Scaling from Circuit Experiment to Real Traffic based on Optimal Velocity Model

A. Nakayama, *Meijo University*
A. Shibata, *KEK*
S. Tadaki, *Saga University*
M. Kikuchi, *Osaka University*
Y. Sugiyama, *Nagoya University*
S. Yukawa, *Osaka University*

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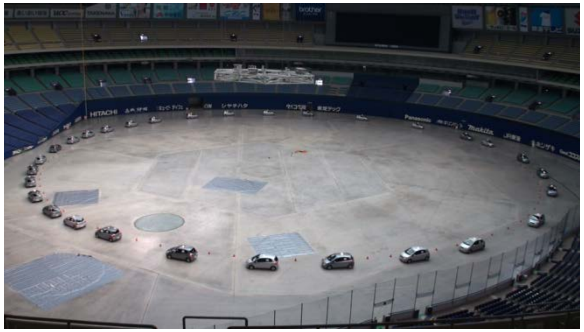
Contents

1. Introduction

History

- Optimal Velocity (OV) model (PRE51, 1995)
  Physical mechanism of jam formation

- 1st experiment (NJP10, 2008)
  Jam occurs without bottleneck.

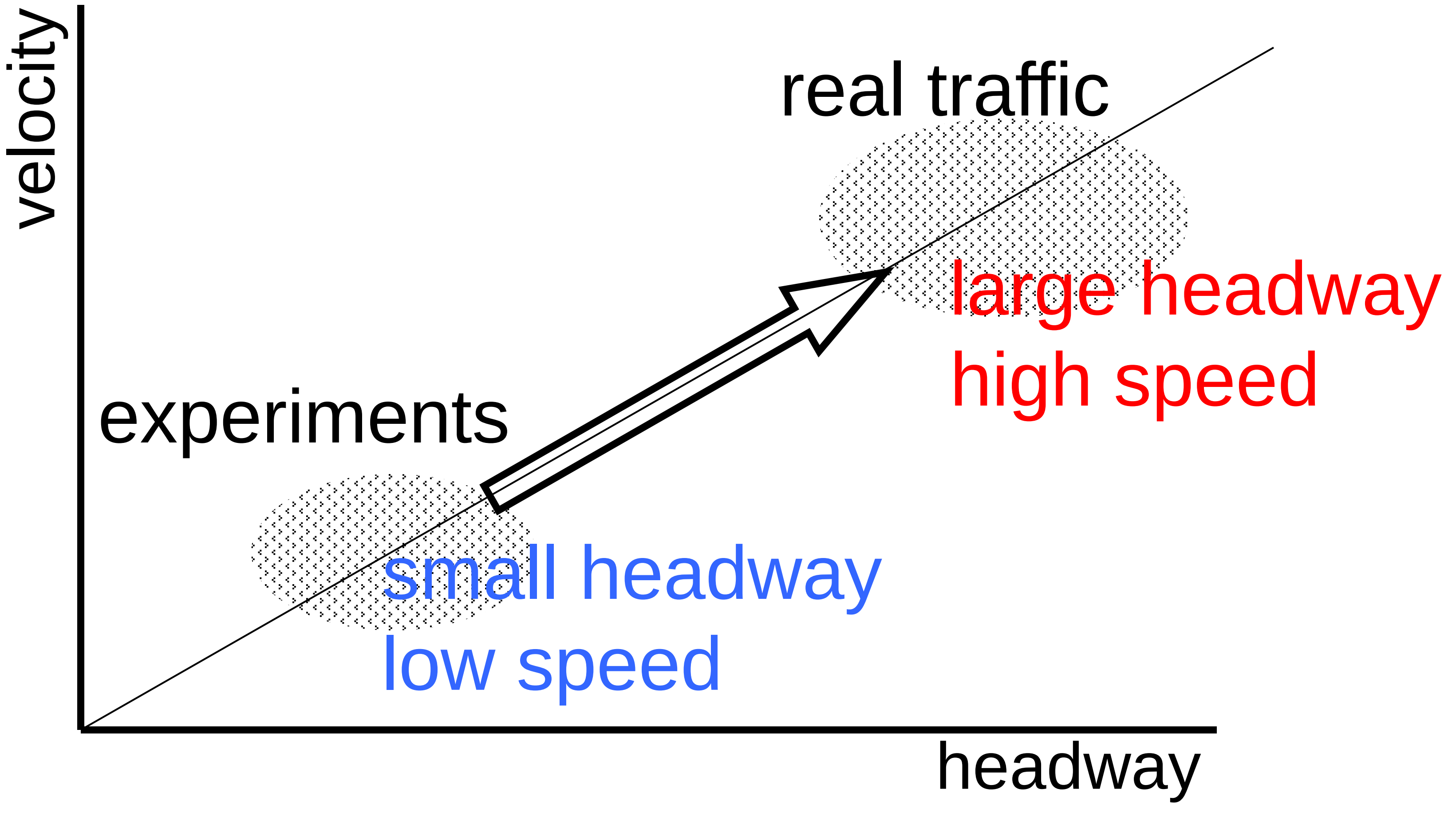


- 2nd experiment (NJP15, 2013)
  Critical density is estimated.

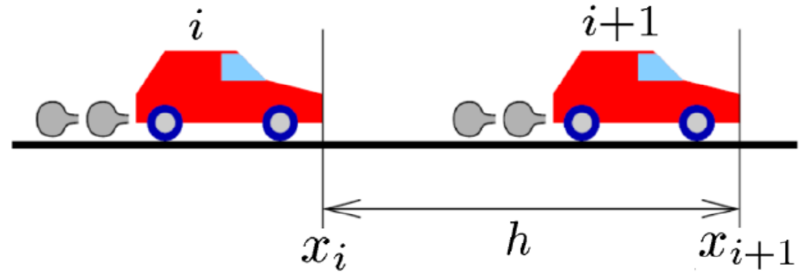
Connection to real traffic

Outline

Critical density varies
with the speed limit of the road.



Review of OV model



Eq. of motion

$$\frac{d^2x_i}{dt^2} = a\left[V(x_{i+1}-x_i) - \frac{dx_i}{dt}\right]$$

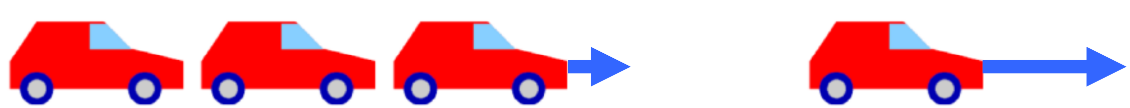
$$V(h) = \underline{\alpha}\tanh[\underline{\beta}(h-\underline{h_0})] + \underline{v_0}$$

$\alpha,\ \beta,\ h_0,\ v_0$ are input parameters.

Sensitivity a is not a free parameter.

$$aT \simeq 1.8$$

Relation between sensitivity and delay time of car motion

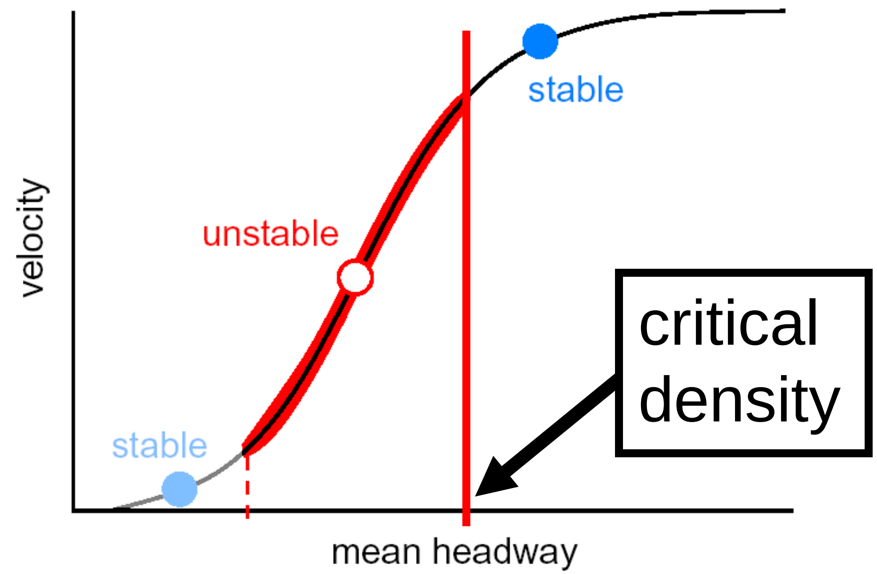


OV function determines the critical density.

