Traffic and Granular Flow'15

From microscopic to macroscopic traffic patterns: different applications of the variational theory

Ludovic Leclercq

Université de Lyon, IFSTTAR / ENTPE, COSYS, LICIT

Outline

- Short historical review
  - Micro to Macro for car-following models
  - Recent results based on homogenisation technics
- The variational theory
- Applications to scaling-up problems
  - Mean traffic states on urban corridors
  - Capacity drops at merges
- Conclusion

SHORT HISTORICAL REVIEW

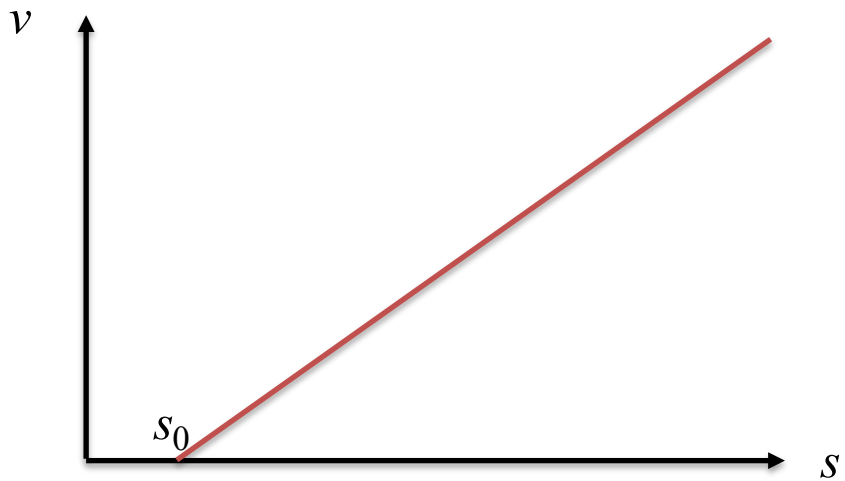
First micro to macro approaches (1)

- 1950-60: Herman, Gazis, Montroll, Chandler…

Car-following rule $$\ddot{x}_{n+1}(t) = \alpha\left[\dot{x}_n(t-T) - \dot{x}_{n+1}(t-T)\right]$$

=> $$\dot{x}_{n+1}(t) = \alpha\left(x_n(t-T) - x_{n+1}(t-T)\right) + v_0$$

(+ steady state) => $$v = \alpha\left(s - s_0\right)$$



Not consistent for high spacings

First micro to macro approaches (2)

Gazis, Herman and Rothery's model:

$$\ddot{x}_{n+1}(t+T) = \alpha_0 \frac{[\dot{x}_n(t) - \dot{x}_{n+1}(t)]}{[x_n(t) - x_{n+1}(t)]^l} [\dot{x}_{n+1}(t+T)]^m$$

l	Equation of State	Reference
	$m=0$	
0	$q=\alpha[1-k/k_j]$ $\alpha=q_m=1/\text{reaction time}$	Chandler et al.[9] Pipes[4]
1	$q=\alpha k \ln[k_j/k]$ $\alpha=$ velocity at optimum flow (u_m)	Greenberg[17] Gazis et al.[13]
3/2	$q=\alpha k[1-(k/k_j)^{1/2}]$ $\alpha=$ velocity at free flow (u_f)	Drew[10]
2	$q=\alpha k[1-k/k_j]$ $\alpha=u_f$	Greenshields[20]
	$m=1$	
2	$q=\alpha k e^{(k/k_0)}$ $\alpha=u_f$; $k_0=$density at optimum flow	Edie[21]
3	$q=\alpha k e-\frac{1}{2}(k/k_0)^2$ $\alpha=u_f$	Drake et al.[22]

[a] Based on May and Keller.[18]

1967

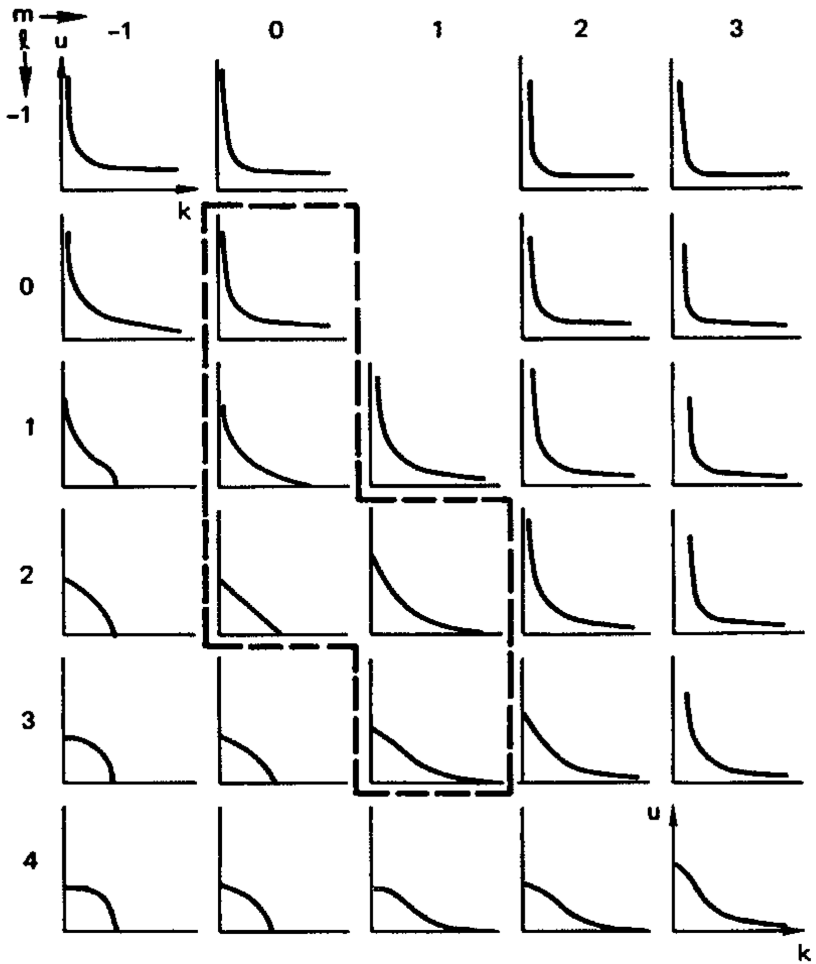


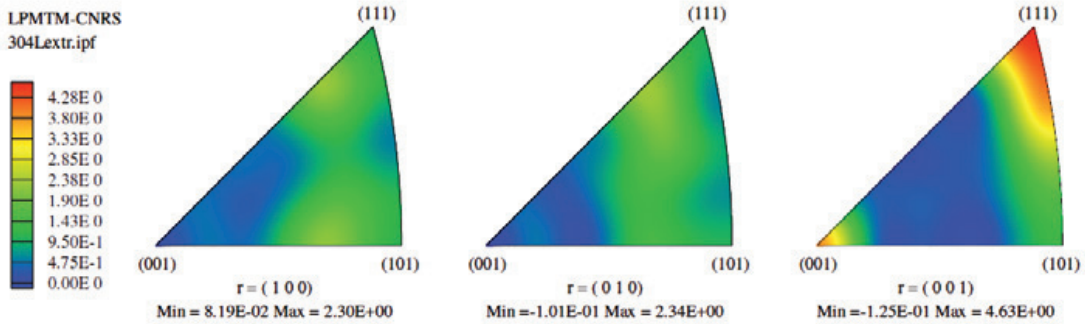
Figure 6.5 Matrix of speed-density relationships for various m, l combinations of the general car-following equation.[18] (Dashed lines enclose limiting values of l and m used in Table 6.5.)

@Gerlough and Huber, Traffic Flow Theory: a monograph, 1975

Homogeneisation technics

A classical method in other engineering field, e.g. materials

(a) 3D microstructure of 304L stainless steel obtained from DCT



(b) Inverse pole figure of the sample regarding to some macroscopic directions r

A. Belkhabbaz et al. / Procedia Engineering 10 (2011) 1883–1888

Determining the mean response of an heterogeneous media using multiscale grids

Application to traffic flow (Monneau & Costesque, 2014)

- Using homogeneisation technics and HJ theory (Monneau & Costeseque, 2014) proves that:
  - The Optimal velocity model (with no delay) is equivalent to the LWR model

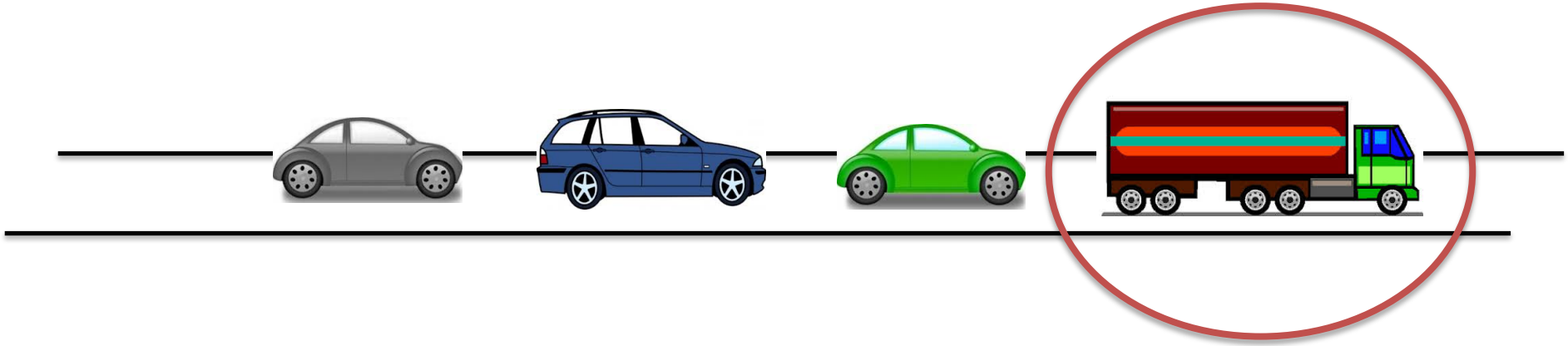
$$\dot{x}_i(s) = F\left(x_{i+1}(s) - x_i(s)\right), \quad \text{for} \quad i \in \mathbb{Z}$$

  - This results can be generalized to:
    - The OVM with (small) delay
    - Multi-anticipative models

$$\dot{x}_i(t+\tau) = F\left((x_{i+j}(t) - x_i(t))_{j=1,\dots,n}, \dot{x}_i(t), (\dot{x}_{i+j}(t))_{j=1,\dots,n}\right), \quad \textit{for all} \quad i \in \mathbb{Z},$$

Application to traffic flow (2)

- Homogeneisation ⇔ steady / stationnary states
- Strongly based on a fixe vehicle order (no overtaking)

