# Boarding of Finite-Size Passengers to an Airplane 

Talk presented at TGF '15

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Thanks to Reinhard Mahnke (Rostock) and Hans Weber (Luleå)

## Air traffic - Vehicular traffic

- The growing need for mobility through the world shows no sign of slowing down.
- The air traffic is a very important part of the global transportation network.
- In distinction from vehicular traffic, the boarding of an airplane is a significant part of the whole transportation process.
- Here we study an airplane boarding model, introduced earlier by Frette and Hemmer (2012), with the aim to test the known theoretical results via Monte Carlo simulations and analysis of blocking relations between the passengers and longest blocking sequences.
- We have tackled an important question about the phase transition in the behavior of the boarding process in a generalized model and questions about scaling exponents and scaling relations.


## Time needed to board an airplane

Recently, an airplane boarding model has been proposed by Hemmer and Frette, Phys. Rev. E 85, 011130 (2012).

- $N$ passengers have reserved seats, but enter the airplane in arbitrary order ( $N$ ! permutations).
- This model shows a power-law scaling behavior $\left\langle t_{b}\right\rangle \propto N^{\alpha}$ of the mean boarding time $\left\langle t_{b}\right\rangle$ depending on $N$. The exponent $\alpha=0.69 \pm 0.01$ was found by Hemmer and Frette, considering system sizes $2 \leq N \leq 16$.
- However, it has been proven analytically and later confirmed by numerical simulations of much larger systems that $\alpha=1 / 2$ is the true exponent, describing the power law at $N \rightarrow \infty$.
- Relative deviations from the asymptotic behavior is described by correction-to-scaling exponent $\theta$. We find numerically $\theta \approx 1 / 3$, but the question about its precise value is non-trivial and merits further investigation.


## Blocking sequences - basic definitions

A passenger must wait for a possibility to move forwards to his/her seat if the motion is blocked by other passengers.
(i) Suppose passenger A takes his/her seat at the $n$-th time step. We say that passenger $A$ has been blocked by passenger $B$, if $B$ is the closest passenger in front of A among those ones, which took seat at the ( $n-1$ )-th time step.
(ii) We depict this blocking relation by drawing an arrow from $B$ to $A$ in the scatter plot (number in queue versus seat number). Nodes and arrows, pointing in certain flow direction, represent a blocking sequence.

The longest blocking sequences have the length $t_{b}$, where $t_{b}$ is the boarding time.

## Blocking sequences - theory and simulations




Figure : The analytical curves represent the longest blocking sequences in the theoretical asymptotic limit $N \rightarrow \infty$. The fluctuating curves represent the longest blocking sequences, extracted from 3 different simulation runs with $N=10^{7}$ passengers (left), as well as from one simulation run with $N=10^{8}$ passengers (right).

## Asymptotic limit and phase transition

A generalized model: $k=\frac{b u}{w}$, where $u$ is the passenger width, $w$ is the distance between successive rows, $b$ is the number of passengers per row.

At $N \rightarrow \infty$ :

$$
\begin{array}{rll}
r(q) & =0 & : \\
r(q) & =-4 e^{k(q-1)}+4 e^{2 k(q-1)}+1 & : \\
q_{0}(k) \leq q \leq q_{0}(k)
\end{array}
$$

where $q_{0}(k)=1-\ln 2 / k$. A phase transition occurs at $k=\ln 2$. The model of Frette and Hemmer corresponds to $k=1$ - the range with two branches.

Corrections to scaling:
$\left\langle t_{b}\right\rangle=c N^{1 / 2}\left[1+\mathcal{O}\left(N^{-\theta}\right)\right]$, where $\theta=1 / 3$ for $k \leq \ln 2$.
$\theta=$ ? for $k>\ln 2$. Our estimate from $N \leq 2^{16}=65536$ : $\theta=0.330 \pm 0.001$.

## A scaling relation



Figure : The probability distribution $P\left(t_{b}\right)$ of the boarding time $t_{b}$ for the model with $N=2{ }^{15}$ passengers. The mean boarding time $\left\langle t_{b}\right\rangle=453.91$ (dashed line) and the asymptotic mean boarding time $\left\langle t_{b}\right\rangle^{\text {as }}=473.13$ (dotted line).

$$
\begin{gathered}
t_{b}-\left\langle t_{b}\right\rangle \sim\left\langle t_{b}\right\rangle^{\text {as }}-\left\langle t_{b}\right\rangle \sim N^{\alpha-\theta} \Rightarrow \operatorname{var}\left(t_{b}\right)=\left\langle\left(t_{b}-\left\langle t_{b}\right\rangle\right)^{2}\right\rangle \sim N^{\gamma}, \\
\gamma=2(\alpha-\theta)=1-2 \theta
\end{gathered}
$$

## Summary and conclusions

- Blocking sequences have been studied by Monte Carlo simulations. The results converge to theoretical asymptotic formula for $N \rightarrow \infty$.
- A phase transition is observed in the airplane boarding process at a certain value of control parameter $k$ in a generalized model. It is related to the change in the correction-to-scaling behavior.
- A scaling relation for the exponents, describing the power-law behavior of the boarding process, have been derived.

