

# Effective Modelling of Traffic Dynamics: Classification and Unification

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Computing

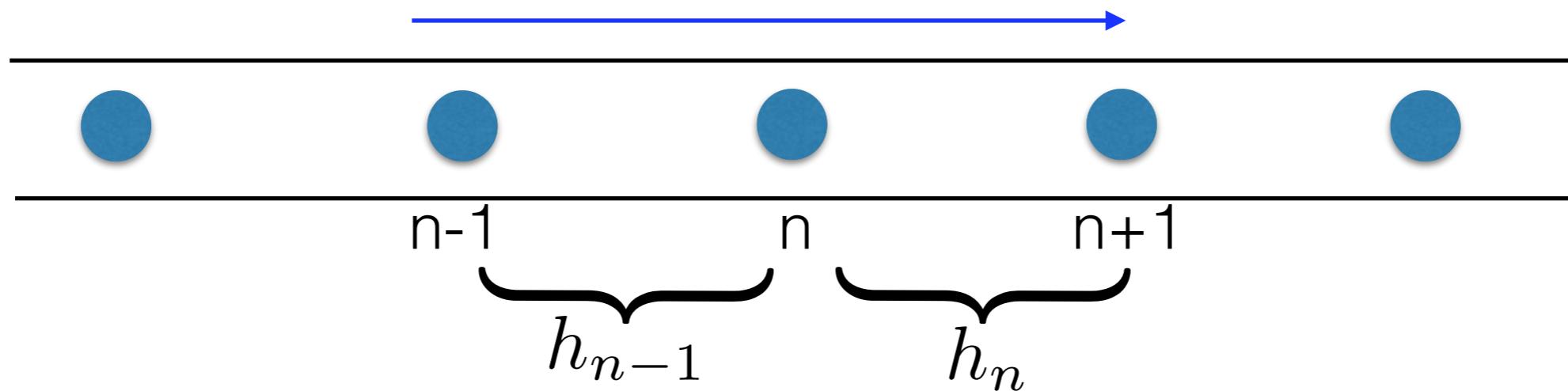
# Outline:

- Overview of the traffic system
- A master model and its controlled expansion
  - Two-phase vs. three-phase traffic models
  - Occam's razor
  - Model extensions and tuning
- Microscopic empirical data collection
  - Video-taping of the traffic flow
  - Image processing and machine learning
  - Data processing and averaging
- Empirical verification of traffic theories and models

# An overview of the traffic system

- theoretical modeling

A One-Dimensional Driven System



$$\Delta v_n = v_{n+1} - v_n$$

nearest neighbour, anisotropic non-linear  
interactions in a dissipative media

# An overview of the traffic system

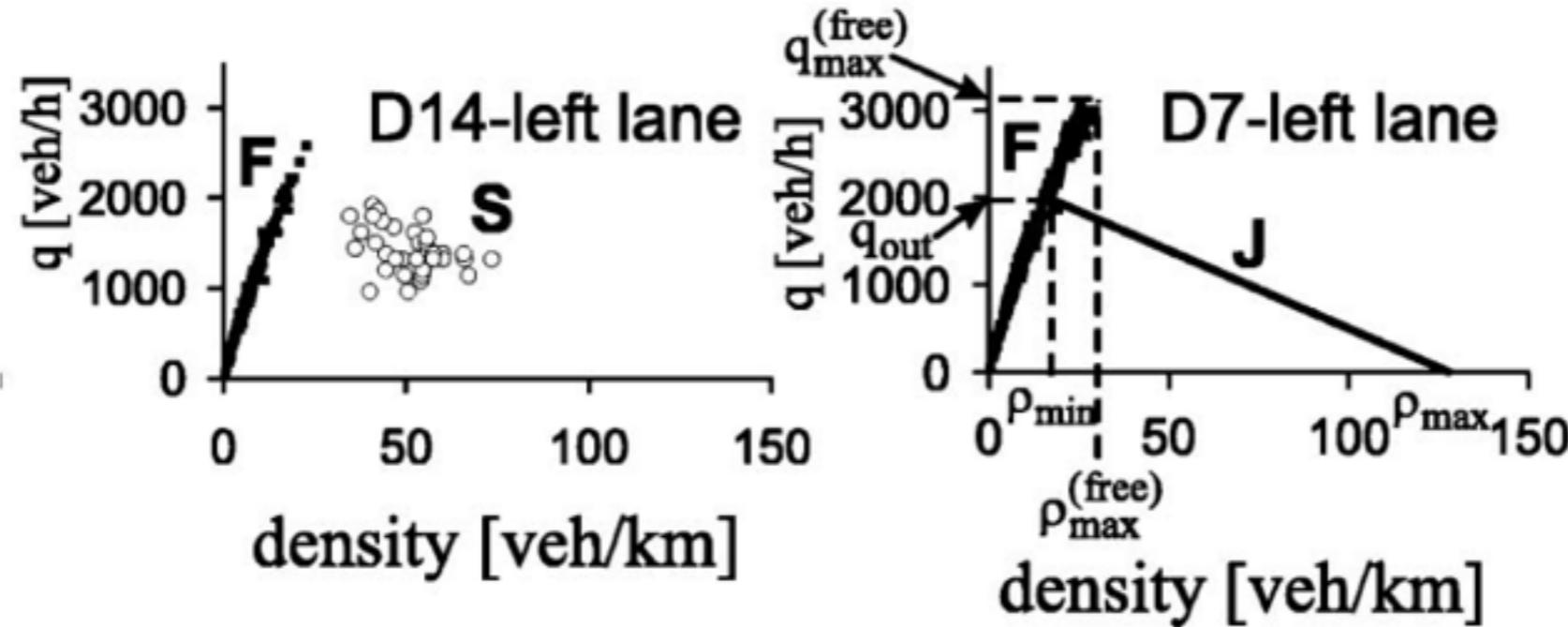
- theoretical modeling



- no symmetry
- non-identical components
- stochasticity and time dependence

# An overview of the traffic system

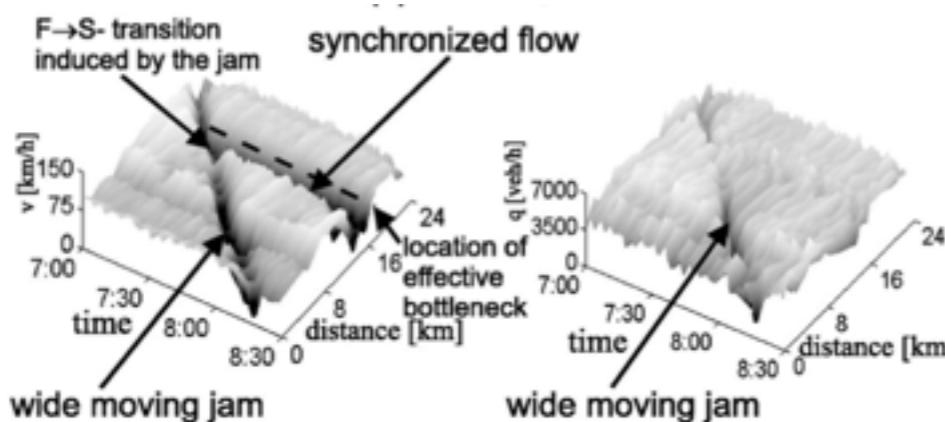
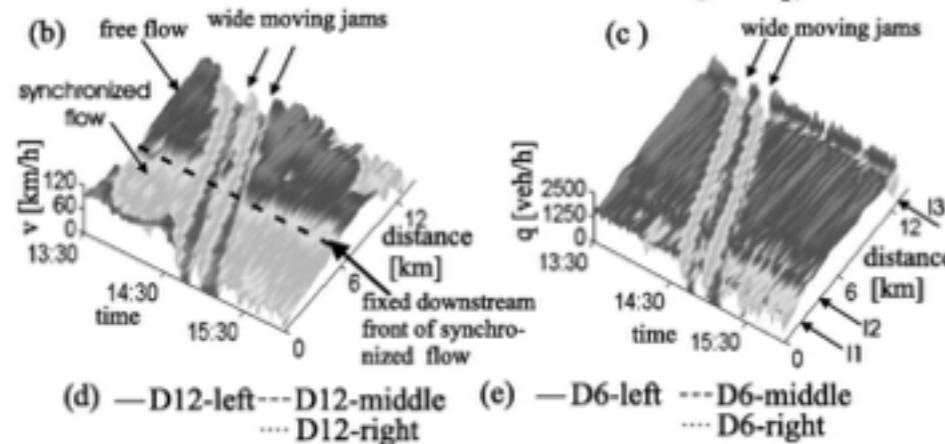
- empirical observations



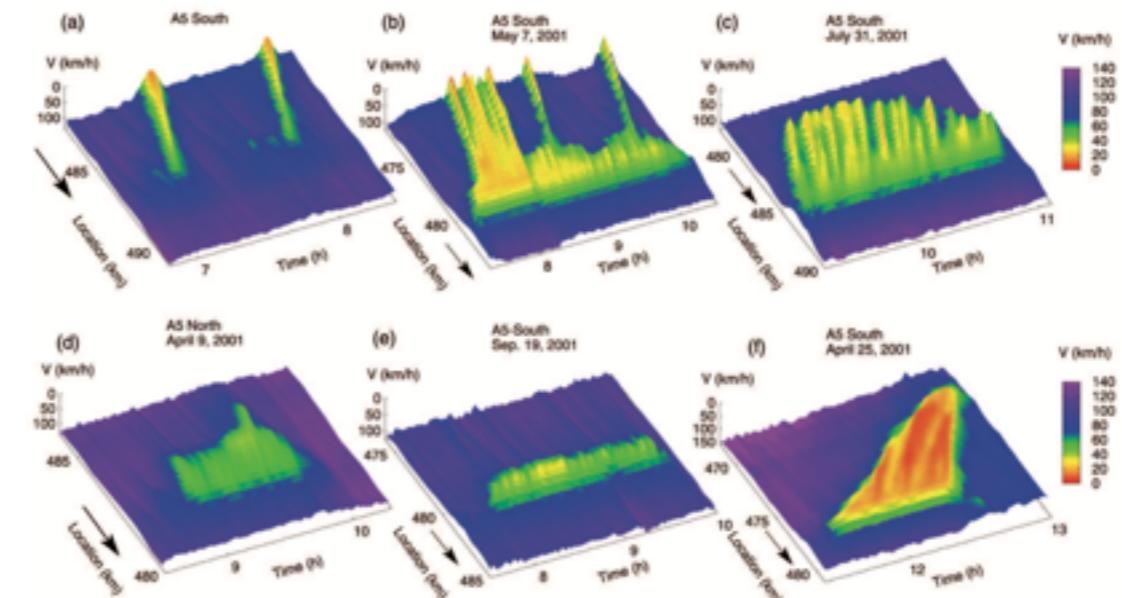
Kerner et.al. 2002

# An overview of the traffic system

- empirical observations



B.S. Kerner, Introduction to Modern Traffic Flow Theory and Control: The Long Road to Three-phase Traffic Theory



D. Helbing et.al., An Analytical Theory of Traffic Flow, A selection of articles by Dirk Helbing reprinted from The European Physical Journal B

# An overview of the traffic system

- empirical observations

| Flow-Density Diagram  | Emergence of the wide moving jams                                   | Congested traffic at an on-ramp bottleneck   |
|---|---|--|
| Pseudo-linear relationship when density is low                              | The congested traffic (“synchronized phase”) can last up to an hour | A wide moving jam passes through the bottleneck unaffected                                   |
| Large scattering of the congested flow-density data points                  | Wide moving jams mostly emerge from the congested traffic           | A wide moving jam may induce or suppress congested traffic at the bottleneck                 |
| The “hysteresis effect”   | “Pinch effect” and the merging of numerous narrow jams              | The region of congested traffic gets smaller with greater bottleneck strength                |
| Significant flow fluctuation at very large vehicle density                  |   | The frequency of the emergence of the moving jams increases with greater bottleneck strength |
| Quantitative features:<br>$V_{max}, \rho_{dj}, \rho_c, \rho_j, Q_c, Q_{dj}$ |   | Downstream front of the congested traffic pinned at the bottleneck                           |

Yang Bo et.al arXiv. 1504.01256

# An overview of the traffic system

- microscopic models

$$a_n(t+T) = cv_n(t+T)^l h_n^{-m} \Delta v_n$$

$$a_n(t+T) = cv_n h_n^{-2} \Delta v_n$$

$$a_n(t+T) = \lambda(h_n - c - T_1 v_n)$$

$$a_n(t+T) = \lambda v_{n+1}^m h_n^{-l} \Delta v_n$$

$$a_n = \lambda_1 \Delta v_n(t-T_1) + \lambda_2 (v_{n+2} - v_n) |_{t-T_2}$$

$$a_n = A \left( 1 - \frac{h_0}{h_n} \right) - \frac{Z^2(-\Delta v_n)}{2(h_n - D)} - kZ(v_n - v_{per})$$

$$Z(x) = (x + |x|)/2$$

B.S. Kerner, Physica A 392 (2013) 5261–5282

$$a_n = \frac{1 + b_1 v_n + b_2 h_n + b_3 v_n h_n + b_4 v_{n+1} + b_5 v_n v_{n+1}}{c_0 + c_1 v_n + c_2 h_n + c_3 v_n h_n + c_4 v_{n+1} + c_5 v_n v_{n+1}}$$

# An overview of the traffic system

- microscopic models

$$a_n = a \left( 1 - \left( \frac{v_n}{v_0} \right)^\delta - \left( \frac{h^*(v_n, \Delta v_n)}{h_n} \right)^2 \right)$$

$$h^*(v, \Delta v) = s_0 + s_1 \sqrt{\frac{v}{v_0}} + T v + \frac{v \Delta v}{2\sqrt{ab}}$$

Helbing et.al 2000

$$a_n = a_0 (V(h_n) - v_n)$$

$$a_n = a_0 (V(h_n) - v_n) - \lambda \Delta v_n \Theta(\Delta v_n) e^{-\frac{h_n - h(v_n)}{R}}$$

$$h(v_n) = d + T v_n$$

$$a_n = a_0 (V(h_n) - v_n + g(\Delta v_n))$$

Bando et.al 1997  
Jiang et.al. 2001  
Gong et.al. 2008

$$g(\Delta v_n) = \begin{cases} \lambda \Theta(\Delta v_n) \Delta v_n \\ \lambda \Delta v_n \\ \lambda_1 \Theta(\Delta v_n) \Delta v_n + \lambda_2 \Theta(-\Delta v_n) \Delta v_n \end{cases}$$

# An overview of the traffic system

- microscopic models

$$a_n = \begin{cases} A^{\text{free}}(V^{\text{free}}(h_n) - v_n) + K(v_n, v_{n+1}) \Delta v_n & v_n \geq v_{\min}^{\text{free}}, h_n > h^{\text{jam}} \\ A^{\text{syn}}(V^{\text{syn}}(h_n) - v_n) + K(v_n, v_{n+1}) \Delta v_n & v_n < v_{\min}^{\text{free}}, h_n > h^{\text{jam}} \\ -K^{\text{jam}}v_n & h_n \leq h^{\text{jam}} \end{cases}$$

$$V^{\text{syn}}(h) = \lambda(h - h^{\text{jam}}) \quad \text{or} \quad \lambda_1(\tanh(\lambda_2(h - h^{\text{jam}})) + \lambda_3(h - h^{\text{jam}}))$$

$$a_n(t + \tau) = \begin{cases} A(V^{\text{free}}(h_n) - v_n) + K(v_n, v_{n+1}) \Delta v_n & h_n > G \text{ and } h_n > h^{\text{jam}} \\ A \min(V^{\text{syn}}(h_n) - v_n, 0) + K(v_n, v_{n+1}) \Delta v_n & h_n \leq G \text{ and } h_n > h^{\text{jam}} \\ -K^{\text{jam}}v_n & h_n \leq h^{\text{jam}} \end{cases}$$

Kerner et.al 2001

# An overview of the traffic system

- microscopic models
  - How do we properly characterise the differences between two traffic models?
  - Is there a standard way of extending an existing traffic model or construction of a new traffic model?
  - Is there a standard way in selecting the best traffic model based on the experimental data?