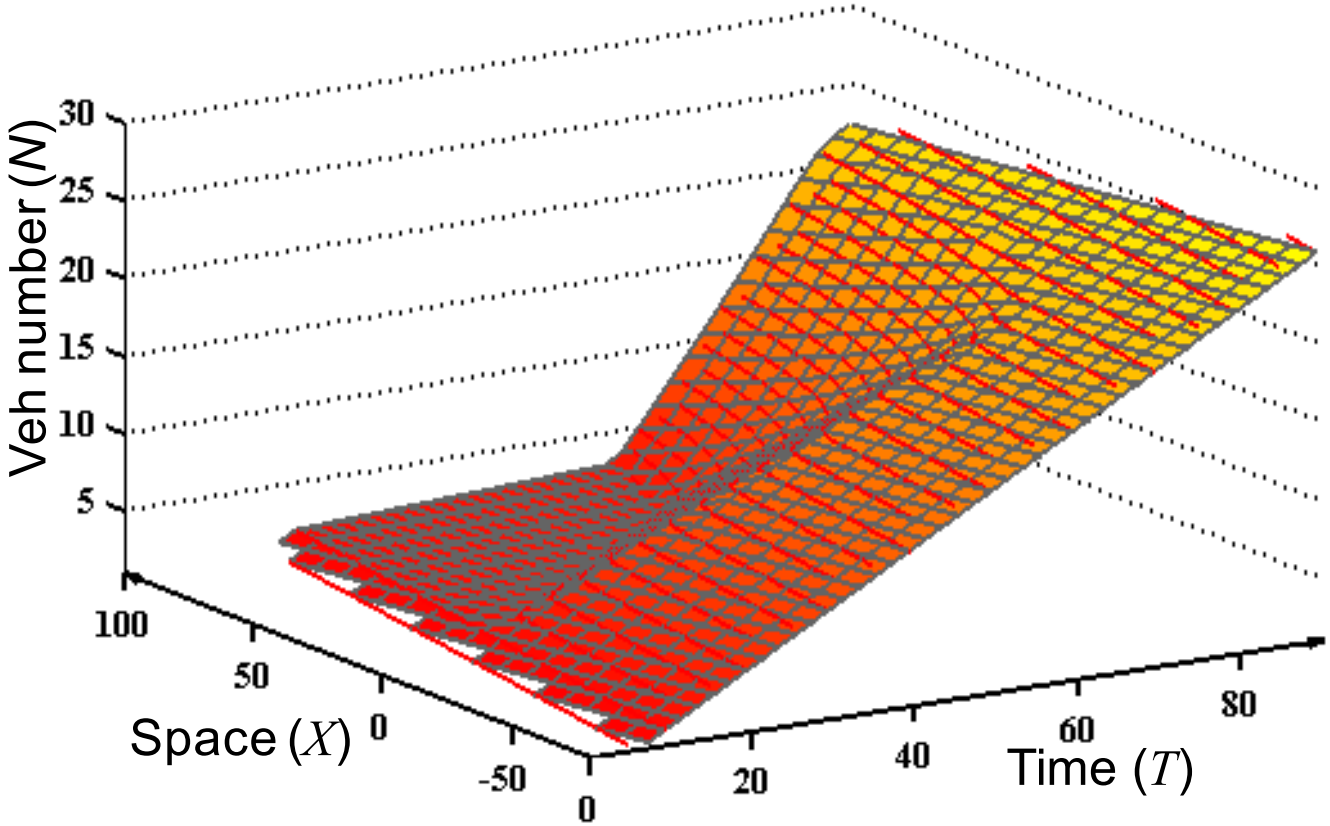


The slowest vehicle becomes predominant

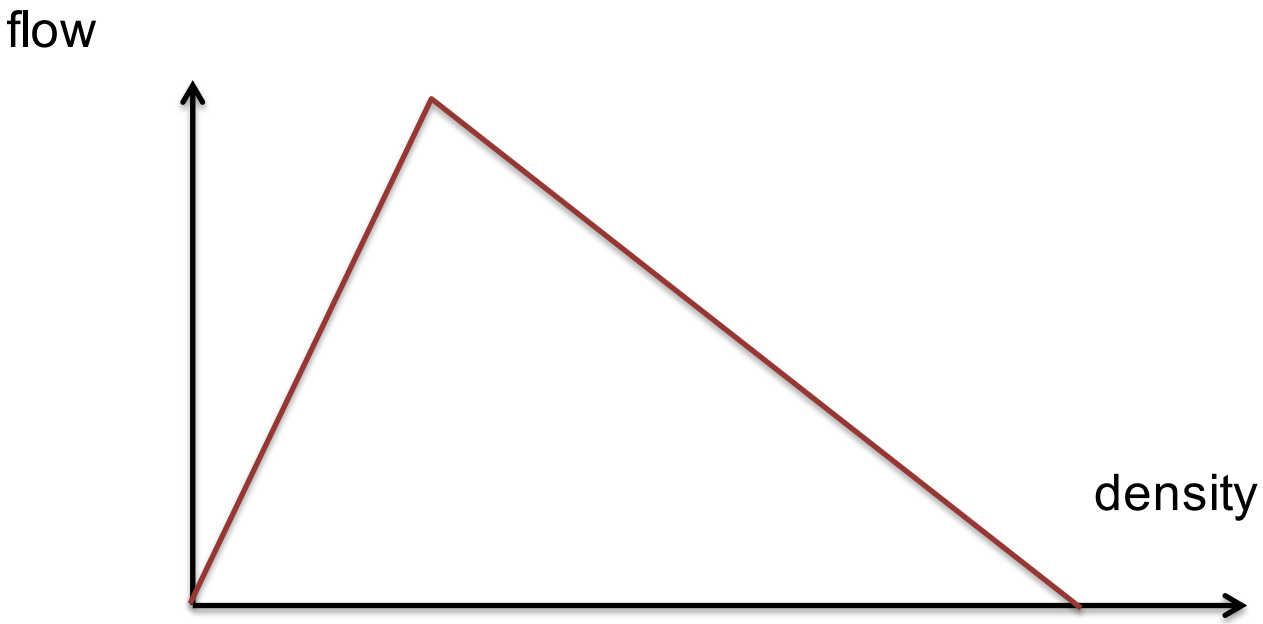
Hardly applicable for transitional phases or unregular patterns

THE VARIATIONAL THEORY

The Moskowitz (Newell)'s function



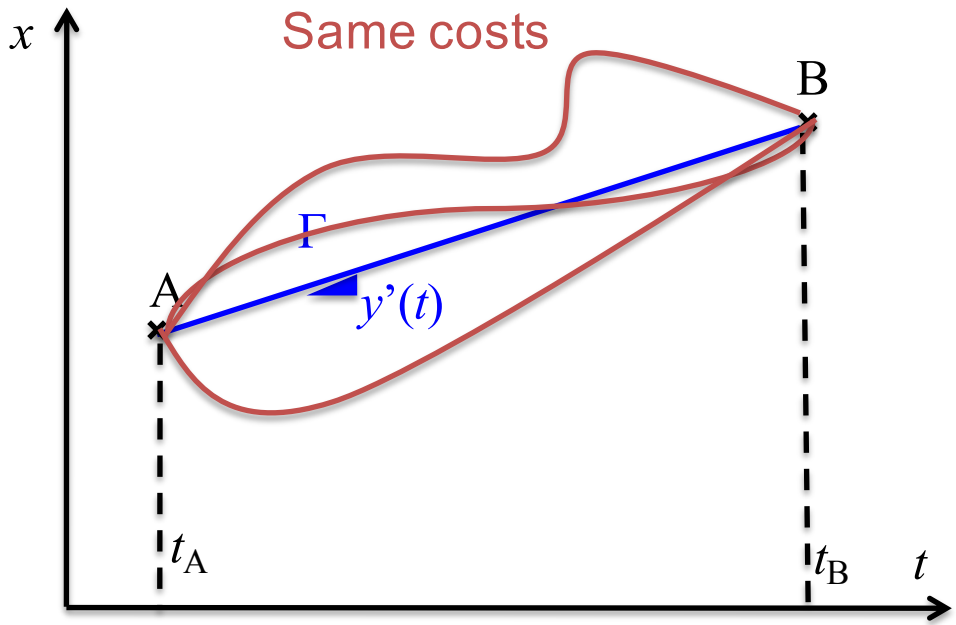
The LWR model as an HJ equation



$$q = Q(k) \quad \Leftrightarrow \quad \frac{\partial N}{dt} = Q\left(-\frac{\partial N}{\partial x}\right)$$

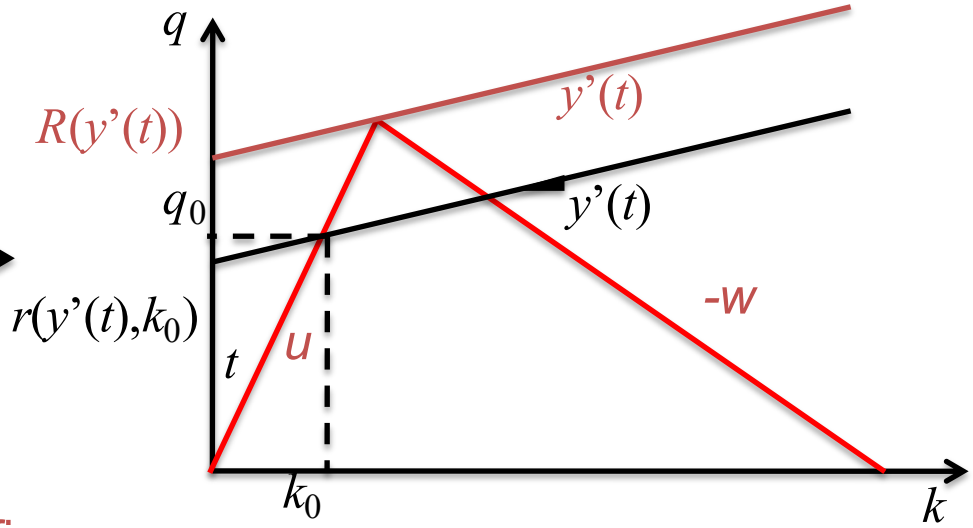
Hamilton-Jacobi equation

General considerations on the variations of *N*



$$\Delta N_{AB} = \int_{t_A}^{t_B} d_t N = \int_{t_A}^{t_B} \partial_t N + y'(t)\partial_x N$$

$$\Delta N_{AB} = \int_{t_A}^{t_B} \underbrace{Q\big(k(t)\big) - y'(t)k(t)}_{r(y'(t),k)}$$



