Network-wide mesoscopic traffic state estimation based on a *variational formulation* of the LWR model and using both *Lagrangian and Eulerian observations*

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Date: 28-10-2015
Location: Den Haag - Nootdorp

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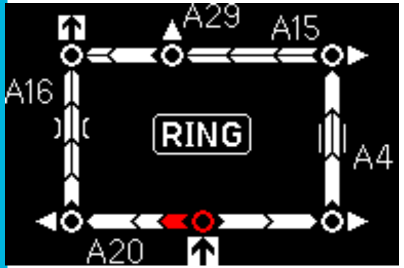
Contents presentation

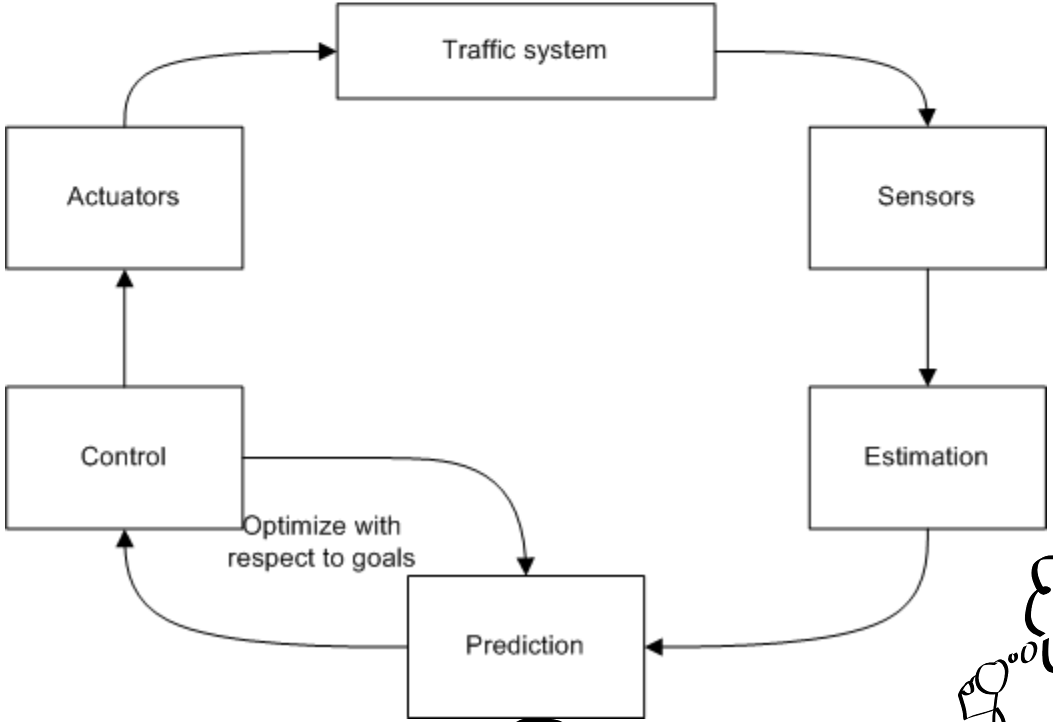
- Introduction
- Formulation
- Methodology
- Follow-up…

- *Introduction & background for traffic management*
- *L-S LWR formulation*
- *Data assimilation methodology*
- *On-going research – experiments, expected results*

Introduction, control cycle current

- Introduction
- Formulation
- Methodology
- Follow-up…





Introduction, control cycle future

- Introduction
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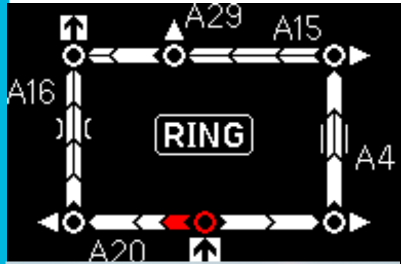
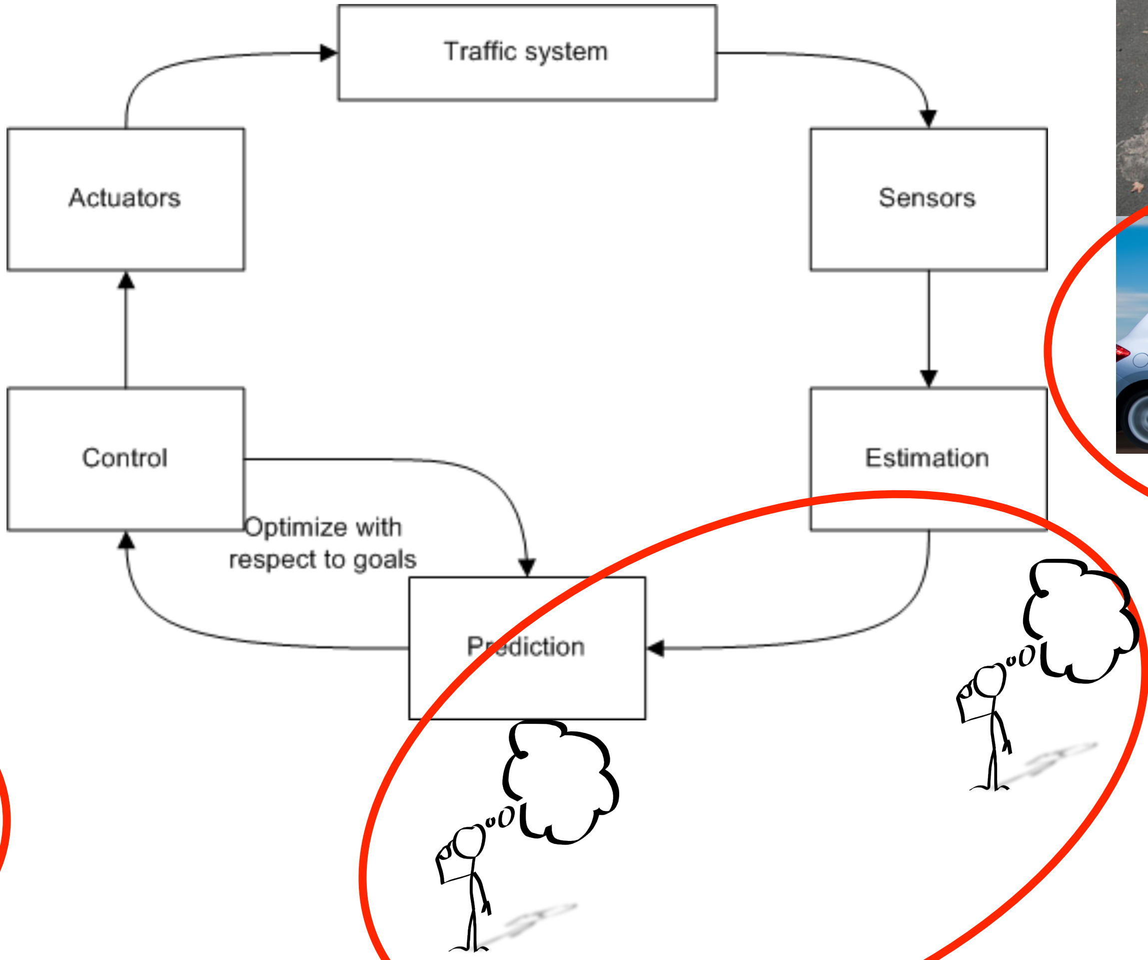
- Follow-up…





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Data assimilation framework

- Introduction
- Formulation
- Methodology
- Follow-up…

Estimation of state: combine real-time measurements and simulation model in order to represent current traffic situation.