stability condition of homogeneous flow

$$\frac{dV}{dh} < \frac{1}{2}a$$

$\downarrow$

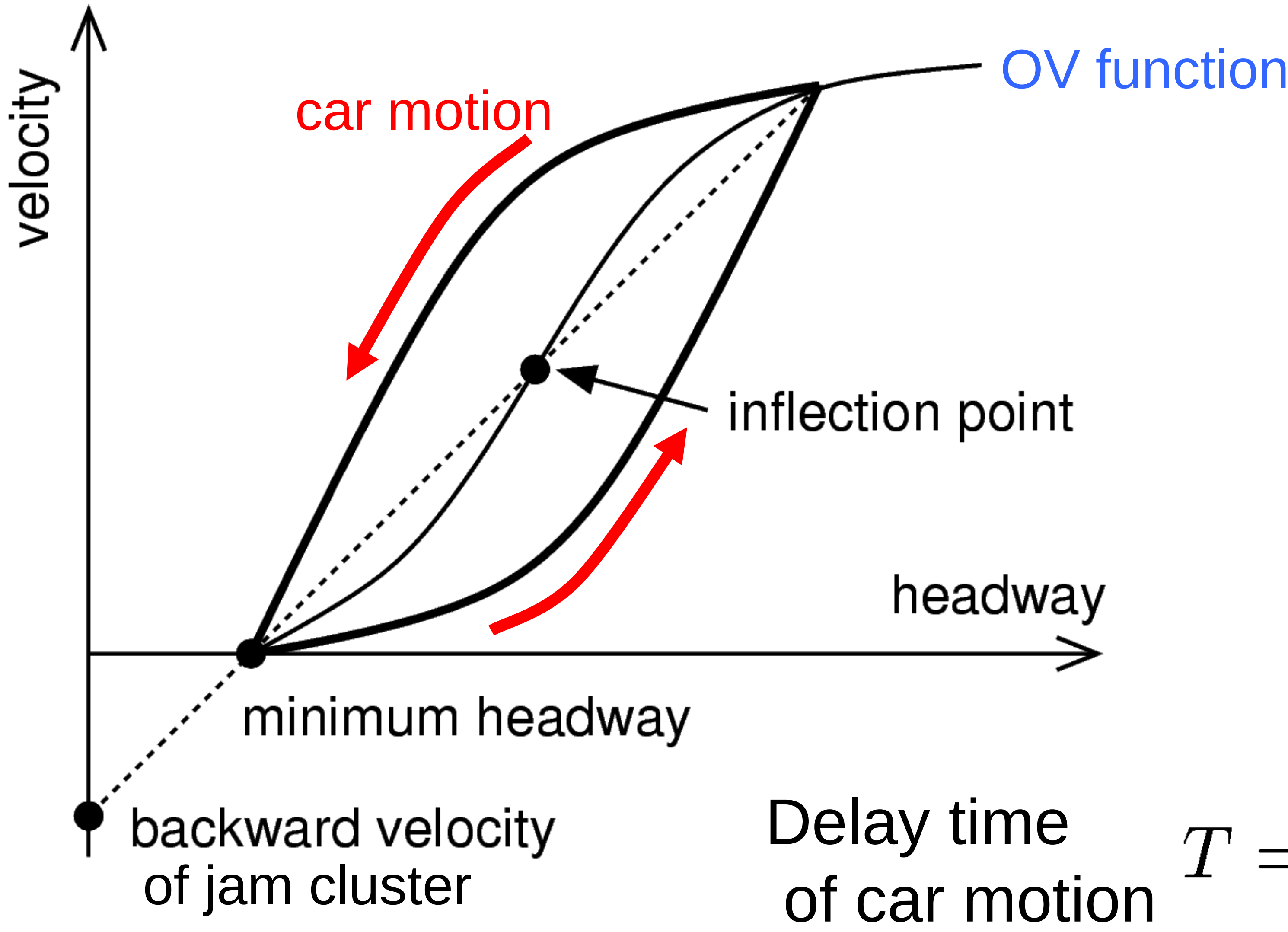


$$\rho_c = \left[\frac{1}{\beta}\cosh^{-1}\sqrt{\frac{2\alpha\beta}{a}} + h_0\right]^{-1}$$

: critical density

The difference in OV function explains the difference in critical density between experiments and real traffic.