Legendre's transformation:

$$r(y'(t),k) \le R(y'(t)) = \sup_k \big(r(y'(t),k)\big)$$

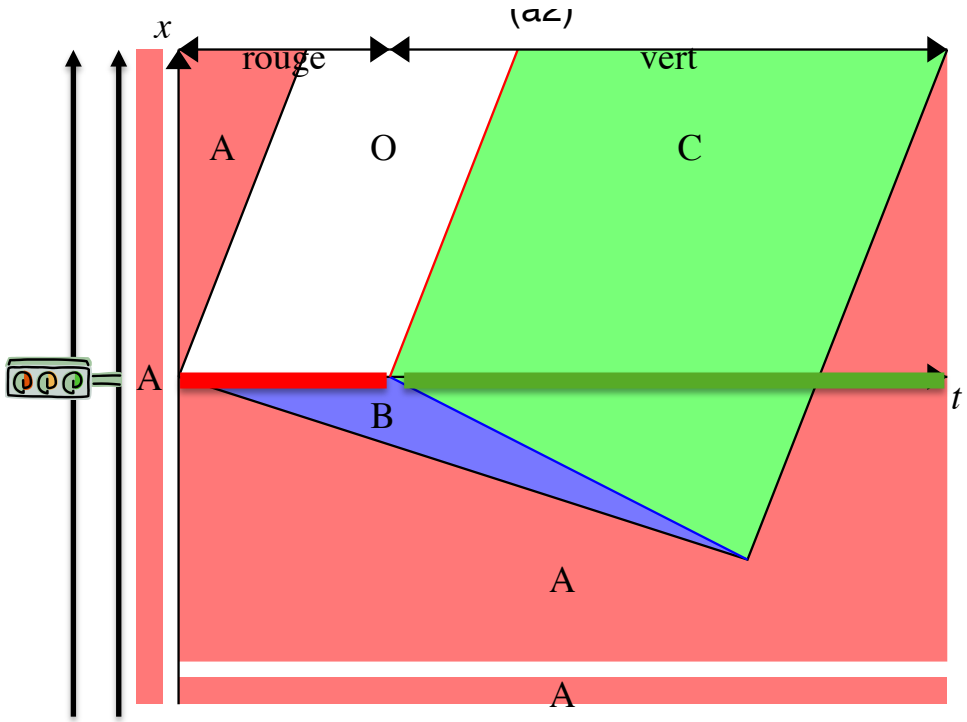
This makes costs independent from traffic states but no longer from the paths

Equality is observed on the optimal wave paths

APPLICATIONS TO SCALING-UP PROBLEMS (1)

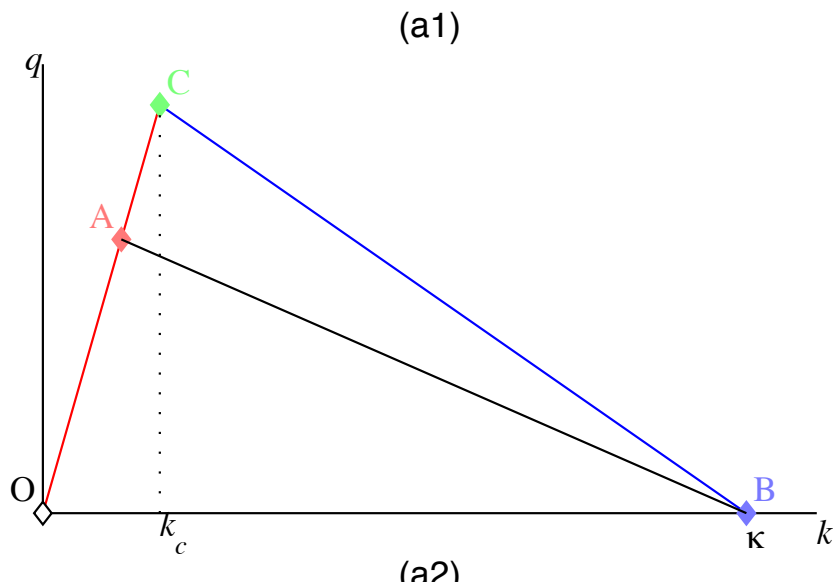
Mean traffic behavior on urban corridors

Traffic dynamics at an isolated signal (1)

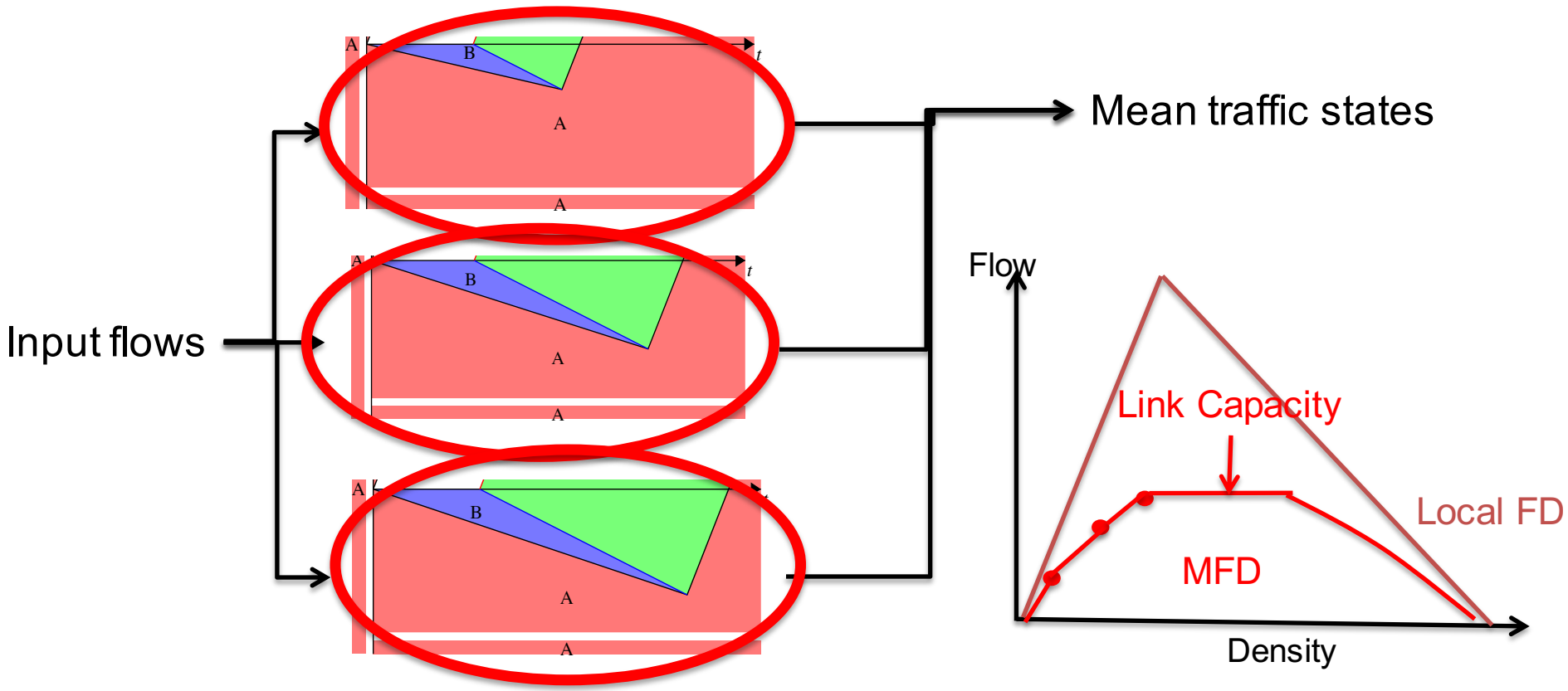


Full description of traffic dynamics

Fundamental Diagram (FD)

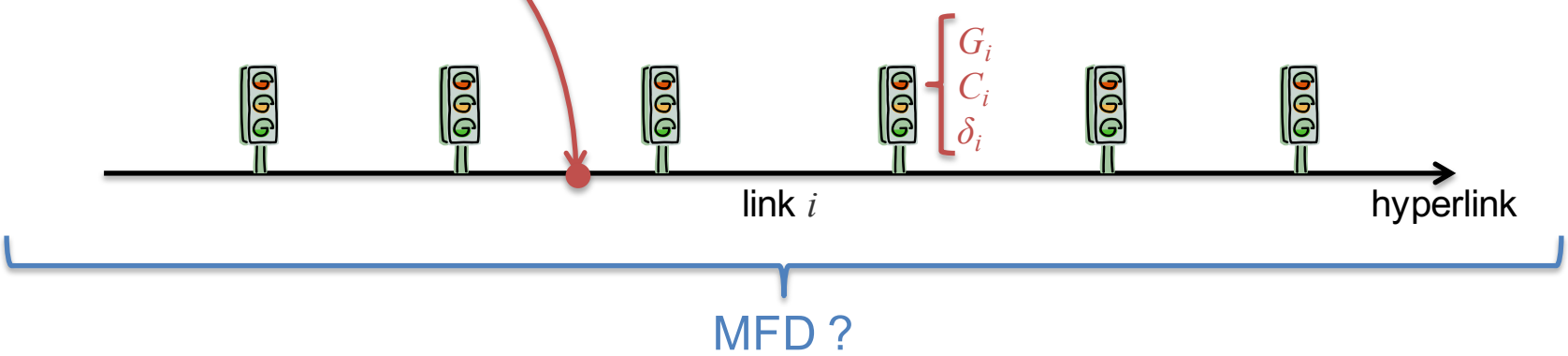


Traffic dynamics at an isolated signal (2)