Also known as data assimilation.

Process Model

Variational LWR

Sensor Model

Fundamental Relation

Data assimilation method

Kalman Filter,
Least square method…

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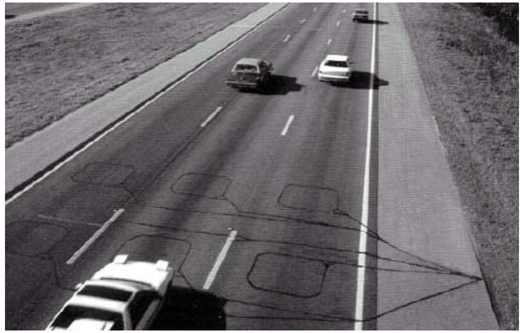
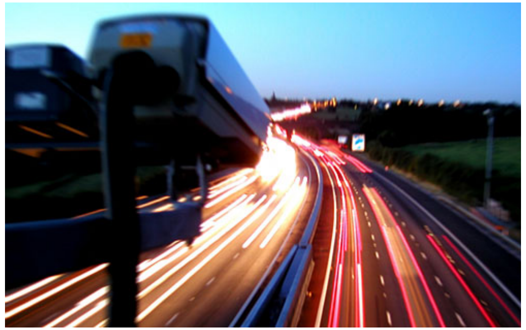
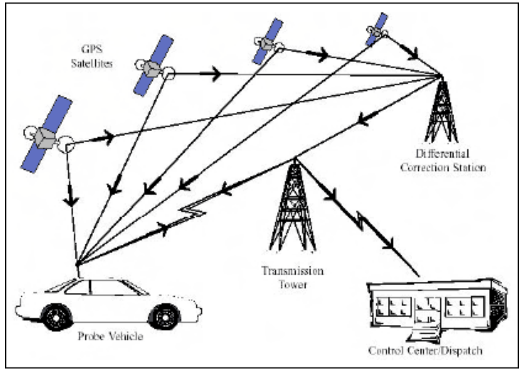
Data assimilation framework, data

- Introduction
- Formulation
- Methodology
- Follow-up…

Fixed location (infrastructure) …

e.g., Loop data have considerable noise and bias



Along with the traffic (vehicles) …

e.g., Probe vehicle data provide x, t, v.



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Data assimilation framework, model

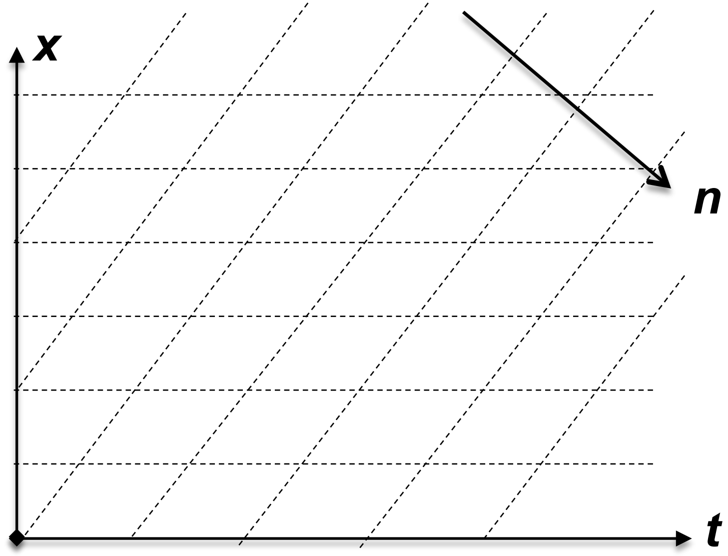
As main traffic flow model type, **traffic flow models at macroscopic or mesoscopic levels** are chosen:

- Describes traffic at a more aggregated level
- Views traffic as a fluid
- Fast computation

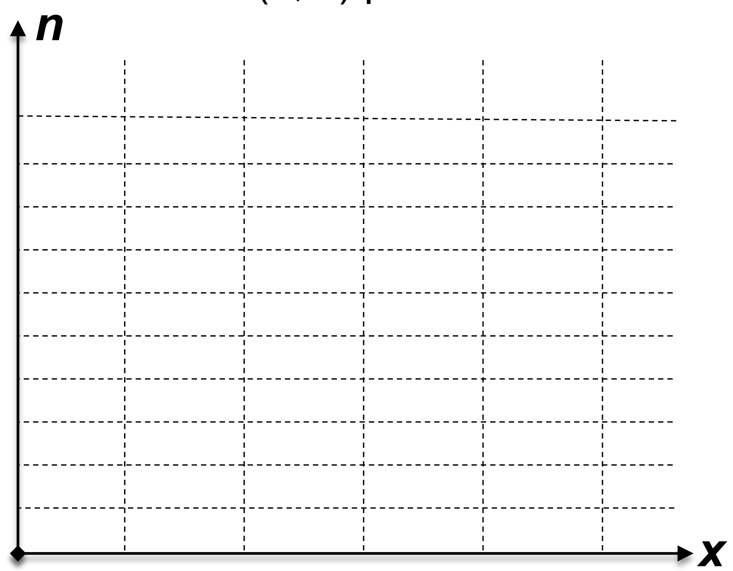
- Discretization: divide network into cells

Foreword – traffic flow model

Eulerian / Lagrangian-time
(x, n, t) plane



Lagrangian-space
(n, x) plane



- ❑ Eulerian formulation: distance and time (x, t) - prevailing
- ❑ Lagrangian-time coordinates: vehicle no - time (n, t)
- ❑ Lagrangian-space coordinates: vehicle no - distance (n, x) – *T* coord.

Mesoscopic: Lagrangian-Space LWR formulation

- Introduction
- Formulation
- Methodology
- Follow-up…

Conservation law (LWR) in Lagrangian-space coordinates

$$\partial_x h - \partial_N(1/V(h)) = 0$$

Variational principle (considering travel time flux)

Daganzo 2005 – Variational theory in Eulerian system
Leclercq et al. 2007 – Variational theory in Lagrangian system
Laval and Leclercq (2013) – Variational theory in *T* system

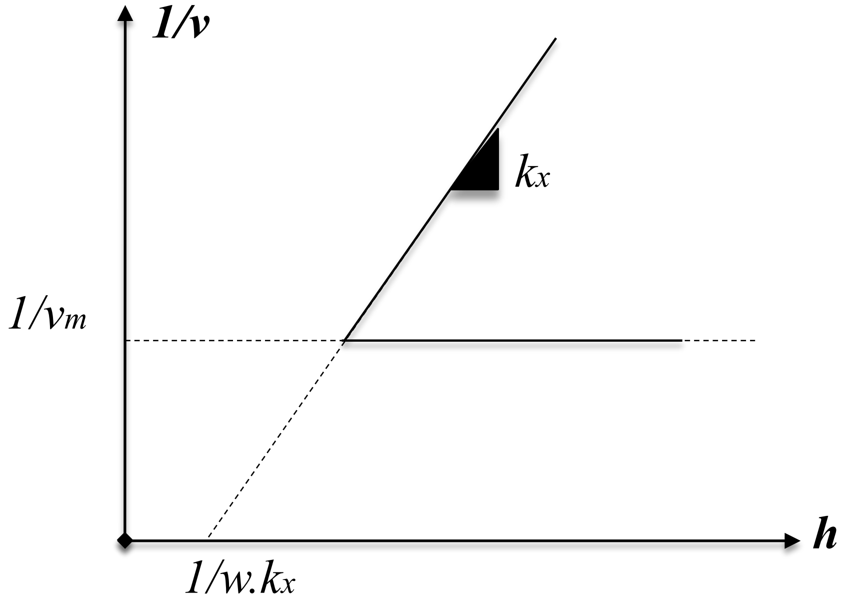
$$\partial_x T = \frac{1}{V(\partial_n T)}$$