# A master model

- a renormalisation-like approach

$$a_n = \mathcal{F}_{n,\{s_i\}}(\{t_i\})$$

# A master model

- a renormalisation-like approach

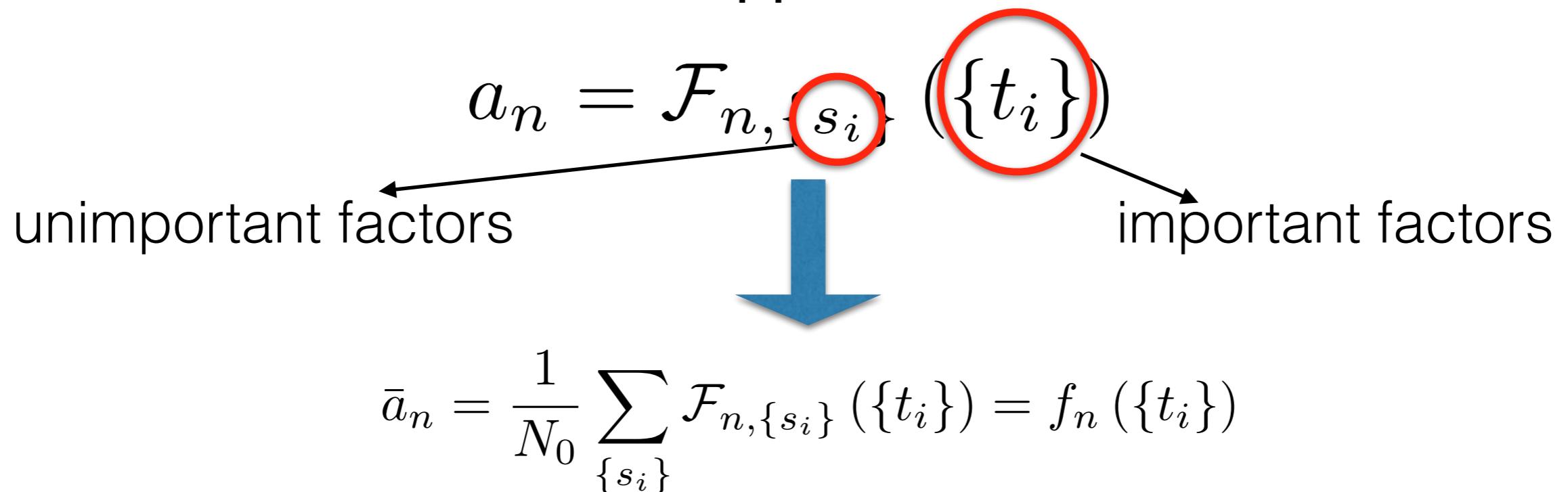
$$a_n = \mathcal{F}_{n, \{s_i\}} (\{t_i\})$$

unimportant factors

important factors

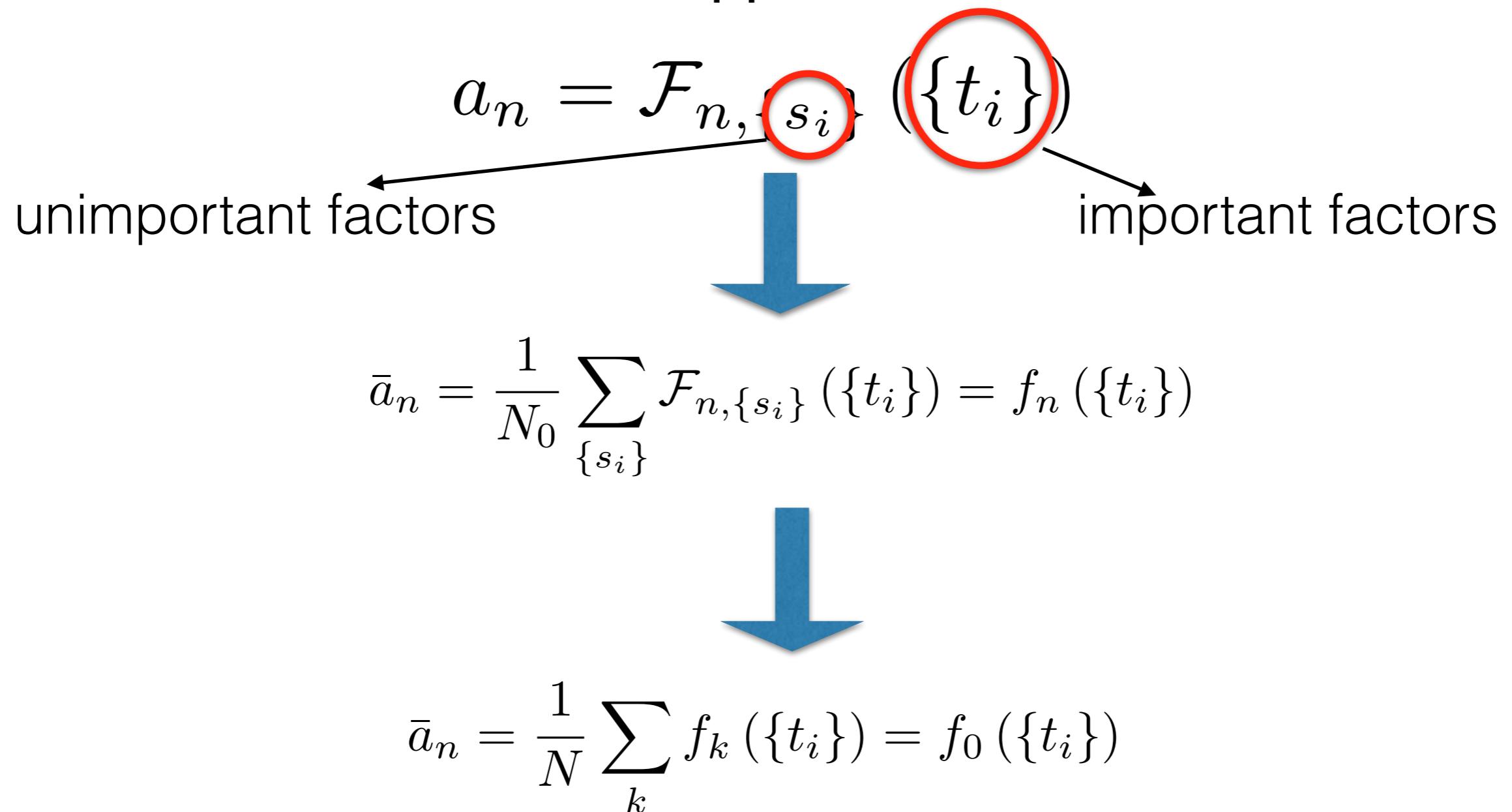
# A master model

- a renormalisation-like approach



# A master model

- a renormalisation-like approach

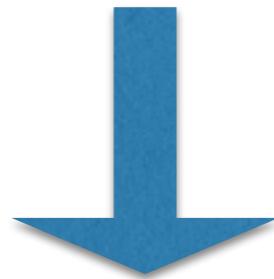


# A master model

- a renormalisation-like approach

$$\bar{a}_n = \frac{1}{N} \sum_k f_k (\{t_i\}) = f_0 (\{t_i\})$$

$$\{t_i\} = \{h_n, \Delta v_n, v_n\}$$

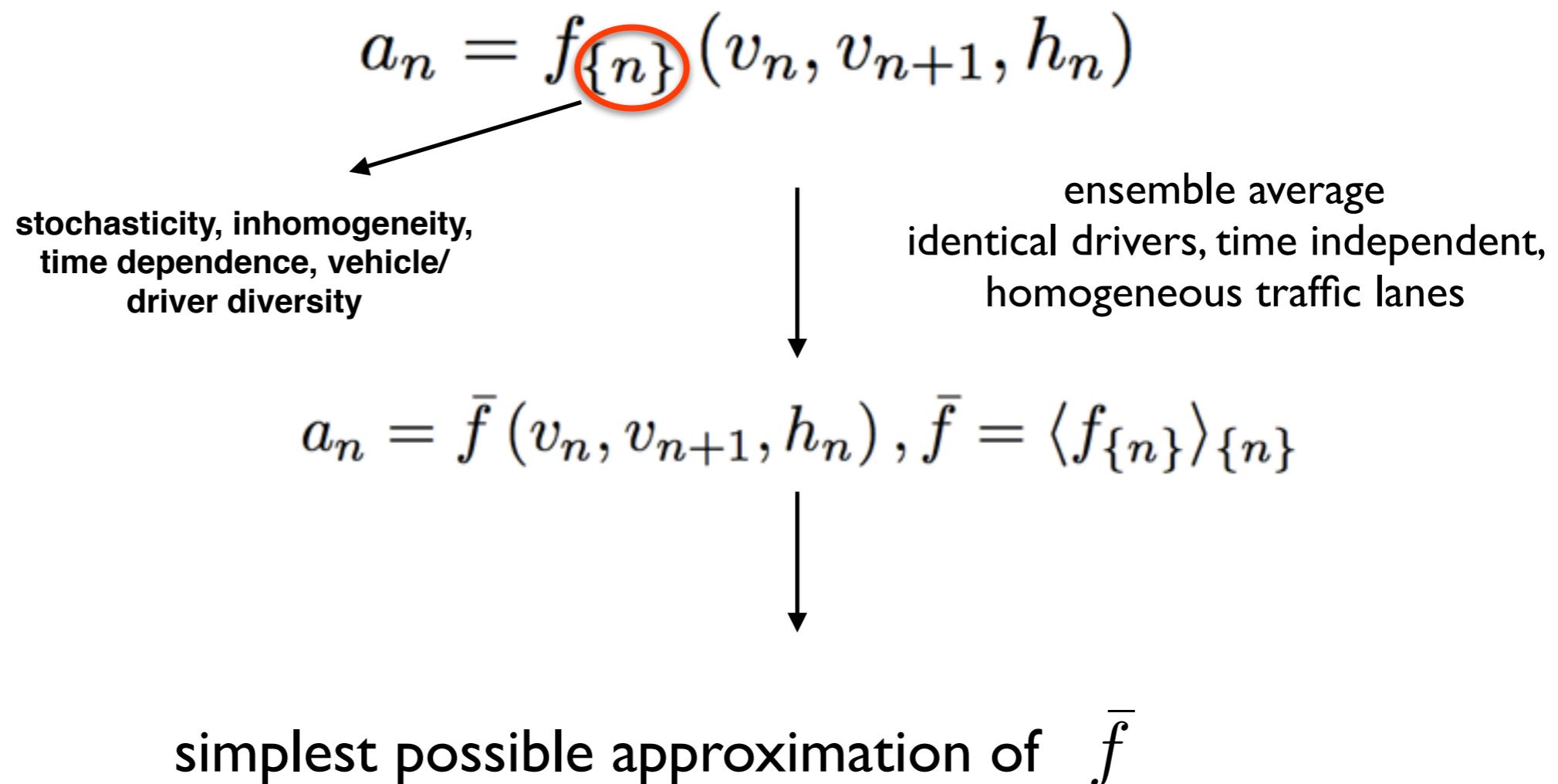


$$a_n = f_0 (h_n, \Delta v_n, v_n)$$

deterministic, time  
independent  
identical drivers

# A master model

- a renormalisation-like approach



# A master model

- a renormalisation-like approach

$$a_n = f_0 (h_n, \Delta v_n, v_n) \longrightarrow f_0 (h_n, 0, v_n) = 0$$



the “ground state” of the traffic dynamics

$$f_0 (h_n, 0, V_{op}) = 0$$

# A master model

- a controlled expansion

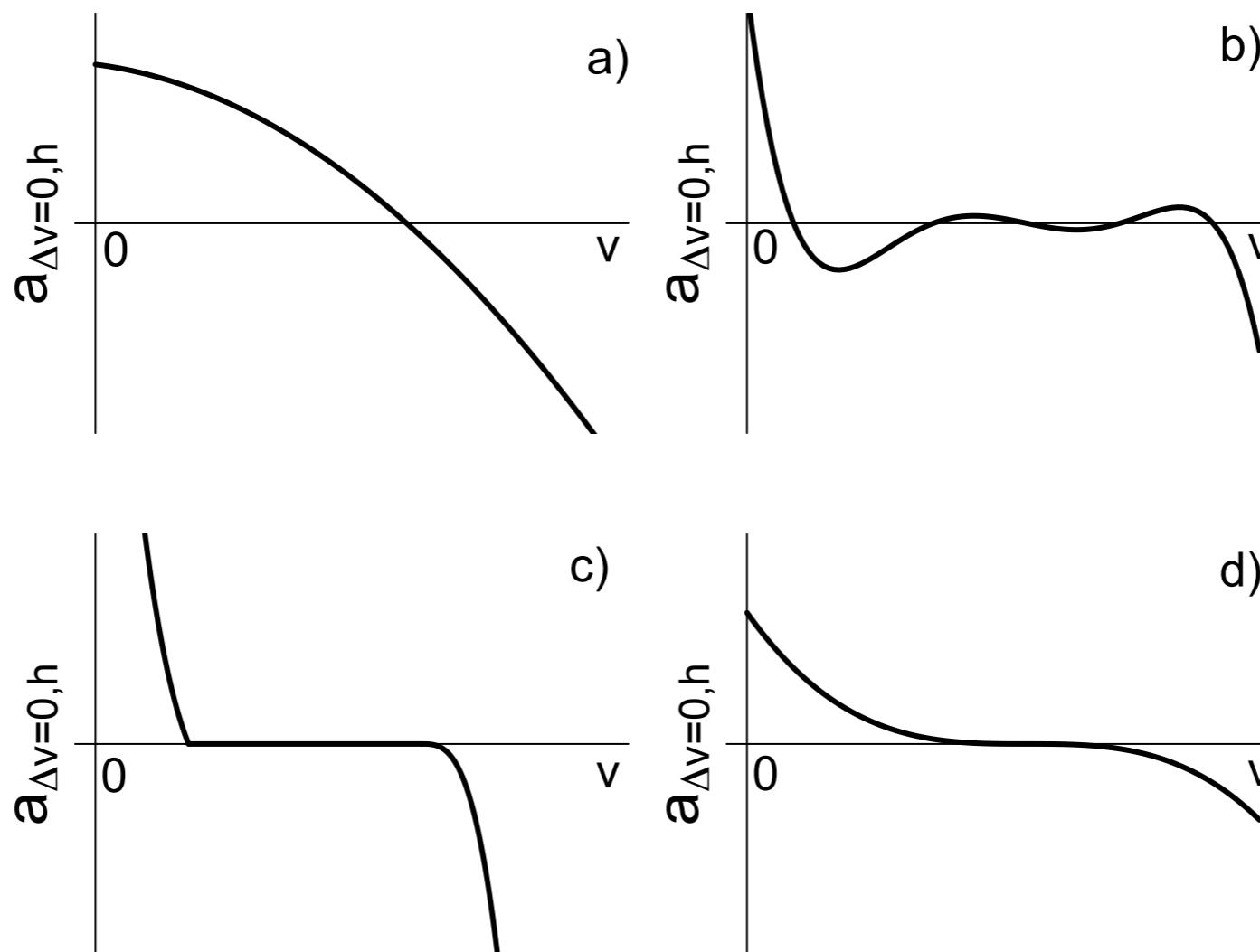
$$\tilde{a}_n = \sum_{p,q} \kappa_{p,q} \left( \tilde{h}_n \right) \left( \tilde{v}_n - V_{op} \left( \tilde{h}_n \right) \right)^p \Delta \tilde{v}_n^q$$

