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A continuous Bayesian network for earth dams' risk assessment: methodology and quantification

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Dams' safety is highly important for authorities around the world. The impacts of a dam failure can be enormous. Models for investigating dam safety are required for helping decision-makers to mitigate the possible adverse consequences of flooding. A model for earth dam safety must specify clearly possible contributing factors, failure modes and potential consequences of dam failure. Probabilistic relations between variables should also be specified. Bayesian networks (BNs) have been identified as tools that would assist dam engineers on assessing risks. BNs are graphical models that facilitate the construction of a joint probability distribution. Most of the time, the variables included in a model for earth dam risk assessment involve continuous quantities. The presence of continuous random variables makes the implementation of discrete BNs difficult. An alternative to discrete BNs is the use of non-parametric continuous BNs, which will be briefly described in this article. As an example, a model for earth dams' safety in the State of Mexico will be discussed. Results regarding the quantification of conditional rank correlations through ratios of unconditional rank correlations have not been presented before and are introduced herein. While the complete application of the model for the State of Mexico is presented in an accompanying paper, here some results regarding model use are shown for demonstration purposes. The methods presented in this article can be applied for investigating risks of failure of civil infrastructures other than earth dams.

Keywords: Bayesian networks; dam safety; correlation; flood frequency; experts' assessment; risk management

Introduction

Embankment or earth dams are among the most abundant structures for retaining water. Donnelly (2006) states that embankment dams are the most common type of water-retaining structures. It is no surprise that these types of dams fail more frequently than others. Figure 1 presents the number of dam failures per 10-year periods from 1891 to 1990 and the proportion of the total number of failures corresponding to embankment dams. It was built with data from International Commission on Large Dams (ICOLD) (1995, pp. 38–45). It may be seen that for every 10-year period, between 50% (1891–1900) and 91.67% (1971–1980) of the failures correspond to embankment dams.

The impacts of a dam collapse can be enormous, encompassing the destruction of private housing, transport and public infrastructure, industrial facilities and agricultural land. The losses may also include human harm and serious disruptions in infrastructure operation, leading to significant total economic damages. Consequently, a model for earth dam safety would be convenient for assessing the associated risks. Such a model must specify clearly possible contributing factors, failure modes and potential consequences of dam failure. Probabilistic relations between variables should also be specified. Although the literature reports studies within the dam industry, most are centred on the analysis of specific collapse models, and only few on mathematical models for dam risk assessment (see, e.g. Federal Emergency Management Agency [FEMA], 2007, 2008). In this sense, Bayesian networks (BNs) can be useful for including the aforementioned factors and consequences.

A BN is a probabilistic graphical model that provides an elegant way of expressing the joint distribution of a large number of interrelated variables. In this case, those representing possible causes of failure, the different failure modes and the possible consequences. A BN consists, in general, of a directed acyclic graph (DAG) and a set of (conditional) distributions. Each node in the graph corresponds to a random variable and the arcs (arrows) represent direct qualitative dependence relationships. The absence of arcs guarantees a set of (conditional) independence facts. The direct predecessors (successors) of a node are called parents (children). An univariate

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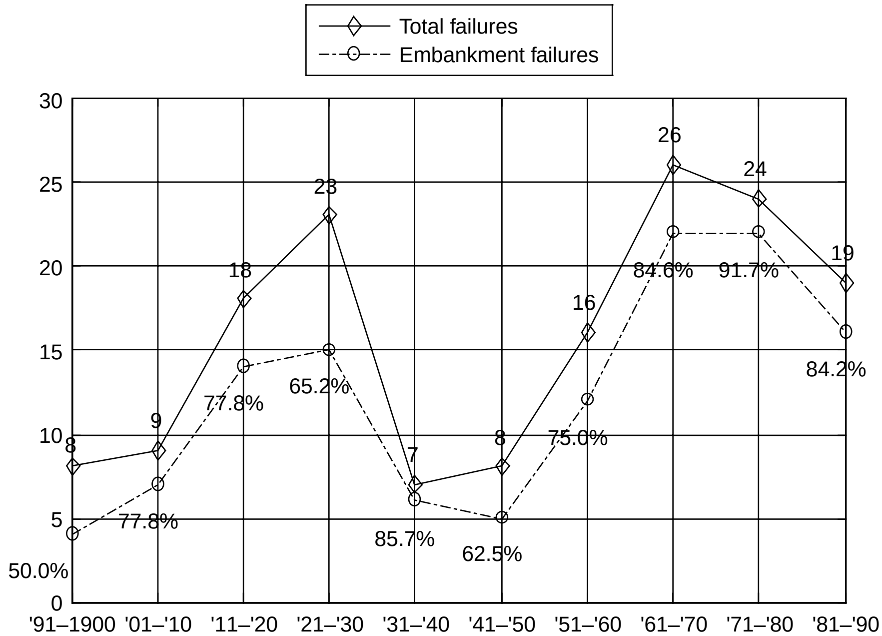

Figure 1. Number of dam failures per 10-year periods from 1891 to 1990, data derived from ICOLD (1995, pp. 38–45).

marginal distribution is specified for each node with no parents, and a conditional distribution is associated with each child node; i.e. the joint distribution over the random variables denoted by the nodes of the graph.

The relatively simple visualisation of the complicated relationships among the random variables is one of the most appealing features a BN model. The main use of BNs is to update distributions of given observations. This is referred to as inference in BNs. They have been successfully used to represent uncertain knowledge, in a consistent probabilistic manner, in a variety of fields. These include health benefits and costs of fish consumption (Jesionek & Cooke, 2007), air transport safety (Ale et al., 2008), among others. For an overview of applications of BNs see Weber, Medina-Oliva, Simon, and Iung (2012).