Mesoscopic: Lagrangian-Space LWR formulation

The passage time *T* of the vehicle *n* at the position *x* follows:

$$T(n,x) = max\left(T(n,x-\Delta x)+\frac{\Delta x}{v_m}, T(n-\Delta n, x+\frac{\Delta n}{k_x})+\frac{\Delta n}{w.k_x}\right)$$

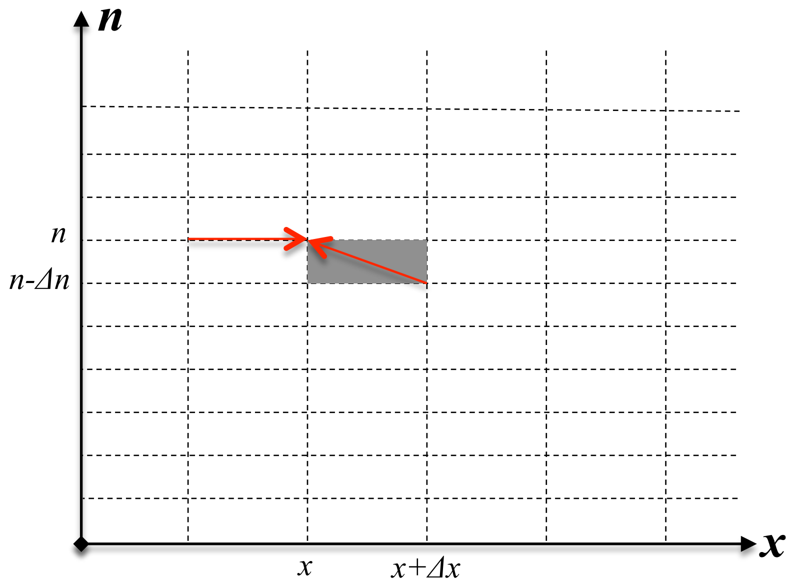
Refer to: Laval and Leclercq (2013)



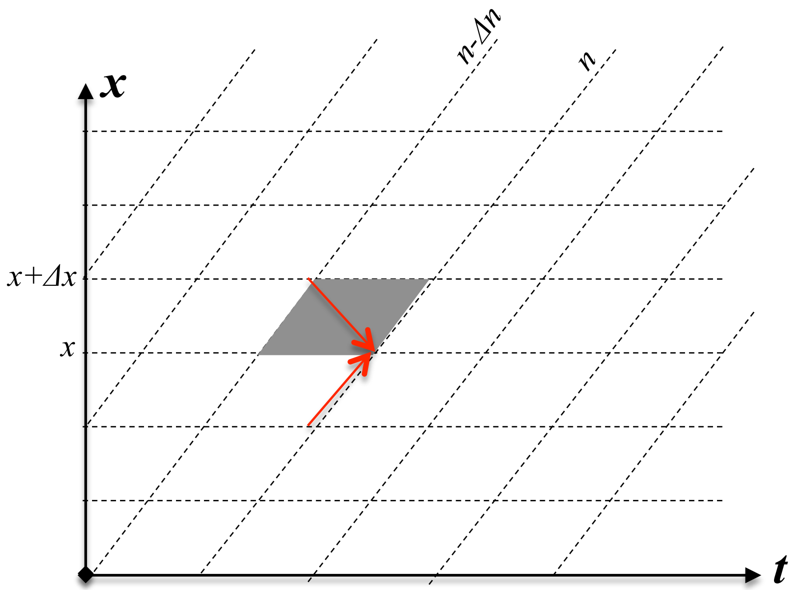
Mesoscopic:
L-S LWR numerical solution (graphical)

- Introduction
- Formulation
- Methodology
- Follow-up…

Mesoscopic grid
(n, x) plane



Mesoscopic grid
(x, t) plane



Advantages of the L-S LWR formulation

- *Variational formulation – simple to implement, numerical accurate*
- *Mesoscopic scale – individual vehicle tracking with macroscopic behavioural rules*
- *Easy to address spatial discontinuities (merges, diverges, lane-drops) & computational efficiency (#node, #veh.)*
- *Convenient for state estimation*

How to incorporate Lagrangian data?

- Introduction
- Formulation
- Methodology
- Follow-up…

- *Duret.et.al.(2016)* have proposed a data assimilation framework with loop data (x fixed)*

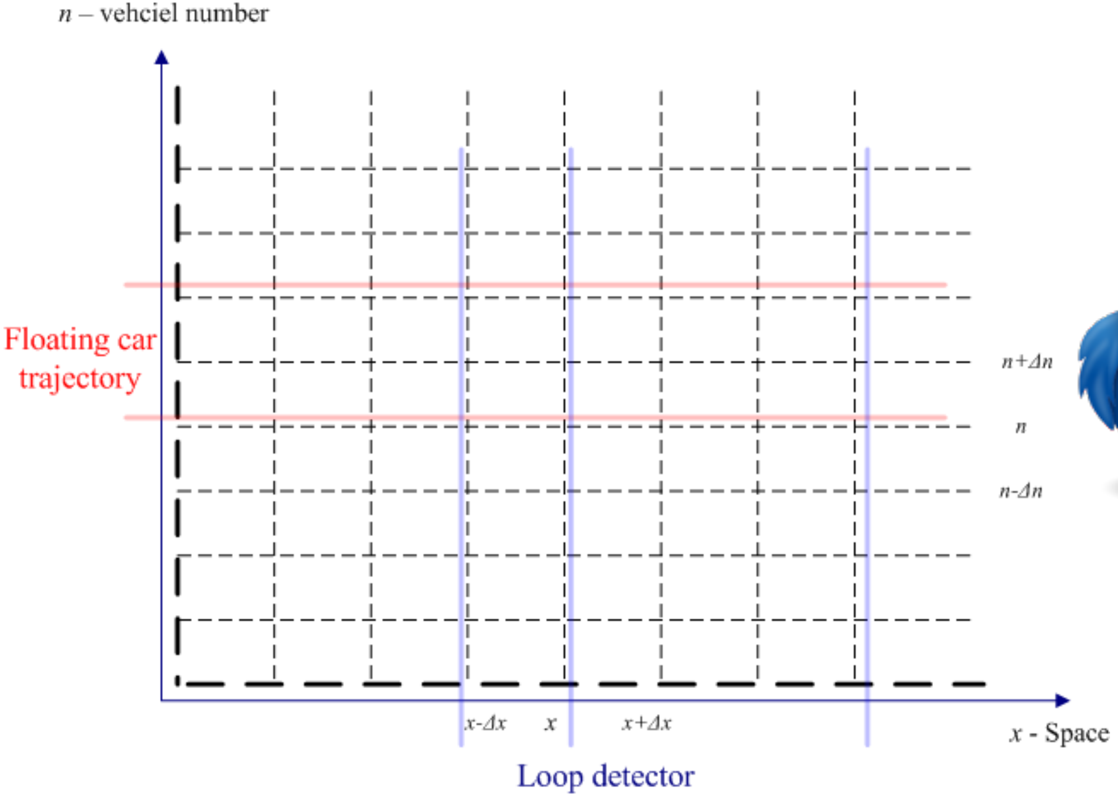
** Duret.et.al.(2016). Data assimilation based on a mesoscopic-LWR modeling framework and loop detector data : methodology and application on a large-scale network*

- *How about incorporating Lagrangian type data (n fixed)?*

- *And why?*

  - More accurate

  - Additional info.



