Jam avoidance with autonomous systems

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Context

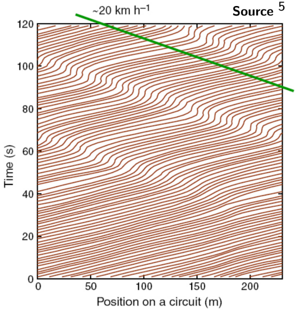
Stop-and-go wave and jam avoidance models

Stop-and-go waves in road traffic flow

- Observed in real flows
- and laboratory conditions

Self-organized phenomena explained by the reaction time of the drivers

- Delayed or relaxed car-following models



→ Development of jam avoidance models to suppress stop-and-go phenomena

[5] Y Sugiyama et al. *New J Phys* 10:033001 (2008)

Summary

We investigate the stability of homogeneous solutions for linear jam avoidance car-following models based on optimal velocity (OV) function

→ Simulation results of a jam

→ Calculus of Lyapunov exponents

Among extended OV models, we observe that autonomous ones (one neighbour in interaction) including speed difference term behave as collective approaches with large number of predecessors taken into account

→ Connection between the vehicles (to implement collective models) may not be necessary to suppress efficiently jamming formation

Overview

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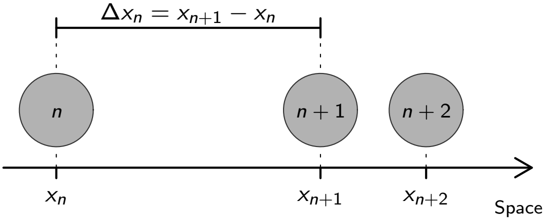
Notations



Notations x_n is the position and Δx_n is the spacing of the vehicle n

Jam avoidance models

Car-following models having **stable** homogeneous solutions[6]

$$\ddot{x}_n(t) = F\big(\Delta x_n(t), \dot{x}_n(t), \Delta \dot{x}_n(t)\big) \qquad \text{(2nd order model)}$$

- Models necessarily based on distance spacings $\Delta x_n = x_{n+1} - x_n$
- Speed difference terms $\Delta \dot{x}_n = \dot{x}_{n+1} - \dot{x}_n$ used to improve the stability

[6] $\Delta x_n(t) = d$ and $\dot{x}_n(t) = v$ for all n and t with $F(d, v, 0) = 0$

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Examples : *Optimal velocity model* (OVM); *Full velocity difference model* (FVDM); *Intelligent driver model* (IDM)...

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Stability analysis

Stable : Convergence to homogeneous solution for any initial configuration[7] (i.e. no stop-and-go)

[7] Linearly stable: Convergence for initial conditions close to homogeneous ones (according to 2nd derivative) Here we manipulate linear models and : Linearly table $\Leftrightarrow$ Stable

[8] Remark : Oppositely, a realistic (i.e. collision-free) jam model should be LSNO and *non*-GS, with long wavelength (see M Treiber and A Kesting *Traffic flow dynamics* Chap 15 Springer 2013)

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- **Locally stable** (LS) : Stability of a vehicle following a leader with constant speed

  LS with no oscillation (LSNO) : Non oscillating convergence (over-damped)

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Jam avoidance model : **LSNO and GS**[8]

$\rightarrow$ Collision-free convergence to homogeneous solutions for any initial condition

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Optimal velocity model (OVM)

Relaxed road traffic flow model based on the spacing[9]

$$\ddot{x}_n(t) = \frac{1}{T}\big(V(\Delta x_n(t)) - \dot{x}_n(t)\big) \tag{1}$$

with OV function $V(\cdot)$ and relaxation time $T > 0$

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- Homogeneous solution *linearly* LSNO and GS if $T < 1/(4V')$

  $\rightarrow$ Jam avoidance model for small relaxation times

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Full velocity difference model (FVDM)

