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Abstract. The kinetic theory approaches to vehicular traffic modeling have given very good results in the understanding of the dynamical phenomena involved. In this work we deal with the kinetic approach modeling of a traffic situation where there are many classes of aggressive drivers. Their aggressivity is characterized through their relaxation times. The reduced Paveri-Fontana equation is taken as a starting point to set the model.
The kinetic equation is taken to write the balance equations for the density and the average speed in each drivers class. In this model we consider that each class of drivers preserve the corresponding aggressivity, in such a way that there will be no adaptation effects. It means that the number of drivers in a class is conserved. Some characteristics of the model are explored with the usual methods.

Outline

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Both give a contribution to the balance in the kinetic equation [7].

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where $\langle ...\rangle_i$ means the average over the f_i distribution function. It should be noticed that $\langle\xi_j\rangle_i - \langle\psi_i\rangle_j = v_j - v_i$ is satisfied.

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$$\frac{\partial \rho_i v_i}{\partial t} + \frac{\partial}{\partial x}(\rho_i v_i^2 + \mathcal{P}_i) = \frac{\rho_i}{\tau_i}(V_i^0 - v_i) + q \sum_j \rho_i \rho_j \left[\langle c\xi_i \rangle_j + \langle c\psi_j \rangle_i\right], \quad (6)$$

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where $V_i^0(x,t)$ comes from the average of the desired speed now taken over the instantaneous speed c, and $\mathcal{P}_i = \int (c - v_i)^2 f_i(c) dc = \rho_i \Theta_i$ is the $i-$class traffic pressure and Θ_i the speed variance.

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$$f_i(c) = \frac{\alpha}{\Gamma(\alpha)} \frac{\rho_i}{v_i} \left(\frac{\alpha c}{v_i} \right)^{\alpha - 1} exp\left(-\frac{\alpha c}{v_i} \right) \tag{7}$$

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It should be noticed that $\rho_i(x,t), \ v_i(x,t)$ in Eq. (7) are local variables.

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• **Between user-class interactions.**

The active and passive interactions between different classes for $(i \neq j)$ can be written in a closed form, however we have found that a simpler and very good approximation for them

$$\mathcal{I}_{ij} = v_i v_j \left\{ 1 - \left(\frac{\alpha+1}{\alpha} \frac{v_j}{v_i} \right) \mathcal{H}(v_i - v_j) + \left(1 - \frac{\alpha+1}{\alpha} \frac{v_i}{v_j} \right) \mathcal{H}(v_i - v_j) \right\}, \quad (10)$$

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In order to begin the analysis we will consider that $v_2 > v_1$ and in such a case the interaction terms simplify so that

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$$\mathcal{I}_{11} = -\Theta_1, \;\; \mathcal{I}_{22} = -\Theta_2, \;\; \mathcal{I}_{12} = 0, \;\; \mathcal{I}_{21} \simeq -(v_2 - v_1)^2. \qquad (11)$$

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where μ_i is a constant analogous of a viscosity and it is different for each class of drivers. It should be said that this hypothesis can be justified in terms of the kinetic model [6, 8].

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$$\rho_1^e v_1^e = \frac{\alpha(\omega - 1)}{\tau_1(1 - p^e)}, \quad \left.\frac{v_2^e}{v_1^e}\right|_{\pm} = \delta \frac{\beta + 2\alpha \pm \sqrt{(\beta + 2\alpha)^2 - 4(1 + \alpha\delta)\frac{\alpha}{\delta}}}{2(1 + \alpha\beta)}. \tag{21}$$

The stability analysis.

It should be noticed that $v_1^* = v_1^e$ and an empirical fundamental diagram is introduced, so $v_1^e = v_1^e(\rho_1)$, besides we define $\gamma_{11}^e = (dv_1/d\rho_1)^e$ a quantity which is always negative.

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where the perturbation has been expanded in modes with a wave vector k and a complex frequency called σ in such a way that the stability condition for the equilibrium state is given by $\mathcal{R}e\sigma > 0$.