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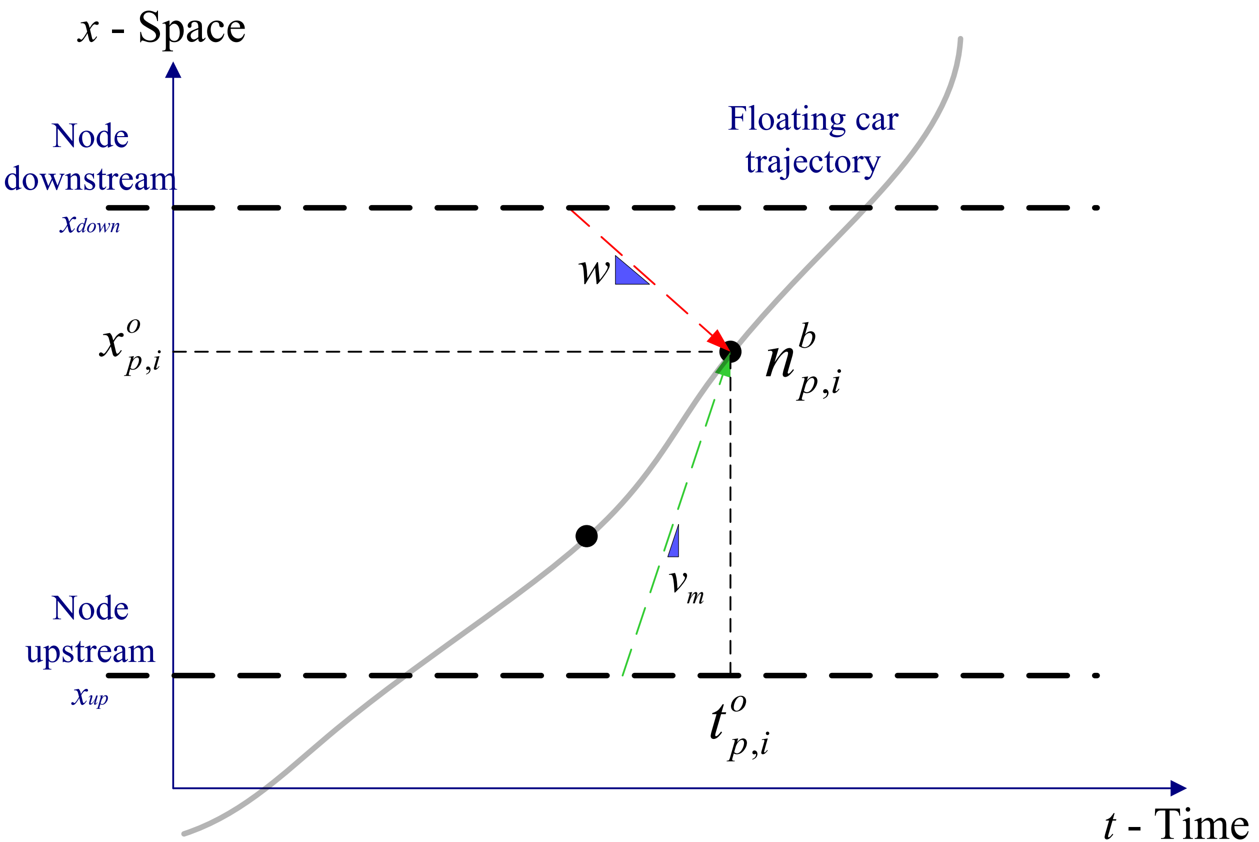
Data assimilation with probe data

Four steps in each sequence:

- *Estimation of local vehicle indexes of probe data*
- *Observation transformation*
  (observation state)
- *Global analysis & Data assimilation*
  (background state + observation state => analysis state)
- *Model update*

Step 1: Estimation of local n indexes

- Introduction
- Formulation
- Methodology
- Follow-up…



$$n^b_{p,i} = \min\left(n_{v,up}, n_{w,down} + k_x\left(x_{down} - x^o_{p,i}\right)\right)$$

$$T\left(n_{v,\mathrm{up}}, x_{up}\right) = t^o_{p,i} - \frac{x^o_{p,i} - x_{up}}{v_m}$$

$$T\left(n_{w,down}, x_{down}\right) = t^o_{p,i} - \frac{x_{down} - x^o_{p,i}}{w}$$

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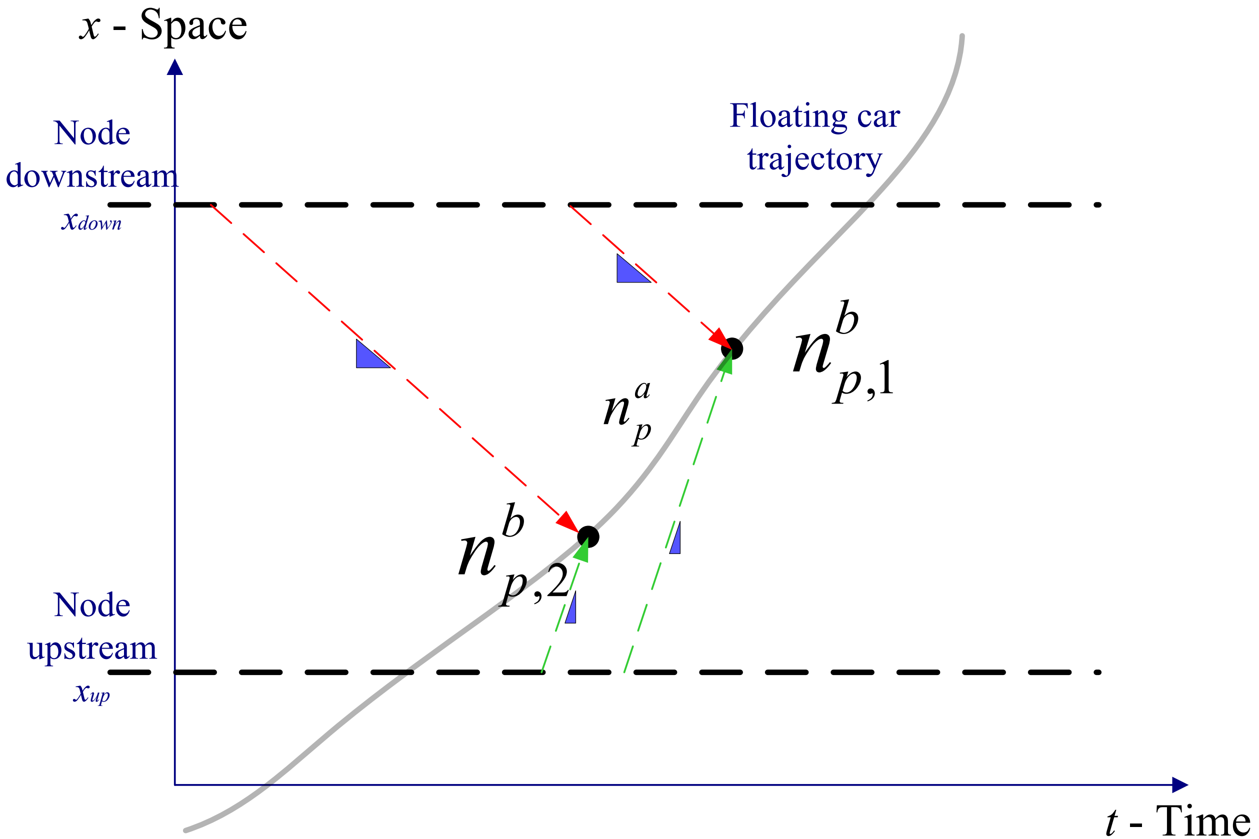
Step 1: Estimation of local n indexes