Introduction of the **speed difference** term[10]

$$\ddot{x}_n(t) = \frac{1}{T_1}\big(V(\Delta x_n(t)) - \dot{x}_n(t)\big) + \frac{1}{T_2}\Delta\dot{x}_n(t) \tag{2}$$

with speed difference $\Delta\dot{x}_n(t) = \dot{x}_{n+1}(t) - \dot{x}_n(t)$ and relaxation times T_1, $T_2 > 0$

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[10] R Jiang et al. *Phys Rev E* 64:017101 (2001), see also W. Helly in *ISTTT* pp. 207 Elsevier (1959)
[11] These conditions are $\quad T < 1/V' \quad$ if $T_1 = T_2 = T$

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with speed difference $\Delta\dot{x}_n(t) = \dot{x}_{n+1}(t) - \dot{x}_n(t)$ and relaxation times $T_1, T_2 > 0$

- Linear LSNO and GS if (resp.)[11] $V' < \frac{1}{4T_1}\left(1 + \frac{T_1}{T_2}\right)^2$ and $V' < \frac{1}{2T_1} + \frac{1}{T_2}$

$\rightarrow$ Stabilization with speed difference (i.e. as $T_2 \rightarrow 0$)

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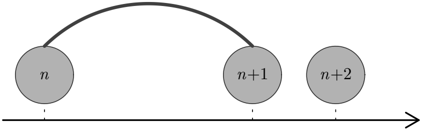
Jam avoidance models

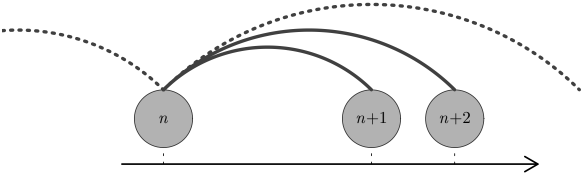
- **Autonomous** models solely based on speed and spacing (OVM) and possibly predecessor speed (FVDM)



Jam avoidance models

- **Autonomous** models solely based on speed and spacing (OVM) and possibly predecessor speed (FVDM)

- **Collective** models depending on spacings and speed of several vehicles in the neighbourhood (connected vehicles)



Collective jam avoidance models

Interaction with $K \geq 1$ **vehicles** in front (with $\Delta x_{n,k} = x_{n+k} - x_n$) :

$$\ddot{x}_n(t) = \sum_{k=1}^{K} \tilde{F}_k\big(\Delta x_{n,k}(t), \dot{x}_n(t), \Delta\dot{x}_{n,k}(t)\big) \qquad (3)$$

[12] H Lenz et al. *Eur Phys J B* 7:331 (1999)

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- *Multi-anticipative model*[12] (MAM) based on spacing distance

$$\tilde{F}_k = \alpha_k \frac{1}{T}\left(V\left(\frac{\Delta x_{n,k}}{k}\right) - \dot{x}_n\right) \tag{4}$$

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$$\tilde{F}_k = \alpha_k \tfrac{1}{T}\left(V\left(\frac{\Delta x_{n,k}}{k}\right) - \dot{x}_n\right) \tag{4}$$

- *Velocity difference multi-anticipative model* (VDMAM) including speed difference

$$\tilde{F}_k = \alpha_k\left[\tfrac{1}{T_1}\left(V\left(\frac{\Delta x_{n,k}}{k}\right) - \dot{x}_n\right) + \tfrac{1}{T_2}\Delta\dot{x}_{n,k}\right] \tag{5}$$

[12] H Lenz et al. *Eur Phys J B* 7:331 (1999)

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Setting of interaction coefficients α_k

Constraint : $\sum_k \alpha_k = 1$

→ OVM (and FVDM) for $K = 1$

→ Weighted mean of acceleration rates with K predecessors

→ Maximization[13] of the stability for **uniform interaction**:

$$\alpha_k = 1/K \tag{6}$$

[13] H Lenz et al. *Eur Phys J B* 7:331 (1999), see also K Hasebe et al. *Phys Rev E* 68:026102 (2003); M Chraibi et al. in *Proc ATT* (2014)

