How To Get A Model In Pedestrian Dynamics To Produce Stop And Go Waves

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28th October 2015

Motivation

Stop and go waves are prominent in

1. car traffic on highways
2. queueing of pedestrians
3. crowded pathways: Hajj 2006 (Helbing et al. (2007))

Stop and Go waves

We want to understand why stop and go waves occur.

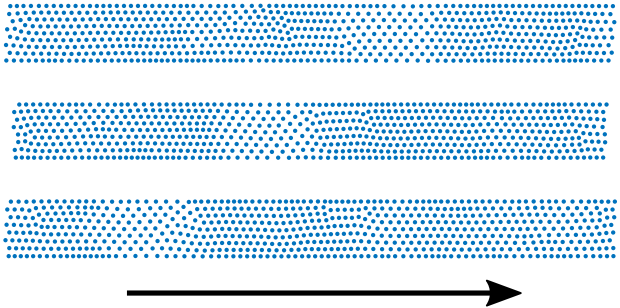
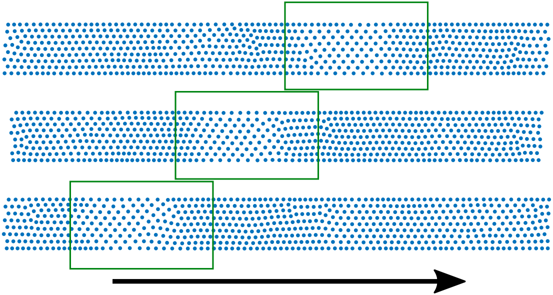


Figure: Three snapshots of positions in a crowded pathway.

Stop and Go waves



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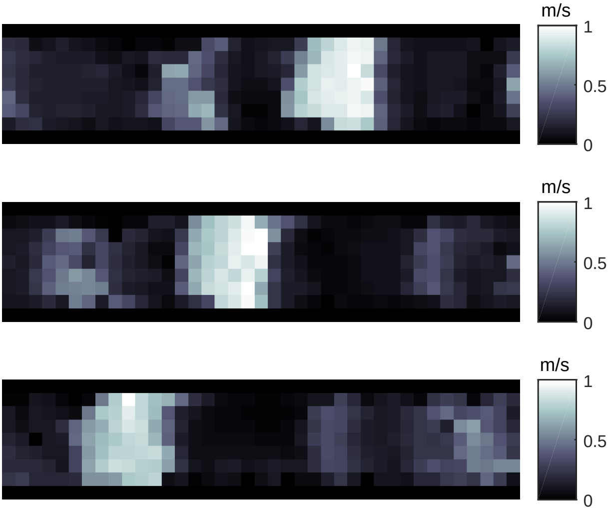
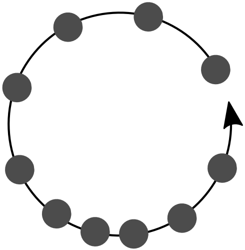


Figure: Three snapshots of speeds in a crowded pathway.



Ring setting

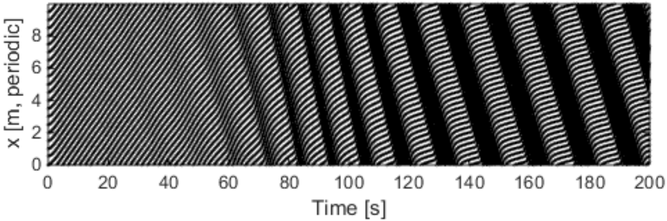


Figure: Positions of 20 pedestrians in the ring setting over time. Stop-and-go waves are clearly visible.

Gradient Navigation Model

Published by Dietrich and Köster (2014).
$\vec{N}_\mu$ is the navigation vector. Interaction with other pedestrians is parametrized with repulsion parameter μ.

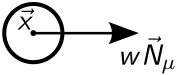


Figure: Notation and setup. $\vec{x} \in \mathbb{R}^2$, $w \in \mathbb{R}$.

$$\text{Motion model for one pedestrian} \begin{cases} \dot{\vec{x}} & = & w\vec{N}_\mu \\ \dot{w} & = & \|\vec{N}_\mu\| - w \end{cases}$$

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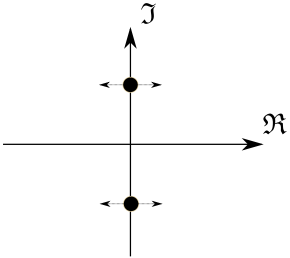
Consider a dynamical system with parameter $\mu \in \mathbb{R}$ and dynamic f:

$$\dot{x} = f_\mu(x), \; x \in \mathbb{R}^n,$$

where (x_0, μ_0) is a steady state so that $f_{\mu_0}(x_0) = 0$. We follow (Guckenheimer and Holmes, 1986, p.150ff) to define a Hopf bifurcation.