- Introduction
- Formulation
- Methodology
- Follow-up…



$$n_p^a = \text{func}\left(n_{p,i}^b\right), i \in I$$

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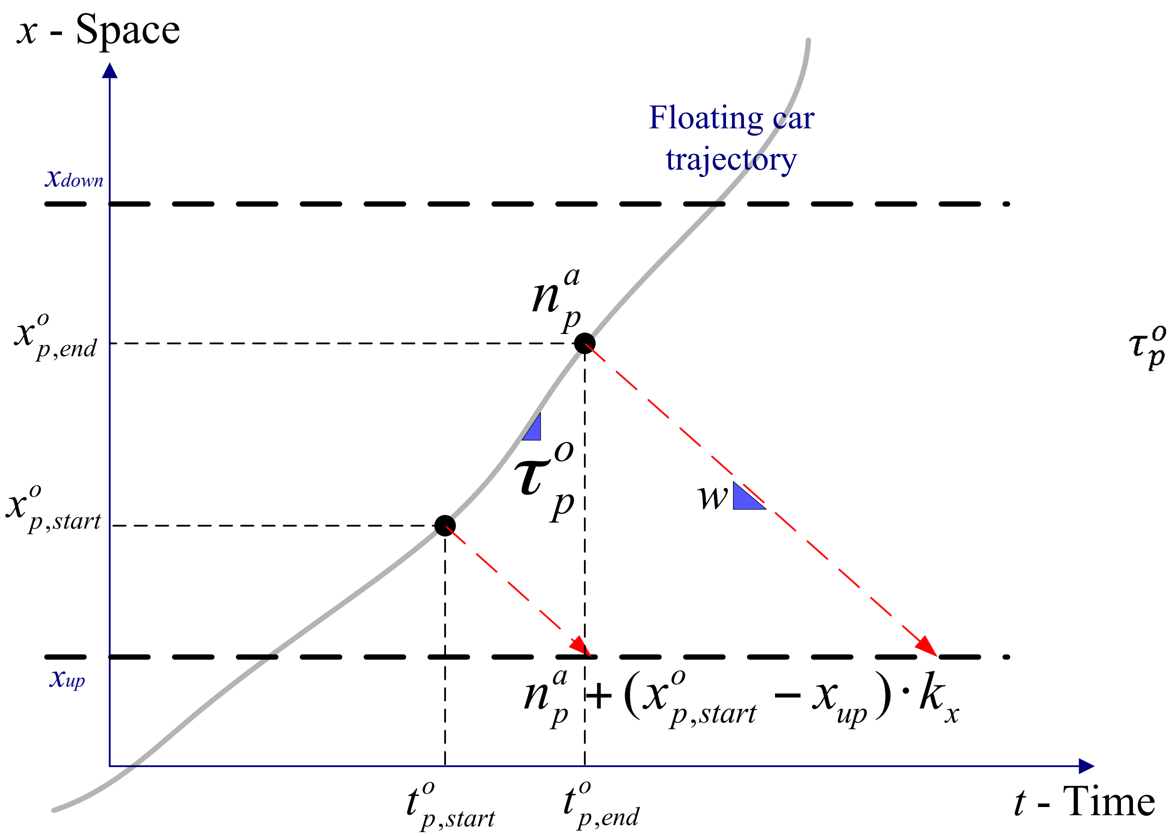
Step 2: Observation transformation (o-state)

- Introduction
- Formulation
- Methodology
- Follow-up…



$$\tau_p^o = \frac{t^o_{p,end} - t^o_{p,start}}{x^o_{p,end} - x^o_{p,start}}$$

$$\mathcal{p}_w(n_p^a) = \{n_p^a + (x^o_{p,start} - x_{up}) \cdot k_x : n_p^a + (x^o_{p,end} - x_{up}) \cdot k_x\}$$

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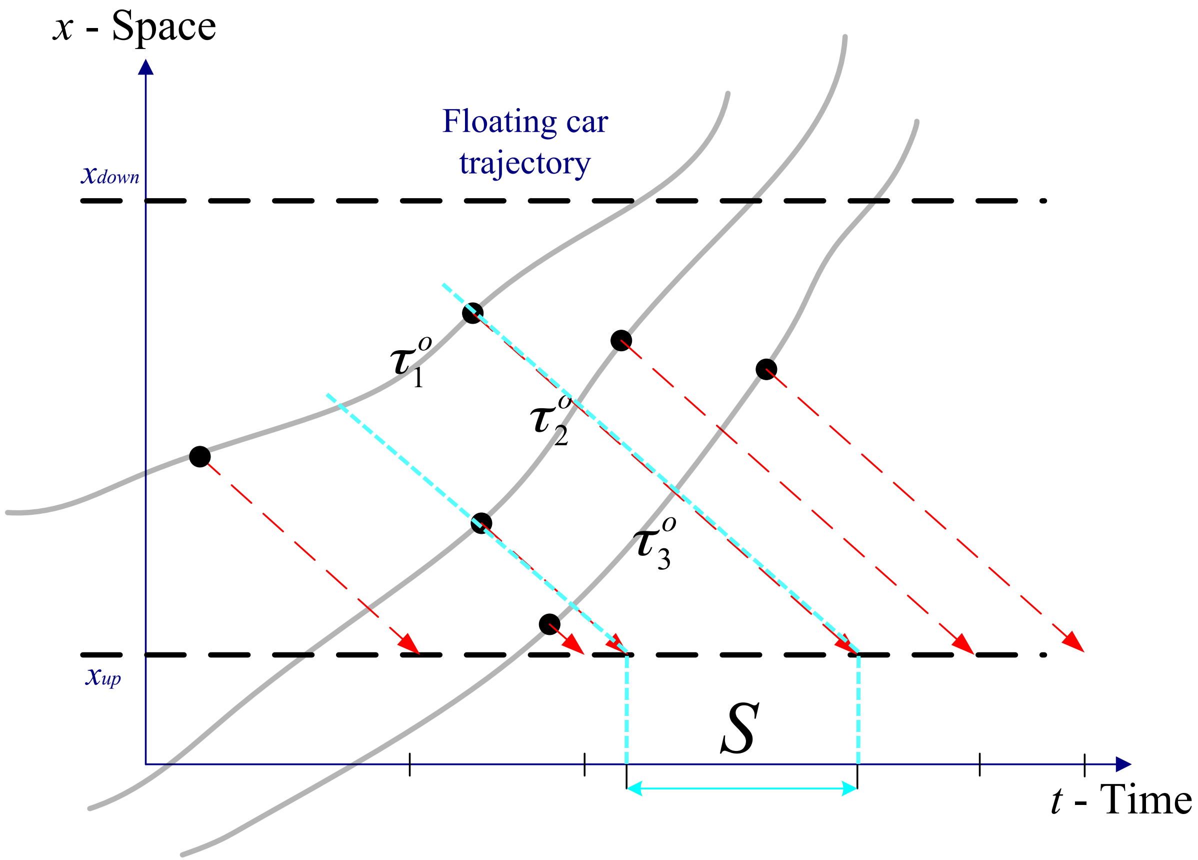
Step 2: Observation transformation (o-state)

