The relationship between the waiting crowd and the average service time

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Main practical question: How many servers are needed?

Outline

- 1. Analytical approach (Kendall's notation)
- 2. Simulation approach with an example
- 3. Empirical data elicitation
- 4. Insights from the empirical data
- 5. Visualization of queuing situations

Kendall's Notation

- Interarrival times
 Distribution function A
- Service times
 Distribution function B
- Number of servers
- Capacity of the system K
- Service discipline
- \rightarrow Queuing system definition A / B / m / K / D



David George Kendall

Performance measures

- Number of waiting customers
- Waiting time of a customer
- Idle and busy time of a server
- Utilization of the server(s)

- Customer-focused perspective
- Server-focused perspective

Analytical solutions for several queuing systems

Analytic solution for ...

- ... M/D/1 queue in 1917 by Erlang.
- ... M/D/k queue in 1920 by Erlang.
- ... M/G/1 queue in 1930 by Pollaczek.
- ... GI/M/k queue in 1953 by Kendall.
- ... GI/G/1 queue in 1957 by Pollaczek.
- ... G/G/1 queue in 1961 by Kingman.
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- Solution for the following queuing systems: supositorio.com



Drawbacks of the analytical approach

 Analytic answers are only available for simpler generic cases.

 \rightarrow No analytic solution, if the queuing situation is quite complex (e.g. strong variation of the arrival rate).

- Analytic answers are idealized answers.
 → E.g. the physical layout of the queuing situation is not taken into account.
- Analytic answer are isolated answers.

 \rightarrow The arrival rate and the service time are exogenous input.



Simulation example with exogenous input (Java-based software Anylogic)



Empirical data elicitation (based on post-evaluation of video data.)

Service times / Average duration of use Bar with beverages: 41,6 seconds Food stand with self-made smoothies: 56,7 seconds Food stand with piglings: 34,9 seconds Mobile toilets: 59 seconds





Simulation example with exogenous input





Animation





⊷ this.utilizationServer1

Bar Example: Varied arrival rate and fixed service time

Service Time = 41,6 seconds with 5 servers 500,00 450,00 400.00 Average Waitingtime 350,00 300,00 250,00 200,00 150,00 100,00 50,00 0.00 0.00 50,00 100,00 150,00 200,00 250,00 300,00 350,00 400,00 450,00 Arrivals / h \rightarrow Tipping points in queuing

 \rightarrow Self-regulation of the system to avoid going beyond the tipping points

At some point we get very quickly to infinite waiting times.

Arrivals/hour = 350 \rightarrow 24 sec. Arrivals/hour = 400 \rightarrow 90 sec. Arrivals/hour = 450 $\rightarrow \infty$ (Infinite)

Interesting finding from the empirical data elicitation





Counterintuitive finding

Increase of queue size leads to an increase of the average service time.

Assumption

Increase of queuing size leads to an increase in the size of orders (grouping and postponment effect) and can not be compensated by an increase of labor efficiency.









Incorporating dynamic feedback in the action selection layer of the simulation



Queuing situations in pedestrian simulation



You can use our post-visualization software with your own data: www.bit.ly/1Nzxduq (https://www.cms.bgu.tum.de/de/forschung/projekte/oip/31-forschung/projekte/463-pedvis)

Comparison between the simulated microworld and the real-world data



Conclusion

- Queuing behavior is of fundamental importance in pedestrian and traffic simulation, because queues define the bottlenecks of your system.
- Generating benchmarks with simulation is often easier and more accurate than coming up with an analytic solution, because boundary conditions can be endogenized.
- Feedback from the length of the queue on the people that start queuing up is essential, as you otherwise may end up several times in situation beyond the tipping point (unrealistic).





Thank you for your attention!