A Hopf bifurcation exists if the following conditions hold:

1. Non-hyperbolicity: $D_x f_{\mu_0}(x_0)$ has exactly two distinct, conjugate, complex eigenvalues with zero real part.
2. Transversality: $\frac{\Re(\lambda(\mu))}{d\mu}(\mu_0) \neq 0$, the eigenvalues cross the imaginary axis with non-zero speed.
3. Genericity: the first Lyapunov coefficient must not be zero.



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Idea of the proof (Disselnkötter (2013)):

1. restrict to 1D ring setting
2. fix one pedestrian and consider system of distances
3. linearize the system at the steady state (equal spacing)
4. compute eigenvalues $\lambda(\mu)$ of the linear system
5. show Hopf conditions
   5.1 Non-hyperbolicity condition
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Analytical results

Periodic solutions, and hence stop and go waves, seem to be generated by intrinsic properties of the model.

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Results with few pedestrians

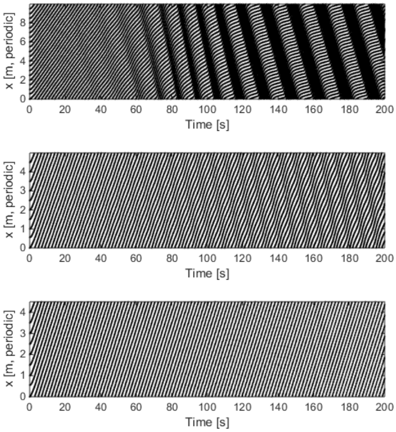
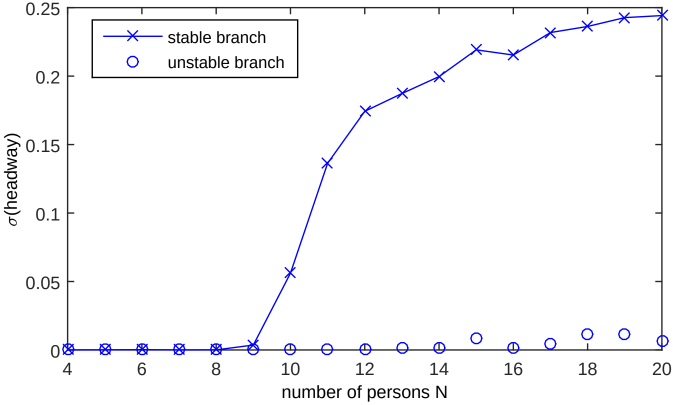


Figure: Positions of 20, 10, and 9 pedestrians over time.

Results with few pedestrians



Conclusion and discussion

What have we seen?

A two-dimensional model produces stop and go waves if confined to a ring setting. With few pedestrians, no stop and go waves seem to occur.

Implications and future work

1. Intrinsic properties of models can generate stop and go waves. Important:
   - Only consider the person in front
   - No overlaps
   - A minimum number of pedestrians
2. Presented techniques can be used to analyse other models.
3. A 2D setting might need a more complicated analysis.

References

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Guckenheimer, J. and Holmes, P. (1986). *Nonlinear oscillations, dynamical systems, and bifurcations of vector fields*. Springer.

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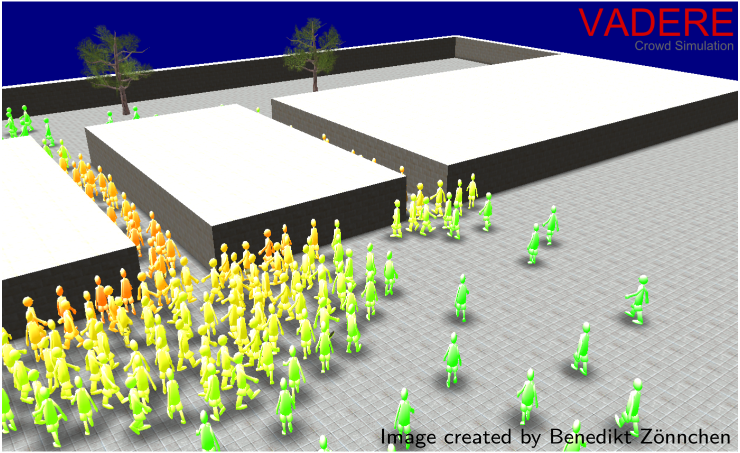
Questions?



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