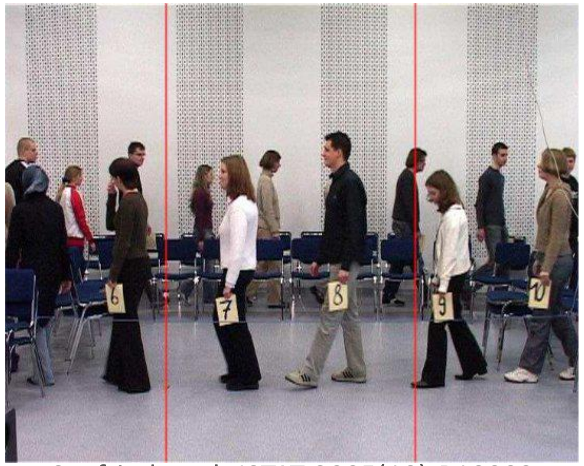
THE INFLECTION POINT OF THE SPEED-DENSITY RELATION AND THE SOCIAL FORCE MODEL

Tobias Kretz, Jochen Lohmiller, Johannes Schlaich
PTV Group

SPEED-DENSITY RELATION OF PEDESTRIAN DYNAMICS

Empirical Foundation

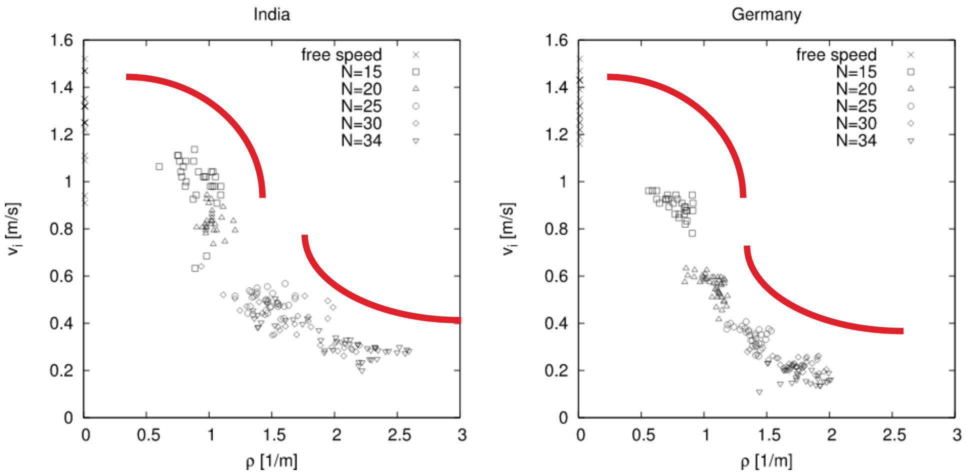
- One-dimensional movement
- Well investigated with different populations



Seyfried et al. JSTAT 2005(10) P10002

- Negative curvature at low densities
- Positive curvature at high densities

- Thus, a function that approximates the data must have an inflection point where curvature is zero.



Chattaraj et al. ACS 12(03) 393 (2009)

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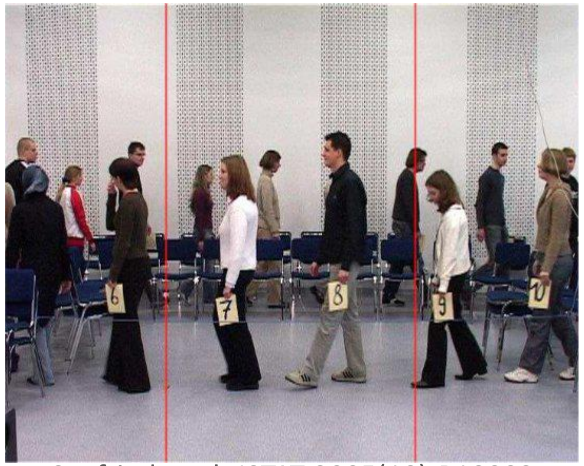
SPEED-DENSITY RELATION OF PEDESTRIAN DYNAMICS

Motivation for further steps

- A function that approximates the data must have an inflection point where curvature is zero.
- A microscopic model should produce data which can be approximated with a function that has an inflection point.
- If a function for the speed-density relation can be analytically derived for the equilibrium state of a microscopic model, this function should exhibit an inflection point.