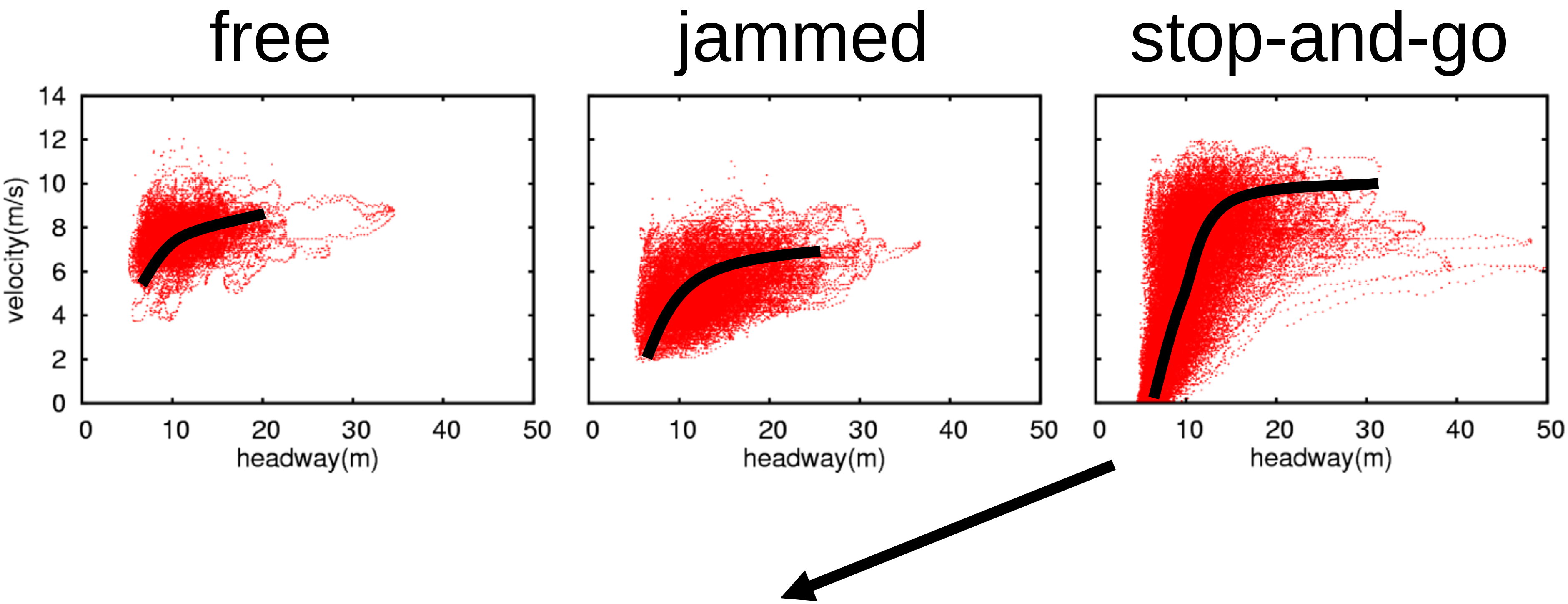
Property of jam in OV model



Delay time
of car motion $T = \dfrac{h_{\mathrm{min}}}{v_{\mathrm{back}}}$

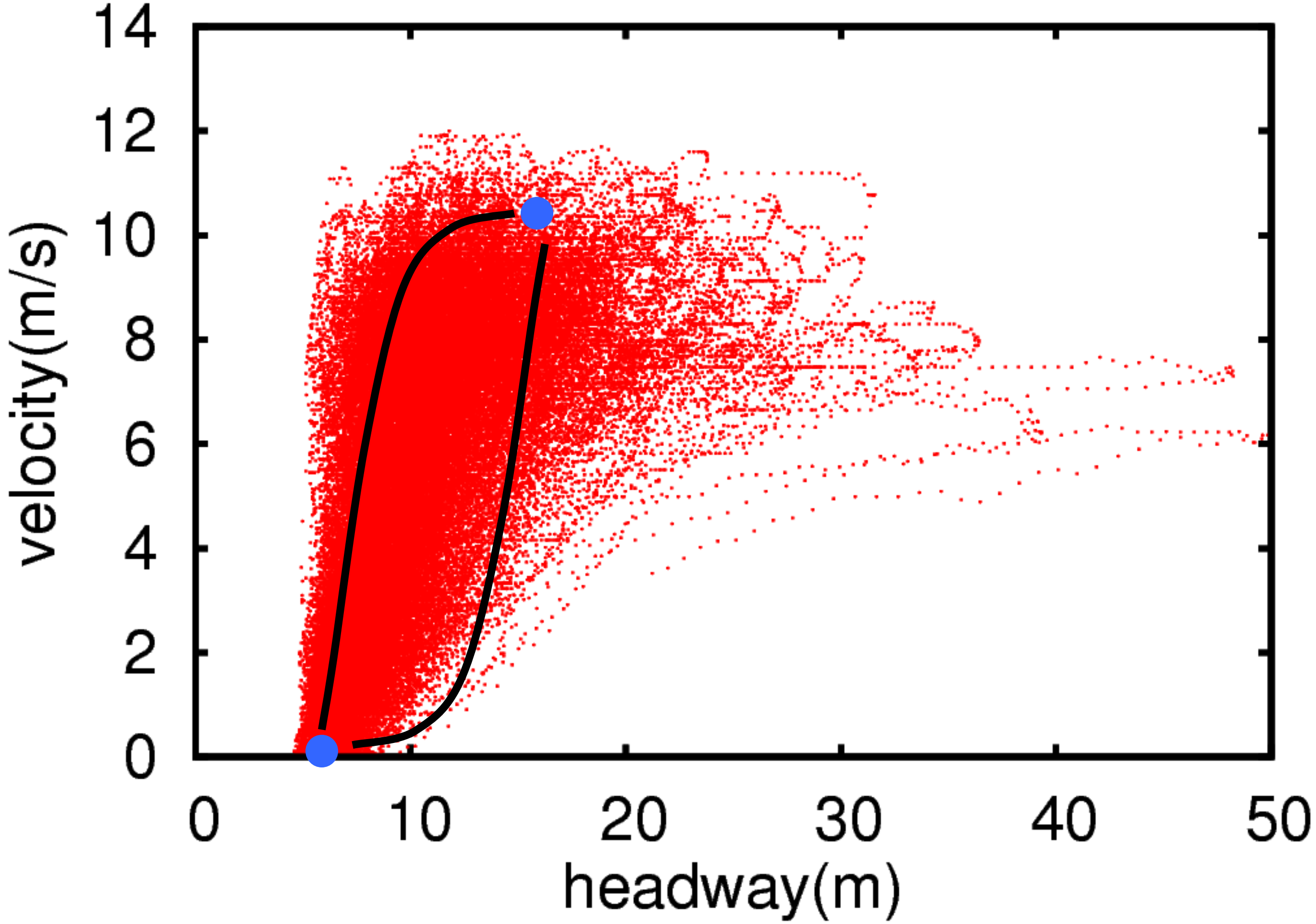
2. Estimation of OV function

from experimental data of headway and velocity



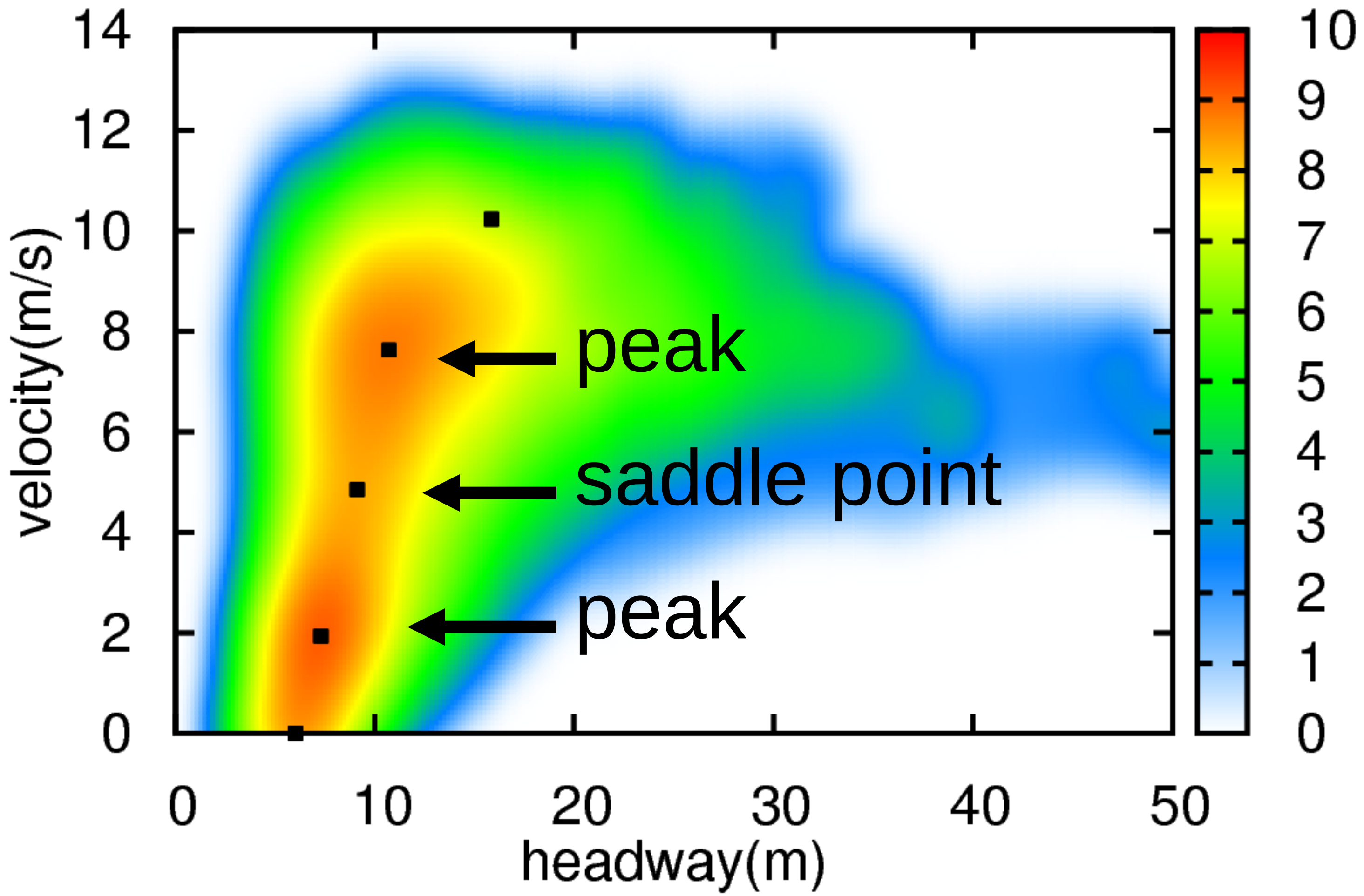
We can determine the OV function only in the case of stop-and-go flow.

Two cusps of the loop can be found from raw data.

