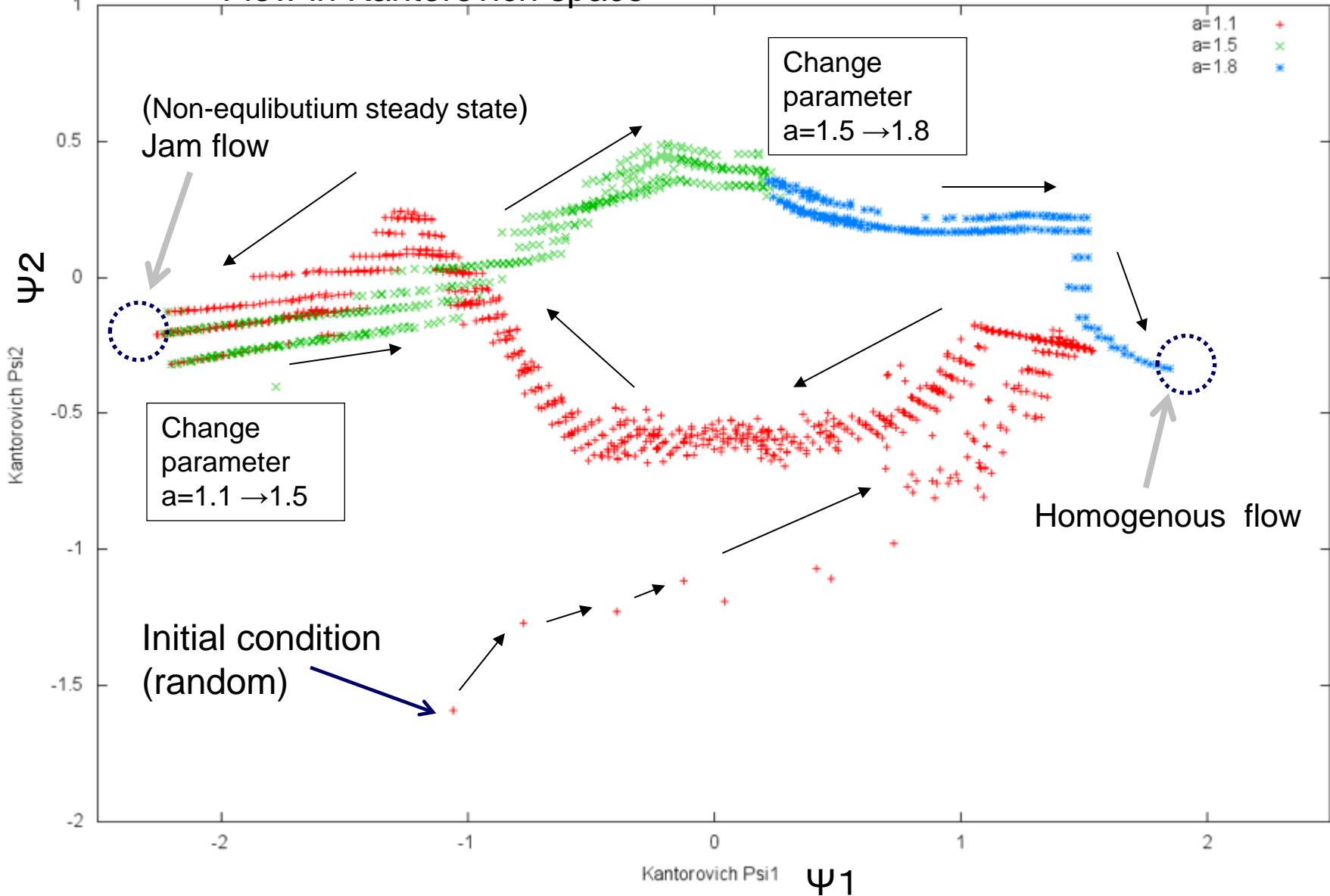
2-3. Time evolution of the forming optimal flow pattern in the Kantorovich metric space