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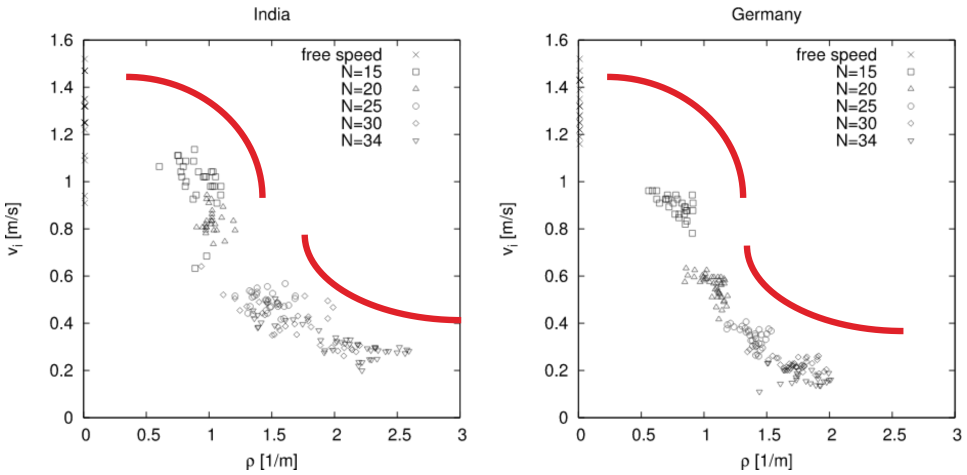

Chattaraj et al. ACS 12(03) 393 (2009)

THE SOCIAL FORCE MODEL

Specifications

- In Johansson et al. ACS 10(02) 271 (2007) three specifications for the Social Force Model were given.
  - Circular Specification considers only relative position (distance in first instance) between pedestrians to compute mutual forces.
  - Elliptical Specification I considers in addition the velocity of the pedestrian that exerts the force.
  - Elliptical Specification II considers in addition (to CS) the relative velocity of both pedestrians.

- The following reasoning is made with the Circular Specification but since relative velocity at equilibrium is zero, it applies to Elliptical Specification II as well.

THE SOCIAL FORCE MODEL

Circular Specification

$$\ddot{\mathbf{x}}_\alpha(t) = \frac{\mathbf{v}_{0\alpha} - \dot{\mathbf{x}}_\alpha(t)}{\tau_\alpha} + \tilde{A}_\alpha \sum_\beta w(\mathbf{x}_\alpha(t), \mathbf{x}_\beta(t), \dot{\mathbf{x}}_\alpha(t), \lambda_\alpha) e^{-\frac{|\mathbf{x}_\beta(t) - \mathbf{x}_\alpha(t)| - R_\alpha - R_\beta}{B_\alpha}} \hat{e}_{\alpha\beta}$$

$$w(\mathbf{x}_\alpha(t), \mathbf{x}_\beta(t), \dot{\mathbf{x}}_\alpha(t), \lambda_\alpha) = \lambda_\alpha + (1 - \lambda_\alpha) \frac{1 + \cos(\theta_{\alpha\beta}(\mathbf{x}_\alpha(t), \mathbf{x}_\beta(t), \dot{\mathbf{x}}_\alpha(t)))}{2}$$

…Boiled down to 1d and identical extrinsic parameters for all pedestrians

$$\ddot{x}_\alpha = \frac{v_0 - \dot{x}_\alpha}{\tau} + A \sum_\beta w(x_\alpha, x_\beta, \lambda) e^{-\frac{d_{\alpha\beta}}{B}}$$

$$d_{\alpha\beta} = |x_\beta - x_\alpha|$$

$$w(x_\alpha, x_\beta, \lambda) = \lambda \text{ if } x_\beta - x_\alpha < 0$$

$$w(x_\alpha, x_\beta, \lambda) = -1 \text{ if } x_\beta - x_\alpha > 0$$

THE SOCIAL FORCE MODEL

Circular Specification for 1d

$$\ddot{x}_\alpha = \frac{v_0 - \dot{x}_\alpha}{\tau} + A \sum_\beta w(x_\alpha, x_\beta, \lambda) e^{-\frac{d_{\alpha\beta}}{B}}$$

$$d_{\alpha\beta} = |x_\beta - x_\alpha|$$

$$w(x_\alpha, x_\beta, \lambda) = \lambda \text{ if } x_\beta - x_\alpha < 0$$

$$w(x_\alpha, x_\beta, \lambda) = -1 \text{ if } x_\beta - x_\alpha > 0$$

At equilibrium speeds do not change anymore and we can resolve this for speed

$$\dot{x}_\alpha = v_0 + \tau A \sum_\beta w(x_\alpha, x_\beta, \lambda) e^{-\frac{d_{\alpha\beta}}{B}}$$