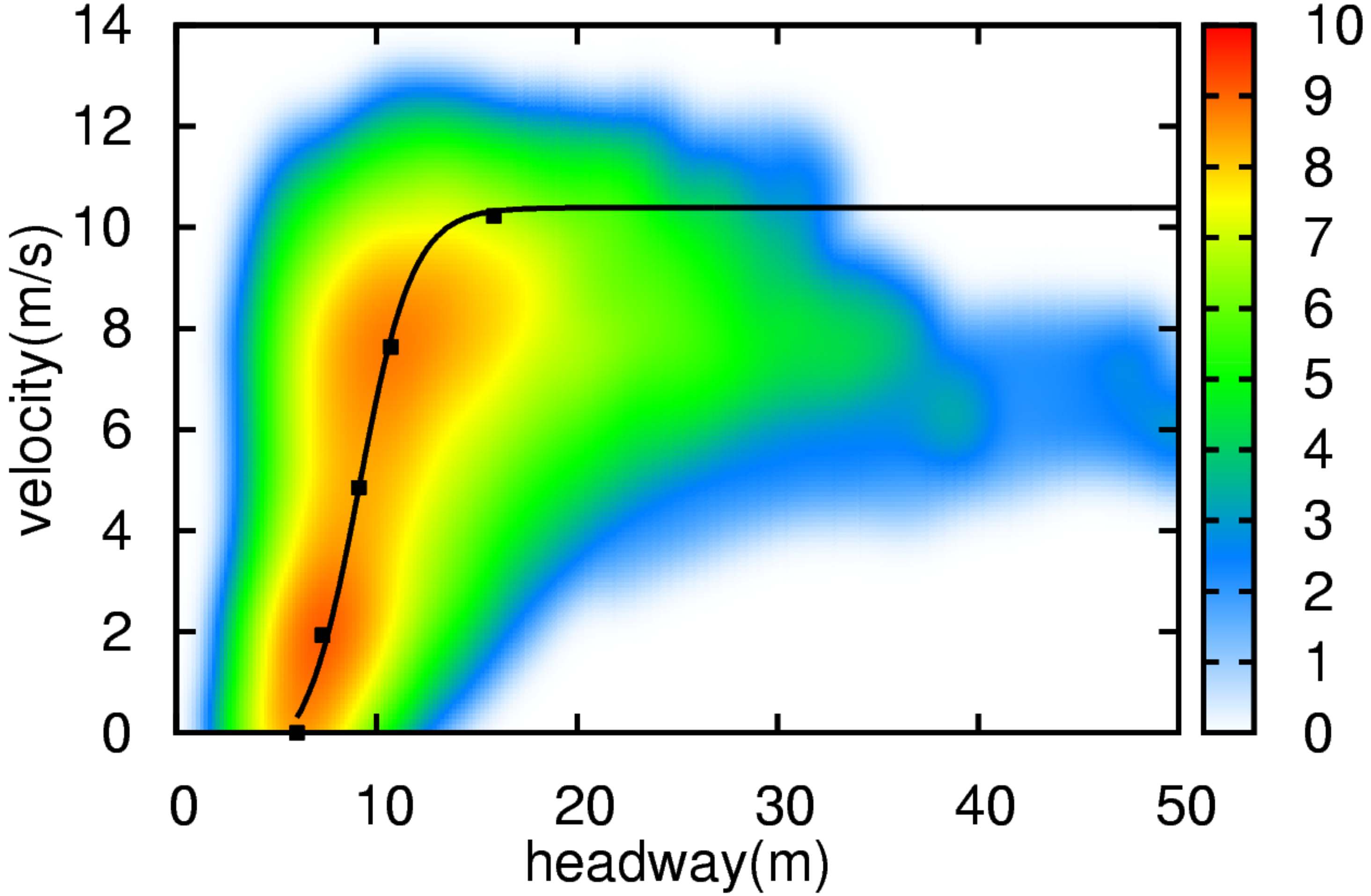


Too much is as bad as too little.

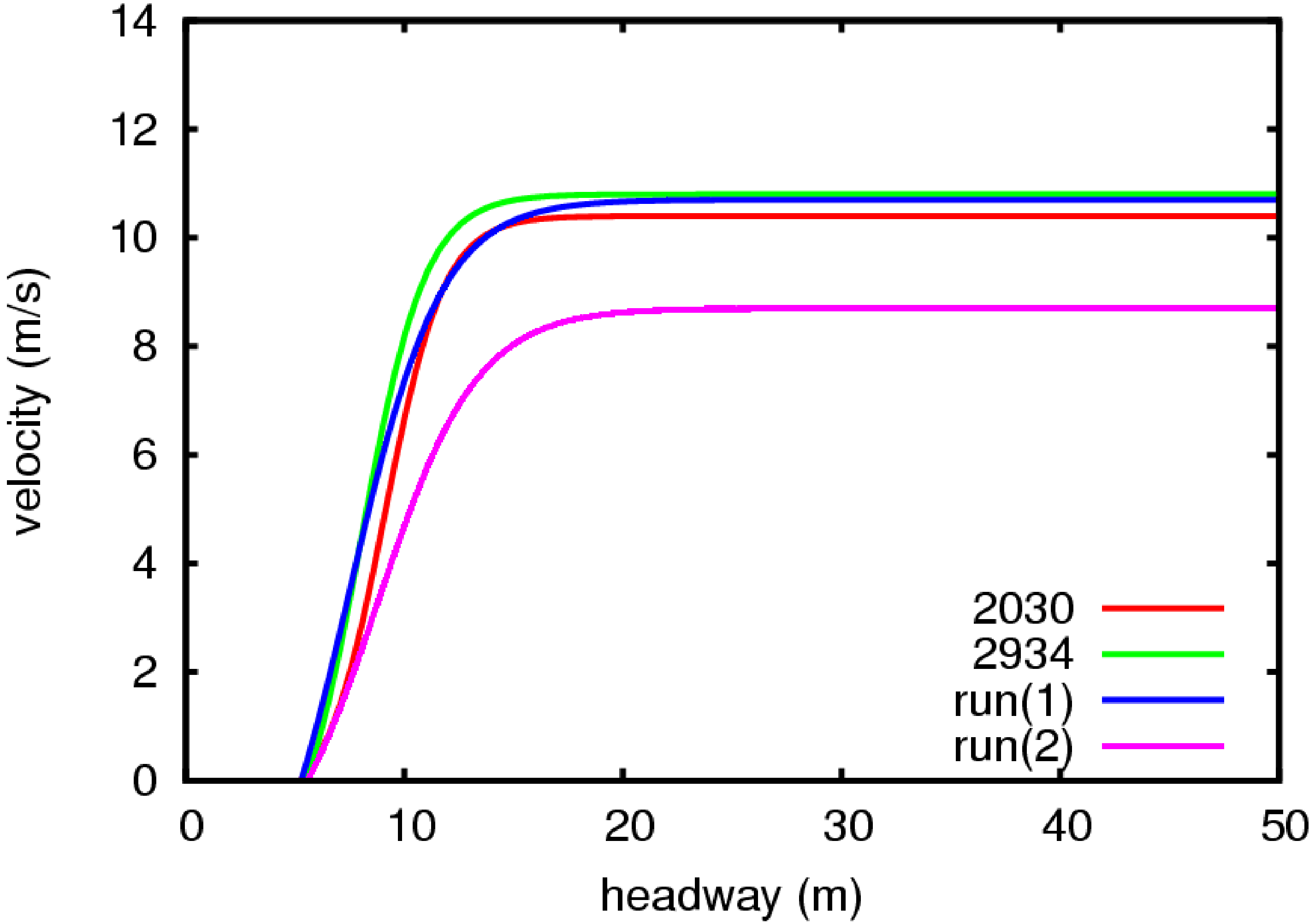
Make smooth distribution to reduce information and select three representative points



Fitting OV function to five points



We obtained OV functions in four (stop-and-go flow) cases.