Conclusions and Discussions

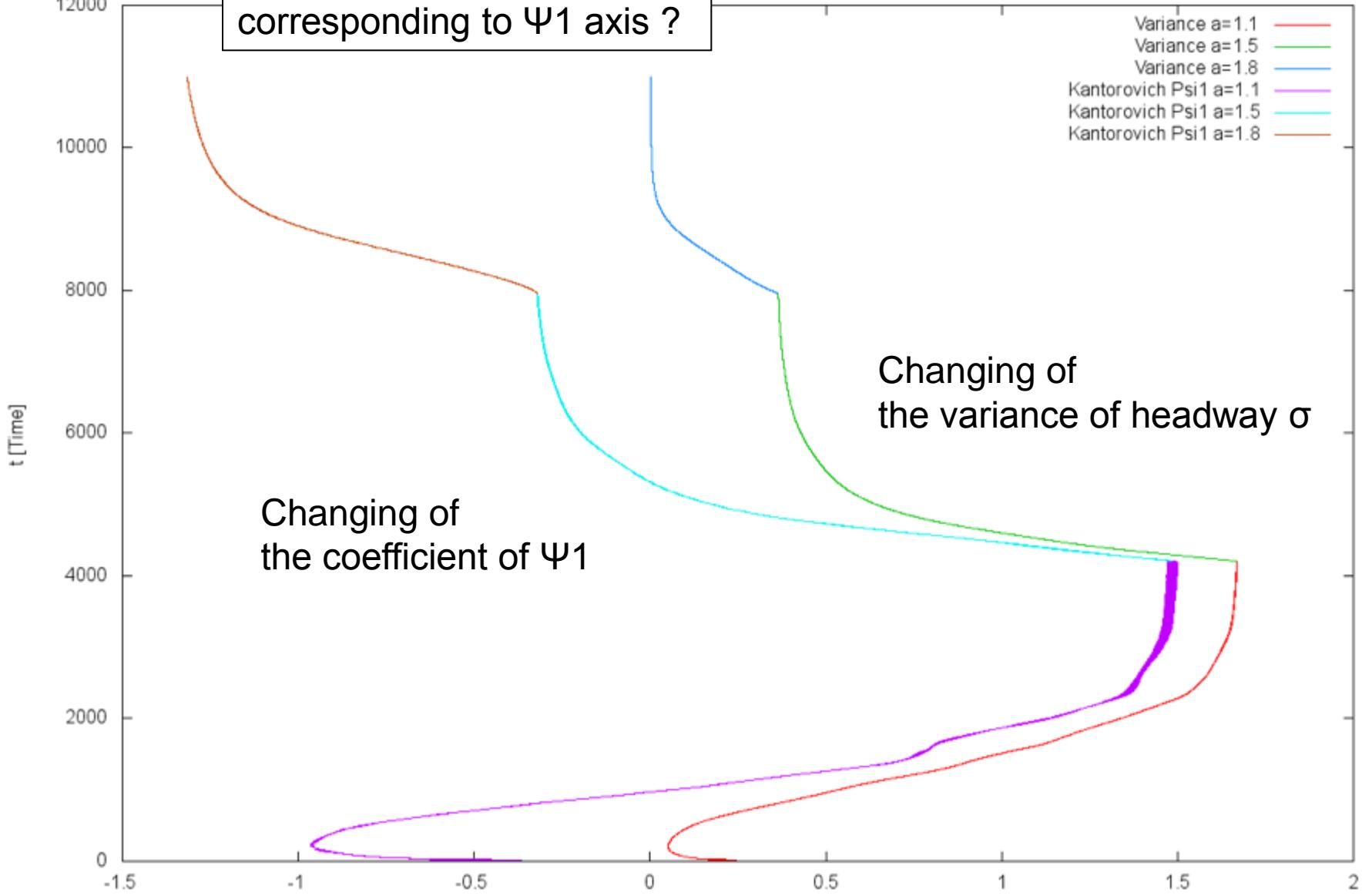
- 2D-OV particles create several flow patterns for a given control parameter.
- Optimal flow of 2D-OV particles form an optimal pattern in a maze. ← Continuous deformation of pattern ↴
- Optimal flows in the maze can be represented as each localized region in a low dimensional **Kantorovich metric space**.
- The high degeneracy of ground state of “thermo dynamical potential” of a few macroscopic variables Ψ_1, Ψ_2 ? in **Kantorovich metric space**.