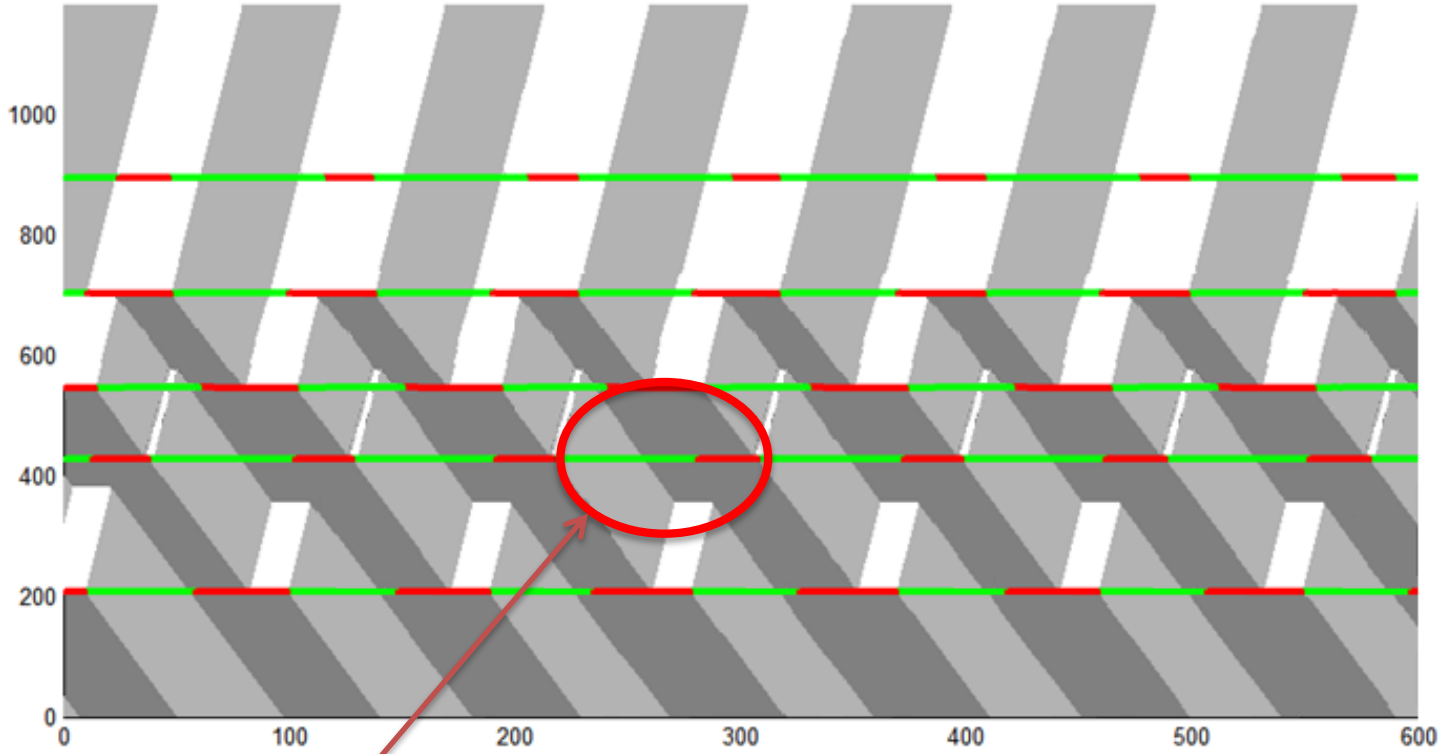


Estimating mean traffic states on corridor

- Definitions
  - corridor: series of m successive links ended by traffic signals
  - Homogeneous traffic conditions, i.e. no or well-balanced turning flow
  - FD parameters: free-flow u, wave speed w, jam density κ



Traffic dynamics for an urban corridor



A new phenomenon can be observed : spillbacks

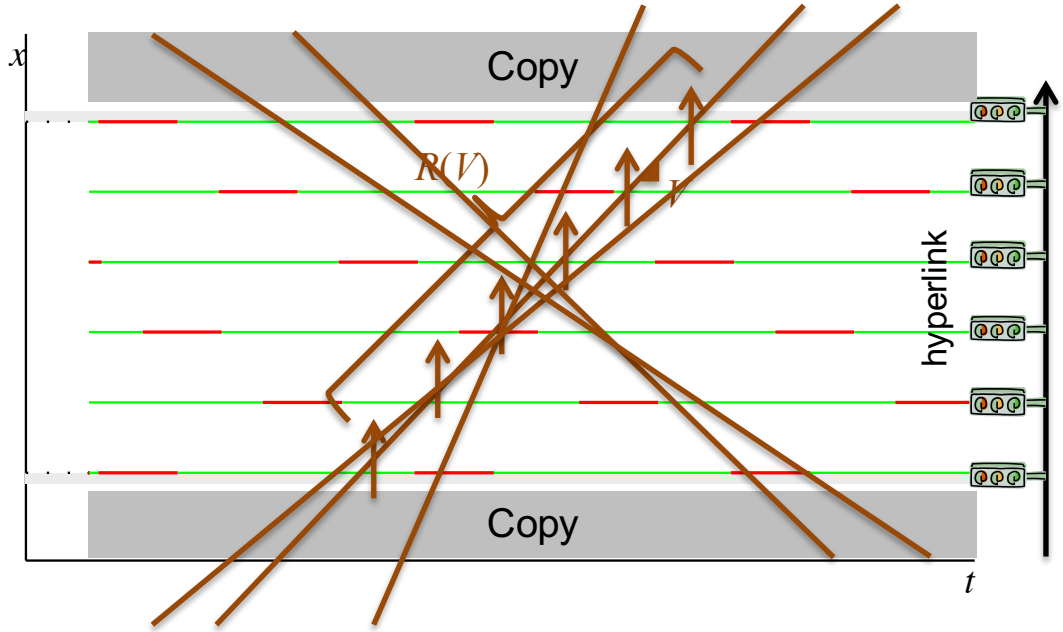
Intersections becomes correlated

Analytical Method (1) – Cuts for a corridor

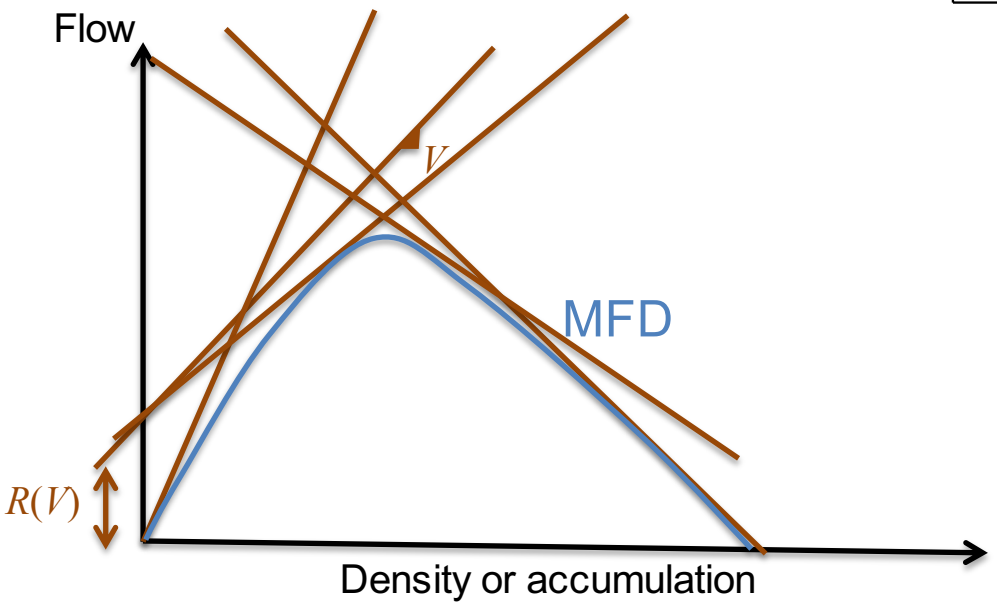
Moving observer (mean speed V)

Minimum overtaking flow $R(V)$

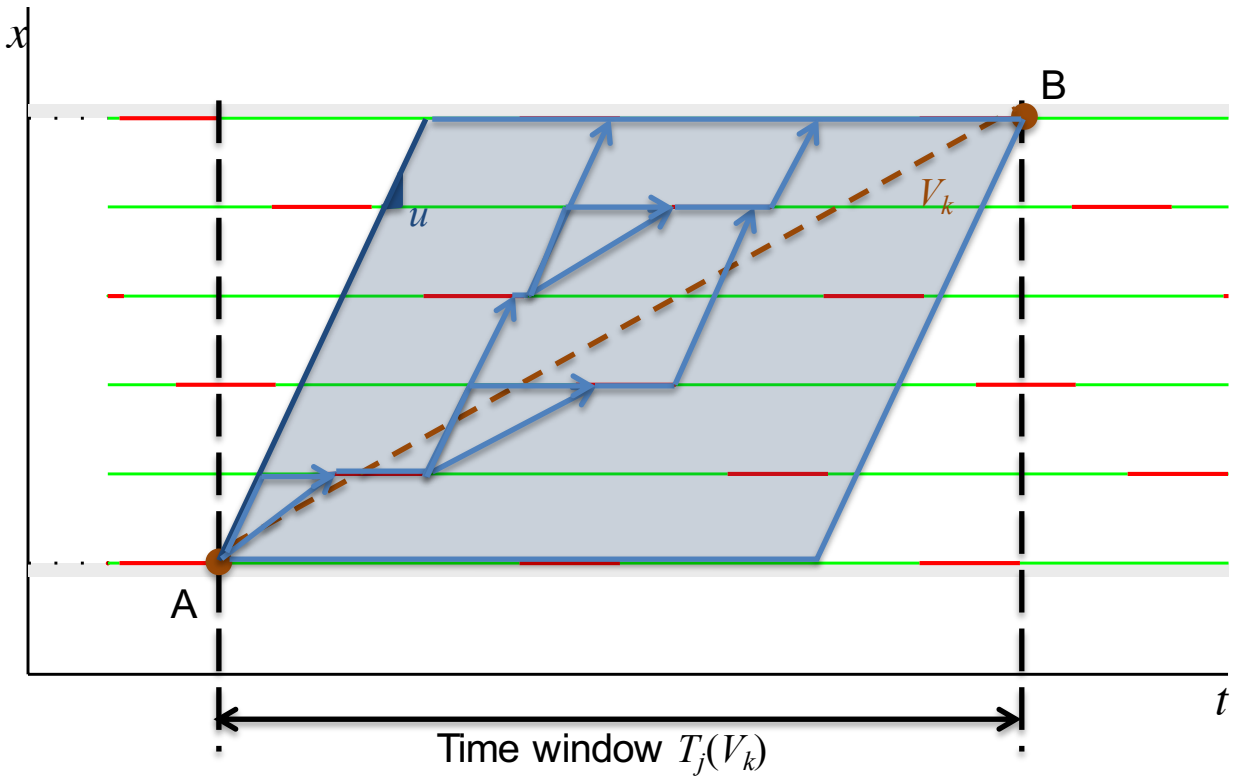
Cut: $Q=\min_V(KV+R(V))$



(Daganzo and Geroliminis, 2008)



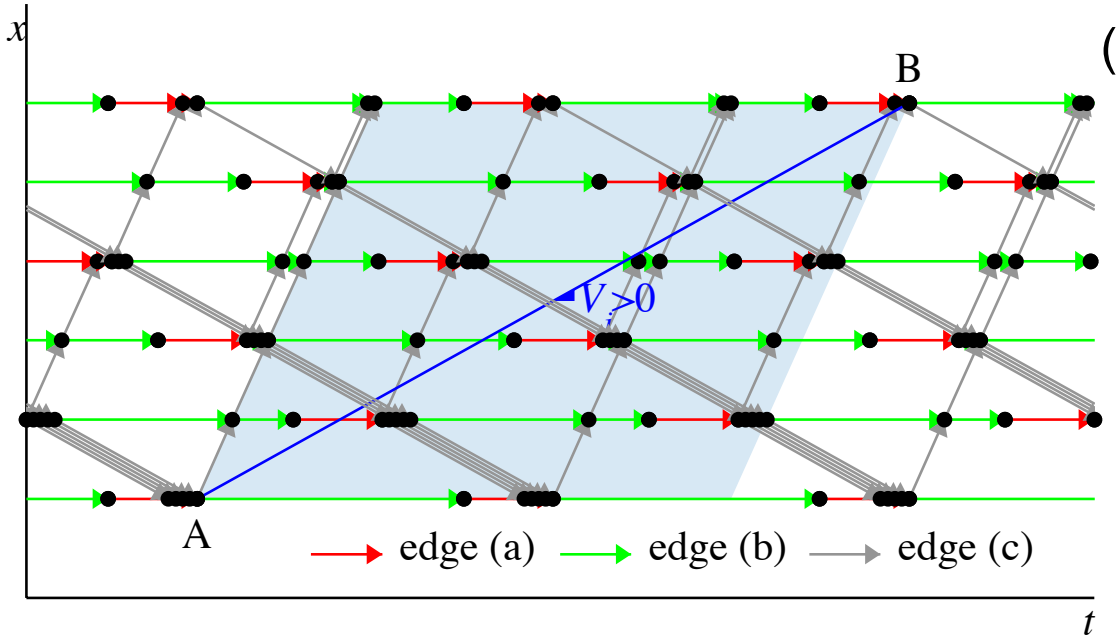
Calculating $R(V)$ with VT



To minimize costs, the observer has to maximize the time spent on red phases – Shortcuts theory (Daganzo and Menendez, 2005)
Internal subpaths with $v<u$ may be replaced at same costs.