3. Scaling to real traffic

The OV function has four parameters.

$$V(h) = \alpha \tanh[\beta(h - h_0)] + v_0$$ (experiment)

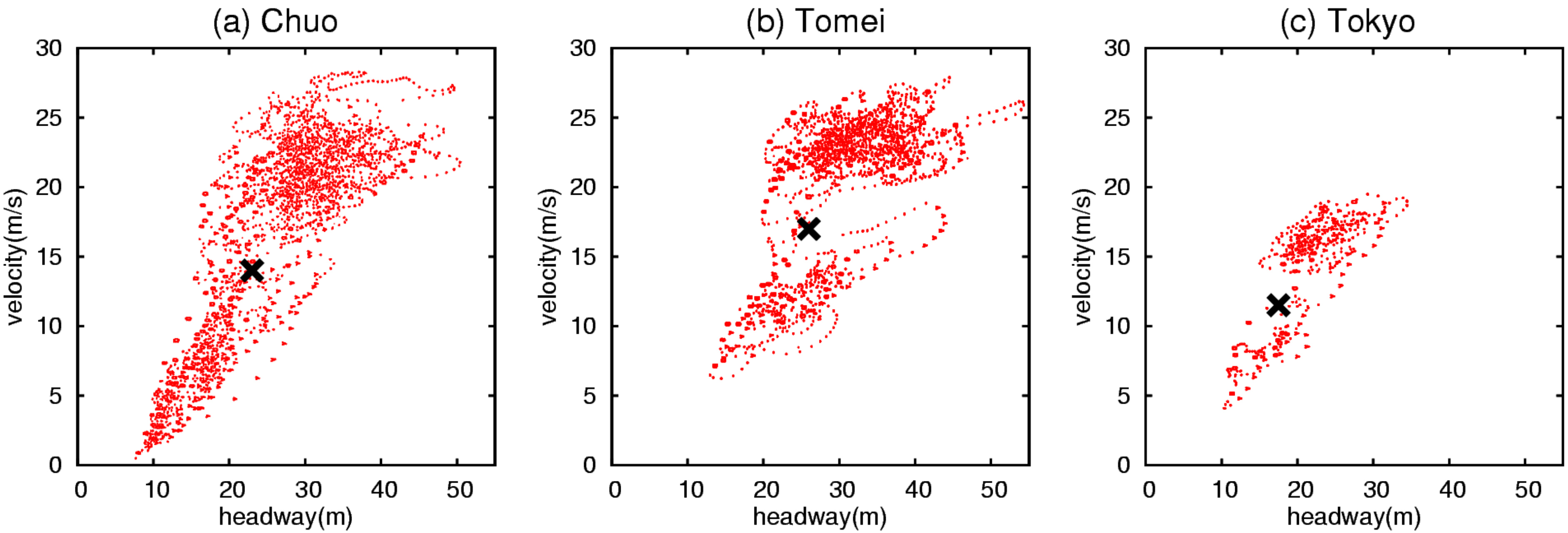
⬍

$$V'(h) = \underline{\alpha'} \tanh[\underline{\beta'}(h - \underline{h'_0})] + \underline{v'_0}$$ (real traffic)

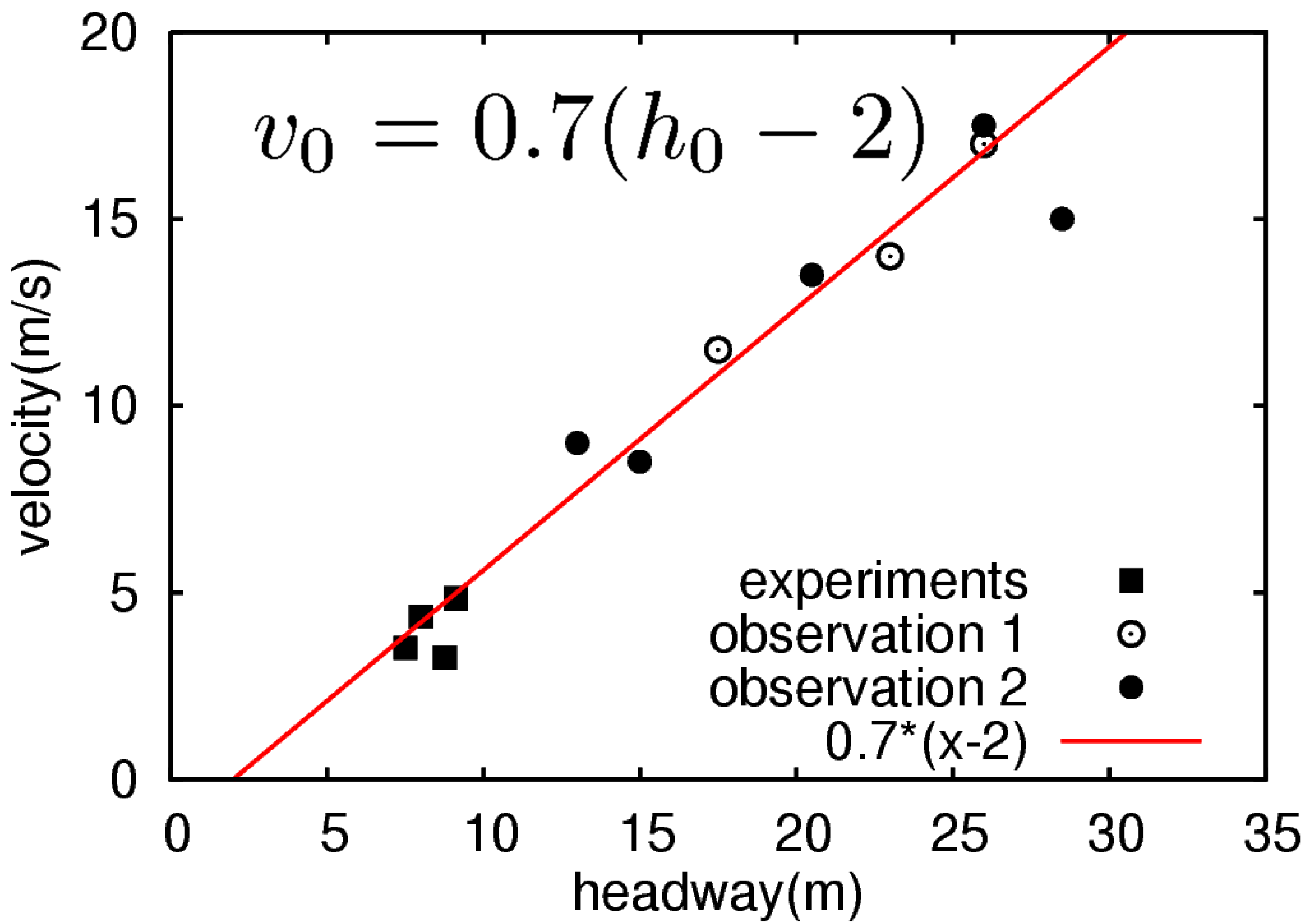
How to express $\alpha', \beta', h'_0, v'_0$ by α, β, h_0, v_0

We found
the inflection point (h_0, v_0) is useful
to define the scaling rule, because

The inflection point can be found, even if stop-and-go flow does not appear.

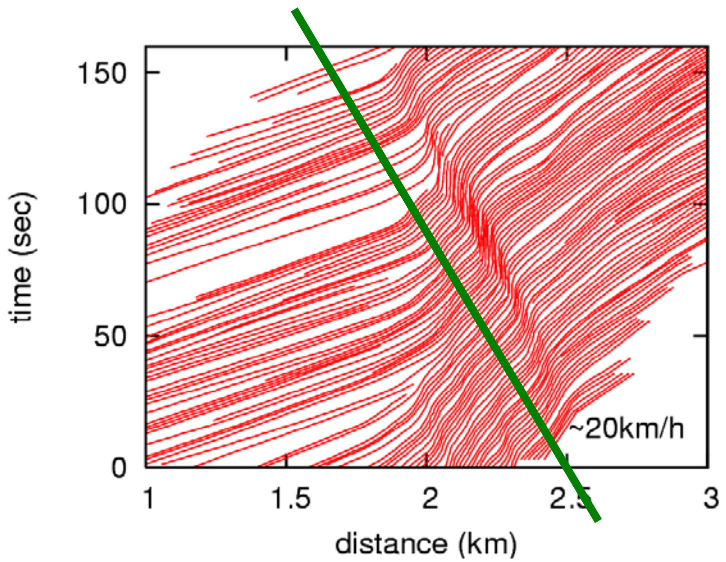


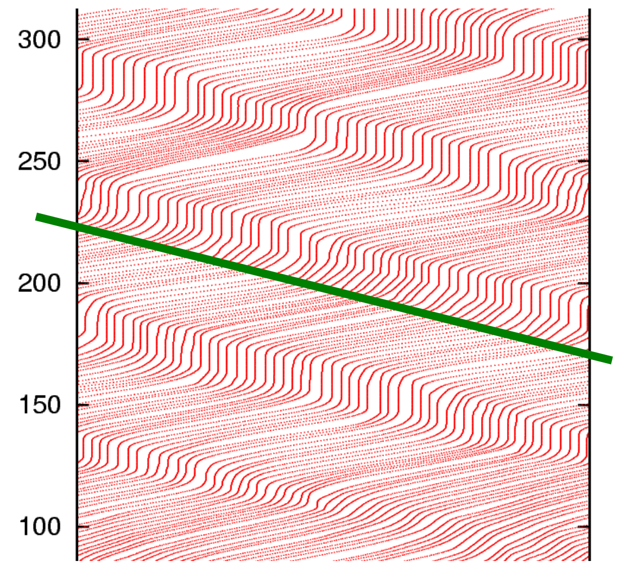
Data shows a relation of inflection points.