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$$\sigma = \sigma_0 + k\sigma_1 + k^2\sigma_2 + \mathcal{O}(k^3), \tag{24}$$

and there will be four different roots, which will be called as Σ_i.

$$\Sigma_1 = ikc_1 + \frac{k^2\tau_1}{\alpha\rho_1^e}\left[c_1^2 - (\alpha+1)(\gamma_{11}^e\rho_1^e)^2\right] \tag{25}$$

$$\Sigma_2 = \frac{1}{\tau_1} + \frac{ik}{\alpha}(2v_1^e + \alpha v_1^e - \alpha\gamma_{11}^e\rho_1^e) + \frac{k^2}{\alpha\rho_1^e}\left[\alpha\mu_1 - \tau_1\rho_1^e((v_1^e)^2 + 2\rho_1^e v_1^e\gamma_{11}^e - \alpha(\rho_1^e\gamma_{11}^e)^2)\right] + \dots \tag{26}$$

$$\begin{aligned}\Sigma_3 &= \frac{ik}{a_4-1}\left[(a_4-1)v_2^e - a_2\rho_2^e\right] + k^2\Sigma_{32}\\ \Sigma_{32} &= -\frac{\tau_2}{\alpha(a_4-1)^3}\left[(a_4-1)^2(v_2^e)^2 - a_2\rho_2^e\left(2v_2^e(a_4-1) + \alpha a_2\rho_2^e\right)\right],\end{aligned} \tag{27}$$

$$\begin{aligned}\Sigma_4 &= \frac{1-a_4}{\tau_2} + \frac{ik}{\alpha(a_4-1)}\left[(a_4-1)v_2^e(2+\alpha) + \alpha a_2\rho_2^e\right] + k^2\Sigma_{42}\\ \Sigma_{42} &= \frac{1}{(a_4-1)^3\alpha\rho_2^e}\left[(a_4-1)\rho_2^e v_2^e\tau_2\left((a_4-1)v_2^e - 2a_2\rho_2^e\right) + \alpha\left[(a_4-1)^3\mu_2 - \rho_2^e\tau_2(a_2\rho_2^e)^2\right)\right]\end{aligned} \tag{28}$$

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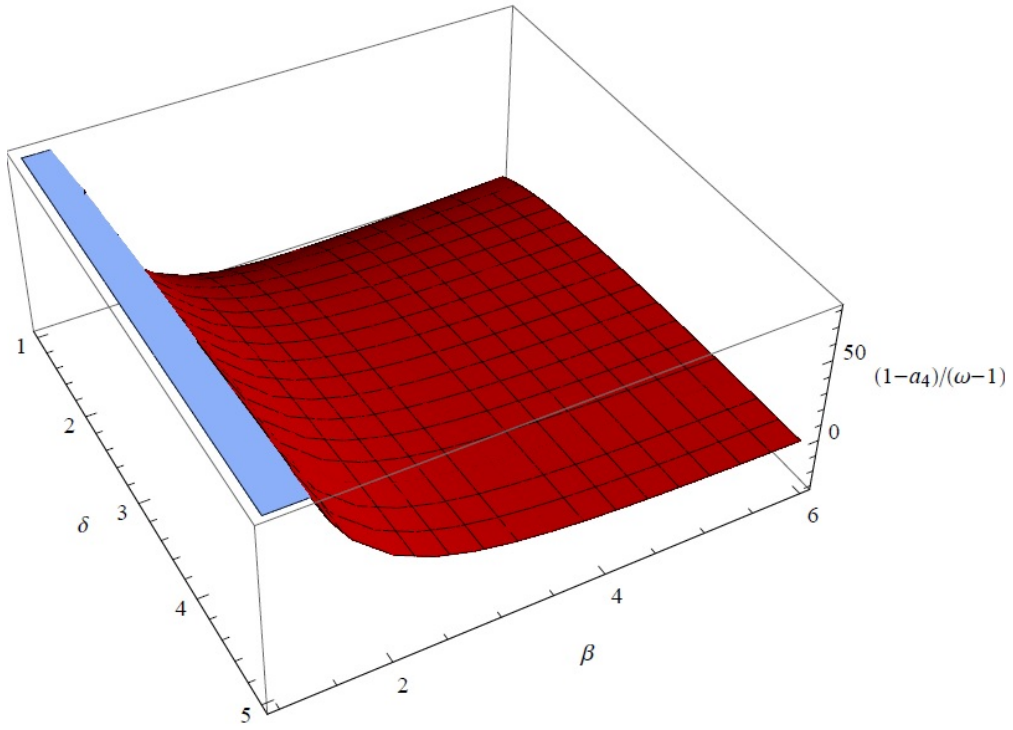