Linear jam avoidance models – Summary

Name	Acronym	Type	Parameter
Optimal velocity	OVM	Autonomous	V', T
Full velocity difference	FVDM	Autonomous	V', T_1, T_2
Multi-anticipative	MAM	Collective	V', T, K
Velocity difference multi-anticipative	VDMAM	Collective	V', T_1, T_2, K

FVDM $\Leftrightarrow$ OVM and VDMAM $\Leftrightarrow$ MAM if $T_2 \to \infty$

MAM $\Leftrightarrow$ OVM and VDMAM $\Leftrightarrow$ FVDM if $K = 1$

Overview

Simulation settings

Simulation of $N = 20$ vehicles from jam initial condition on 405 m line with periodic boundaries

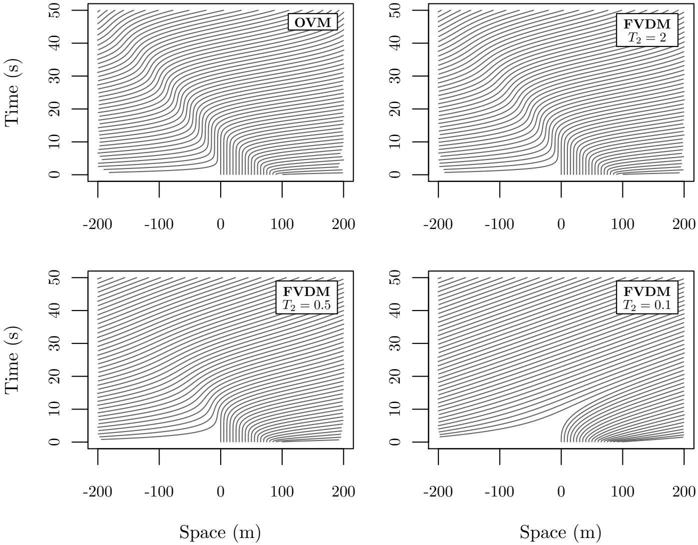
- Simulation with explicit Euler scheme with time step 0.001 s
- Parameter settings (fix) : $V' = 1\ \mathrm{s}^{-1}$; $T = T_1 = 0.25$ s
  (tested) : $T_2 = 2,\ 0.5,\ 0.1$ s ; $K = 2,\ 4,\ 10$ veh

Speed of convergence of the system to the uniform solution quantifies by spacing standard deviation (Lyapunov function)

$$\sigma_{\Delta x} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(\Delta x_n - \Delta\bar{x}_n\right)^2} \qquad \text{with} \quad \Delta\bar{x}_n = \frac{1}{N}\sum_{n=1}^{N}\Delta x_n \tag{7}$$