A graph can simply be constructed to explore all the possible paths

Analytical Method (2) - The sufficient graph



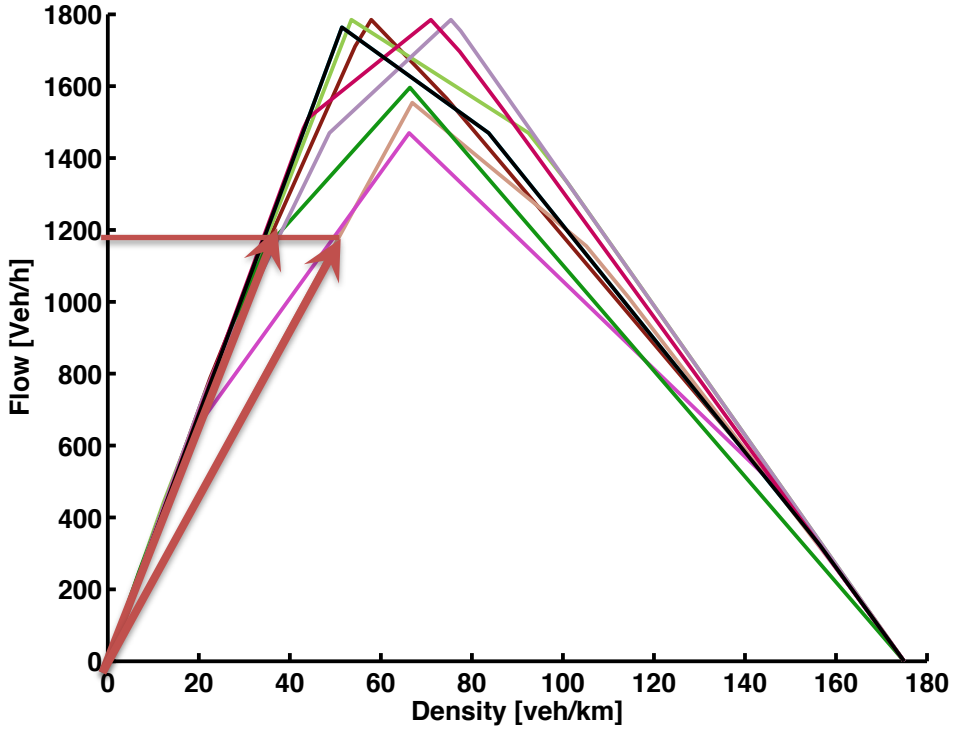
(Leclercq and Geroliminis, 2013)

The estimation is tight only if the network is homegenously loaded

The cost $R(V)$ for different V can be calculated using a sufficient graph defined by three kinds of edges:

- edge (a): red phase (cost 0)
- edge (b): green phase (cost $Q_{\max}$)
- edge (c): path with speed u or $-w$ that starts at the end of red phases (cost 0 or $w\kappa$)

Influence of the time lag between signals



δ=0 s

δ=10 s

δ=20 s

δ=30 s

δ=40 s

δ=50 s

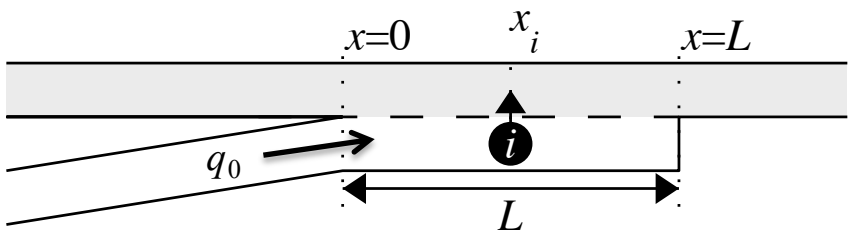
δ=60 s

δ=70 s

APPLICATIONS TO SCALING-UP PROBLEMS (2)

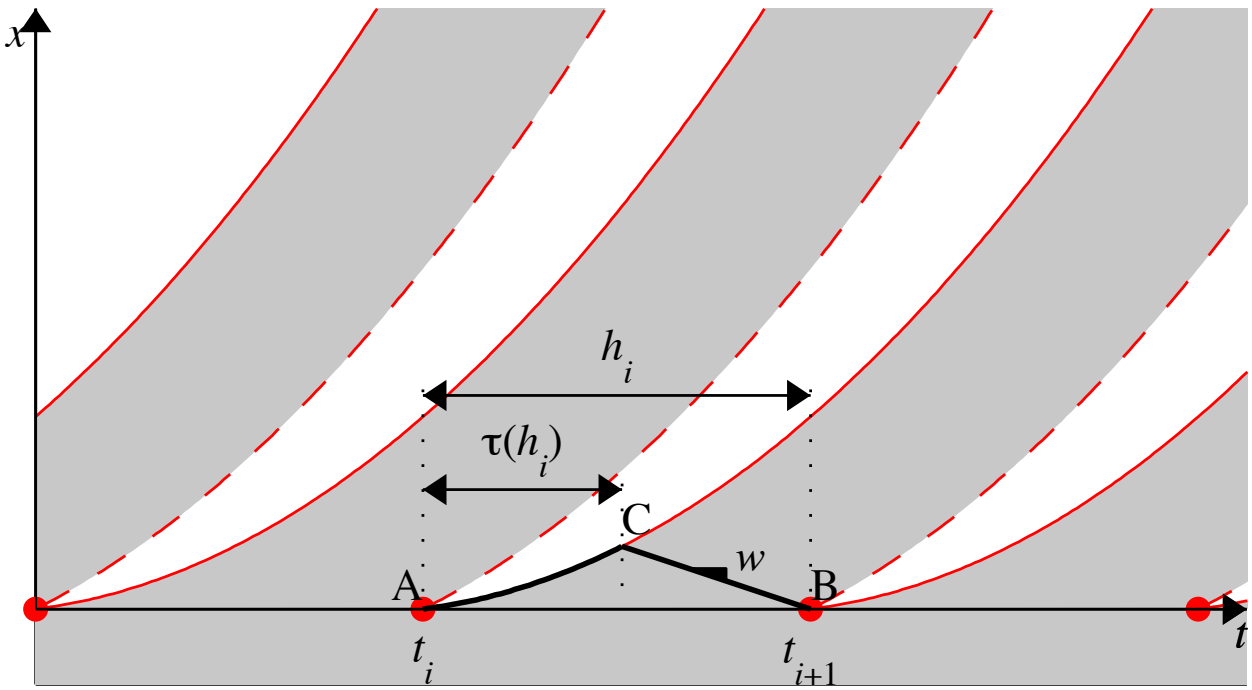
Capacity drop at freeway merges

Hypothesis



- Network:
  - The main road has only one lane
  - The inserting flow is equal to q_0
  - LWR model and triangular fundamental diagram (free-flow speed u wave speed w and jam density κ)
- Insertions:
  - Time between two insertions: $H(h_0=1/q_0, s_H)$
  - Inserting positions are uniformly distributed between 0 and L
  - Vehicles insert at speed v_0 with an acceleration a and a jam density κ
  - Inserting vehicles behave as moving bottlenecks on target lane

Case 1: L=0 and s_H>0



The effective capacity C is given by :

$$C = \sum_{i=1}^{n} \left(N_{i+1} - N_i \right) / \sum_{i=1}^{n} h_i \quad \text{with} \quad n \to +\infty$$

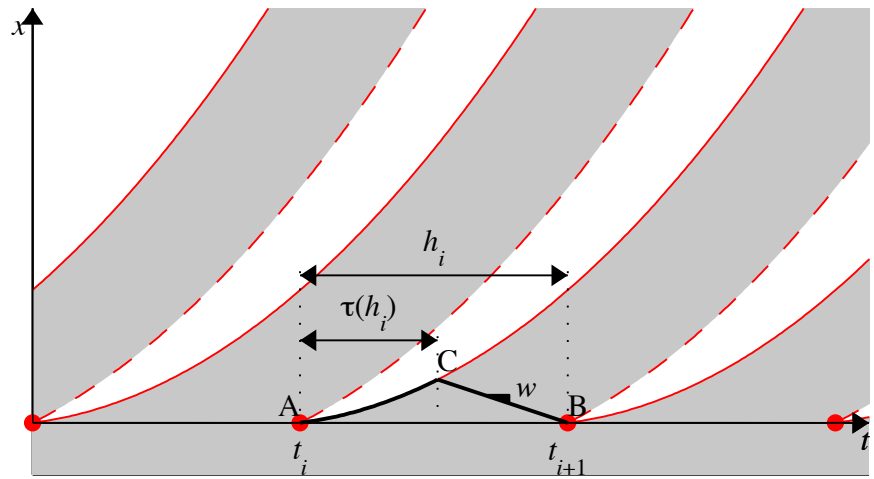
Simplest homogeneous case (L=0, s_H>0)

