UNIVERSITÄT DUISBURG ESSEN

Open-Minded

Fractal Analysis Of Empirical And Simulated Traffic Time Series

Thomas Zaksek, Michael Schreckenberg

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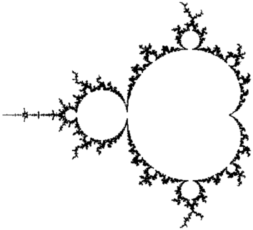
Outline

- Fractal behavior of time series
- Multifractal analysis of time series
- Traffic time series and multifractal behavior

Fractals

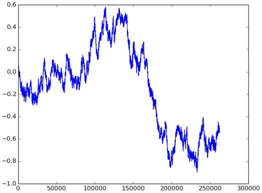
Recapitulation: Fractals

- Exact, statistical or qualitative self similiar behavior over all or many scales
- A fractal (fractional) dimension greater than their topological dimension
- Multifractals: Spectrum of different fractal dimensions



Fractal Time Series

- Time series can not only describe systems with fractal behavoir (i.e complex systems, strange attractors), but be fractal by itself
- Fractal behavoir can be monofractal or multifractal
- Examples: path of brownian motion, time series of fully developed turbulence, stock market time series like S&P500, heartbeat time series

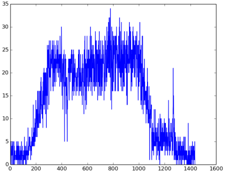


Traffic Time Series

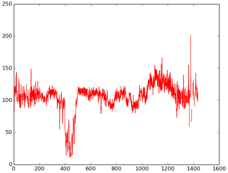
Typical time series of traffic flow measured by loop detectors

- Typical variables: flow, velocity, and density over a time interval
- Different classes of observed vehicles (passenger, motorcycles trucks/transports, public transport)

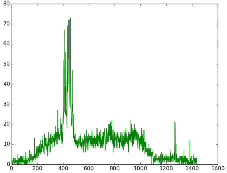
Flow



Velocity



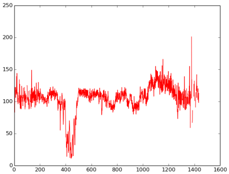
Density

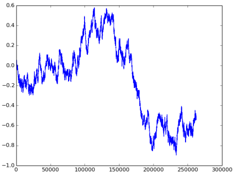


Traffic Time Series

- Exhibit complex behavior, different “phases” (syncronized traffic flow, jamming)
- Nonstationary, trends over many scales
- Resemble time series of fractal or multifractal systems like random walks or time series from hydrodynamics
- Some velocity time series have been shown to exhibit multifractal behavior

velocity time series vs random walk





Multifractal analysis of time series

A framework for multifractal analysis of time series...

1. Divide the timeseries into intervals over a range of scales
2. Calculate roughness grain exponents α for each interval
3. For each scale, consider scaling functions τ
4. Normalize the time series
5. Repeat 1-3 with the normalized series
6. Calculate the Legendre Transform τ^* for each α and τ at different scales and analyse the graphs of $\tau^*(\alpha)$

Dyadic Scales

As an alternative of just varying the interval size, we analyse the time series at dyadic scales:

- For an integer n, we divide the series into 2^n nonoverlapping, equal sized intervals.
- The length of each interval is 2^{-n}
- Each dyadic interval is contained in exactly one dyadic interval of twice the size and spanned by two intervals half the size
- The set of all intervals for given n: ξ_n

Calculating grain exponents α with MDFA

1. "Profile" the time series:

$$X_p(i) = \sum_{i=1}^{N} X_i - \langle X \rangle$$

   for a time series X_i with length N

2. Calculate the Coefficients

$$D(I_(k)) = \left(\frac{1}{\# I_n(k)} \sum_{\chi \in I_n(k)} \left(X_p(\chi) - P_i^{dp} \right)^p \right)^{1/p}$$

   where I is a dyadic interval, $\# I$ is the cardinality of the points in I, P_i^{dp} is a polynomial fit of degree dp fitting the $X \in I$