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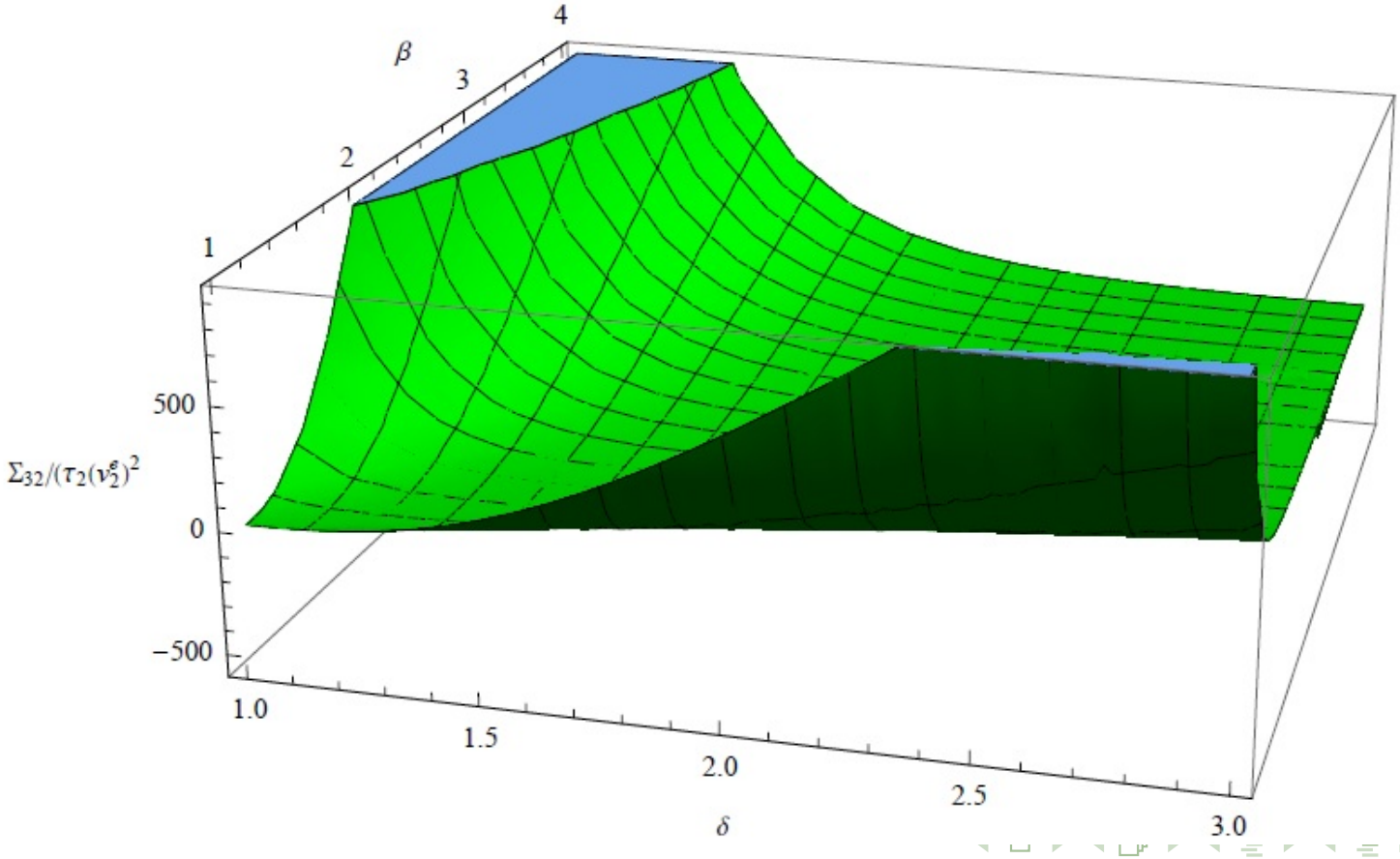
And the real part in Σ_{32} must be positive to obtain stability in the mode corresponding to the density in class 2.



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