THE SOCIAL FORCE MODEL

Circular Specification for 1d at equilibrium, resolved for speed

$$\dot{x}_\alpha = v_0 + \tau A \sum_\beta w(x_\alpha, x_\beta, \lambda) e^{-\frac{d_{\alpha\beta}}{B}}$$

With identical extrinsic parameters for all pedestrians the distance from each pedestrian to its leader must be identical d_0: and the distance to the second, third, etc. next pedestrian must be

$$d_{\alpha\beta n} = n d_0$$

And the sum can be rewritten

$$\dot{x}_\alpha = v_0 - (1-\lambda)\tau A \sum_{n=1}^{\infty} e^{-\frac{n d_0}{B}}$$

THE SOCIAL FORCE MODEL

Equilibrium speed of Circular Specification for 1d at equilibrium in dependence of equilibrium distance d_0 between pedestrians:

$$\dot{x}_\alpha = v_0 - (1-\lambda)\tau A \sum_{n=1}^{\infty} e^{-\frac{nd_0}{B}}$$

This is a geometric series and the solution can directly be given:

$$\begin{aligned}\dot{x}_\alpha &= v_0 - (1-\lambda)\tau A \left(\frac{1}{1-e^{-\frac{d_0}{B}}} - 1 \right) \\ &= v_0 - (1-\lambda)\tau A \frac{1}{e^{\frac{d_0}{B}} - 1} \\ &= v_0 - (1-\lambda)\tau A \frac{1}{e^{\frac{1}{B\rho}} - 1}\end{aligned}$$

Where in the last step it is made use of $\rho = 1/d_0$ to write the equation in terms of density.

THE SOCIAL FORCE MODEL

Speed-Density relation of the Circular Specification at equilibrium for a homogeneous population

$$v(\rho) = v_0 - (1-\lambda)\tau A \frac{1}{e^{\frac{1}{B\rho}} - 1}$$

Derivatives

$$\frac{\partial v(\rho)}{\partial \rho} = -(1-\lambda)\tau A \frac{e^{\frac{1}{B\rho}}}{B\rho^2 (e^{\frac{1}{B\rho}} - 1)^2}$$

$$\frac{\partial^2 v(\rho)}{\partial \rho^2} = (1-\lambda)\tau A e^{\frac{1}{B\rho}} \frac{(2B\rho - 1)e^{\frac{1}{B\rho}} - (2B\rho + 1)}{B^2 \rho^4 (e^{\frac{1}{B\rho}} - 1)^3}$$

There is no (real) solution for

$$(2B\rho_i - 1)e^{\frac{1}{B\rho_i}} - (2B\rho_i + 1) = 0$$

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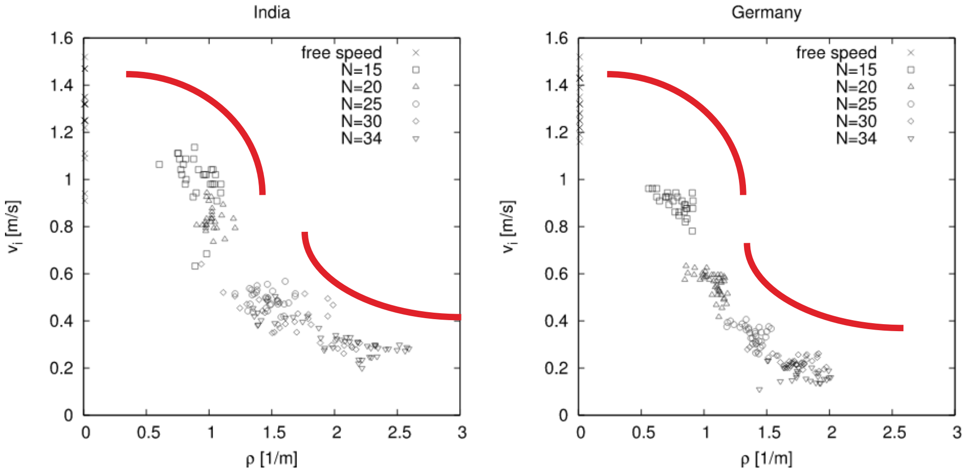
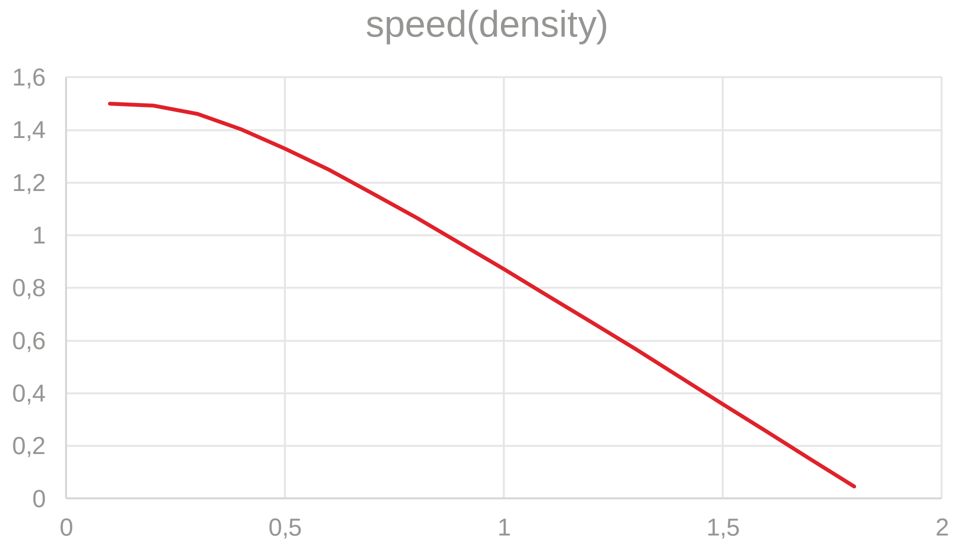
SPEED-DENSITY RELATION OF PEDESTRIAN DYNAMICS