Calculating grain exponents α with MDFA

3. $\alpha_{n,k}$ is the exponent such that D_x shows power law behavior to the interval size:

$$\alpha_{n,k} = \frac{\log(D_x(I_n(k)))}{\log(2^{-n})}$$

Legendre Spectrum

- Calculate the Legendre Spectrum:

$$\tau^*_{X,n}(\alpha) = \inf_{q \in \mathbb{R}} \{\alpha q - t_{x,n}(q)\}$$

with

$$t_{x,n}(q) = -\frac{1}{n} \log_2 \sum_{I \in \xi_n} 2^{-nq\alpha_X(I)}$$

- Calculate τ^* for a range of $q \in [-50, 50]$

Normalization

Due to method and choosen scaling with dyadic intervals, the grain exponents α drift with scale n. It is necessary to normalize the signal.

- First linearly regress with the following ansatz:

$$n\big(1 - \tau_{X,n}(1)\big) = A + nD$$

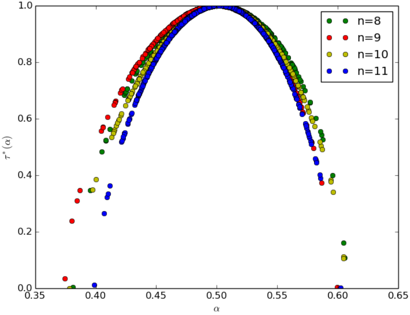
- Find a factor C that zeros A for an arbitrary n_0:

$$n_0\big(1 - \tau_{X/C,n_0}(1)\big) = n_0 D$$

- Recalculate α and τ with the normalized time series X/C
- D estimate for the most likely fractal dimension of X

Legendre Spectrum

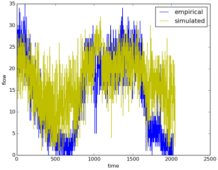
- Analyse the multifractal spectrum $\tau^*(\alpha)$ over α at different scales *n*
- Example: Path of wiener process with 2^{20} steps (theoretically expected $\alpha = 0.5$)



Analysing Traffic Time Series

Empirical and simulated traffic time series in comparison:

- Detector data from the A3 motorway in germany north of cologne, middle lane
- Simulation model:
  - A multilane Nagel-Schreckenberg cellular automaton variety
  - Tuned every minute with empirical data
  - Simulating the freeway network of North-Rhine Westphalia (Germany)



Analysing Traffic Time Series - Motivation

Analyse and compare multifractal spectra of

- Empirical and simulated traffic time series
- Passenger and transport traffic time series
- Traffic time series at different scales (around one day and one week)

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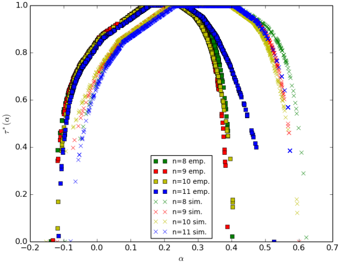
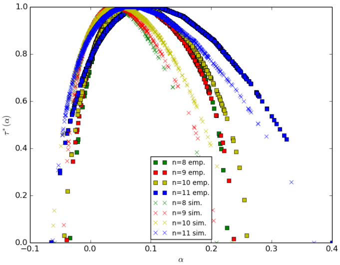
- Comparison of multifractal spectrum for velocity time series (empirical vs. simulated, 16384 datapoints)



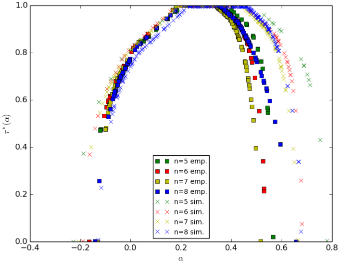
Results

- Comparison of multifractal spectrum for traffic flow time series (empirical vs. simulated, 16384 datapoints)



Results

- Comparison of multifractal spectrum for velocity time series (empirical vs. simulated, 2048 datapoints)



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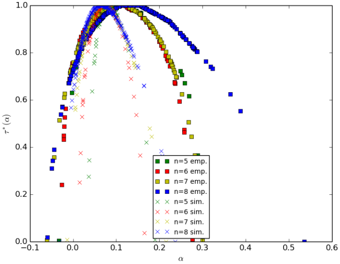