Backward velocities of jam clusters are

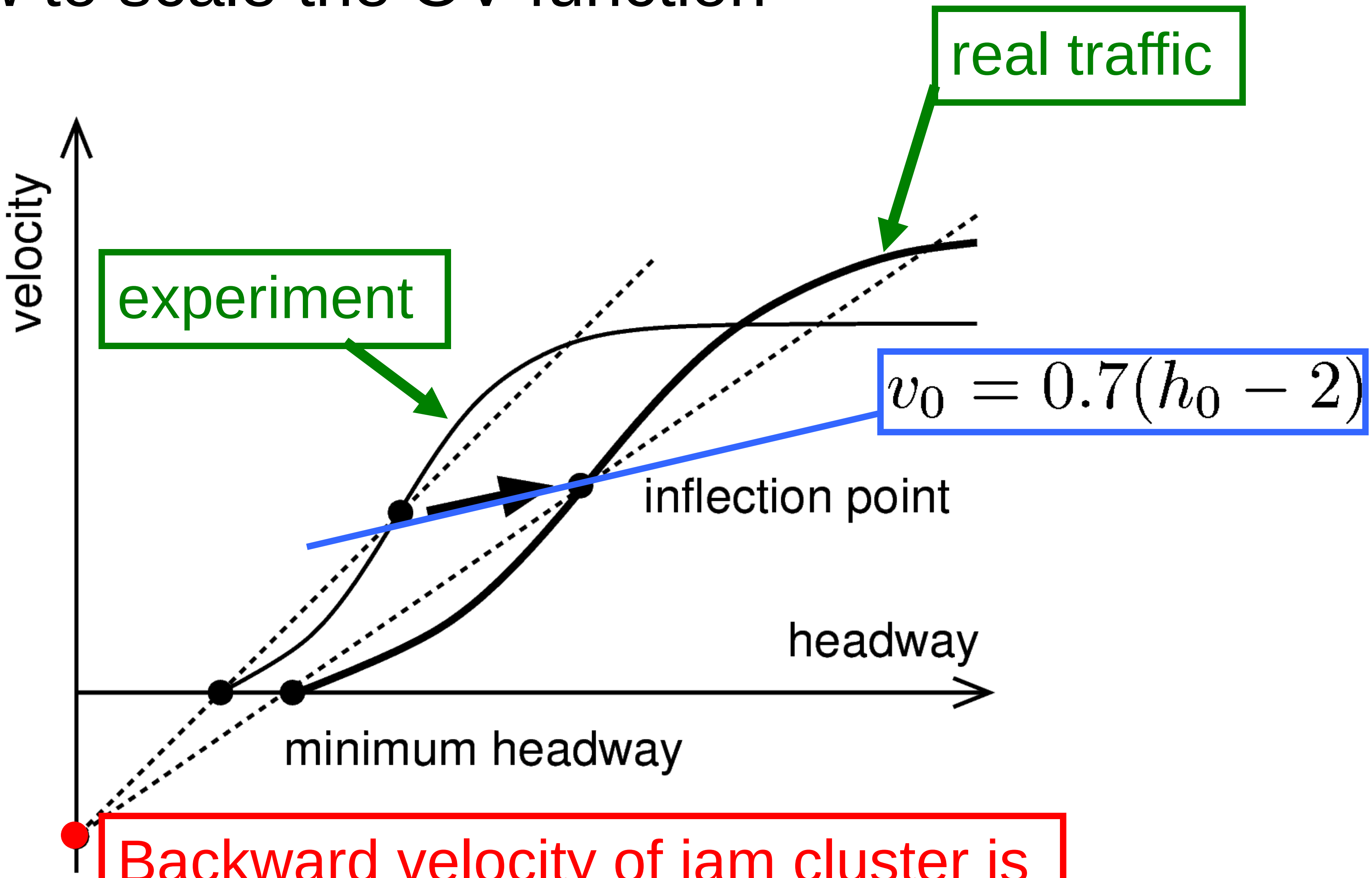
～20km/h in real traffic ～6m/s in experiments





We assume that the backward velocity is common to the experiments and real traffic.

How to scale the OV function



Backward velocity of jam cluster is common to both. (6m/s～20km/h)

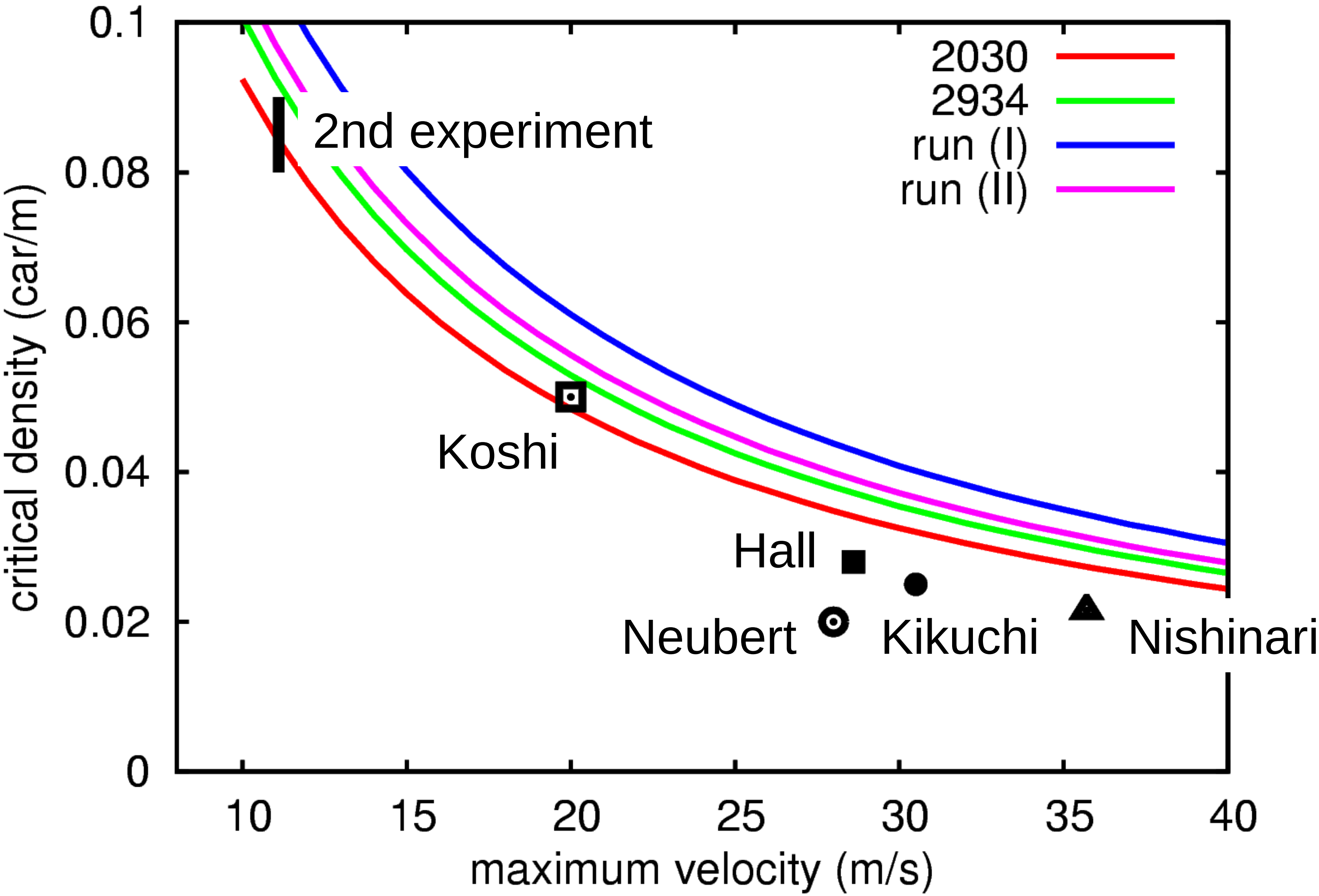
Relations between $\underbrace{\alpha', \beta', h'_0, v'_0}_{\text{(real traffic)}}$ and $\underbrace{\alpha, \beta, h_0, v_0}_{\text{(experiment)}}$

$$V(h) = \alpha \tanh[\beta(h - h_0)] + v_0$$

$$\boxed{\begin{array}{ll} \alpha' = \dfrac{v'_0}{v_0}\alpha & h'_{\min} = \dfrac{v_{\mathrm{back}}}{v'_0 + v_{\mathrm{back}}} h'_0 \\ \beta' = \dfrac{h_0 - h_{\min}}{h'_0 - h'_{\min}}\beta & v'_0 = 0.7(h'_0 - 2) \end{array}}$$

We choose maximum velocity $(\alpha' + v'_0)$ as a scaling parameter.