- Introduction
- Formulation
- Methodology
- Follow-up…



$$\hat{\tau}_S^o = \left(\sum_{p \in P_S}^{N} \tau_p^o\right) \Big/ N$$

$$h_S^o = H(\hat{\tau}_S^o)$$

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Step 3: Global analysis (b+o => a state)

- Introduction
- Formulation
- Methodology
- Follow-up…

$$h_S^b = \frac{\max\left(T\left(n \middle| n \in S, x_{up}\right)\right) - \min\left(T\left(n \middle| n \in S, x_{up}\right)\right)}{Card(S)} = \frac{\Delta T_S}{Card(S)}$$

$$r_S^b = \begin{cases} 0, if\ T\left(n^*, x_{up}\right) - T\left(n^*, x_{up} - \Delta x\right) = \dfrac{\Delta x}{v_m} \\ 1, otherwise. \end{cases}$$

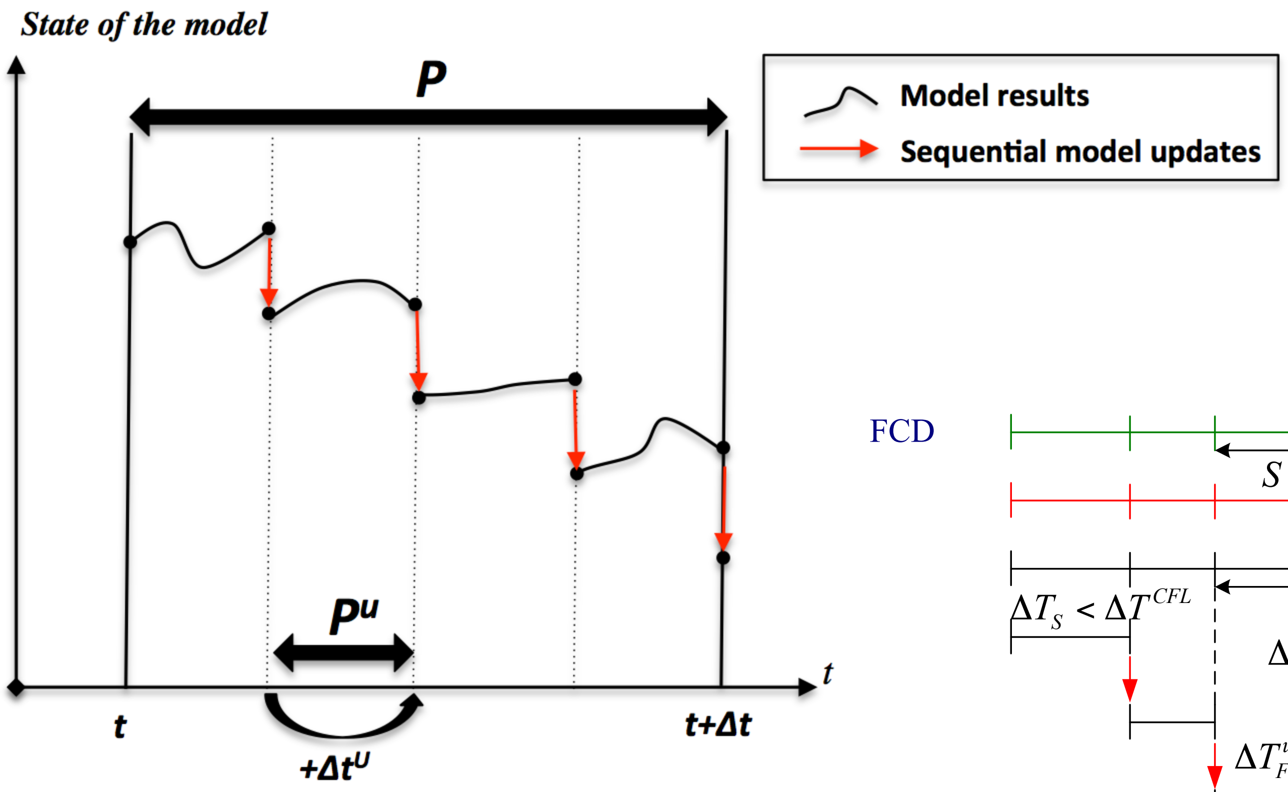
Determine analysis state (based on o-state and b-state)
By data assimilation - e.g., Kalman filter, least square method…

$$h_S^a = h_S^b + W^h \cdot \left(h_S^o - h_S^b\right)$$

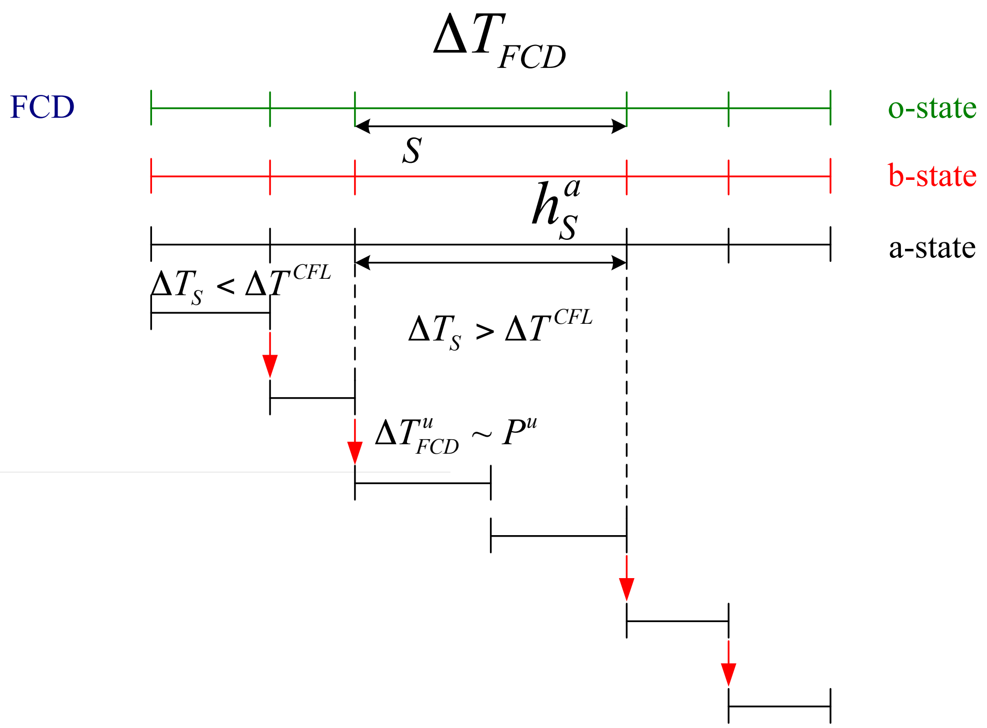
$$r^a = r^o = 1$$

Step 4: Model update – CFL condition

- Introduction
- Formulation
- Methodology
- Follow-up…



Source: Duret.et.al.(2016)