Empirical Data



Circular Specification of the Social Force Model at equilibrium for a homogeneous population



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CAN THIS BE FIXED?

Returning to equilibrium speed of Circular Specification for 1d at equilibrium in dependence of equilibrium distance d_0 between pedestrians:

$$\dot{x}_\alpha = v_0 - (1-\lambda)\tau A \sum_{n=1}^{\infty} e^{-\frac{nd_0}{B}}$$

Imagine only nearest neighbors exert a force mutually (as is usually the case in car following models) . Then only the case of *n=1* is considered and the rest of the sum is neglected.

$$\dot{x}_\alpha = v_0 - (1-\lambda)\tau A e^{-\frac{1}{B\rho}}$$

We can write this unnecessarily complicated

$$\dot{x}_\alpha = v_0 - (1-\lambda)\tau A \frac{1}{e^{\frac{1}{B\rho}} - 0}$$

And see in this way that the only difference to the full model is a zero instead of a one in the denominator.

THE SOCIAL FORCE MODEL

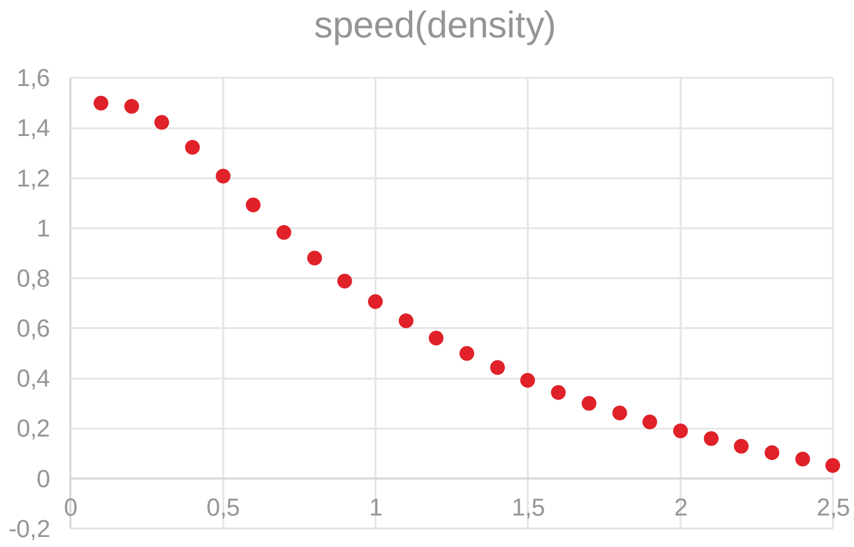
Speed-Density relation of the FULL Circular Specification at equilibrium for a homogeneous population

$$v(\rho) = v_0 - (1-\lambda)\tau A \frac{1}{e^{\frac{1}{B\rho}} - 1}$$

And when only nearest neighbors exert a force mutually

$$v(\rho) = v_0 - (1-\lambda)\tau A \frac{1}{e^{\frac{1}{B\rho}} - 0}$$

This has an inflection point



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THE SOCIAL FORCE MODEL

Speed-Density relation of the FULL Circular Specification at equilibrium for a homogeneous population

$$v(\rho) = v_0 - (1-\lambda)\tau A \frac{1}{e^{\frac{1}{B\rho}} - 1}$$

And when only nearest neighbors exert a force mutually

$$v(\rho) = v_0 - (1-\lambda)\tau A \frac{1}{e^{\frac{1}{B\rho}} - 0}$$

This has an inflection point

And by the way has exactly the mathematical form of the Kladek formula which Weidmann used to approximate the pedestrian fundamental diagram.