$$\kappa_{p,q} \left( \tilde{h}_n \right) = \frac{1}{p!q!} \left. \frac{\partial^{p+q} \tilde{f}}{\partial^p \tilde{v}_n \partial^q \Delta \tilde{v}_n} \right|_{\begin{array}{l} \tilde{v}_n = \tilde{V}_{op}(\tilde{h}_n) \\ \Delta \tilde{v}_n = 0 \end{array}}$$

Yang Bo et.al PRE 92, 042802 (2015)

# A new perspective in modelling

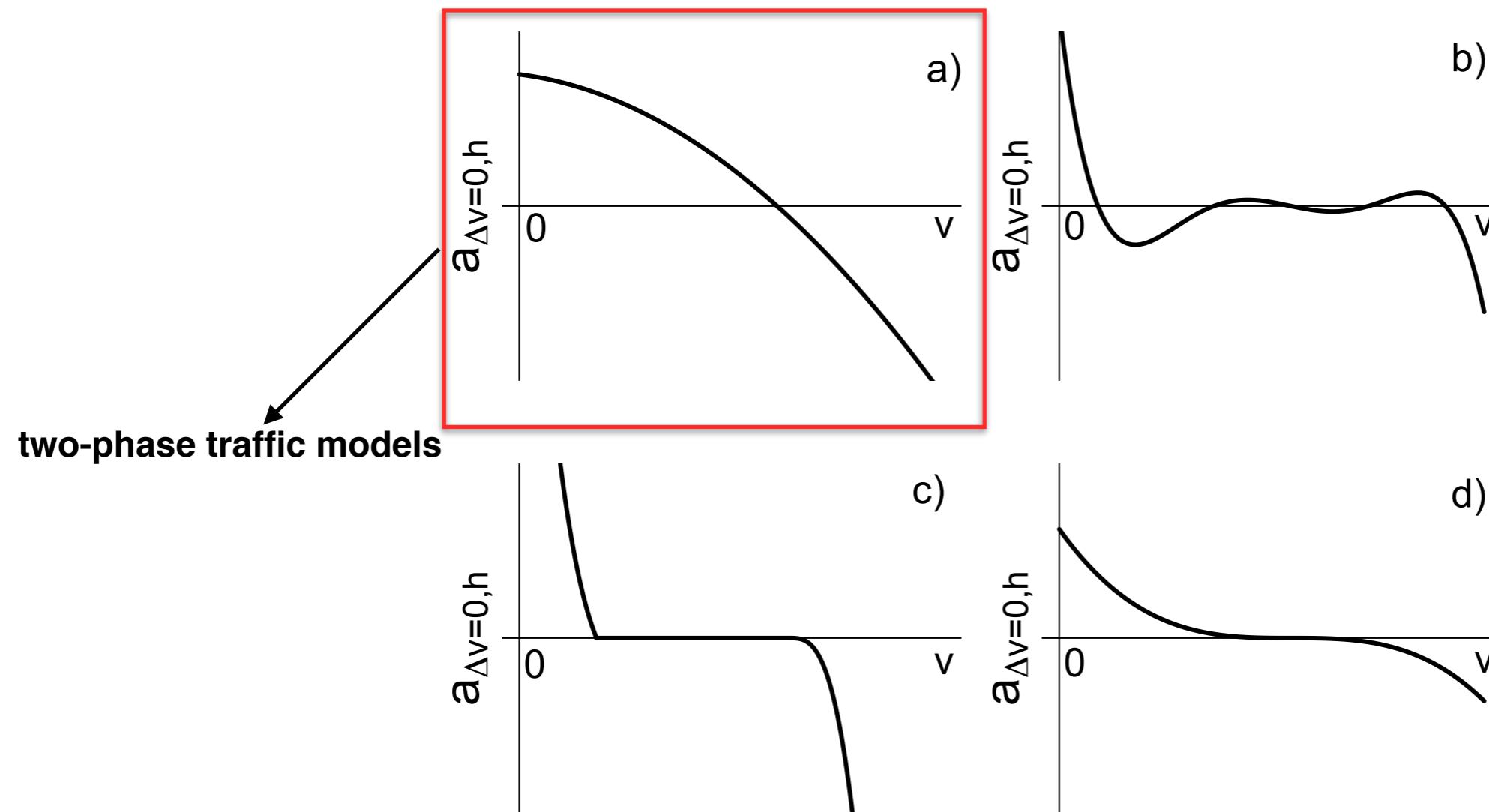
- a universal mathematical structure



Yang Bo et.al PRE 92, 042802 (2015)

# A new perspective in modelling

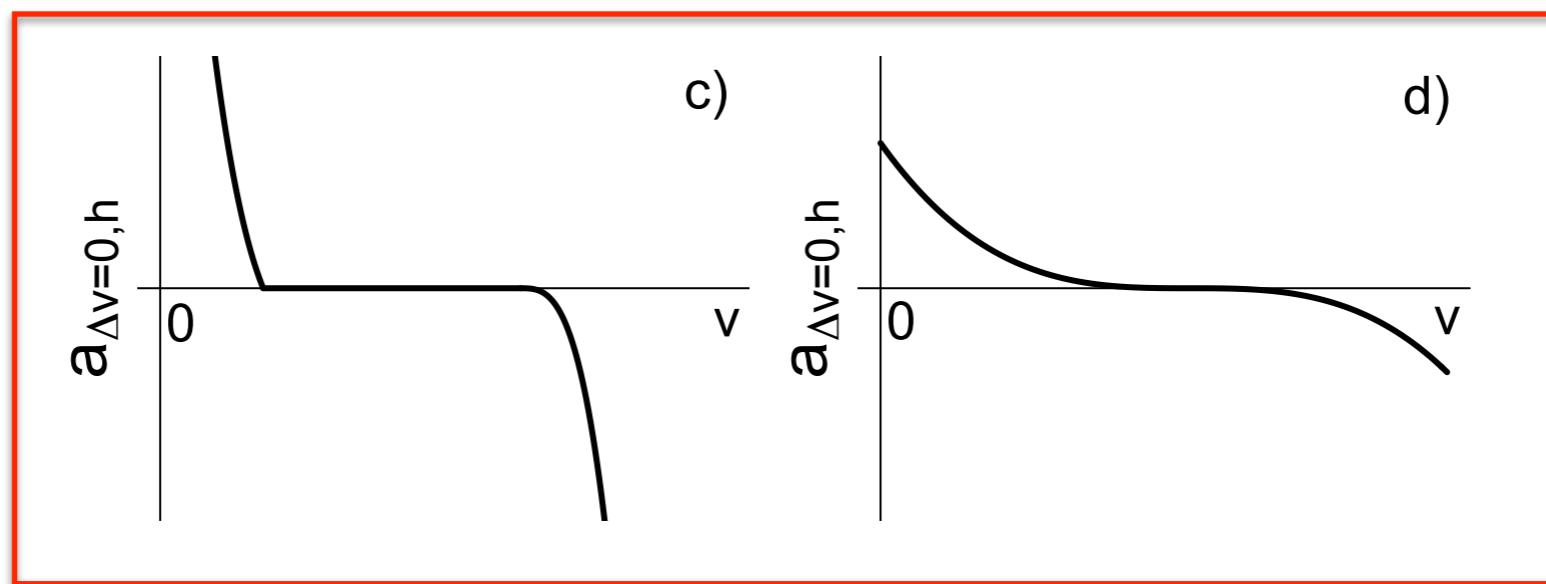
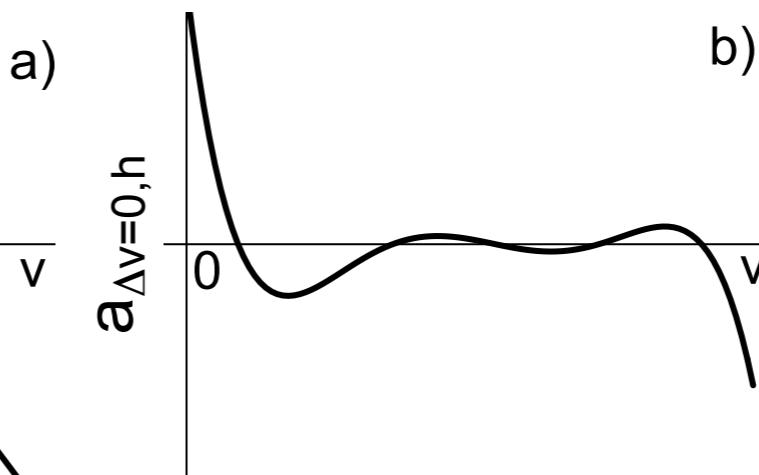
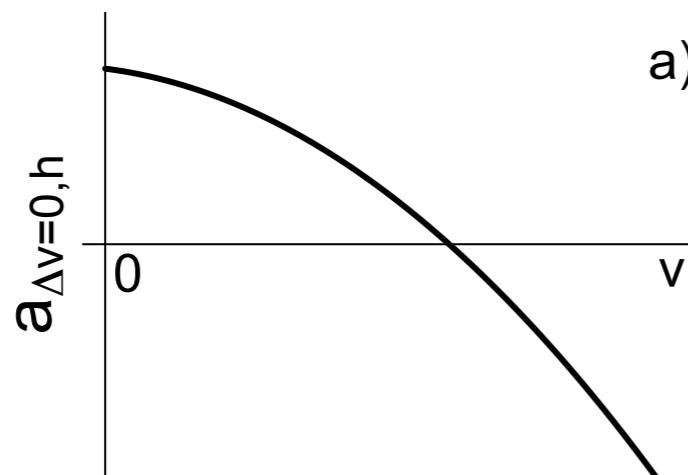
- a universal mathematical structure



Yang Bo et.al PRE 92, 042802 (2015)

# A new perspective in modelling

- a universal mathematical structure
- two-phase vs. three-phase



$$\tilde{a}_n \sim (\tilde{v}_n - V_{op}(\tilde{h}_n))^p$$

Yang Bo et.al PRE 92, 042802 (2015)

# A new perspective in modelling

- a universal mathematical structure
- two-phase vs. three-phase
  - All microscopic traffic models are defined by the optimal velocity (OV) and a set of expansion coefficients (EC).
  - the two-phase and three-phase traffic models can be unified by a “common language”.
  - The simplification of OV and ECs can be empirically verified.

# A new perspective in modelling

- special cases - IDM

$$a_n = a \left( 1 - \left( \frac{v_n}{v_0} \right)^\delta - \left( \frac{h^*(v_n, \Delta v_n)}{h_n} \right)^2 \right)$$

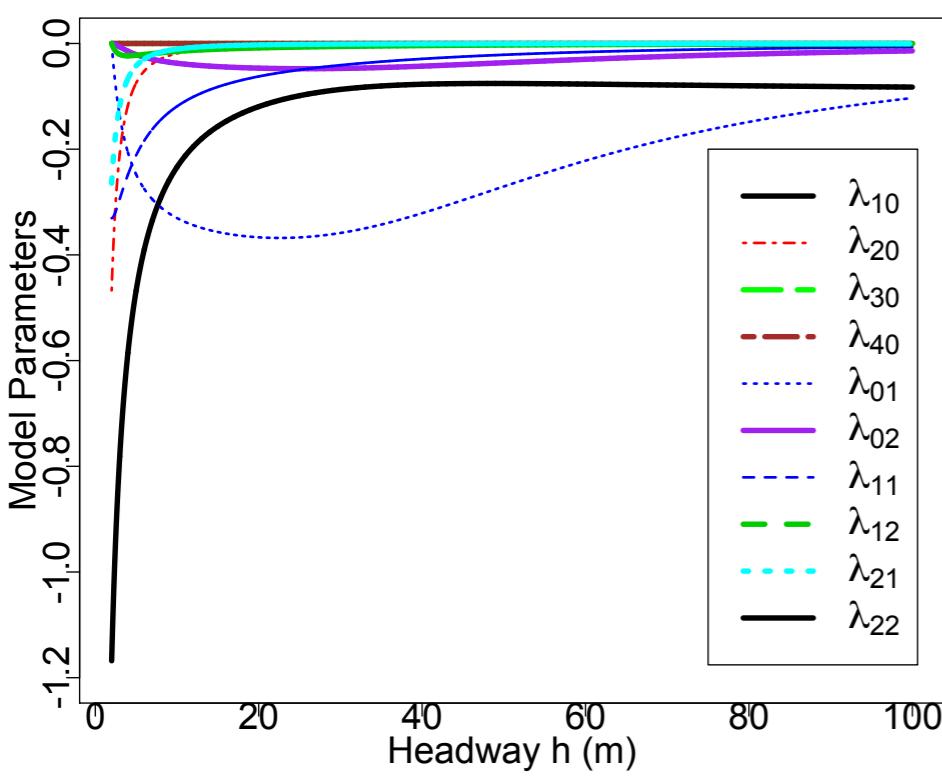


$$a_n = \sum_{p=1, q=0}^{p=4, q=2} \lambda_{p,q} (v_n - V_{op}(h_n))^p \Delta v_n^q$$



$$a_n = \lambda_{10}(h_n)(v_n - V_{op}) + \lambda_{01}(h_n)\Delta v_n$$

Yang Bo et.al PRE 92, 042802 (2015)