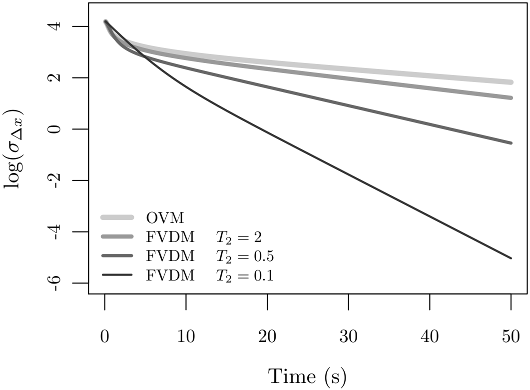
Trajectories

OVM and FVDM



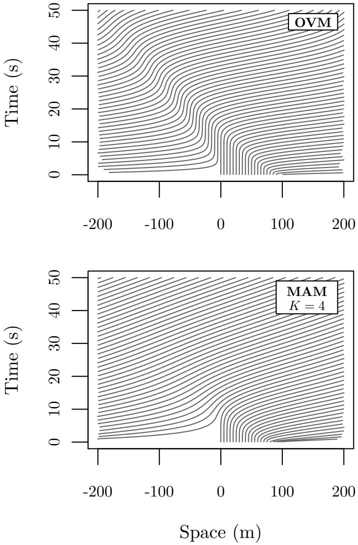
Spacing standard deviation

OVM and FVDM



Trajectories

OVM and MAM



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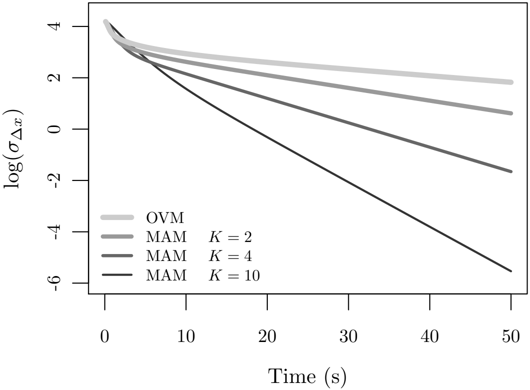
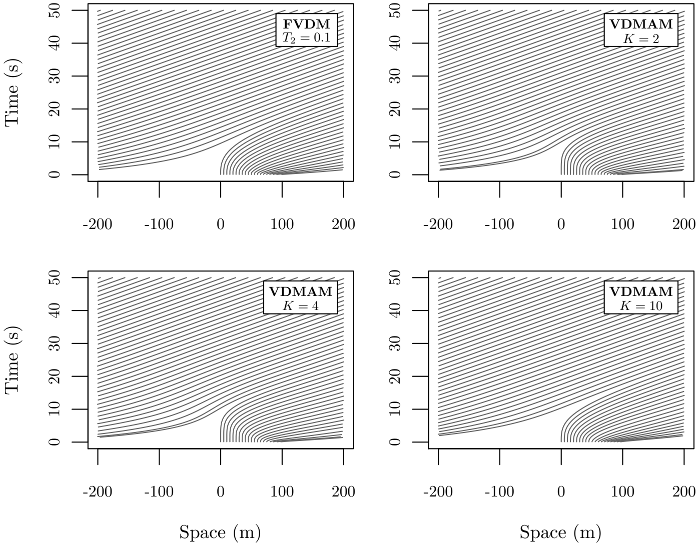


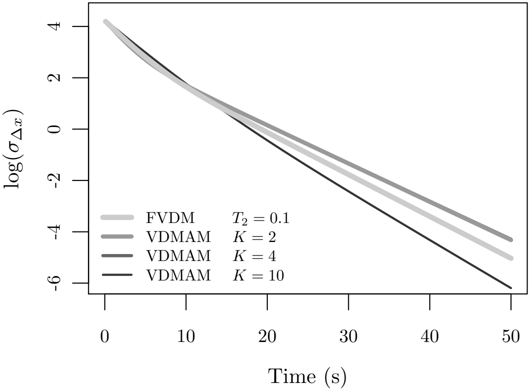
Trajectories

FVDM and VDMAM ($T_2 = 0.1$)



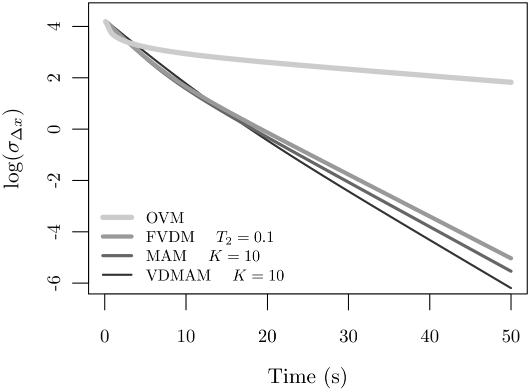
Spacing standard deviation

FVDM and VDMAM ($T_2 = 0.1$)



Spacing standard deviation

OVM, FVDM, MAM and VDMAM



Overview

Lyapunov exponents

Solution of linear systems are linear combinations (LC) of exponentials

$$x_n(t) = \mathrm{LC}\big(\exp(\lambda_l t), t\exp(\lambda_l t)\big) \tag{8}$$

with (λ_l) the Lyapunov exponents (in $[\mathrm{t}^{-1}]$)

