A FORCE-BASED MODEL TO REPRODUCE STOP-AND-GO WAVES

IN PEDESTRIAN DYNAMICS

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MOTIVATION

STOP-AND-GO WAVES - EXPERIMENTS (1D)

- Typically 1D phenomenon
- Unlike in traffic there is no extensive database

- Cellular automata: Discrete in space
- First-order models (velocity models):
  - Rules defining the velocity

$$x_n' = V(t)$$

- Second-order models (force-based models):
  - Newtonian mechanics: Definition of the acceleration

$$x_n'' = \underbrace{g(x_n')}_{\text{driving term}} + \underbrace{f(x_n', \Delta x_n', \Delta x_n, \cdots)}_{\text{repulsive term}}$$

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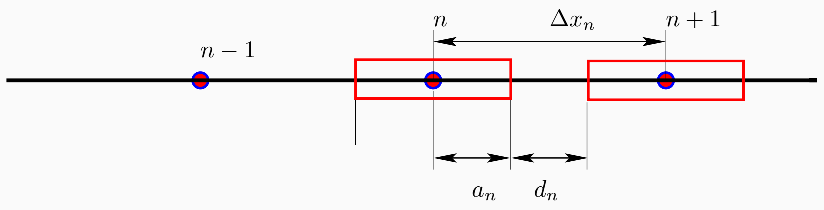
Investigation of stop-and-go waves with force-based models

STABILITY ANALYSIS

- One-dimensional system with closed boundary conditions



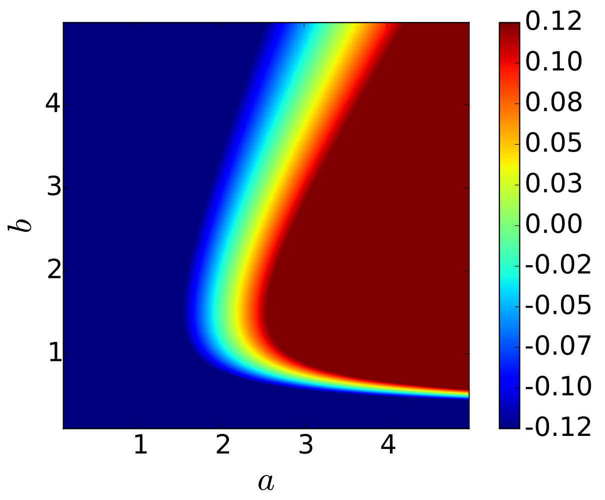
- Normal distribution of N-agents with zero-speed.
- Position of one agent (e.g. the first) is slightly perturbed

$$x_0 \leftarrow x_0 + \epsilon.$$

- Find out under which conditions can we *hope* to reproduce stop-and-go waves
- How does the perturbation ϵ behave with time? (decays to zero? increases to some limit?)
- Define analytic conditions that guarantees [1] reproduction of stop-and-go waves

[1] in theory at least!

Stability region for a (well-known) force-based model [2]



[2] Chraibi et al. Phys. Rev. E 92, 042809 (2015)

MODEL PROPOSITION

The generic equation of motion

$$x_n'' = \frac{v_0 - x_n'}{\tau} + \underbrace{f(x_n', \Delta x_n', \Delta x_n, \cdots)}_{\text{repulsive term}}$$

can be rewritten as

$$x_n'' = \frac{v_0' - x_n'}{\tau},$$

with $v_0' = \tau f + v_0 \leqslant v_0$.

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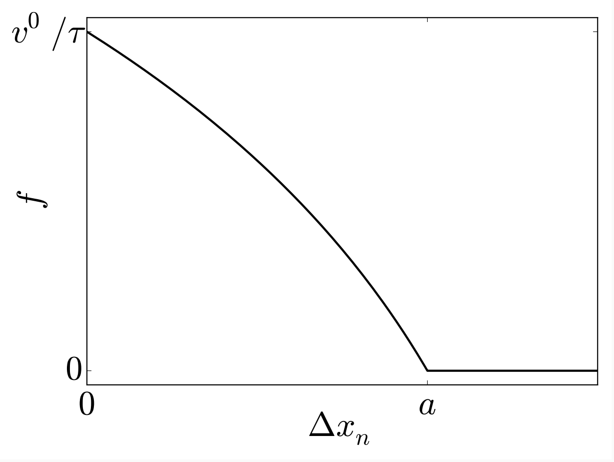

$$f(\Delta x_n, x'_n, x'_{n+1}) = -\frac{v_0}{\tau} \log\Big(c \cdot R_n + 1\Big), \tag{1}$$

with

$$R_n = r_\varepsilon\Big(\frac{\Delta x_n}{a_n + a_{n+1}} - 1\Big), \qquad c = e - 1. \tag{2}$$

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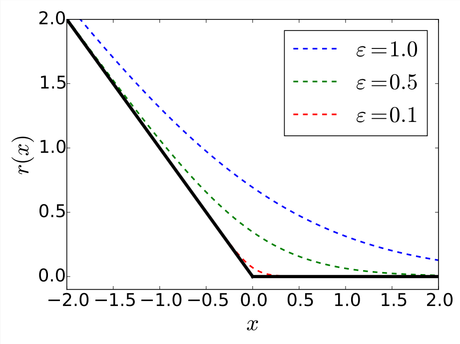
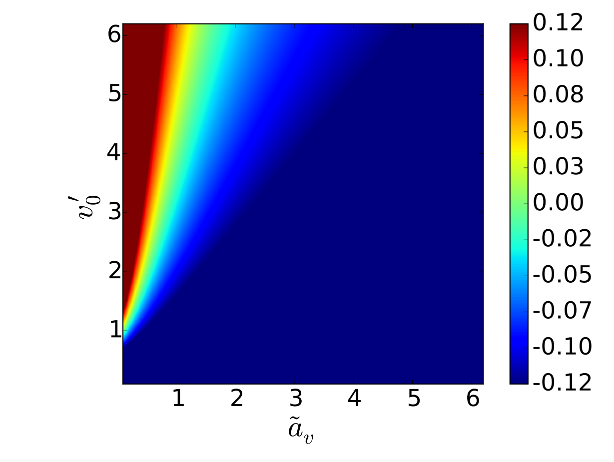