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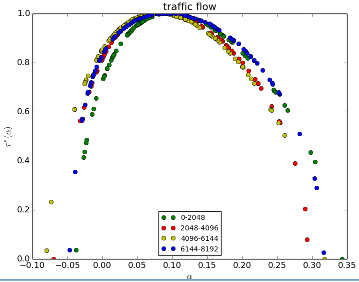
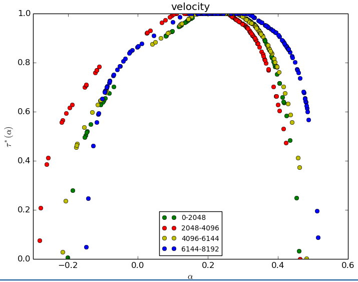
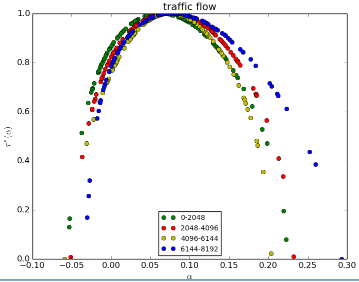
- Computing α, $\tau(\alpha)$ and the likeliest fractal dimension estimator D_g for succeeding time series of 2048 empirical measurements
- $D_{g,flow}$ = 1.916, 1.931, 1.933, 1.927
- $D_{g,vel}$ = 1.829, 1.894, 1.834, 1.874

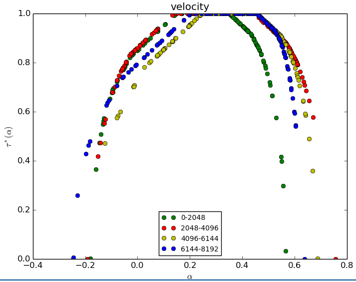




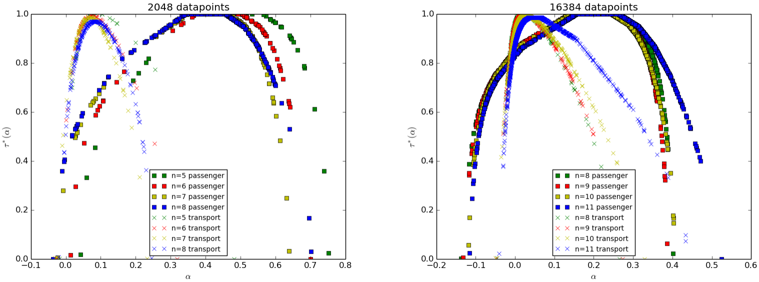
Results

- Computing α, $\tau(\alpha)$ and the likeliest fractal dimension estimator D_g for succeeding time series of 2048 simulated data points
- $D_{g,flow}$ = 1.946, 1.936, 1.932, 1.933
- $D_{g,vel}$ = 1.851, 1.861, 1.754, 1.805





- Comparing multifractal spectra of empirical passenger and transport traffic velocity time series



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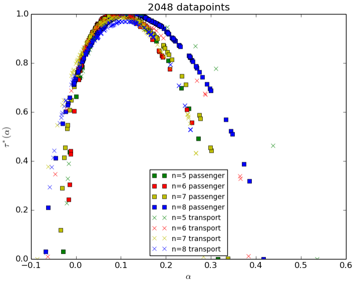

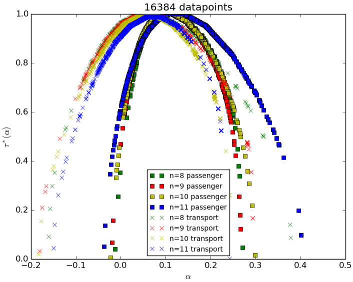

Conclusions

- Both empirical and simulated time series show signatures of fractal and multifractal behavior
- Estimated fractal dimensions of traffic time series about $D_g = 1.8 - 1.9$ (slightly larger than Brownian Motion)
- Simulated time series shows stronger multifractal behavior in velocity time series and weaker multifractal behavior in traffic flow
- Multifractal spectra show little difference over multiple intervals (about 1.5 days each)
- Spectrum of transport velocity time series shows monofractal behavior and slightly larger fractal dimension estimate
- Multifractal spectrum of transport traffic flow similar to passenger traffic flow

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Thank you for your attention