THE SOCIAL FORCE MODEL

Speed-Density relation of the FULL Circular Specification at equilibrium for a homogeneous population

$$v(\rho) = v_0 - (1-\lambda)\tau A \frac{1}{e^{\frac{1}{B\rho}} - 1}$$

And when only nearest neighbors exert a force mutually

$$v(\rho) = v_0 - (1-\lambda)\tau A \frac{1}{e^{\frac{1}{B\rho}} - 0}$$

But can a model of pedestrian dynamics with only nearest neighbor interactions be realistic?

What if we write with a $0 < k < 1$?

$$v_k(\rho) = v_0 - (1-\lambda)\tau A \frac{1}{e^{\frac{1}{B\rho}} - k}$$

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THE K-EXTENDED SOCIAL FORCE MODEL

Hypothetical speed-density relation

$$v_k(\rho) = v_0 - (1-\lambda)\tau A \frac{1}{e^{\frac{1}{B\rho}} - k}$$

Derivatives

$$\frac{\partial v_k(\rho)}{\partial \rho} = -(1-\lambda)\tau A \frac{e^{\frac{1}{B\rho}}}{B\rho^2 (e^{\frac{1}{B\rho}} - k)^2}$$

$$\frac{\partial^2 v_k(\rho)}{\partial \rho^2} = (1-\lambda)\tau A e^{\frac{1}{B\rho}} \frac{(2B\rho - 1)e^{\frac{1}{B\rho}} - k(2B\rho + 1)}{B^2 \rho^4 (e^{\frac{1}{B\rho}} - k)^3}$$

For the inflection point it is required that

$$(2B\rho_i - 1)e^{\frac{1}{B\rho_i}} - k(2B\rho_i + 1) = 0$$

THE K-EXTENDED SOCIAL FORCE MODEL

For the inflection point it is required that

$$(2B\rho_i - 1)e^{\frac{1}{B\rho_i}} - k(2B\rho_i + 1) = 0$$

Numerical solutions for different values of *k*:

k	$B\rho_i$	k	$B\rho_i$	k	$B\rho_i$
0.0	0.500	0.90	0.981	0.99	2.049
0.1	0.515	0.91	1.013	0.999	4.379
0.2	0.531	0.92	1.051	0.9999	9.416
0.3	0.551	0.93	1.096	0.99999	20.28
0.4	0.576	0.94	1.151	0.999999	43.68
0.5	0.606	0.95	1.219	0.9999999	94.10
0.6	0.646	0.96	1.309	0.99999999	202.7
0.7	0.703	0.97	1.435	0.999999999	436.8
0.8	0.793	0.98	1.635	0.9999999999	941.0

THE K-EXTENDED SOCIAL FORCE MODEL

Is there a microscopic model (a modification of the Social Force Model) associated with the hypothetical speed density relation?

Yes, the summation and simplification steps can be undone carrying parameter *k* along. The resulting model equation is

$$\ddot{x} = \frac{v_0 - v}{\tau} - (1-\lambda)A \sum_{n=1}^{\infty} k^{n-1} e^{-\frac{d_{\alpha\beta}}{B}}$$

The force between two pedestrians reduces with distance and additionally with the neighborhood relation.

- The force between direct neighbors is not suppressed
- The force to/from the second next neighbor is suppressed with a factor *k*.
- The force to/from the third next neighbor is suppressed with a factor k^2.
- And so on