Step 4: Model update

- Introduction
- Formulation
- Methodology
- Follow-up…

$$\delta n = \Delta T_S \cdot \left|\frac{1}{h_S^a} - \frac{1}{h_S^b}\right|$$

	$r^b = 0$	$r^b = 1$
$r^a = r^o = 1$	$h^a > h^b$ ……..(1) $h^a \leq h^b$ ……..(2)	$h^a > h^b$ ……..(3) $h^a \leq h^b$ ……..(4)

(1) Vehicle delaying

(2) Contradiction

(3) Vehicle delaying → similar to (1)

(4) Vehicle advancing

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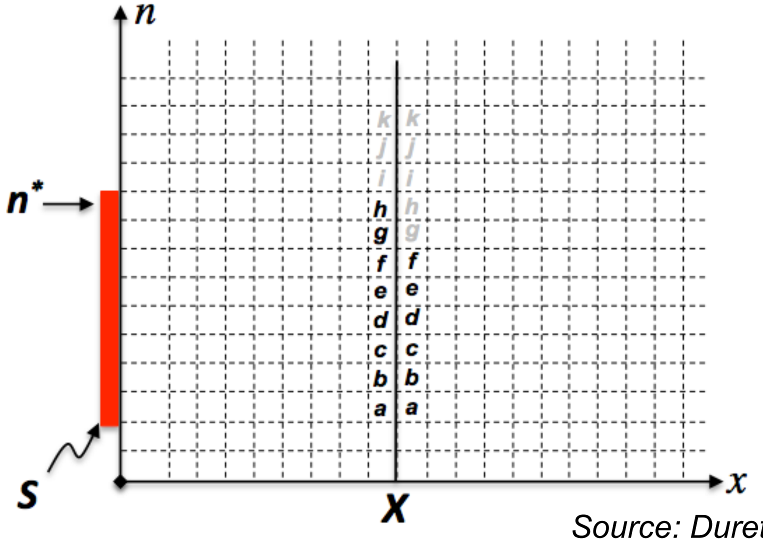
Step 4: Model update (1)

$$\delta n = \Delta T_S \cdot \left| \frac{1}{h_S^a} - \frac{1}{h_S^b} \right|$$

	$r^b = 0$	$r^b = 1$
$r^a = r^o = 1$	$h^a > h^b$……..(1) $h^a \leq h^b$……..(2)	$h^a > h^b$……..(3) $h^a \leq h^b$……..(4)

(1) Vehicle delaying

$$r^a = 1, r^b = 0 \text{ and } h^a > h^b$$



Source: Duret.et.al.(2016)

$$T(n, x_{up}) = T(n-1, x_{up}) + h_S^a, \forall n \in \left\{ \underbrace{S}_{\text{initial set of veh.}} \setminus \underbrace{[n^* - \delta n + 1 : n^*]}_{\text{veh. delayed}} \right\}$$

Step 4: Model update (2)

$$\delta n = \Delta T_S \cdot |\frac{1}{h_S^a} - \frac{1}{h_S^b}|$$

- Introduction
- Formulation
- Methodology
- Follow-up…

	$r^b = 0$	$r^b = 1$
$r^a = r^o = 1$	$h^a > h^b$……..(1) $h^a \leq h^b$……..(2)	$h^a > h^b$……..(3) $h^a \leq h^b$……..(4)

(2) Contradiction

$$r^a = 1, r^b = 0 \text{ and } h^a \leq h^b$$

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Step 4: Model update (3)

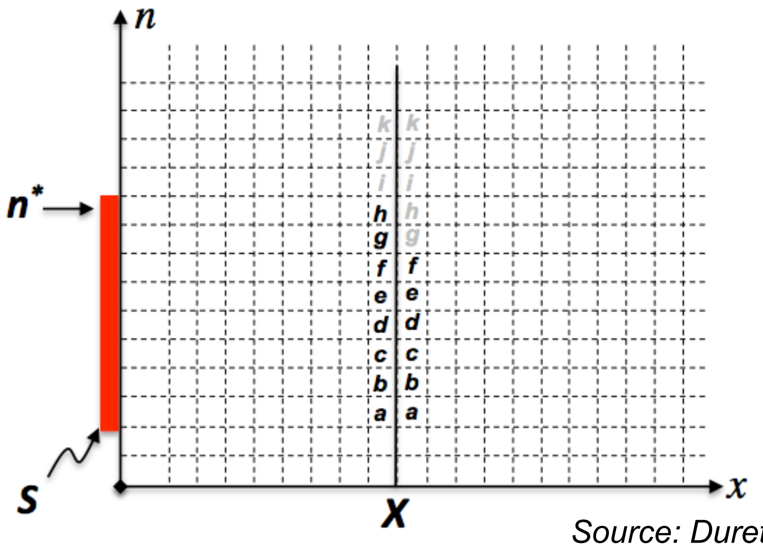
$$\delta n = \Delta T_S \cdot \left| \frac{1}{h_S^a} - \frac{1}{h_S^b} \right|$$

- Introduction
- Formulation
- Methodology
- Follow-up…

	$r^b = 0$	$r^b = 1$
$r^a = r^o = 1$	$h^a > h^b$……..(1) $h^a \leq h^b$……..(2)	$h^a > h^b$……..(3) $h^a \leq h^b$……..(4)

(3) Vehicle delaying

$$r^a = r^b = 1 \text{ and } h^a > h^b$$



Source: Duret.et.al.(2016)

$$T(n, x_{up}) = T(n-1, x_{up}) + h_S^a, \forall n \in \left\{ \underbrace{S}_{\text{initial set of veh.}} \setminus \underbrace{[n^* - \delta n + 1 : n^*]}_{\text{veh. delayed}} \right\}$$

Step 4: Model update (4)