# A new perspective in modelling

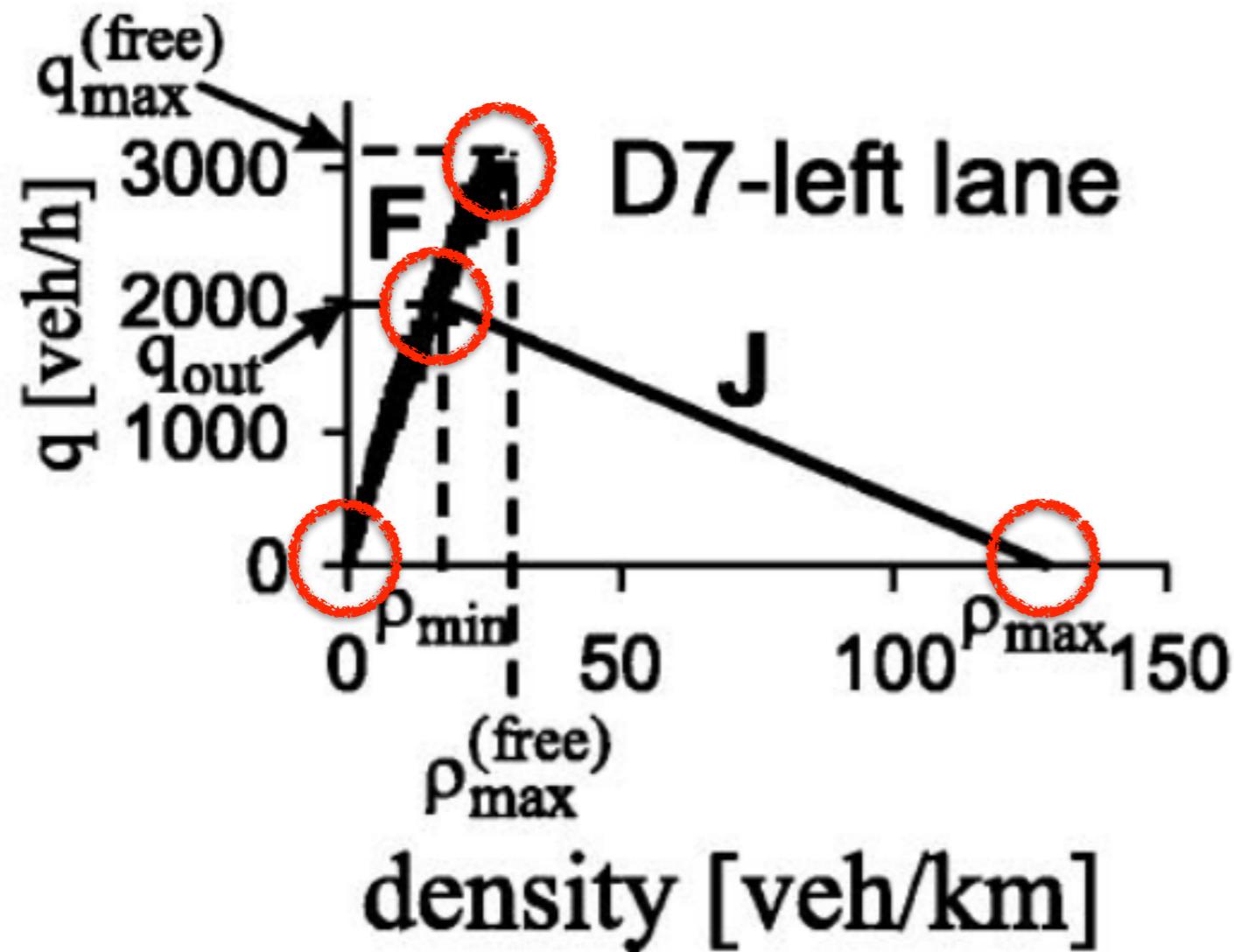
- special cases - Occam's razor

**The best model should be as simple as possible (but not simpler)**

$$a_n = \sum_{p,q} \kappa_{p,q} (h_n) (v_n - V_{op})^p \Delta v_n^q$$

# A new perspective in modelling

- special cases - Occam's razor
- Model tuning



$$a_n = \kappa (V_{op}(h_n) - v_n) + g(\Delta v_n)$$

$$g(\Delta v_n) = \lambda_1 \Delta v_n + \lambda_2 |\Delta v_n|$$

$h_{\max}, h_{\min}, n_0$

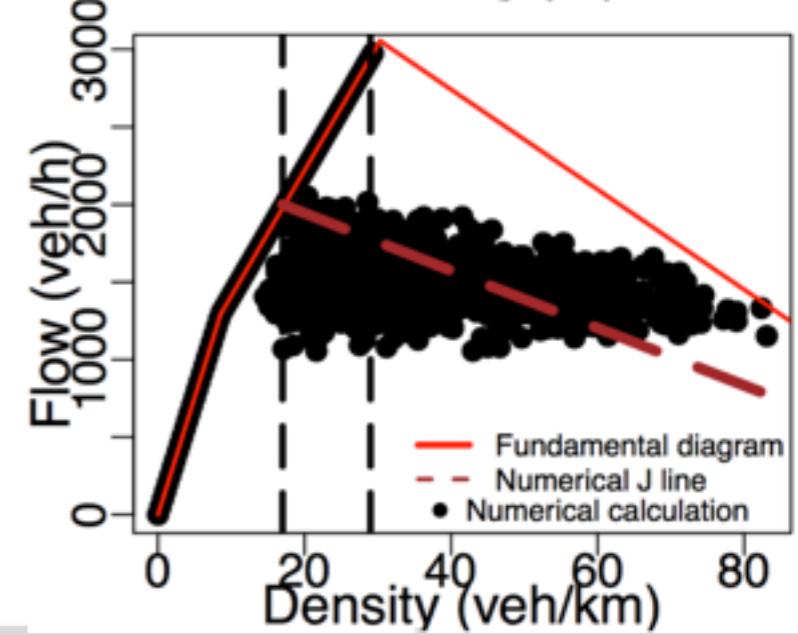
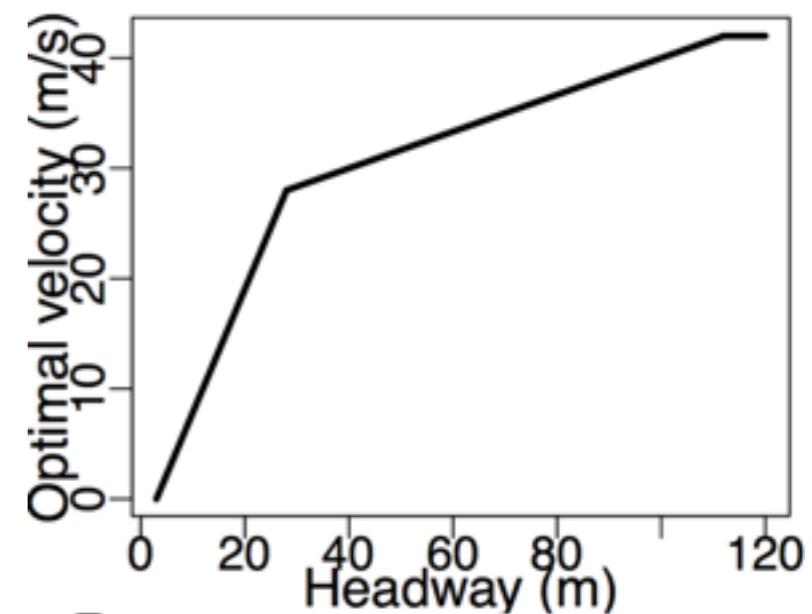
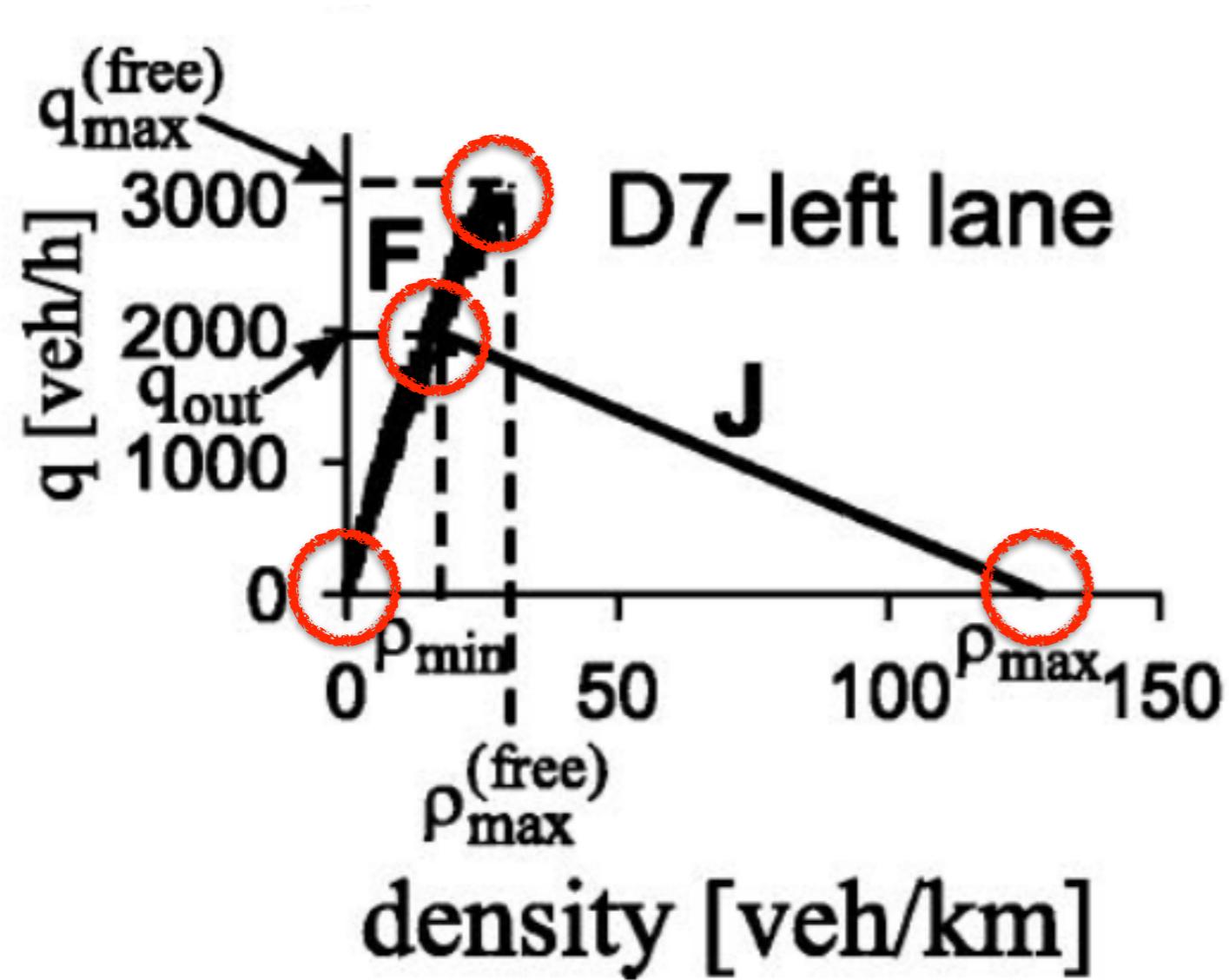
emergent quantities  
from non-linear  
interactions

Yang Bo et.al arXiv:1504.01256

Yang Bo et.al arXiv:1407.3177

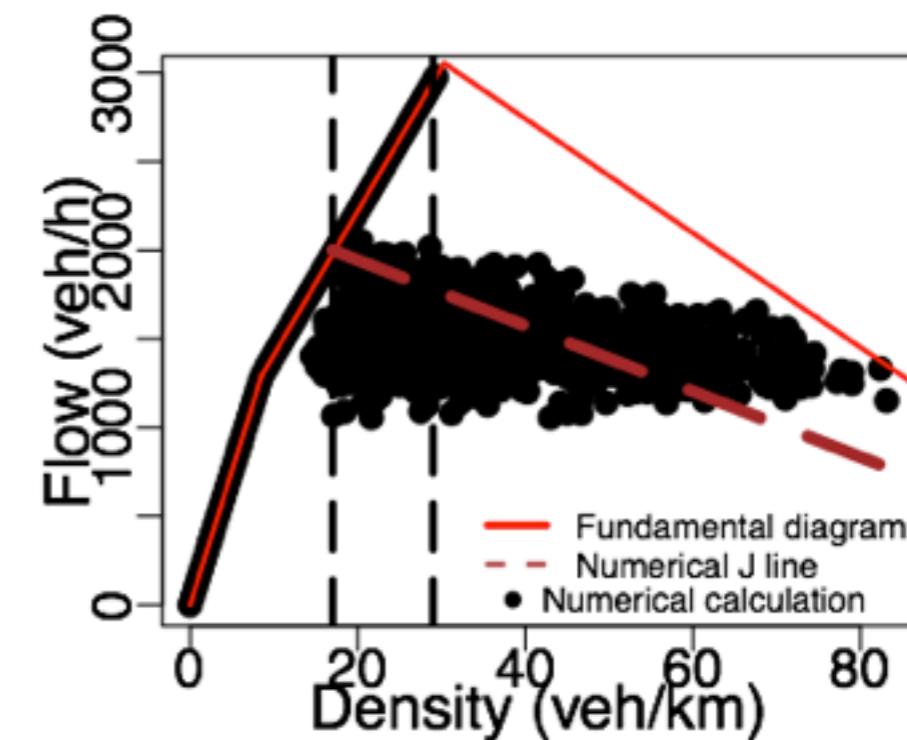
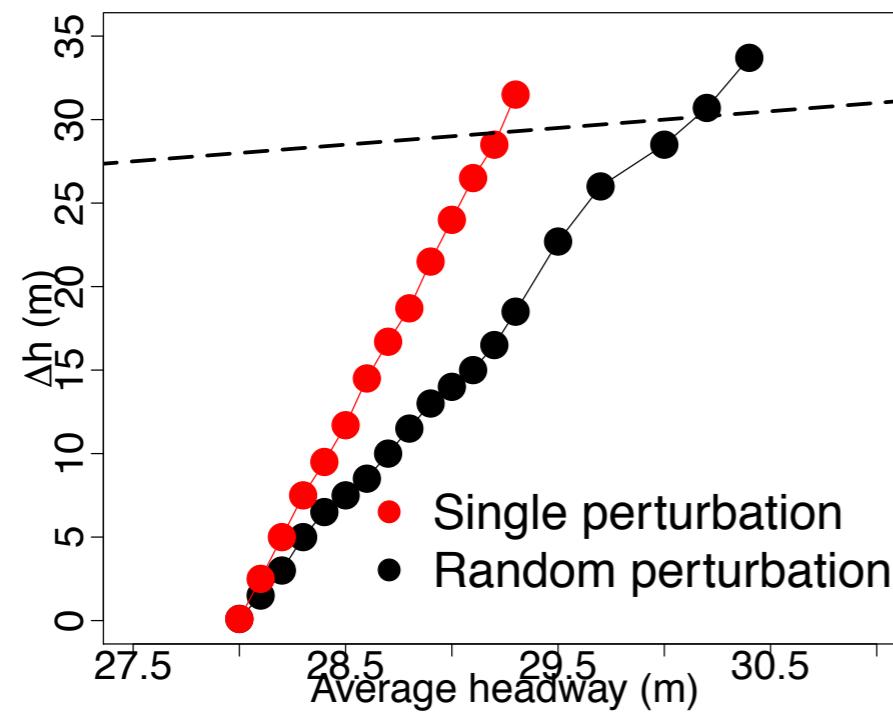
# A new perspective in modelling