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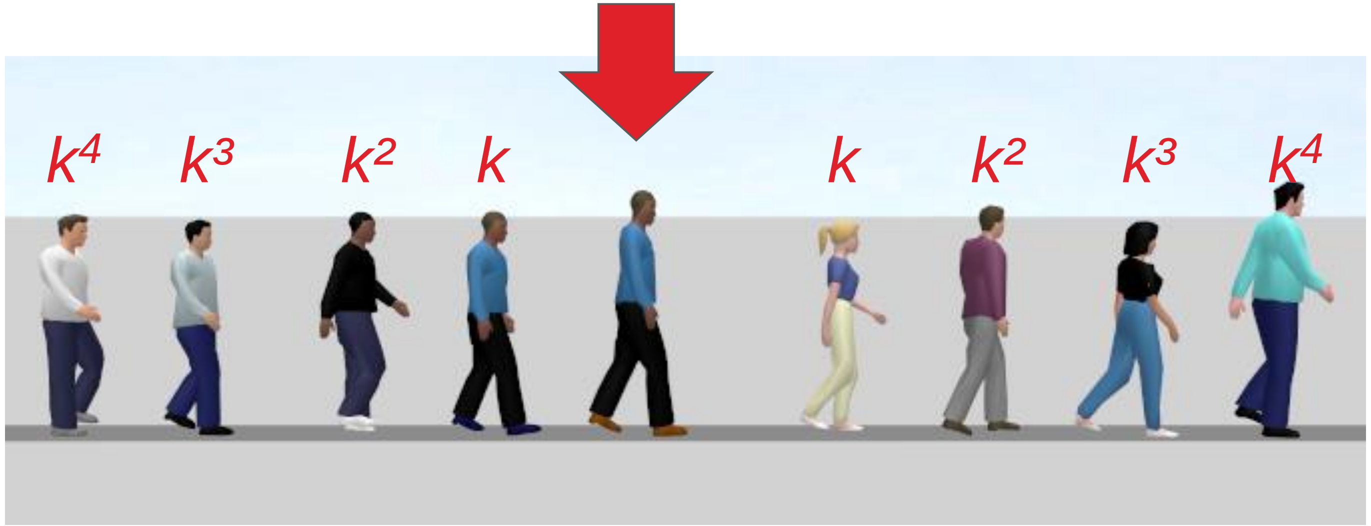
THE K-EXTENDED SOCIAL FORCE MODEL



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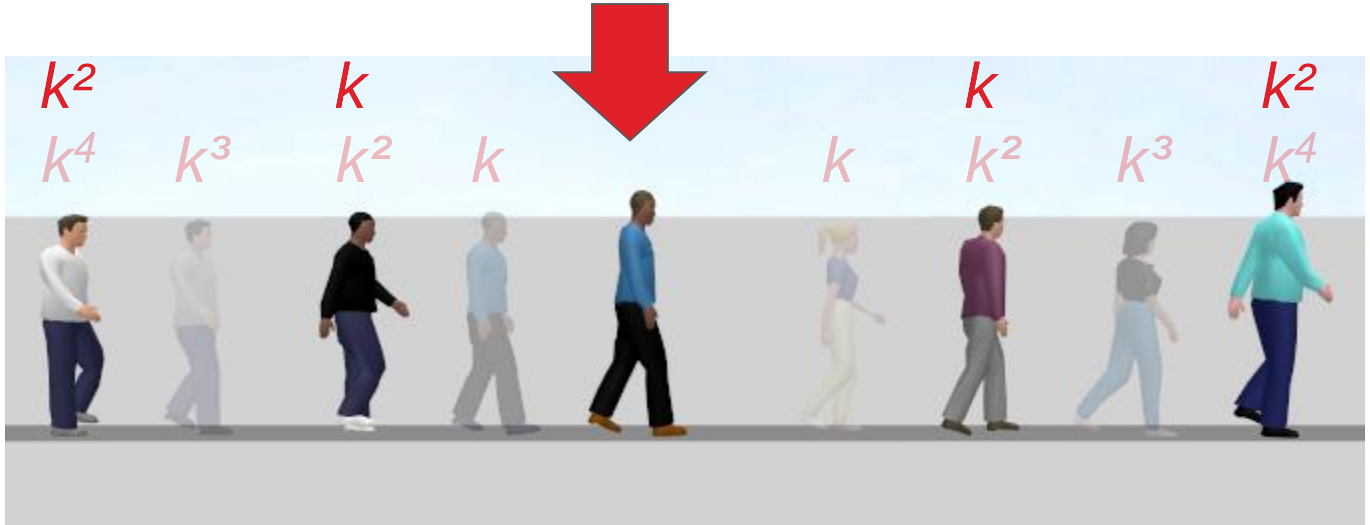
THE K-EXTENDED SOCIAL FORCE MODEL