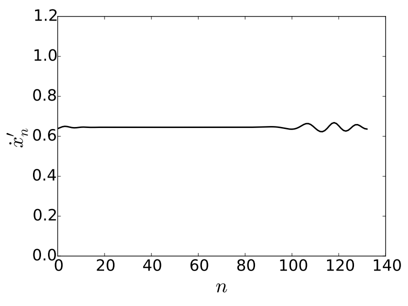
The model has two parameters

- Desired speed
- Velocity dependence of the "space requirement"

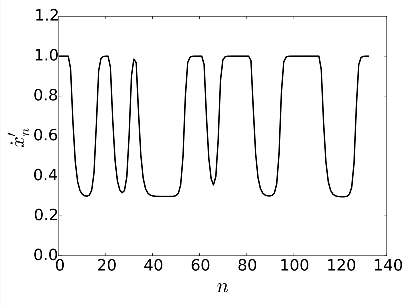


RESULT

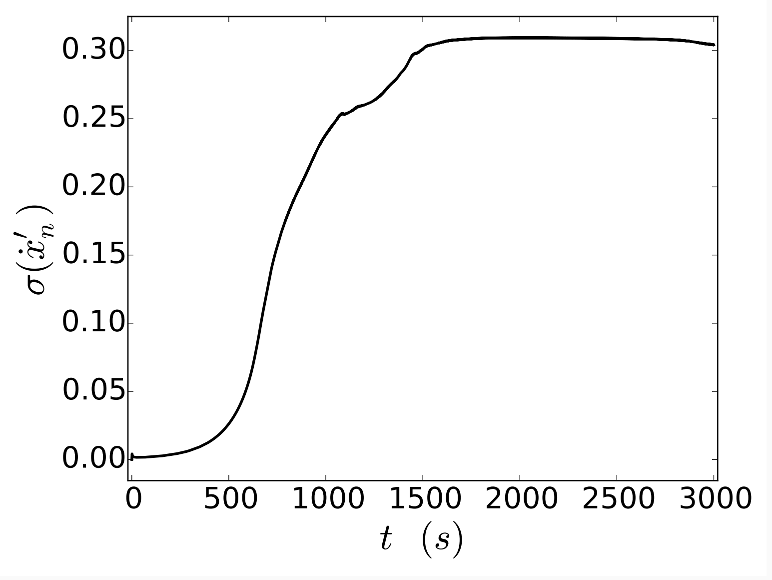
Speed of pedestrians at different time steps

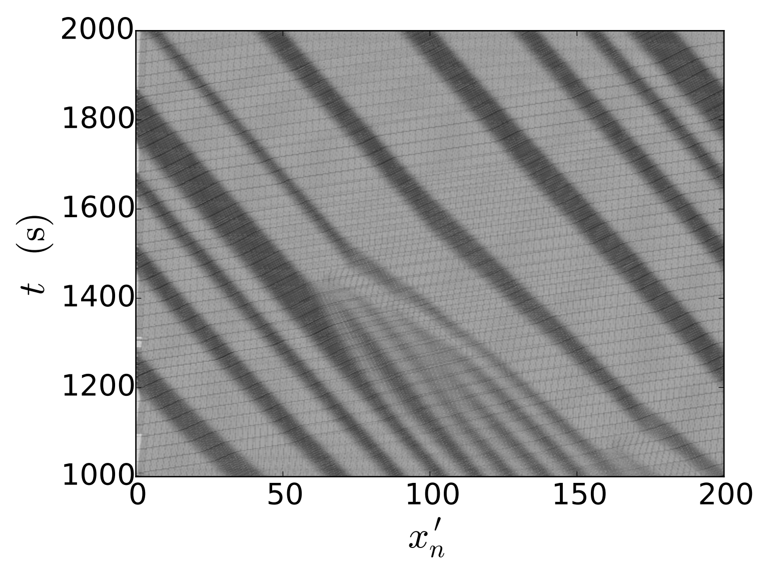


$t = 300\,s$



$t = 2000\,s$


0.30
0.25
0.20
0.15
0.10
0.05
0.00
$\sigma(\dot{x}'_n)$
0
500
1000
1500
2000
2500
3000
$t \quad (s)$



2000
1800
1600
1400
1200
1000
t (s)
0
50
100
150
200
x'_n

LIMITATIONS OF THE MODEL

As a force-based model with some *intrinsic* problems that are quite difficult to solve in a *satisfactory* manner

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- Immediate stopping of pedestrians is not instantaneous: Stopping after delay lead to collisions
- Additional components are necessary to “stabilize” the model (e.g. relative velocities, physical forces, collision-avoidance algorithms, …)

- Maybe a shift to first-order models gives the answer?
- Models are easy to control
- See talk by Antoine Tordeux in Session 8A (**Friday, 09.50**)
  Collision-free first order model for pedestrian dynamics

Thank you for your attention!