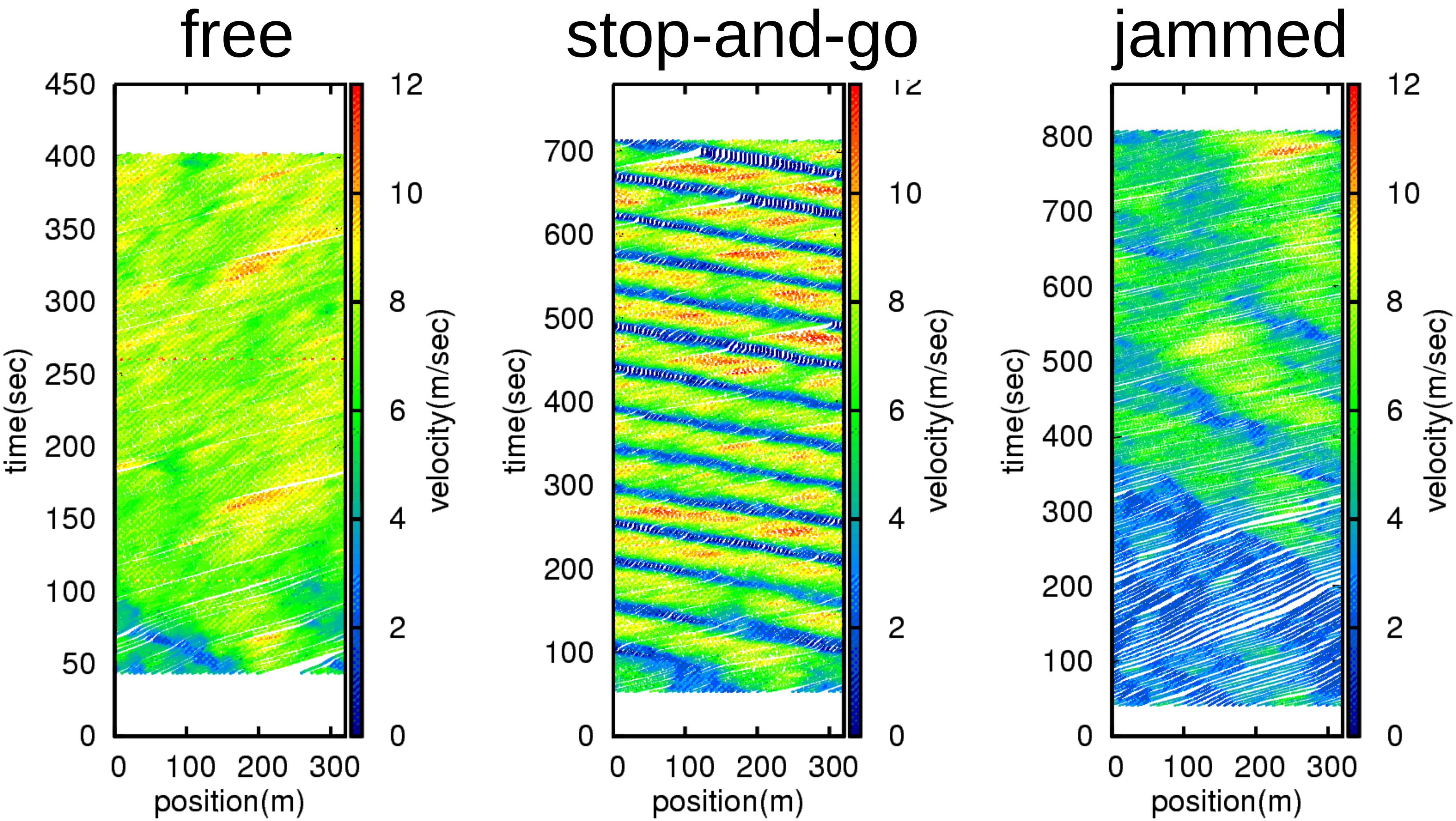
Critical density



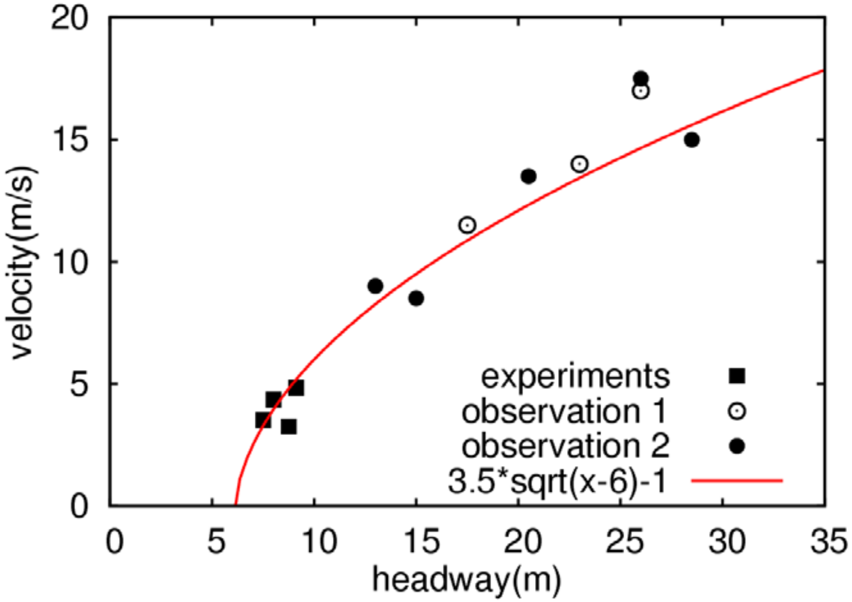
4. Summary

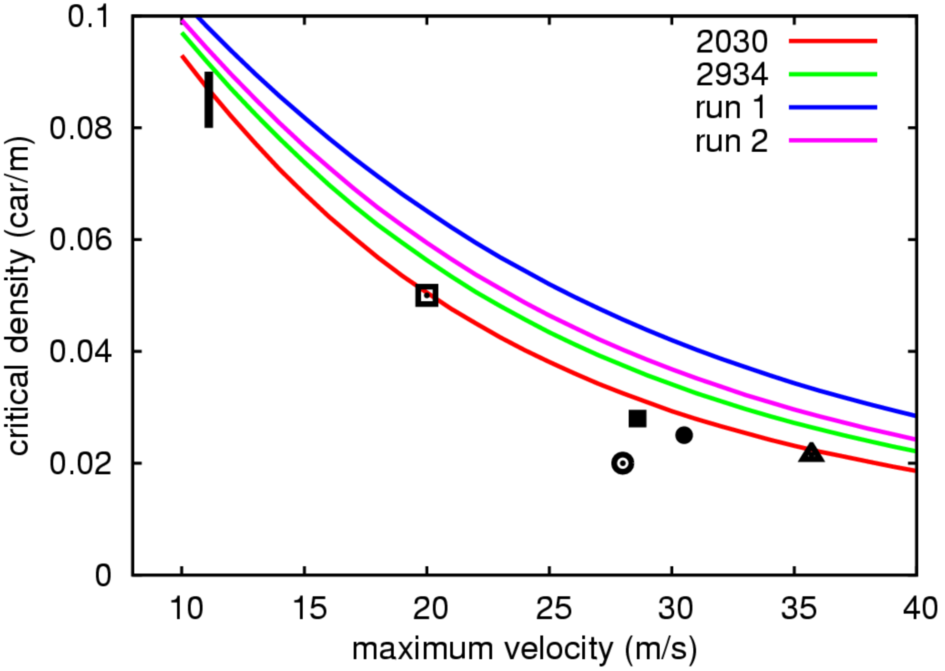
- OV functions are determined by experiments.
- We propose a scaling rule from experiments to real traffic.
- The agreement of estimated critical density with observed values is fair.
- Both experiments and real traffic are explained by the OV model.