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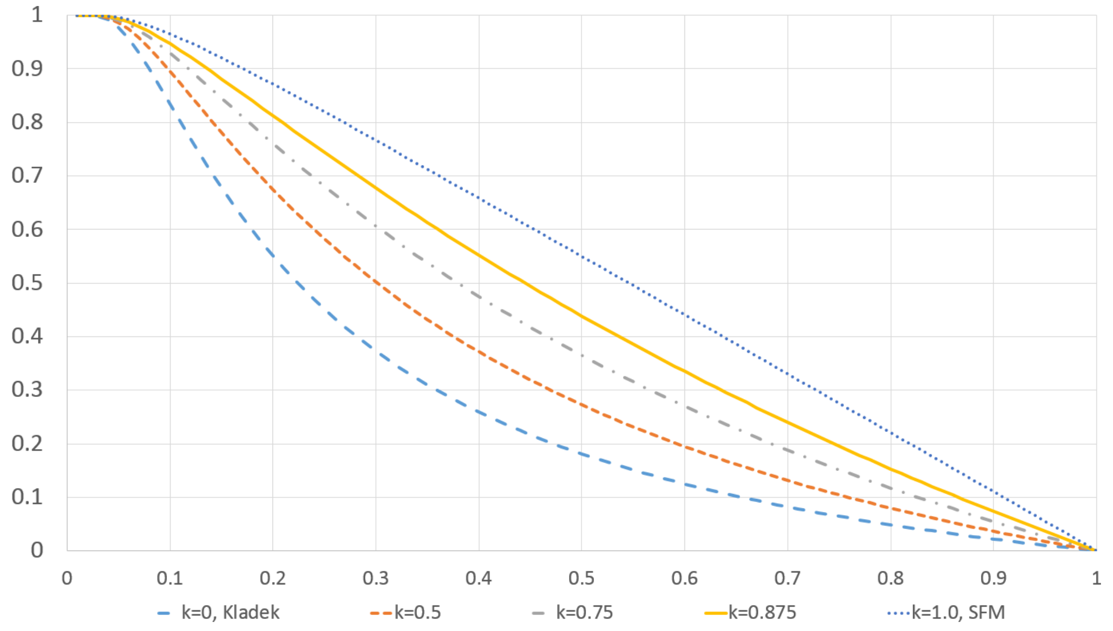
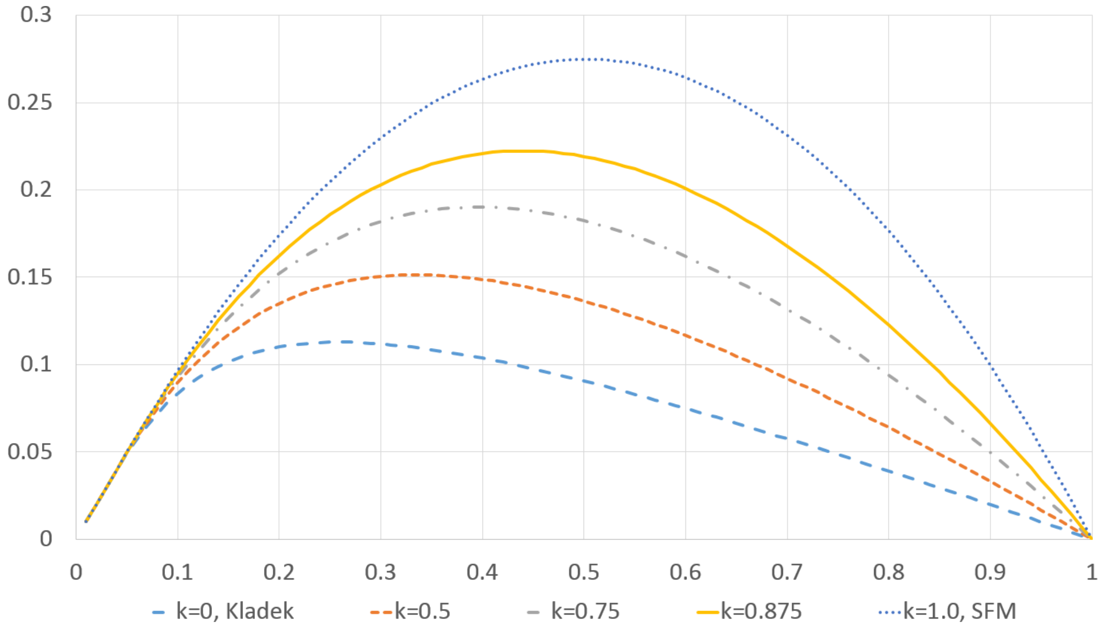
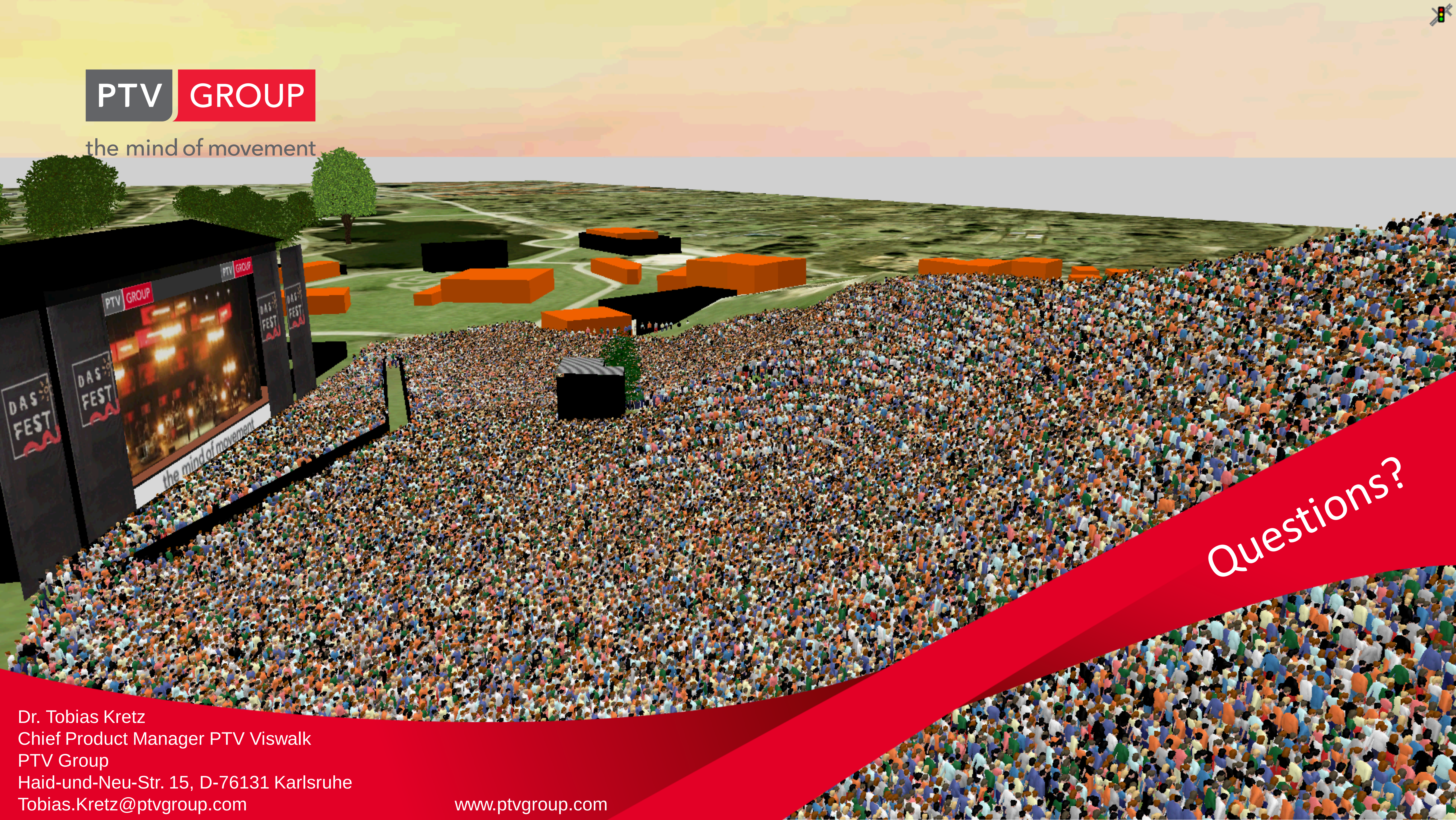
SOME DIAGRAMS





FUTURE DIRECTIONS

- Extension to 2d
  - Who is “next”? Who is “next to next”? …
    - One pedestrian per “n”? One pedestrian per “n” in each slice of a fan? Voronoi?
  - Movement order in equilibrium?
  - *v(density)* becomes more complicated, because in 2d *density* ~ d_0^2
  - Eventually the force in an infinite plane might diverge.
- Is sorting by distance the best option in a micro-model? One could think of other parameters, for example:
  - Absolute value of force
  - Time to collision
- What if there is just a cut-off to the number of pedestrians considered?
  - Compare (multi-anticipative) car-following models
- Comparison to vehicles and bicycles → TRB 2016



Questions?

Dr. Tobias Kretz
Chief Product Manager PTV Viswalk
PTV Group
Haid-und-Neu-Str. 15, D-76131 Karlsruhe
Tobias.Kretz@ptvgroup.com

www.ptvgroup.com