Flow in Kantorovich space

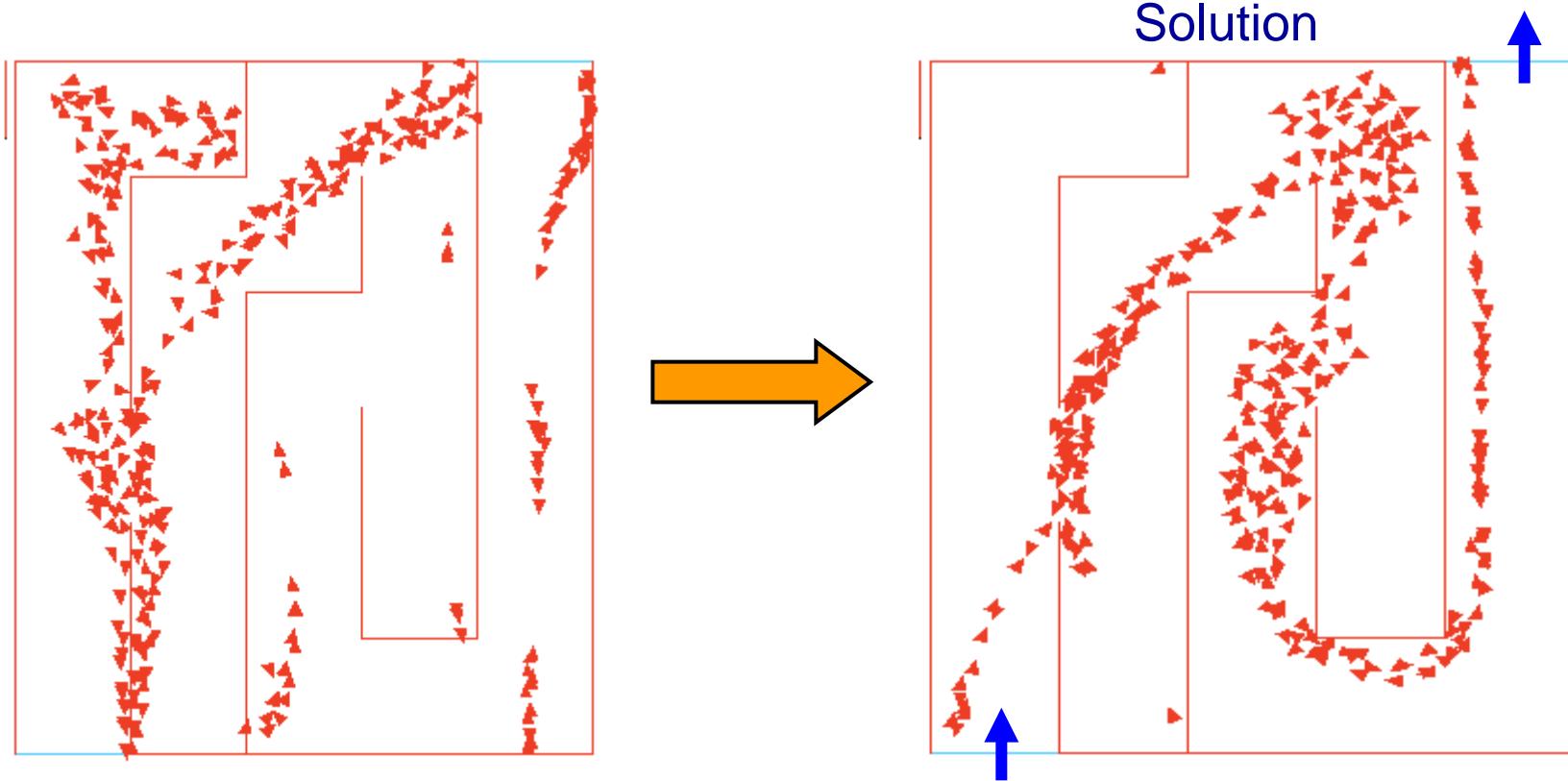


Ψ_1 axis is important !

What variable is corresponding to Ψ_1 axis ?