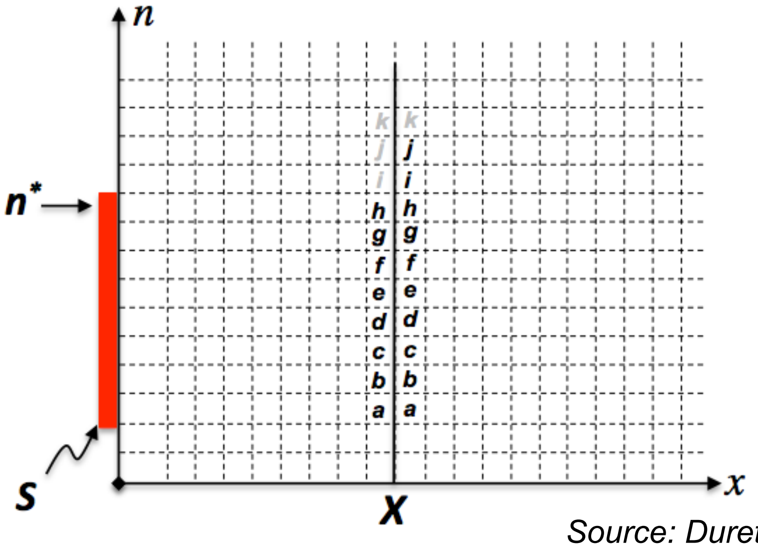
- Introduction
- Formulation
- Methodology
- Follow-up…

$$\delta n = \Delta T_S \cdot \left| \frac{1}{h_S^a} - \frac{1}{h_S^b} \right|$$

	$r^b = 0$	$r^b = 1$
$r^a = r^o = 1$	$h^a > h^b$ ....(1) $h^a \leq h^b$ ....(2)	$h^a > h^b$ ....(3) $h^a \leq h^b$ ....(4)

(4) Vehicle advancing

$$r^a = r^b = 1 \text{ and } h^a \leq h^b$$

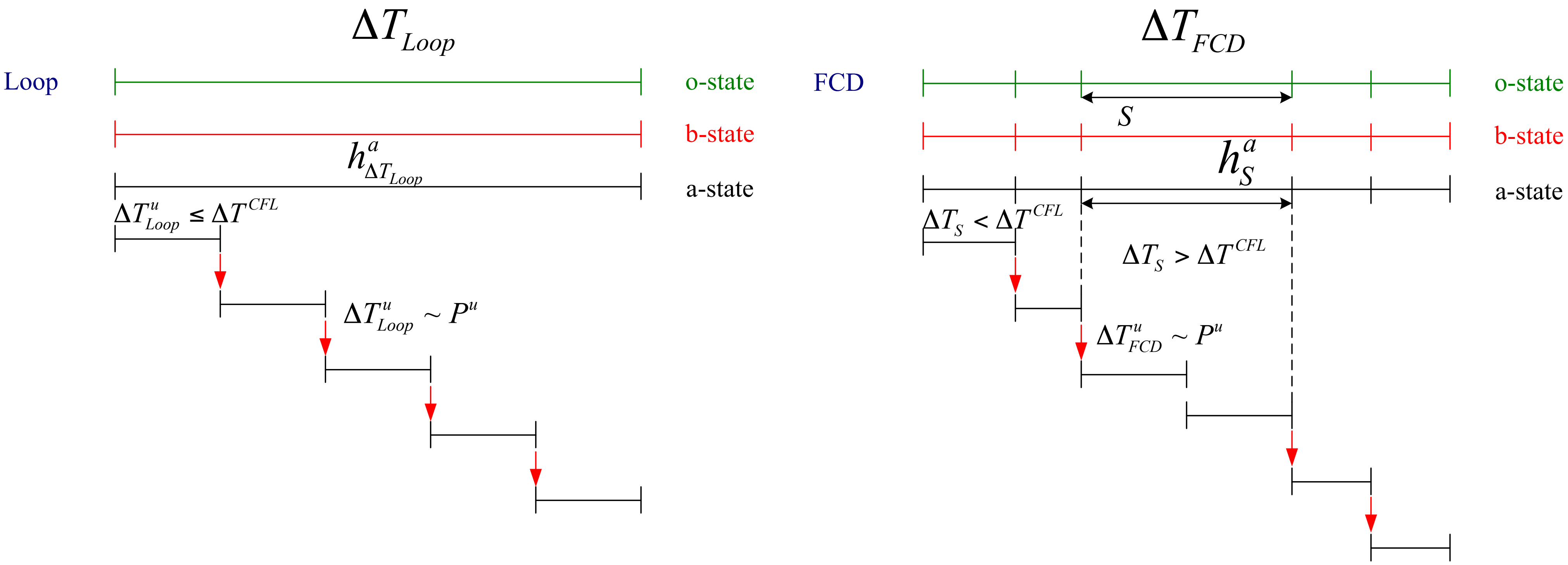


Source: Duret.et.al.(2016)

$$T(n, x_{up}) = T(n-1, x_{up}) + h_S^a, \forall n \in \left\{ \underbrace{S}_{\text{initial set of veh.}} \cup \underbrace{[n^* + 1 : n^* + \delta n]}_{\text{veh. advanced}} \right\}$$

Comparison DA with Loop and FCD

- Introduction
- Formulation
- Methodology
- Follow-up…

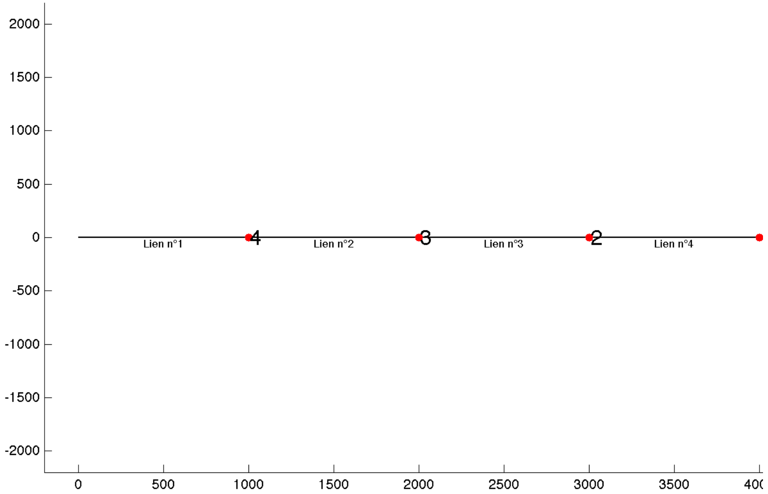


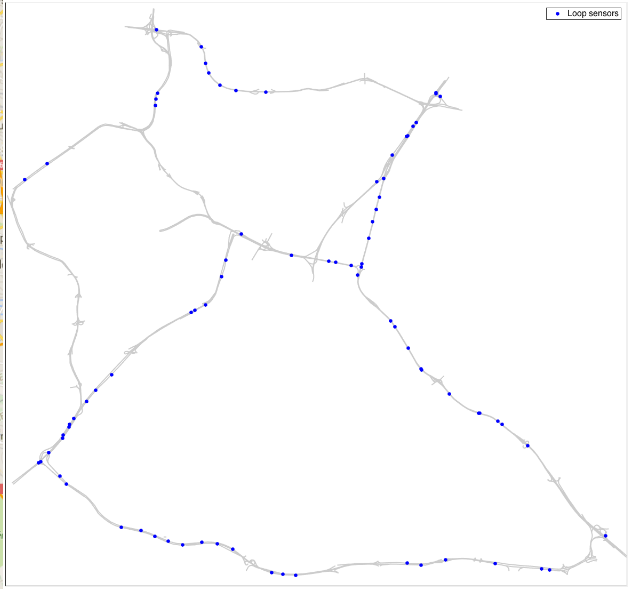
Future validation: experimental studies

- Introduction
- Formulation
- Methodology
- Follow-up…

- *Mesoscopic simulation platform: validation done with loop*
- *Addition with FCD data for validation*

- *5 node case in a synthetic network, with both loop and FCD*





More to come….

Thank you for listening!