- special cases - Occam's razor
- Model tuning



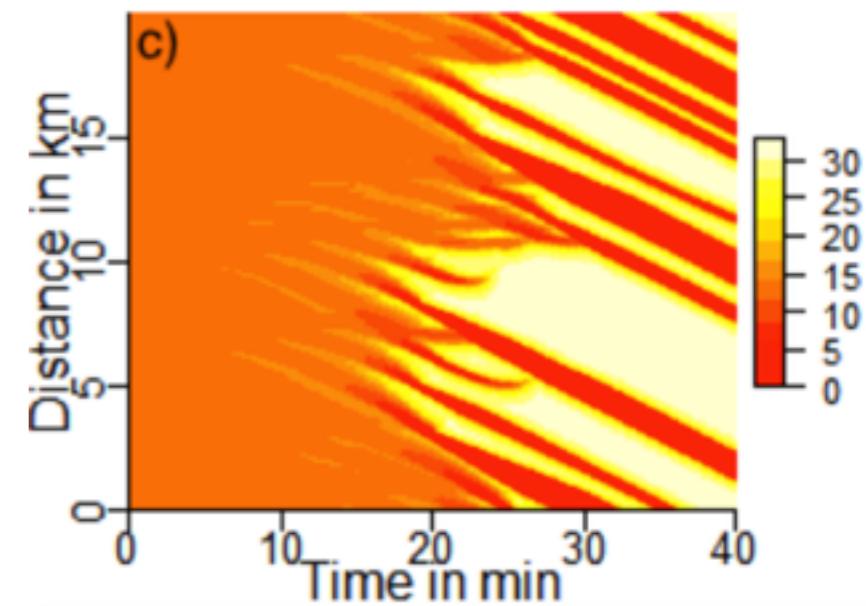
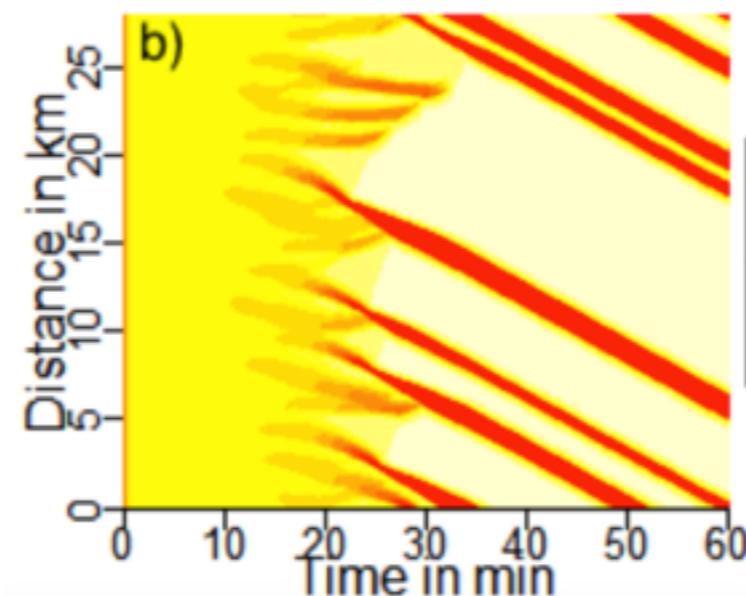
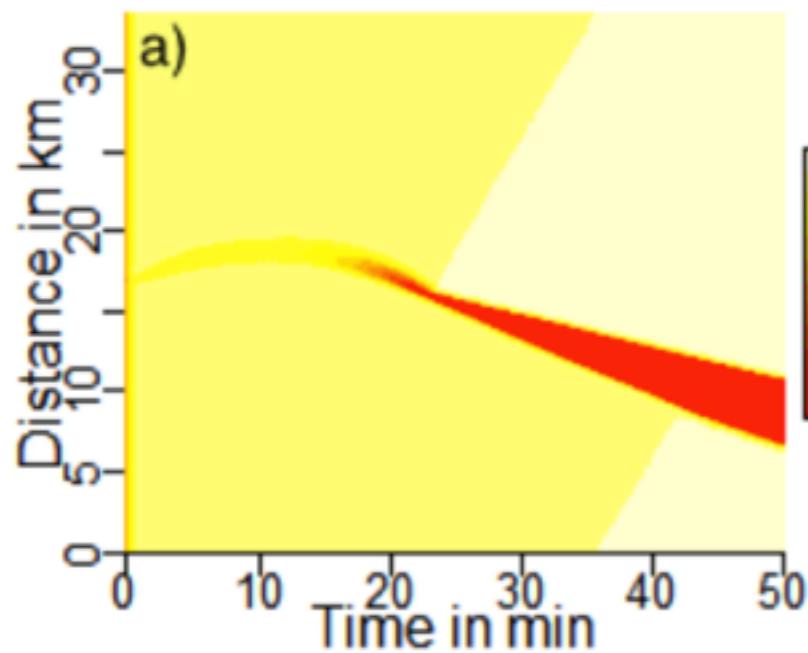
# A new perspective in modelling

- special cases - Occam's razor
- numerical simulation



# A new perspective in modelling

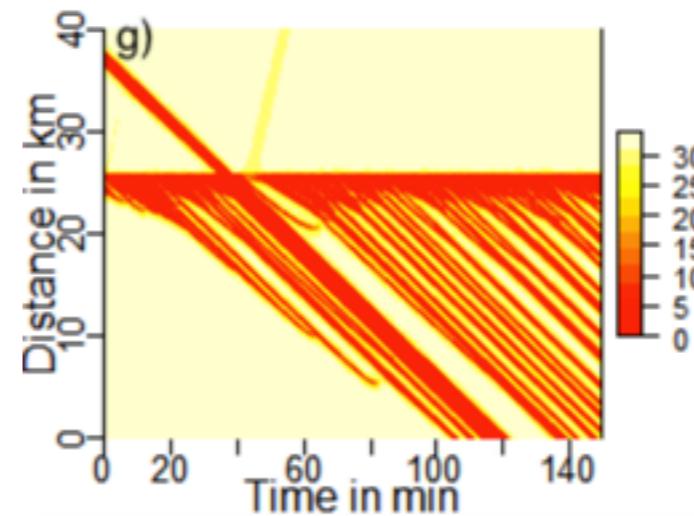
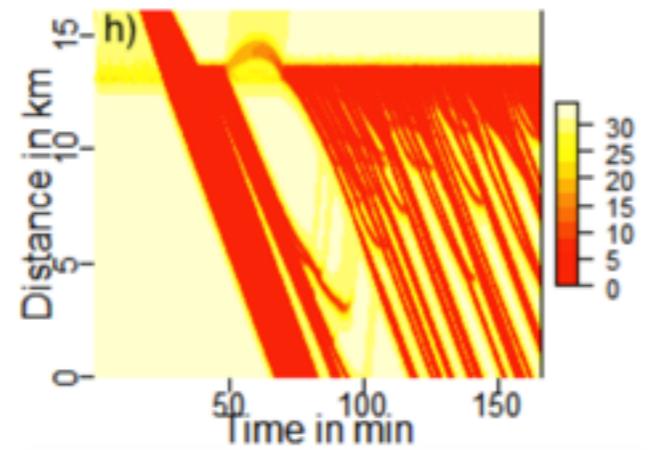
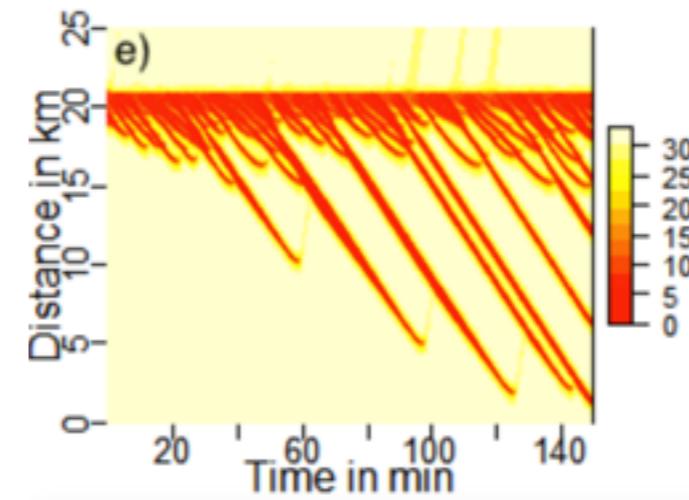
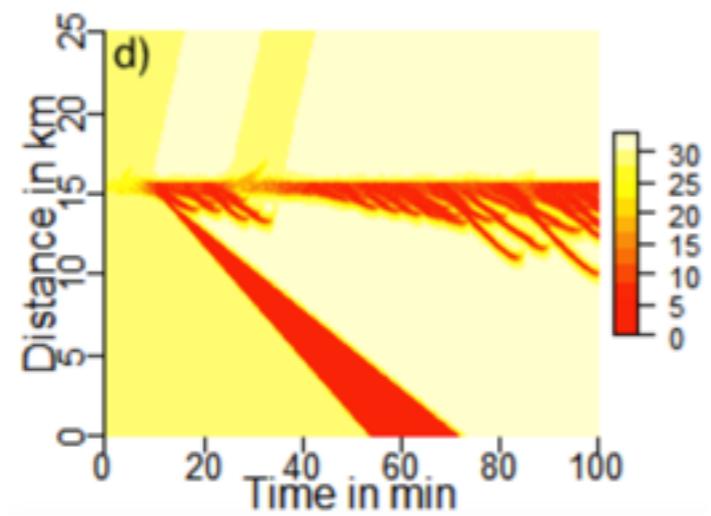
- special cases - Occam's razor
- numerical simulation



Yang Bo et.al arXiv. 1504.01256

# A new perspective in modelling

- special cases - Occam's razor
- numerical simulation



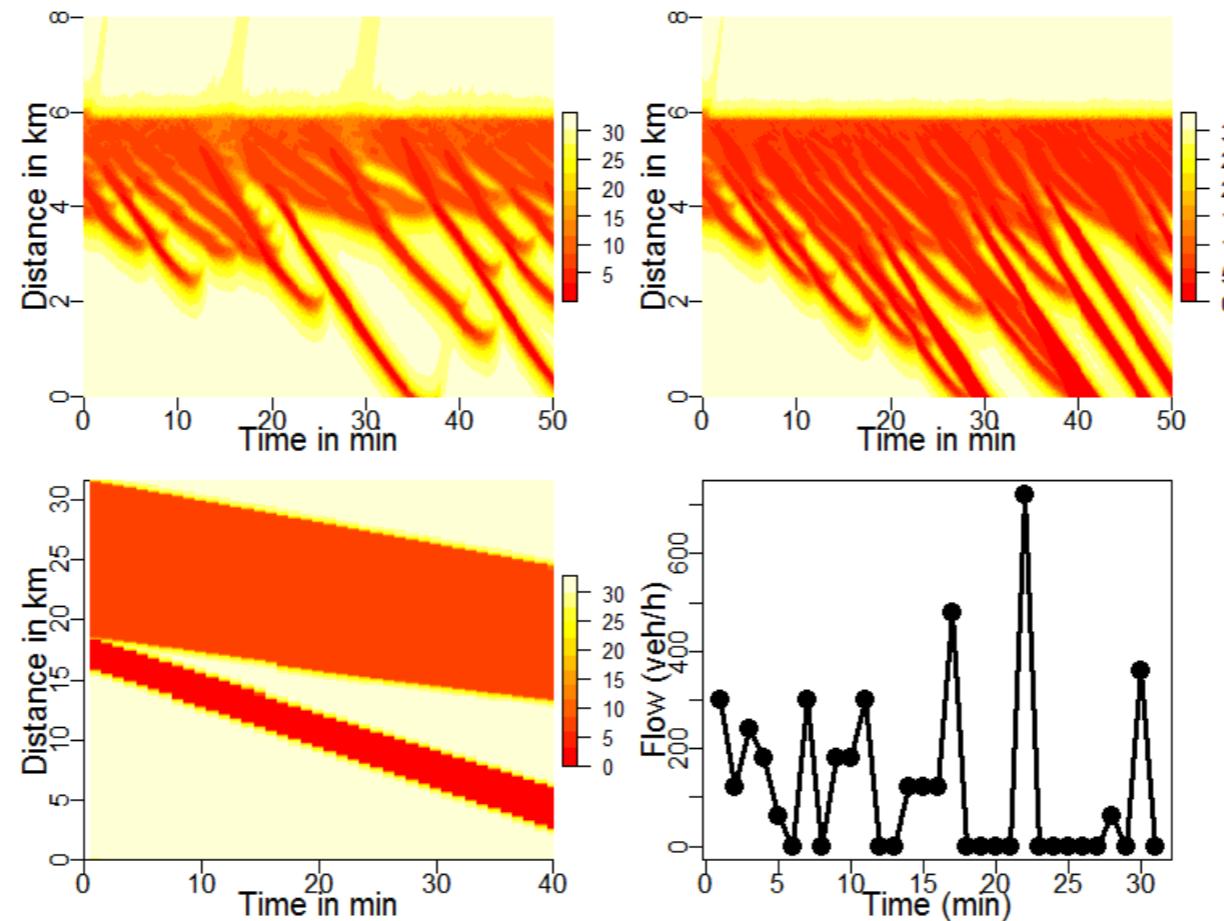
interplay of congested flow upstream of a bottleneck and the wide moving jams

Yang Bo et.al arXiv. 1504.01256

# A new perspective in modelling

- special cases - Occam's razor
- numerical simulation

frequency of jam evolutions and the width of the “synchronized flow”