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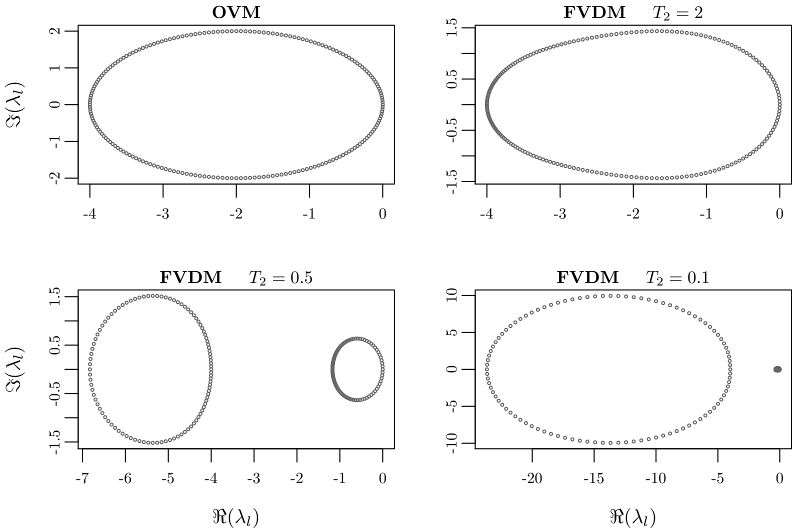
- Here $\lambda_l = \frac{1}{2}\sum_{k=0}^{K}\beta_k\iota_l^k \pm \frac{1}{2}\Big[\Big(\sum_{k=0}^{K}\beta_k\iota_l^k\Big)^2 - 4\sum_{k=1}^{K}\alpha_k(1-\iota_l^k)\Big]^{1/2}$

  with $\iota_l = \exp(2i\pi l/N)$, $\alpha_k = \frac{1}{kT_1}\frac{V'}{K}$, $\beta_0 = -\frac{1}{T_1} - \frac{1}{T_2}$ and $\beta_k = -\frac{1}{KT_2}$