- Effective capacity

$$C = \sum_{i=1}^{n} w\kappa\left(\tau\left(h_i\right) - h_i\right) / \sum_{i=1}^{n} h_i$$

$$C = w\kappa\left(1 - \sum_{i=1}^{n} \tau(h_i) / \sum_{i=1}^{n} h_i\right)$$

$$\tau(h_i) = -\frac{w + v_0}{a} + \frac{1}{a}\sqrt{(w + v_0)^2 + 2awh_i}$$



- Law of large numbers

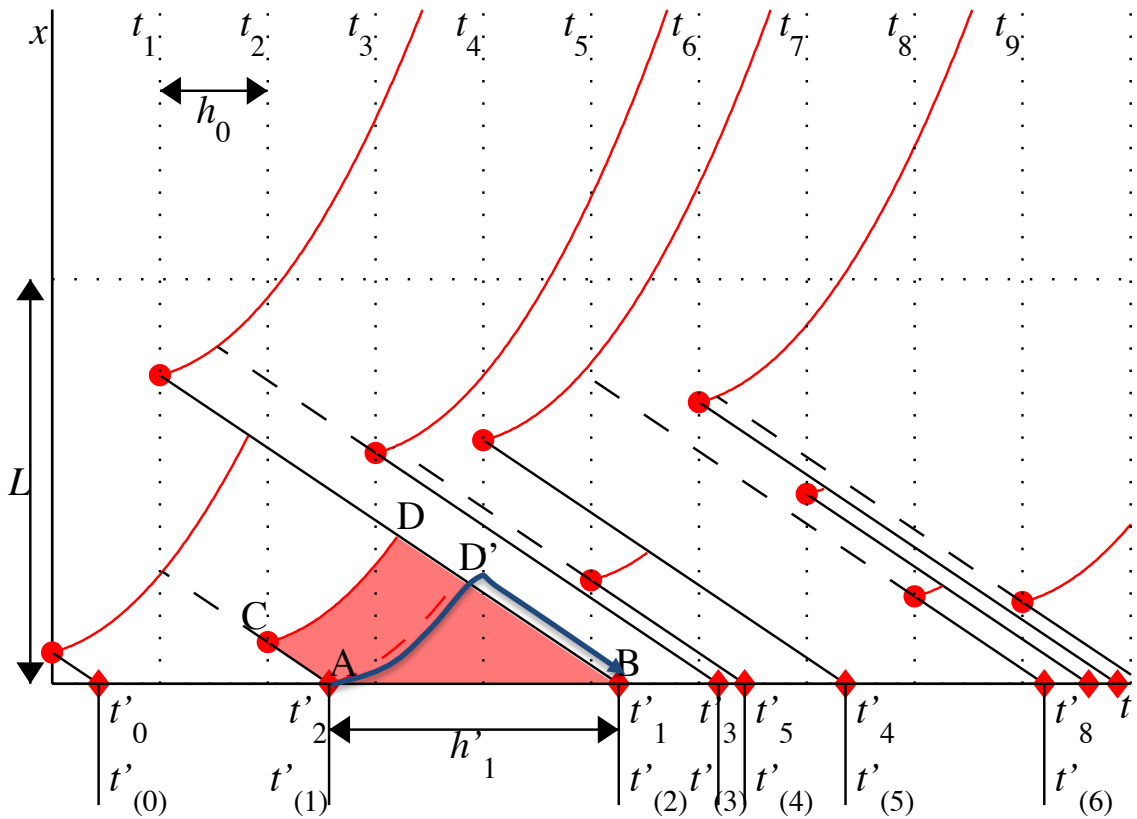
$$(1/n)\sum_{i=1}^{n} h_i \to E(h_i) \text{ and } (1/n)\sum_{i=1}^{n} \tau(h_i) \to E\left(\tau(h_i)\right)$$

- Second order Taylor approximation

$$E\left(\tau(h_i)\right) \simeq \tau\left(E\left(h_i\right)\right) + \frac{1}{2}s^2 \frac{\partial^2 \tau}{\partial h_i^2}\left(E(h_i)\right) = \tau\left(h_0\right) - \frac{as^2w^2}{2\left(\left(w + v_0\right)^2 + 2awh_0\right)^{3/2}}$$

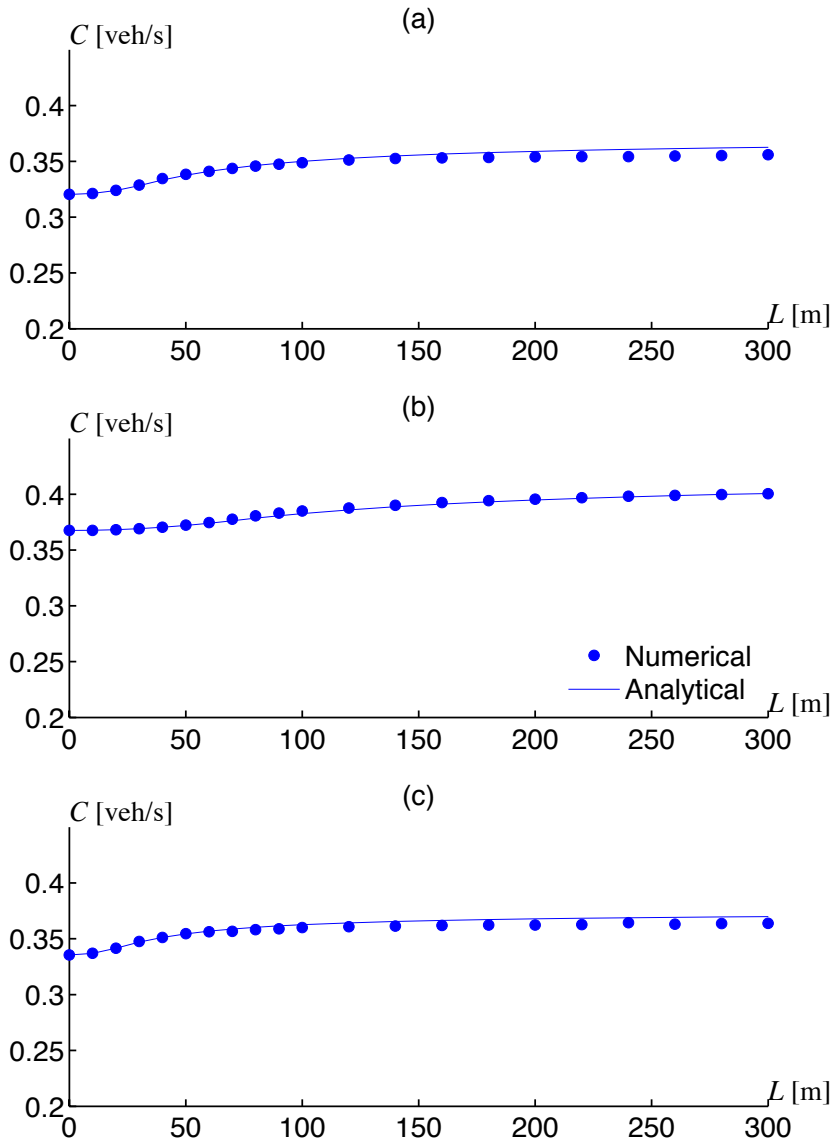
(Leclercq et al, part B, 2011)

Case 2: $L>0$ and $s_H>0$ – no interactions



Same problem as case 1 by switching inserting times and $t'_{()}$ at $x=0$!