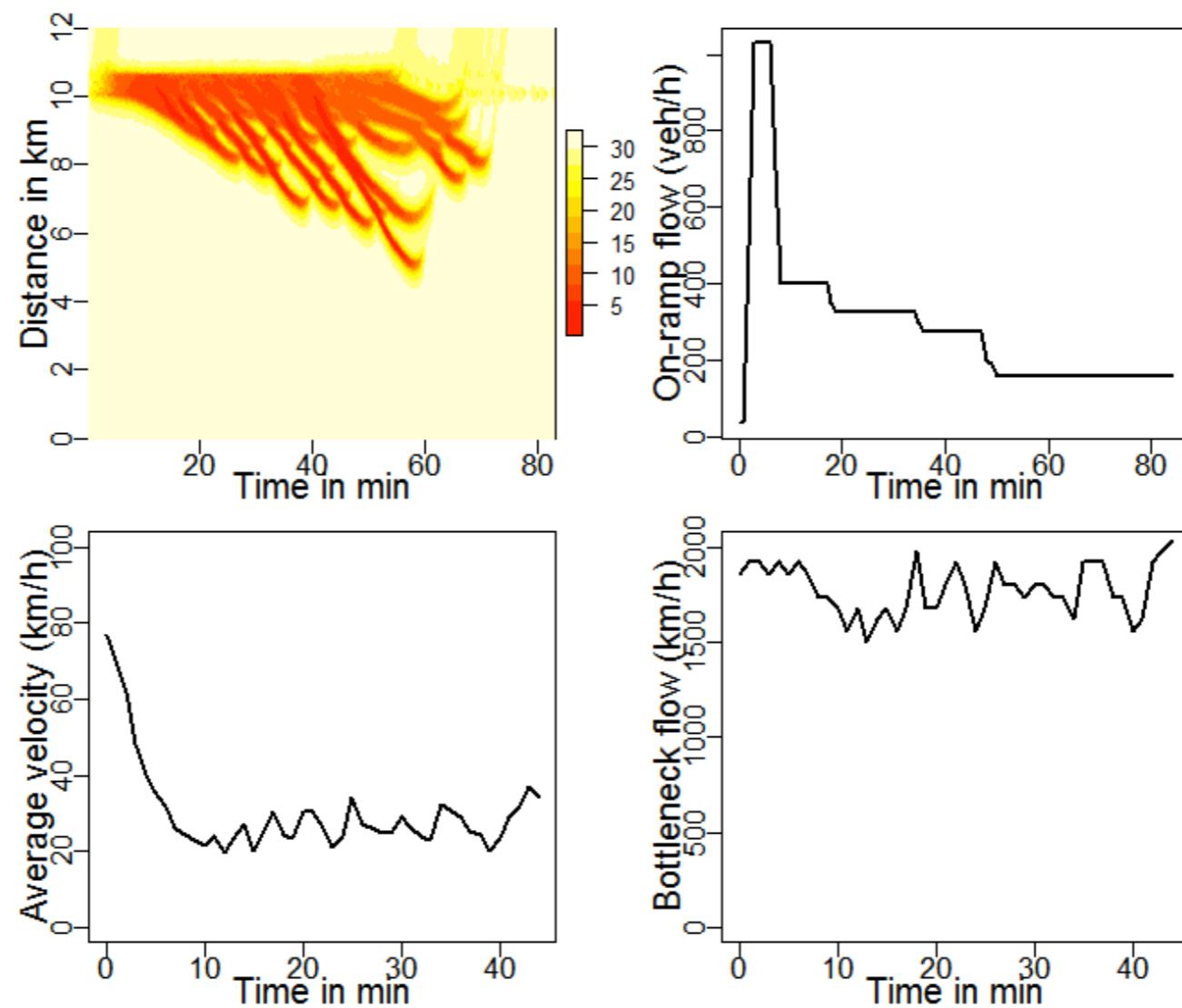
robustness of the wide moving jams

flow fluctuation of the dense traffic

Yang Bo et.al arXiv. 1504.01256

# A new perspective in modelling

- special cases - Occam's razor
- numerical simulation



arbitrary profile of the congested flow

# A new perspective in modelling

- systematic extension of the model
  - More realistic optimal velocity function
  - Making the coefficients of expansion density dependent
  - Include higher order terms

# Empirical verification of microscopic traffic models from the detailed acceleration patterns

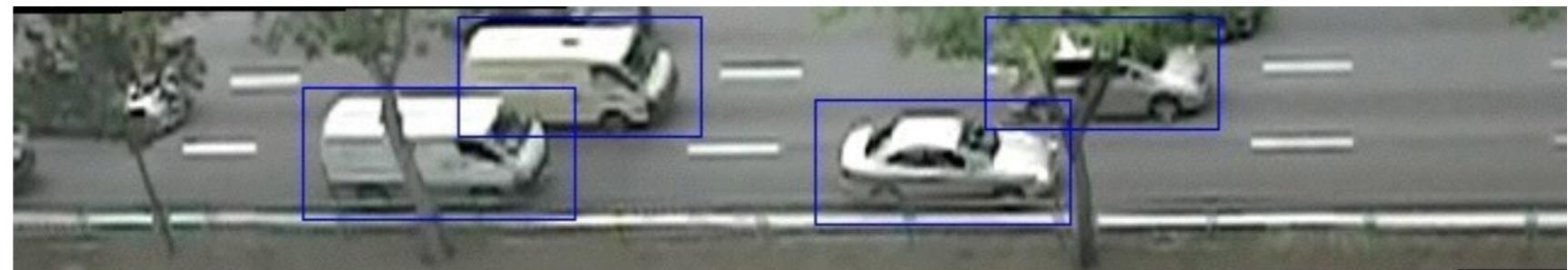
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Institute of High Performance Computing  
Ji Wei Yoon  
UC Berkeley



Institute of  
High Performance  
Computing

# Empirical verifications

- understanding human driving behaviours



- Video-taping of the traffic flow with high frame-rate camera
- Machine learning to identify vehicles in the video
- Virtual sensors to measure the velocity, acceleration as well as headway and approach velocity
- Data aggregation and statistical averaging



# Empirical verifications

- image processing and machine learning

**Positive Samples**

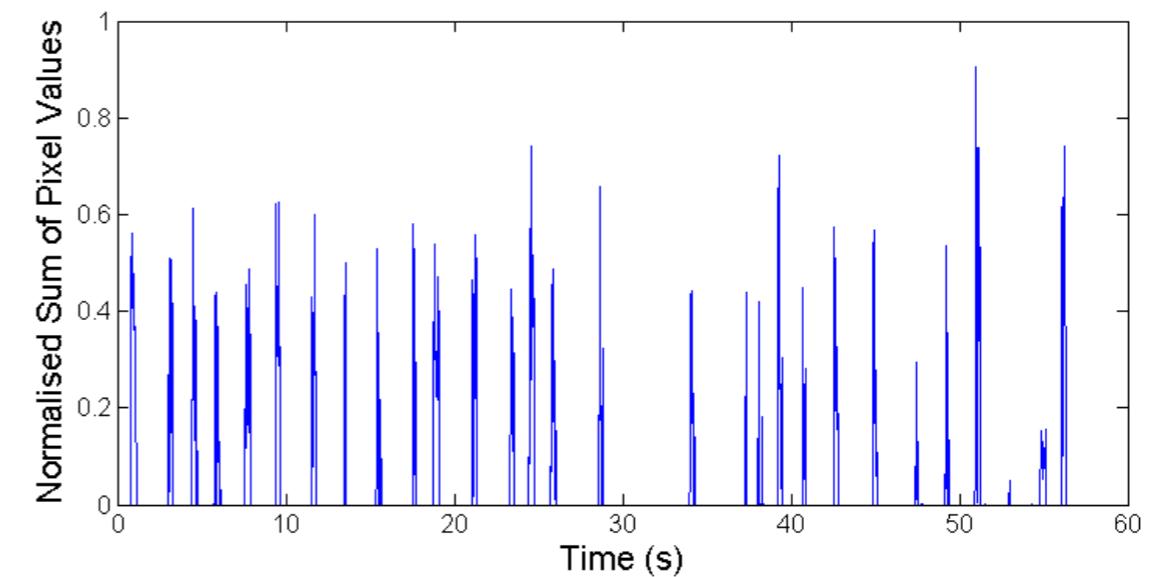
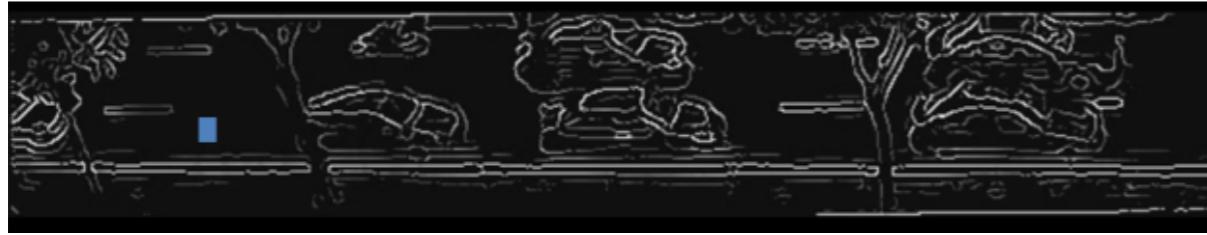