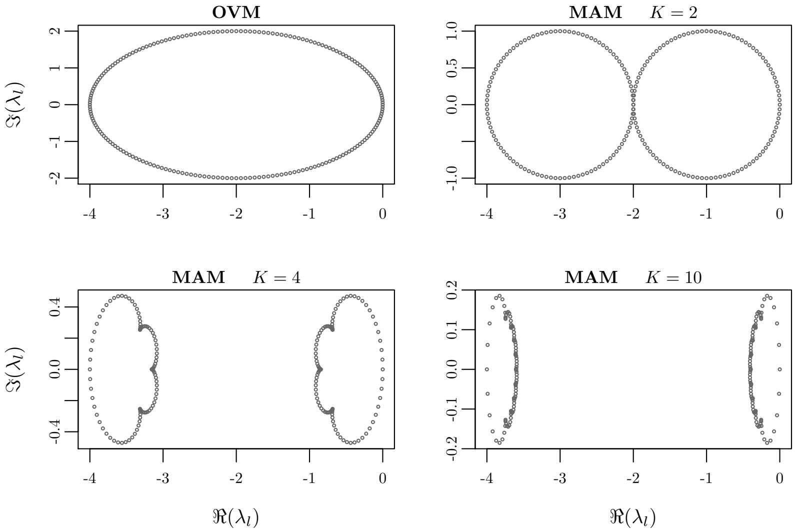
Lyapunov exponents

FDVM : Double mode pattern as $T_2 \to 0$



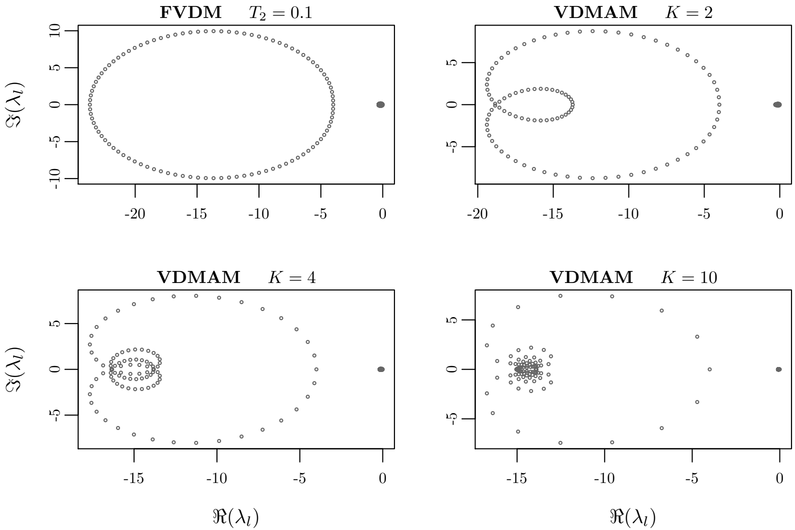
Lyapunov exponents

MAM : Double mode pattern as $K \to \infty$



Lyapunov exponents

VDMAM : Remains double mode as $K \to \infty$



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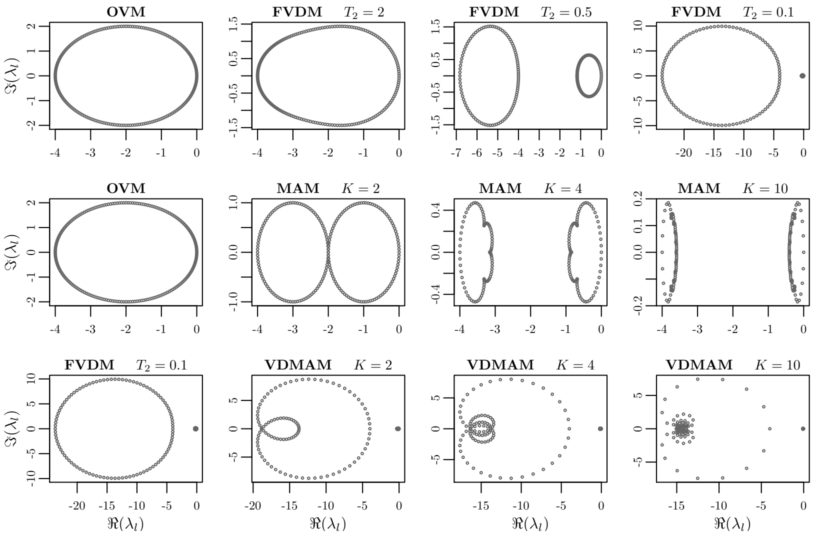

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Conclusion

Lyapunov exponents and simulation of the jam experience shown that

- Comparable behaviors of ‘more stable’ multi-anticipative (collective) and full velocity difference (autonomous) models as (resp.) $K \to \infty$ and $T_2 \to 0$ (cf.[14])
- Few improvement of the stability with multi-anticipation if speed difference is taken into account

[14] K Hasebe et al. Equivalence response among extended optimal velocity models *PRE* 69:017103 (2004)

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→ To be confirmed... (impacts of system size, density, initial condition; interpretation of the Lyapunov exponents)

→ Non-linear models are not considered

[14] K Hasebe et al. Equivalence response among extended optimal velocity models *PRE* 69:017103 (2004)

Outlook

Estimation of the speed difference by using spacing time-differences (delayed feedback model)

$$\ddot{x}_n(t) = \frac{1}{T_1}\big(V(\Delta x_n(t)) - \dot{x}_n(t)\big) + \frac{1}{T_2}\frac{1}{\delta}\big(\Delta x_n(t) - \Delta x_n(t-\delta)\big) \tag{9}$$

with time interval δ such that $0 < \delta \leq T_2$

$\rightarrow$ **Feasible model** only based on spacing sequence (vehicles not connected)

$\rightarrow$ LSNO and GS expected to be the same as FVDM (at least at the limit $\delta \rightarrow 0$)