Resulting analytical curves

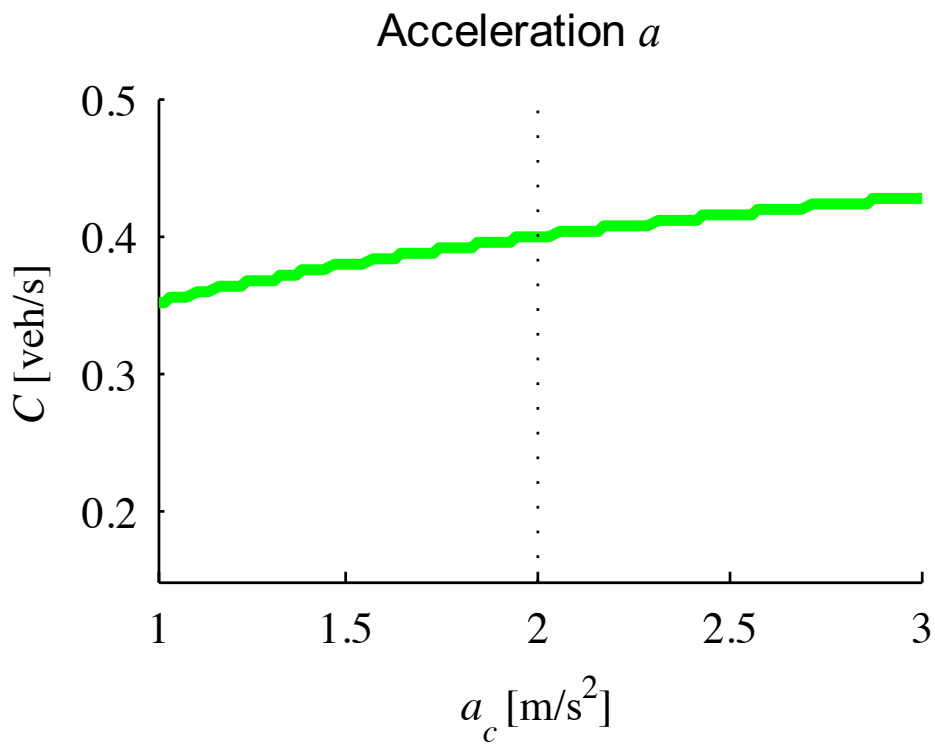


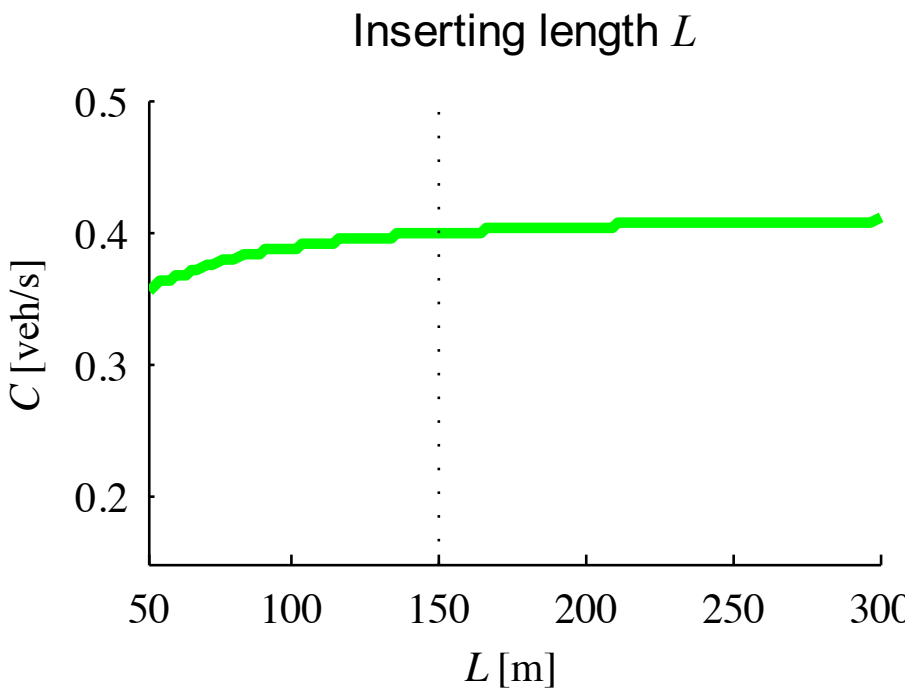
q_0=0.174 veh/s

q_0=0.08 veh/s

q_0=0.26 veh/s

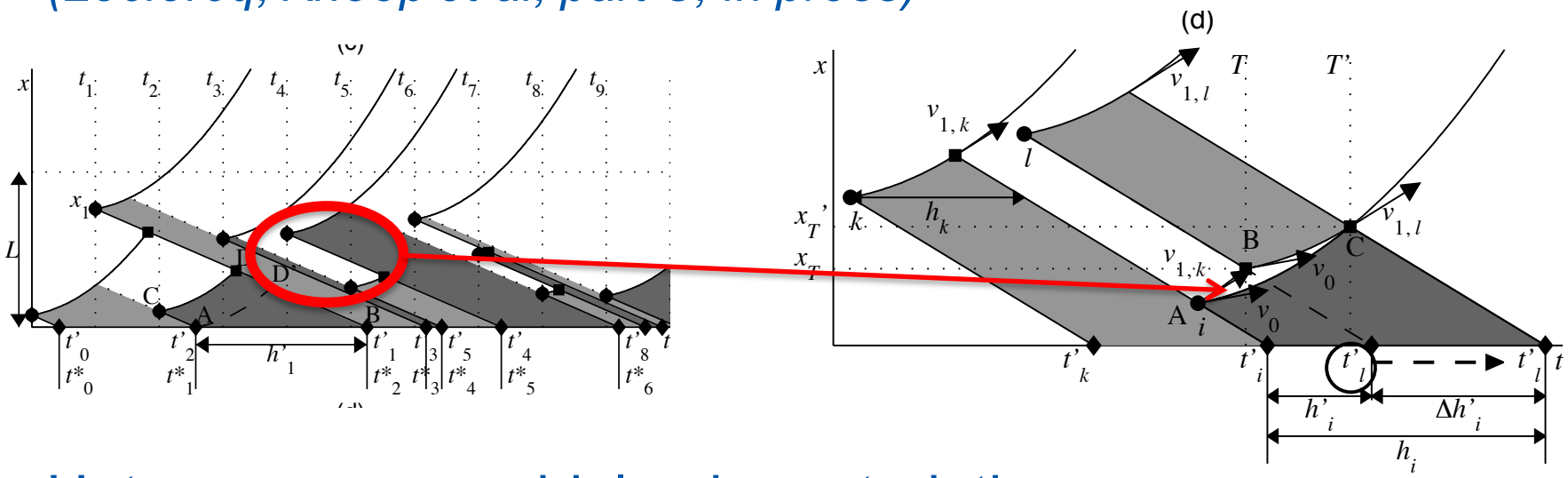
Sensitivity analysis





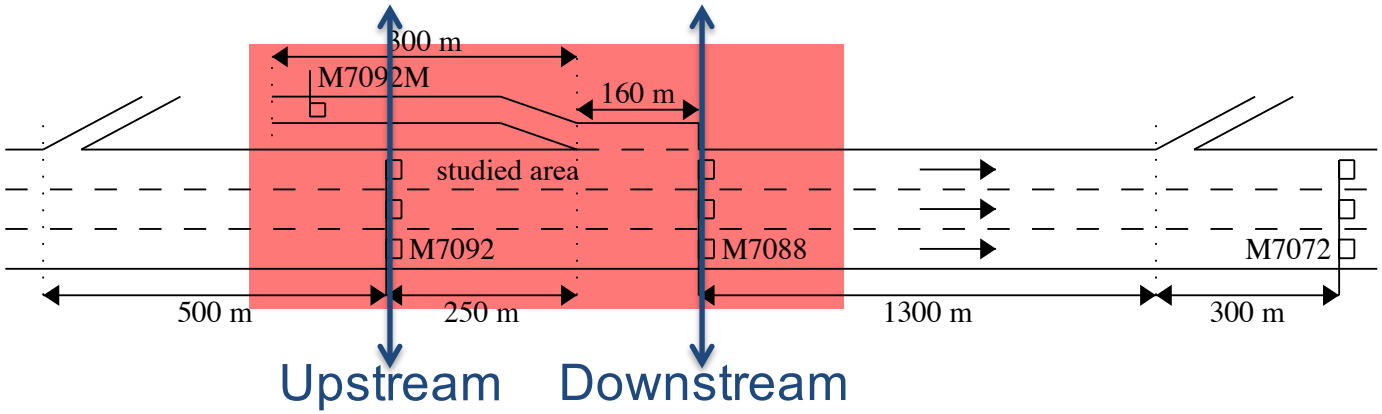
Extended framework

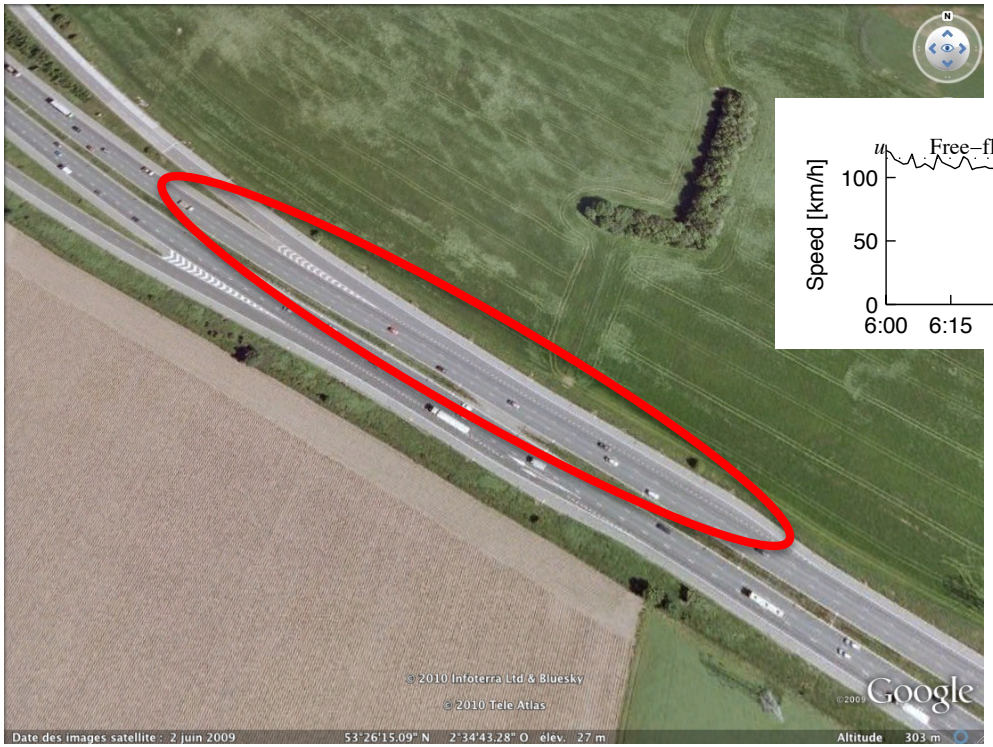
- Refined description of traffic dynamics (interactions between waves and voids)
  (Leclercq, Knoop et al, part C, in press)

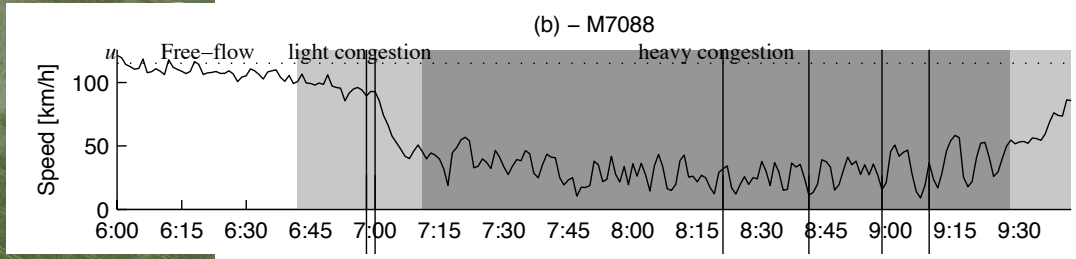


- Heterogeneous vehicle characteristics
  (Leclercq et al, part C, in press)
- Multilane on freeways
  (Leclercq et al, TRB2016)

Experimental site (M6 – England)