● A trial to more difficult maze -1 ($a=20$, $c=1$)

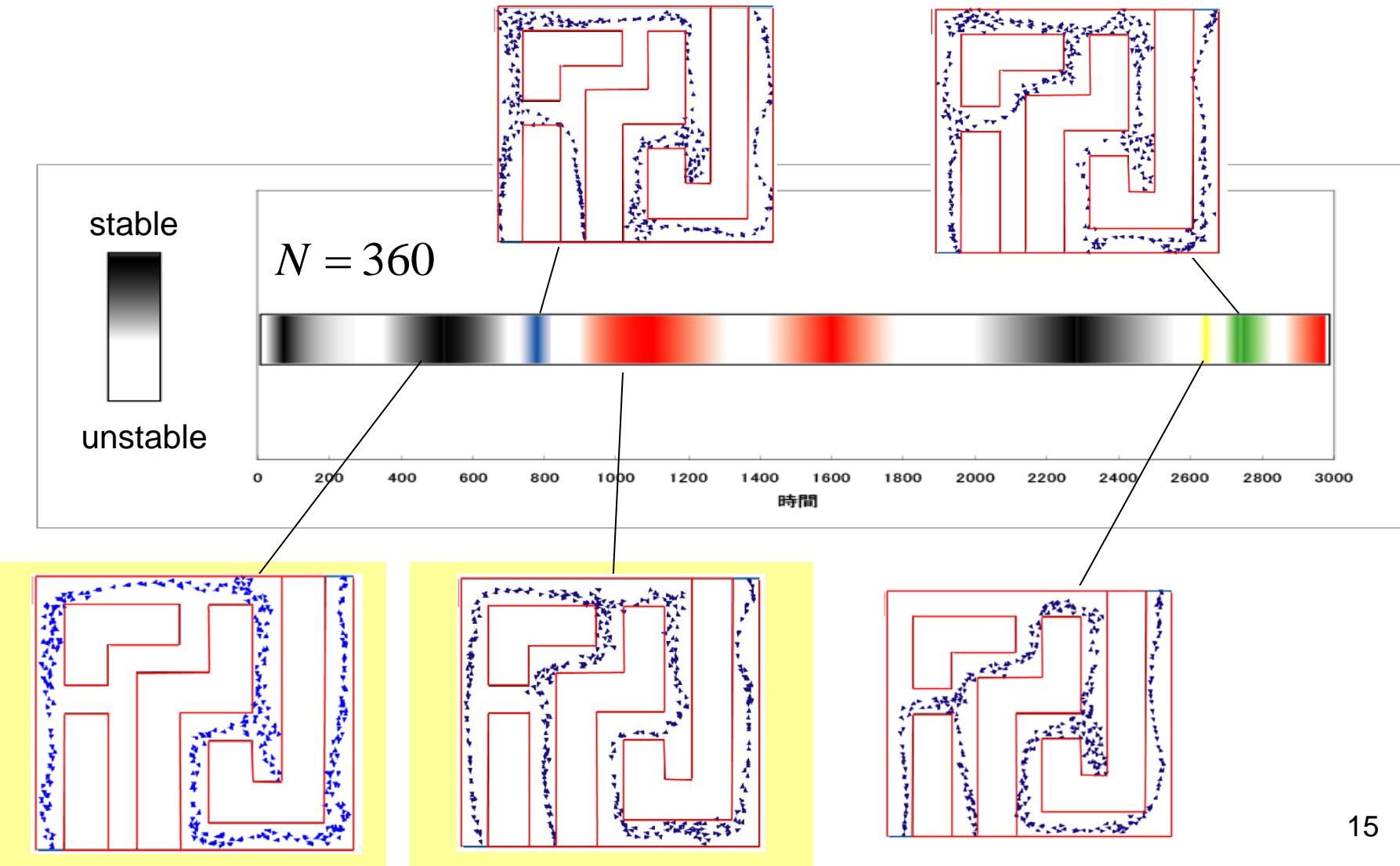


of particles 330

- Solution is formed as a quasi stable state.
- Optimal # of particles is needed.

● Another trial to more difficult maze -2

Life time and frequency of appearance of patterns



References

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