**Negative Samples**



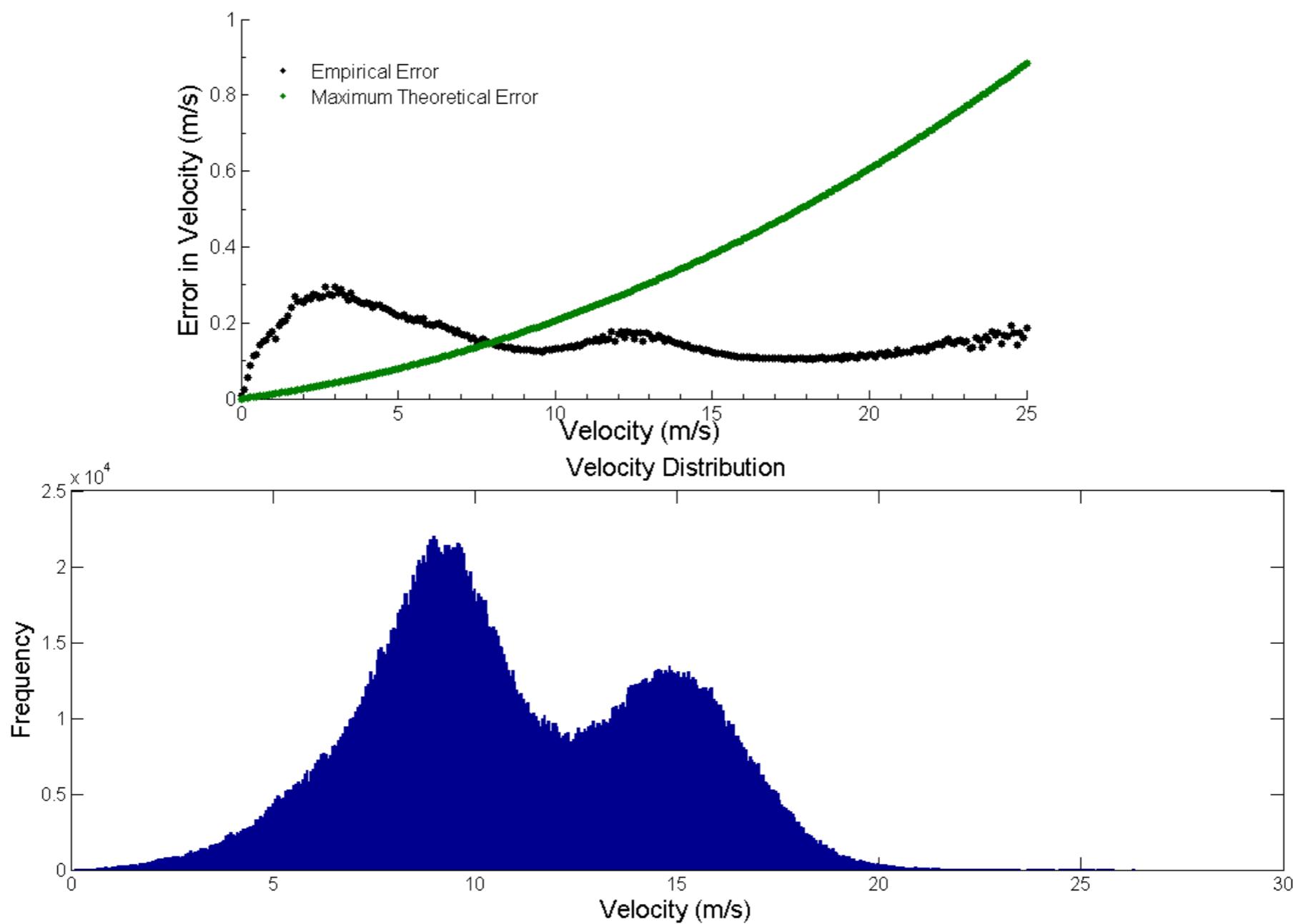
# Empirical verifications

- edge detections and quantitative measurements



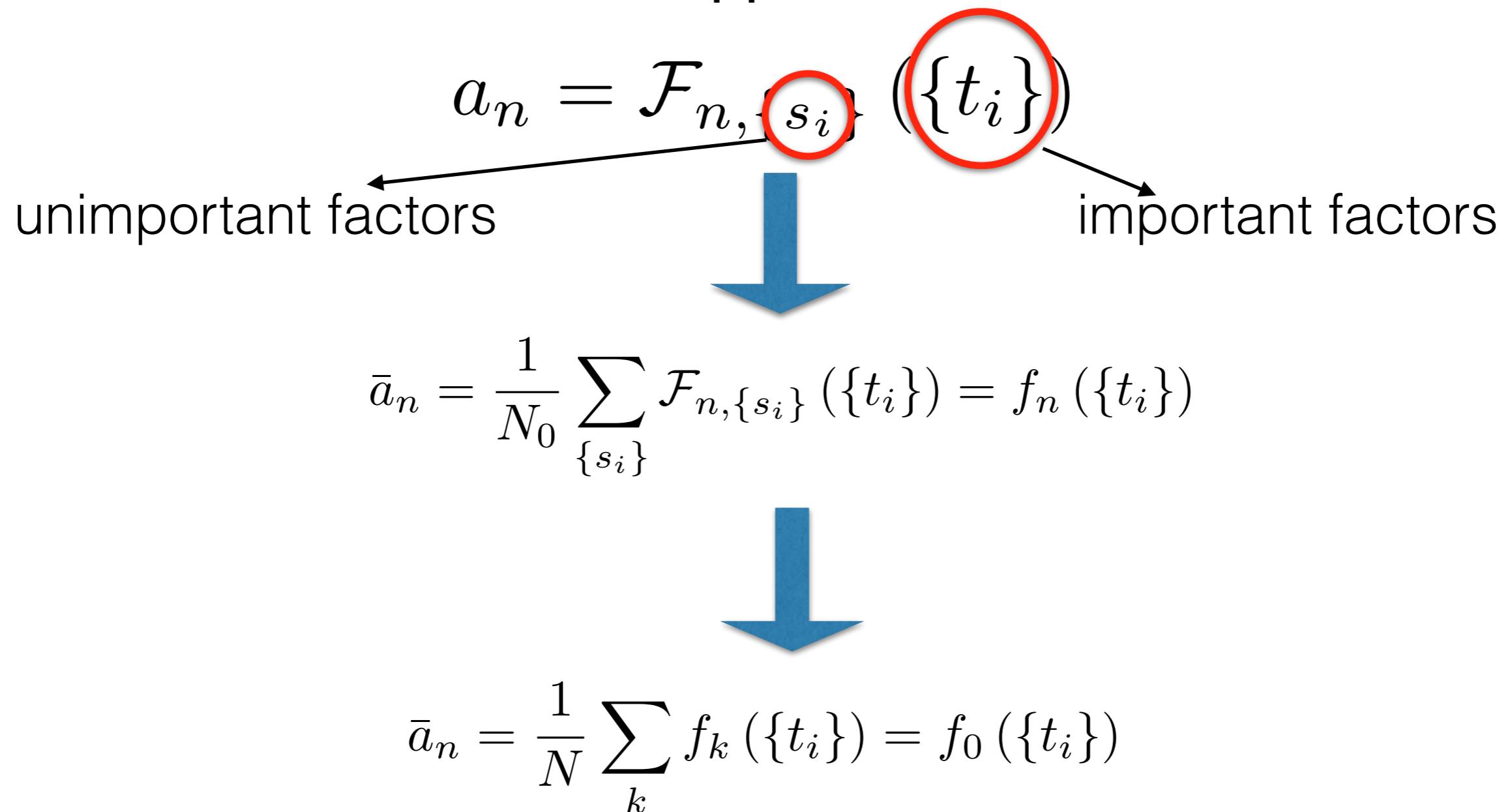
# Empirical verifications

- systematic errors



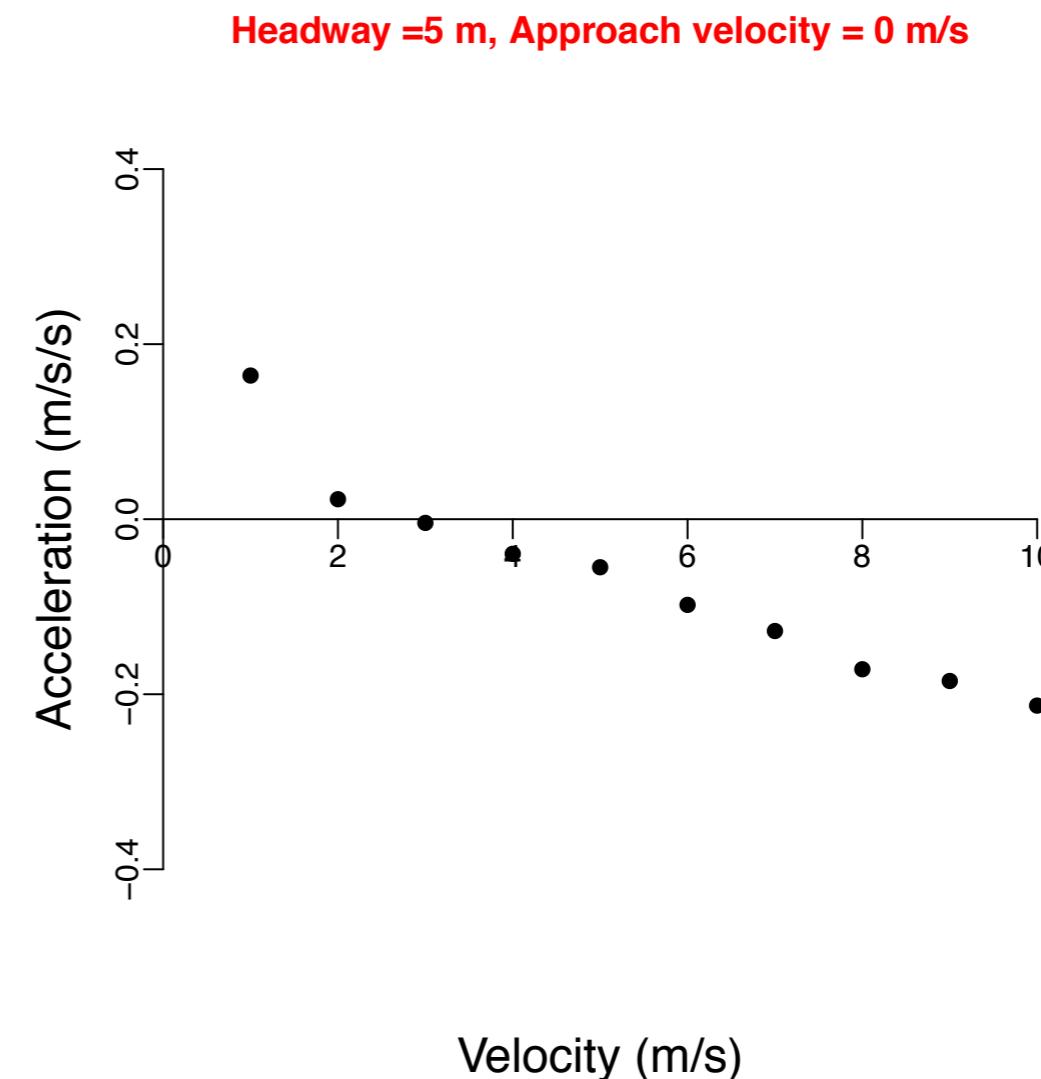
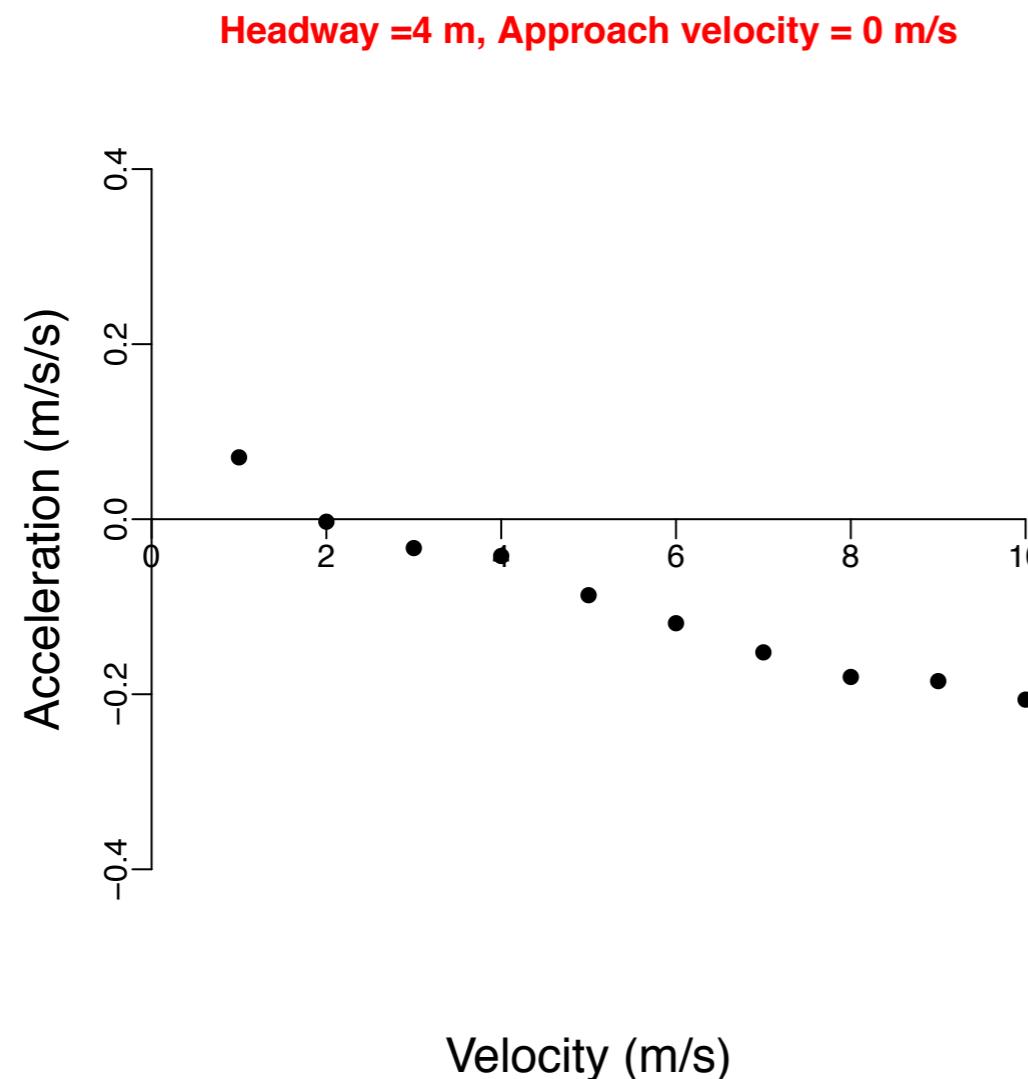
# A master model