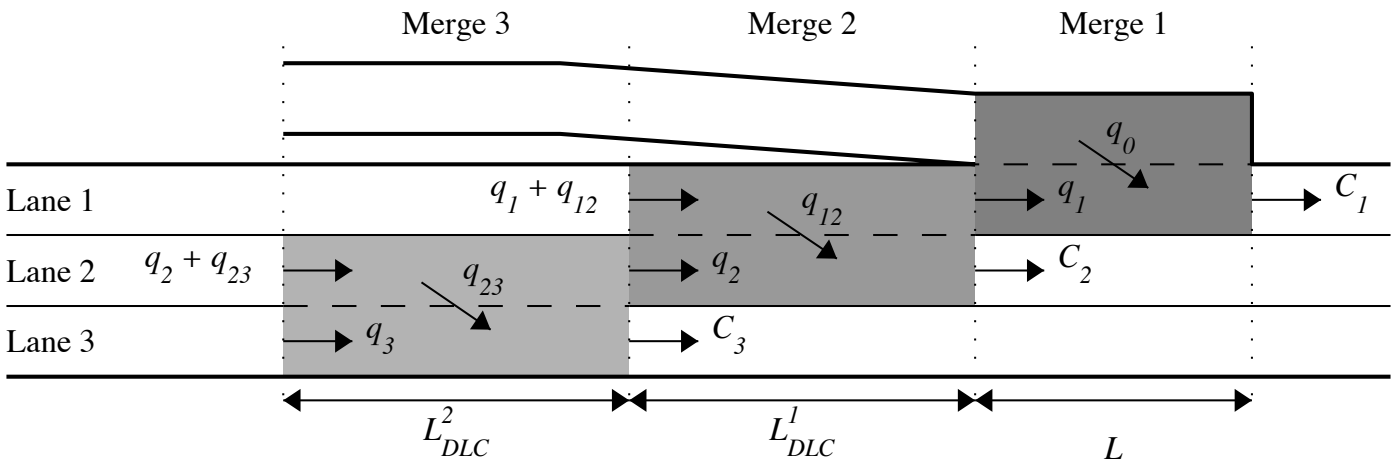






6 days of observations
17 periods (20 min) of heavy congestion

Extended sketch of the model



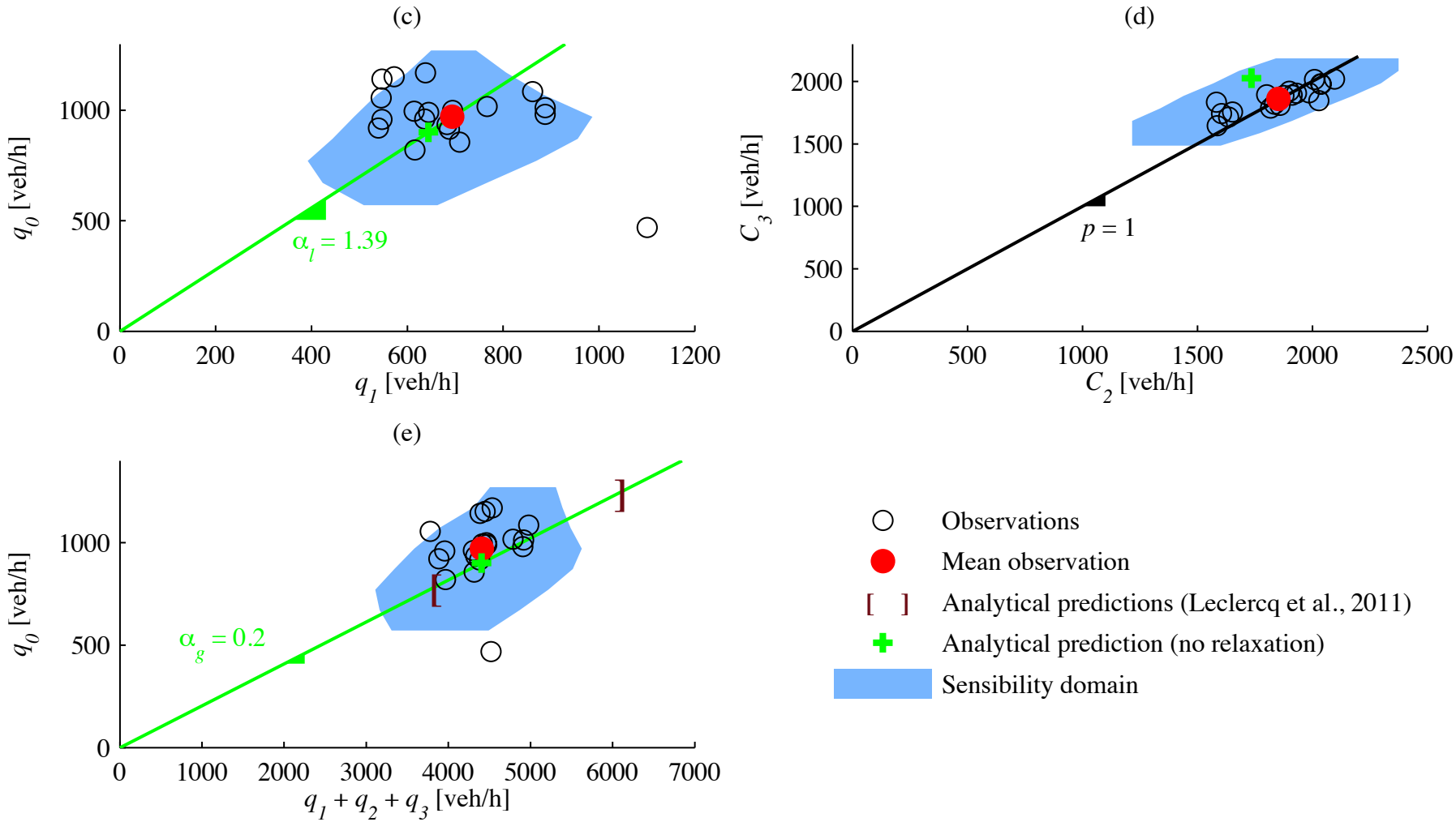
Rough calibration:

-FD (per lane): u=115 km/h, w=20 km/h, κ=145 veh/km
-a=1.8 m/s^2; τ_1=τ_2=3 s;
-L=160 m ; L^2_{DLC}=L^1_{DLC}=100 m

L^2_{DLC}=L^1_{DLC}

τ_1=τ_2

Experimental results



CONCLUSION

Conclusion

- Variational theory is a powerful to determine mean traffic states from a microscopic description of the physical mechanisms
- Variational theory is by nature an integrating operator
- A real challenge is to operate micro to macro transformations for transitional phases
- Consistent integration is very important for hierarchical modeling and to adress large scale problems

Thank you for your attention

Are you interested in collaborating on an very exiting project ?.....

MAGnUM: A Multiscale and Multimodal Traffic Modelling Approach for Sustainable Management of Urban Mobility

European Research Council

Established by the European Commission

RESIDUALS

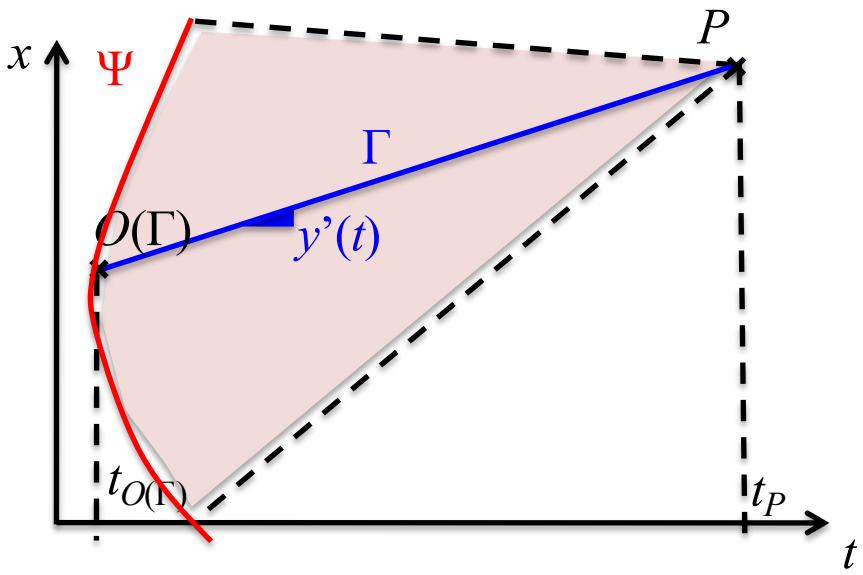
Variational theory (VT) in Eulerian – General basis

HJ Equation: $q = Q(k) \quad \Leftrightarrow \quad \partial_t k = Q\left(-\partial_x k\right)$

General expression for the solutions:

$$N_P = \min_{\Gamma \in D_P}\left(N_{O(\Gamma)} + \Delta(\Gamma)\right)$$

$$\Delta(\Gamma) = \int_{t_{O(\Gamma)}}^{t_P} r(y'(t),k)\,dt$$



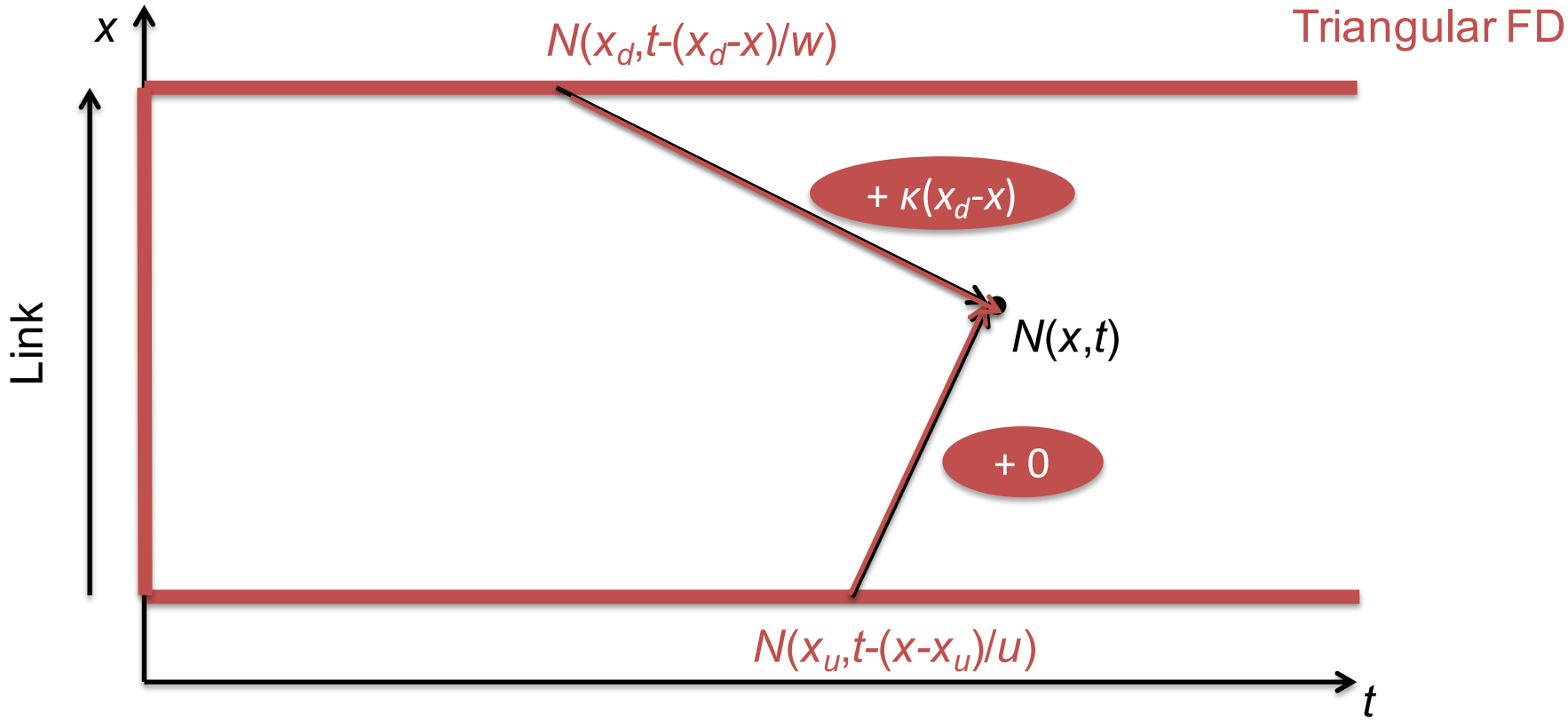
Key VT result using the Legendre's transformation

$$N_P = \min_{\Gamma \in D_P}\left(N_{O(\Gamma)} + \Delta'(\Gamma)\right)$$

$$\Delta'(\Gamma) = \int_{t_{O(\Gamma)}}^{t_P} R(y'(t))\,dt$$

VT is really useful with PWL FD
(and especially triangular one)

VT in Eulerian – The Highway Problem



$$N(x,t) = \min\left[\underbrace{N\left(x_u, t - \frac{(x - x_u)}{u}\right)}_{free-flow}, \underbrace{N\left(x_d, t - \frac{(x_d - x)}{w}\right) + \kappa(x_d - x)}_{congestion}\right]$$

Newell's model (1993) !!!