Spacetime diagram of experiments

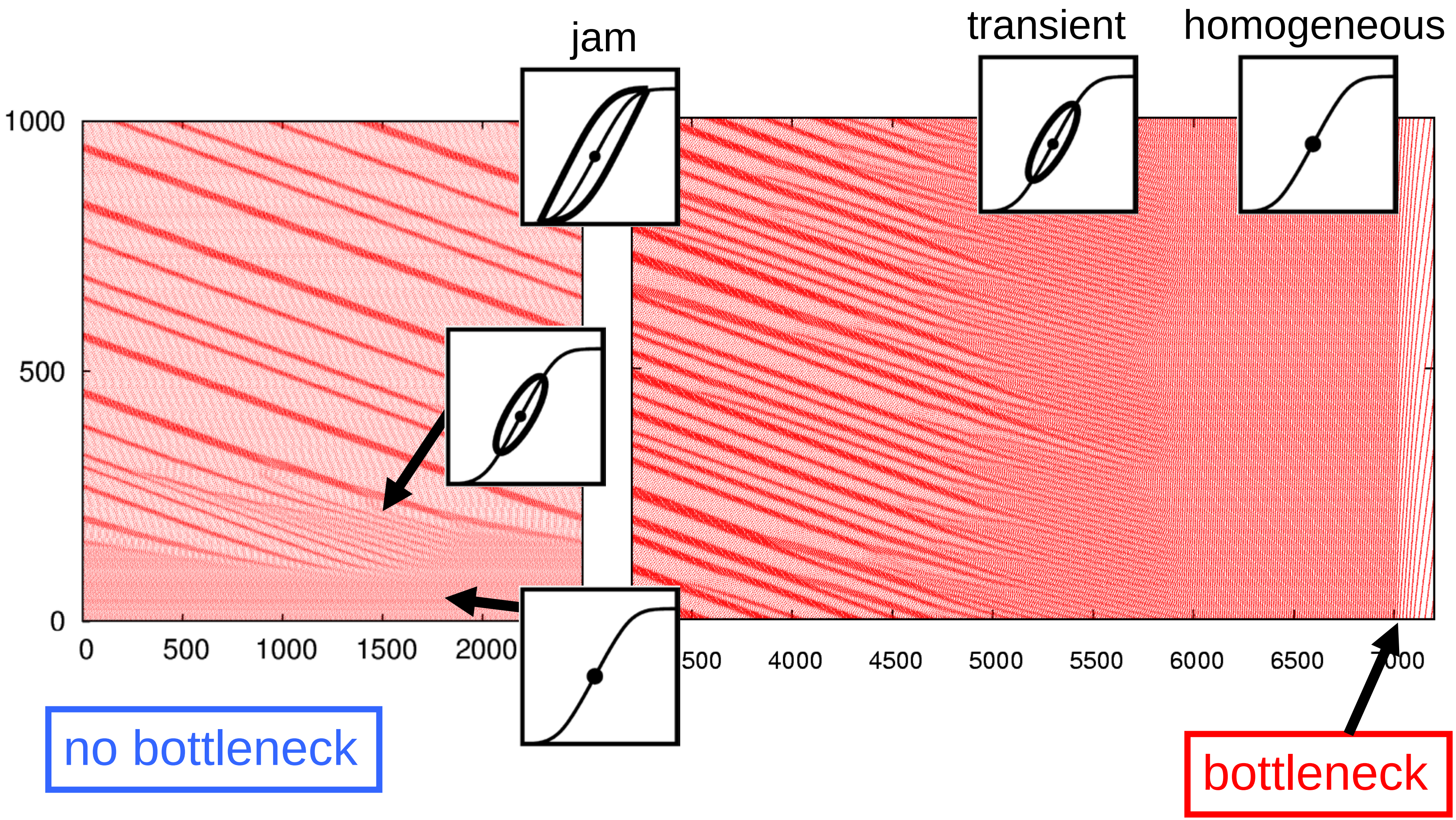


Other choice of the relation of inflection points





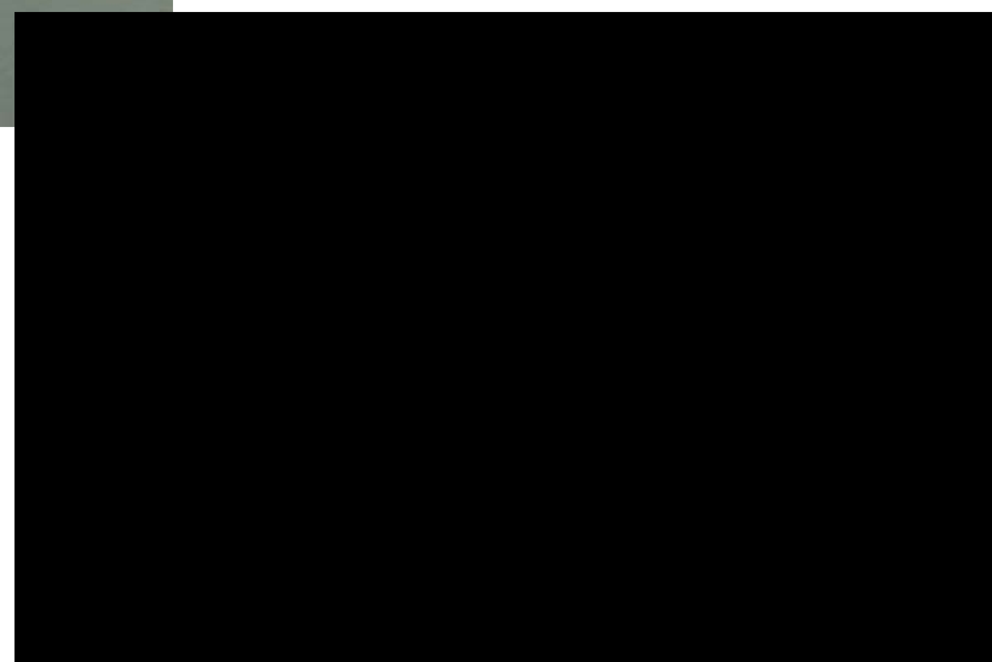
Unstable case



Experiments

on a ground

indoor baseball field



Critical density can be written
as a function of maximum velocity $\alpha' + v'_0$

$$\rho_c = \left[\frac{1}{\beta'}\cosh^{-1}\sqrt{\frac{2\alpha'\beta'}{a'}} + h'_0\right]^{-1}$$ (cars/m)

$$\left(a'T' = 1.8\ , \quad T' = \frac{h'_{\mathrm{min}}}{v_{\mathrm{back}}}\right)$$

$$\left(\begin{array}{ll} \alpha' = \frac{v'_0}{v_0}\alpha & h'_{\mathrm{min}} = \dfrac{v_{\mathrm{back}}}{v'_0 + v_{\mathrm{back}}} h'_0 \\ \beta' = \dfrac{h_0 - h_{\mathrm{min}}}{h'_0 - h'_{\mathrm{min}}}\beta & v'_0 = 0.7(h'_0 - 2) \end{array}\right)$$

- Optimal Velocity model (PRE51, 1995)
  Physical mechanism of jam formation
  is a kind of dynamical phase transition.

criticisms

- Simulation. Not real cars.
- Too simple. Reality must be more complicated.
- Real jam appears near bottleneck.
- 1st experiment (NJP10, 2008)
  Jam occurs without bottleneck.
- 2nd experiment (NJP15, 2013)
  Density is a control parameter.
  Critical density is estimated.

Answer to “Not real cars”.