- a renormalisation-like approach



# Empirical verifications

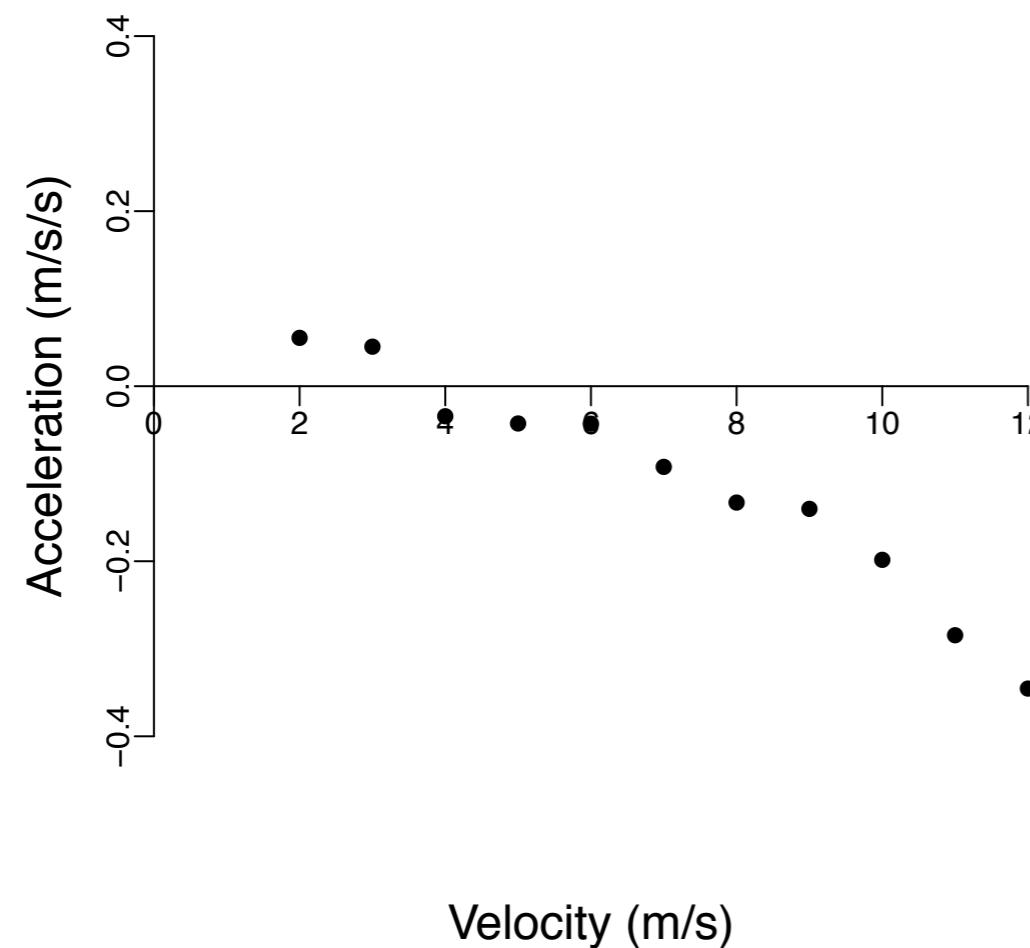
- tentative results



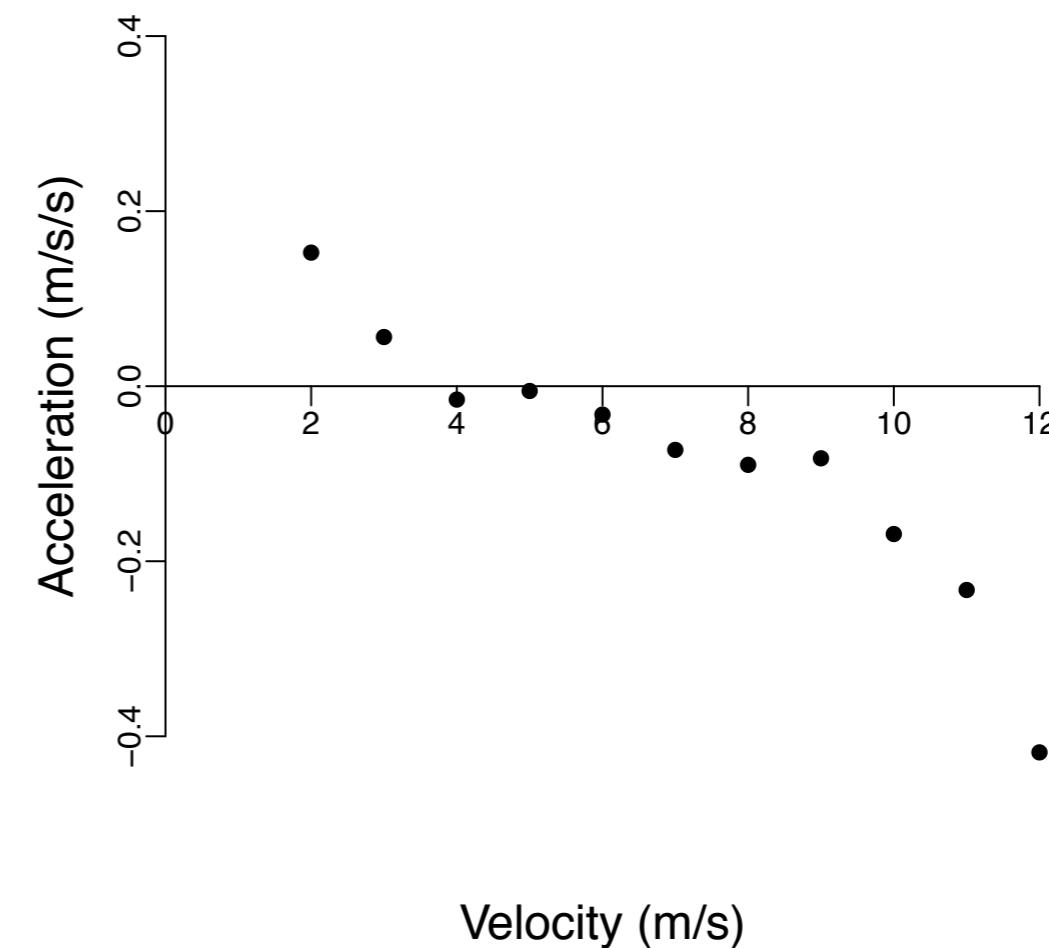
# Empirical verifications

- tentative results

Headway =6 m, Approach velocity = 0 m/s



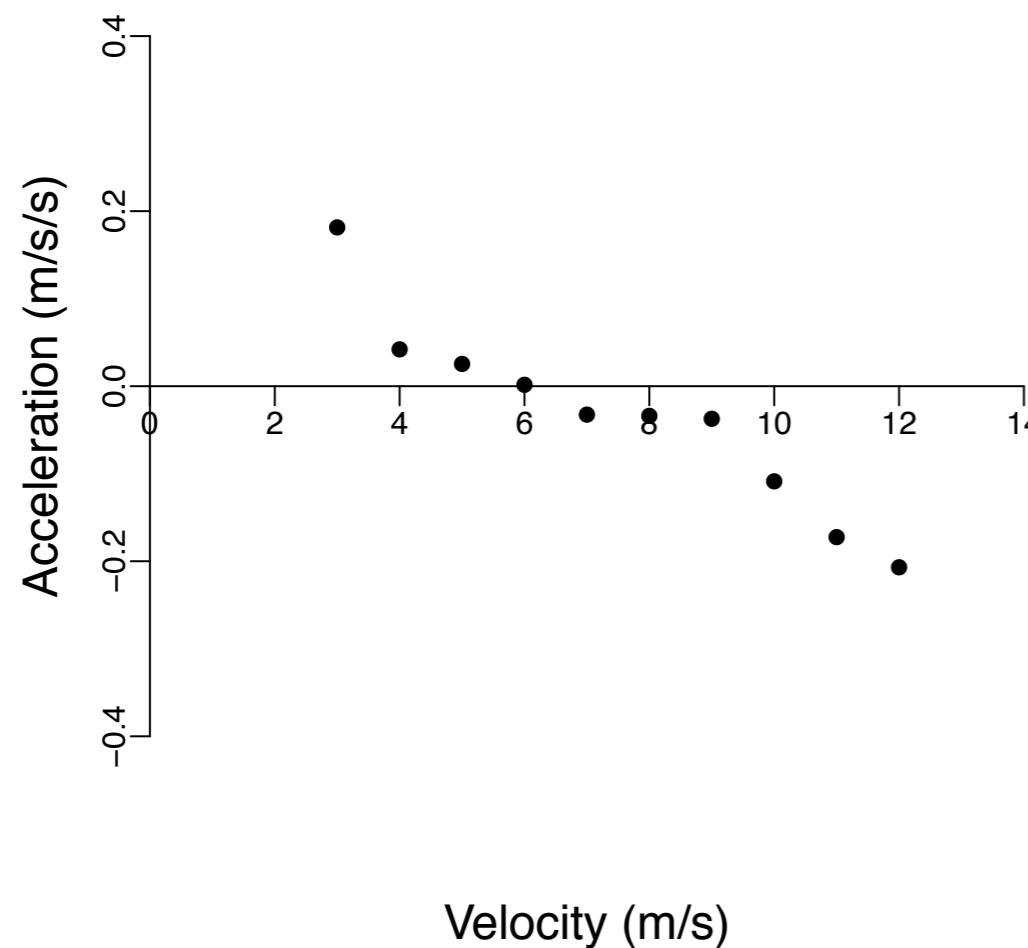
Headway =7 m, Approach velocity = 0 m/s



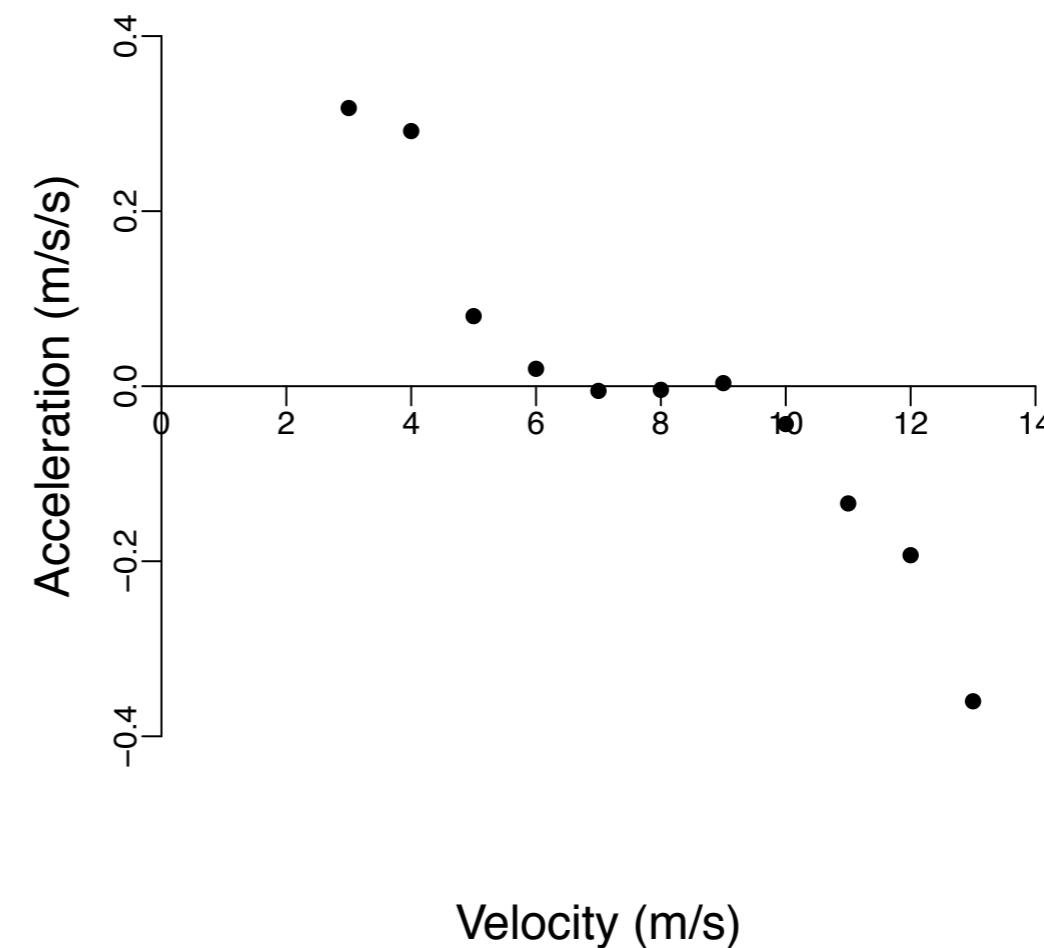
# Empirical verifications

- tentative results

Headway =8 m, Approach velocity = 0 m/s

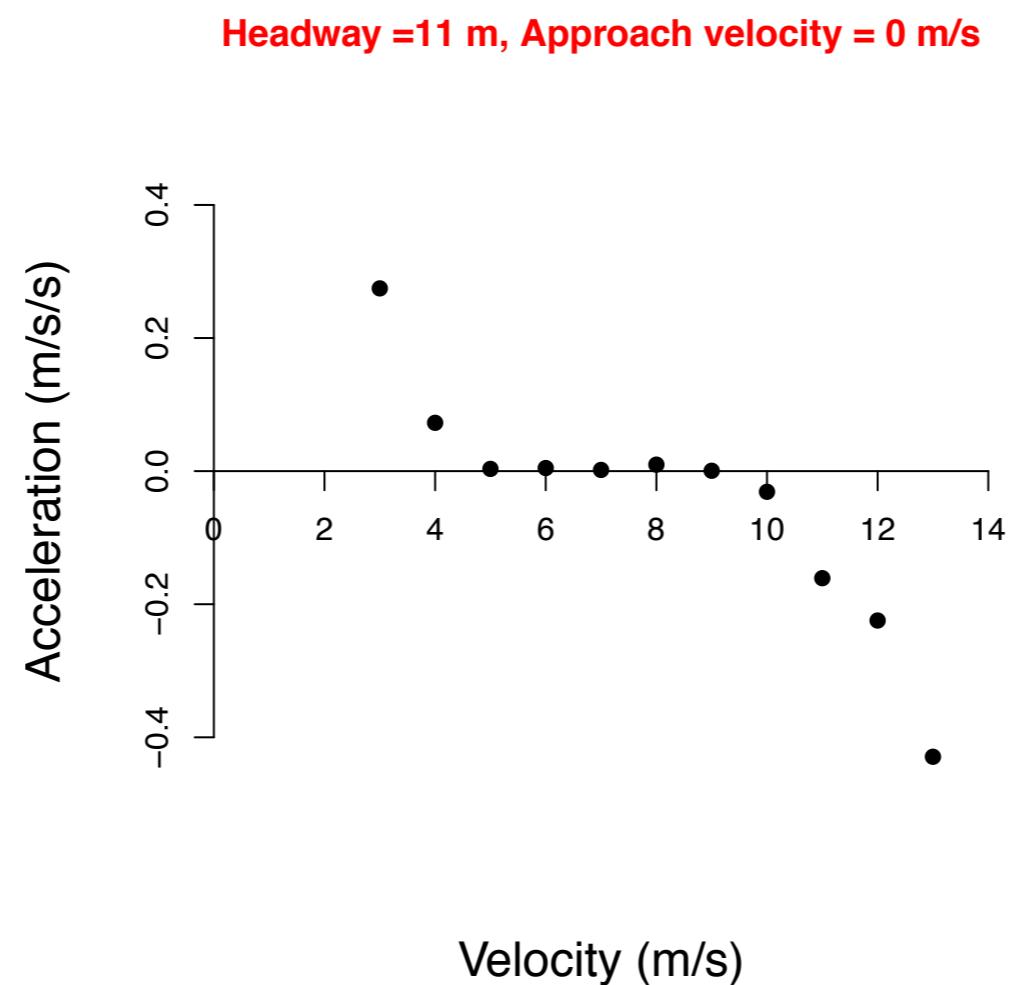


Headway =10 m, Approach velocity = 0 m/s



# Empirical verifications

- tentative results



# Empirical verifications

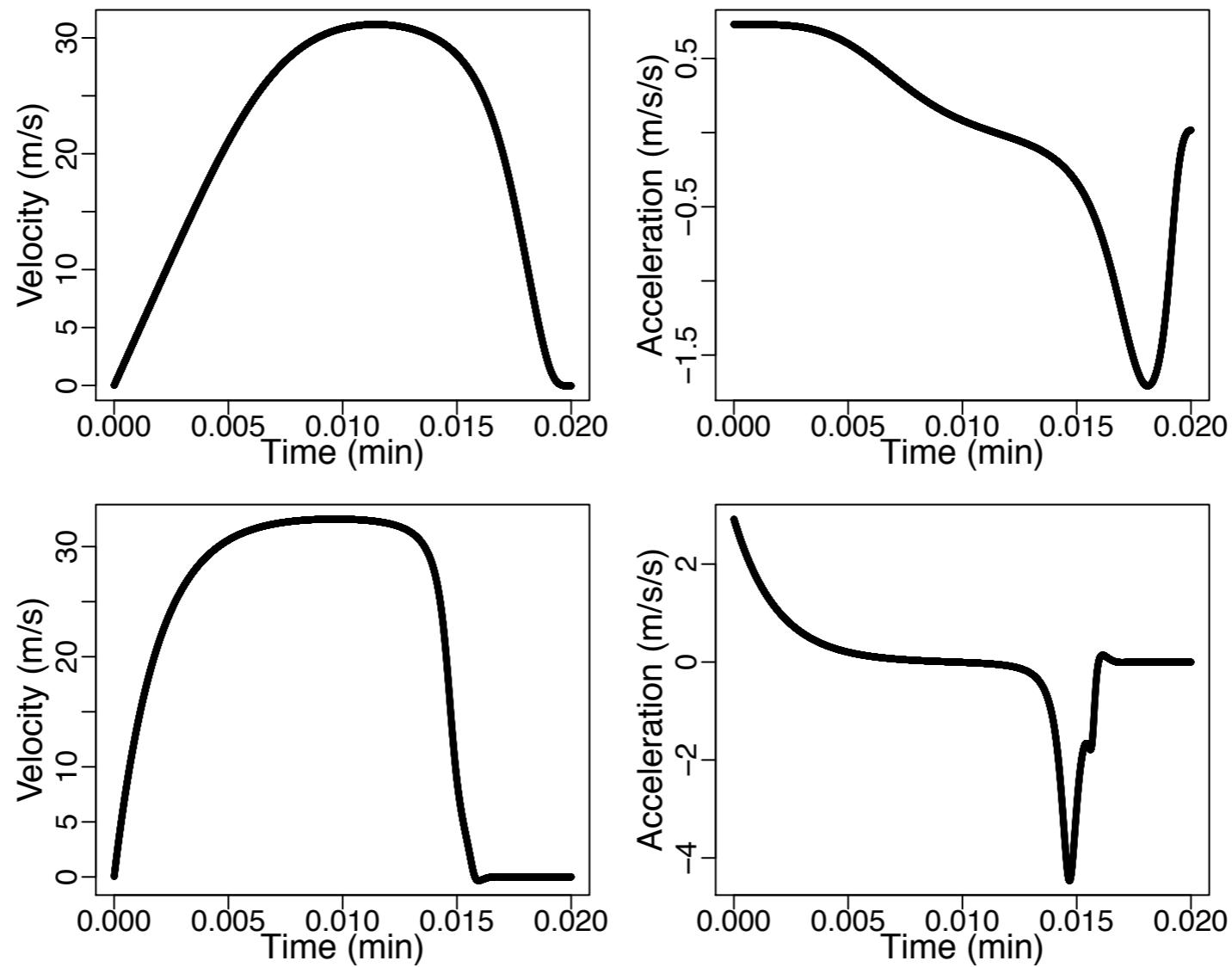
- tentative results - summary

- The renormalisation-like procedure does yield well-defined master microscopic model from the microscopic empirical data
- The empirical data shows evidence of the multitude of steady-states (or at least very long lasting states) that can be characterised as the states in the “synchronized phase”
- There are strong dependence of the expansion of coefficients on the density.

# Thank you very much!

# A new perspective in modelling

- special cases - IDM



Yang Bo et.al PRE 92, 042802 (2015)