With the use of BNs, contributing factors, failure modes and possible consequences of dam failure (to carry out risk assessments) may be specified in an intuitive and meaningful way. In this article, a continuous BN for earth dams risk assessment is presented. Dealing with continuous or hybrid (discrete and continuous) BNs imposes challenges with respect to both inference mechanisms and quantification. A review of four methods for hybrid BNs with their advantages and disadvantages is presented in Langseth, Nielsen, Rumí, and Salmerón (2009). Another more recent approach dealing with hybrid BNs combines them with structural reliability methods to create a new computational framework called enhanced BN (eBN) (Straub & Der Kiureghian, 2010).

With these ideas in mind, there are three key objectives of the research. The first purpose is to briefly describe the particular class of BNs that will be dealt with, the so-called non-parametric Bayesian networks (NPBNs)[1] (Hanea, Kurowicka, & Cooke, 2006; Kurowicka & Cooke, 2005). A comparison of NPBNs with other techniques for hybrid BNs is out of the scope of this article. The reader may, however, find a discussion in this context in Hanea and Morales-Napoles (2012) where nine real applications of NPBNs are discussed.

Additionally, in this area of research some techniques for the quantification of NPBNs from, but not limited to, the expert judgement methodology in the absence of data will be reviewed. In fact, the second objective of this article is to present an alternative approach to the one discussed in Morales, Kurowicka, and Roelen (2008) and used in Hanea, Jagtman, and Ale (2012) for the quantification of NPBNs from domain experts. Although not conclusive, previous results indicate that the most accurate way to obtain a subjective measure of bivariate dependence is simply to ask experts to estimate the correlation between the two variables in question (Clemen, Fischer, & Winkler, 2000). Partly motivated by this observation, the alternative approach for quantification of BNs from domain experts presented here is based on estimating directly rank correlations as opposed to estimating them from probabilistic statements.

While the actual application to earth dams' safety in Mexico is presented in the accompanying paper (Delgado-Hernández, Morales-Nápoles, De León Escobedo, & Arteaga-Arcos, 2012), the last objective is thus to provide the methodological framework for the application presented there. However, the methods described in this document are general and may also be used for risk assessment of similar civil infrastructures. The concepts presented should be especially useful for readers that decide to apply the NPBN methodology for risk evaluation. Therefore, one of the main contributions of this work is to provide a methodological guide for the construction and understanding of results of NPBNs for specific applications.

Concepts and definitions

Representing multivariate probability distributions for certain phenomena can be a challenging task. Perhaps the multivariate model which is most widely used is the joint normal distribution. However, many phenomena behave far from normal. This is one of the reasons for researchers to look for alternative models such as copulas. Copulas are part of the building blocks of the graphical models that will be used in this article, and for that reason basic concepts and definitions regarding them are introduced first. The book by Nelsen (1998) is useful to gain a general view within the subject. The book by Joe (1997) is also an important reference on the area. Specifically, bivariate copulas will be of special interest for this research. Note that from now onwards bivariate copula(s) will be referred to as copula(s) throughout the paper unless otherwise specified.

The bivariate copula of two random variables X and Y with cumulative distribution functions F_X and F_Y, respectively, is the function C such that their joint distribution can be written as:

$$F_{X,Y}(x,y) = C_\theta(F_X(x), F_Y(y)).$$

Thus, a copula is a joint distribution on the unit square with uniform univariate margins. Measures of association such as the rank correlation may be expressed in terms of copula (Nelsen, 1998). Notice that the copula is indexed by the parameter θ, which is related to measures of dependence such as the rank correlation coefficient.

Dependence measures

The product moment correlation of random variables X and Y with finite expectations $E(X)$, $E(Y)$ and finite variances $\mathrm{var}(X)$, $\mathrm{var}(Y)$ is

$$\rho(X,Y) = \frac{E(XY) - E(X)E(Y)}{\sqrt{\mathrm{var}(X)\mathrm{var}(Y)}}.$$

Whenever possible $\rho(X,Y)$ will be denoted as $\rho_{X,Y}$. The rank correlation of random variables X and Y with cumulative distribution functions F_X and F_Y is

$$\begin{aligned} r(X,Y) &= \rho(F_X(x), F_Y(y)) \\ &= \frac{E(F_X(x)F_Y(y)) - E(F_X(x))E(F_Y(y))}{\sqrt{\mathrm{var}(F_X(x))\mathrm{var}(F_Y(y))}}. \end{aligned}$$

The rank correlation is the dependence measure of interest and can be defined as the product moment correlation of the ranks of variables X and Y, and measures the strength of monotonic relationship between variables. As before, whenever possible, $r(X,Y)$ will be denoted as $r_{X,Y}$. Rank correlations may be realised by copulas and may be expressed in the following way:

$$r(X,Y) = 12\int_0^1\int_0^1 C_\theta\big(u,v\big)\mathrm{d}u\mathrm{d}v - 3,$$

where C_θ is the copula joining variables X and Y. The conditional rank correlation of X and Y given Z is

$$r(X,Y|Z) = r(\tilde{X},\tilde{Y})$$

in which $(\tilde{X},\tilde{Y})$ has the distribution of (X,Y) given $\mathbf{Z} = \mathbf{z}$. Notice that when $\mathbf{Z}$ is a random vector, then $\mathbf{Z} = \mathbf{z}$ means $Z_1 = z_1, \ldots, Z_n = z_n$ for n conditioning variables. Again, whenever possible the conditional rank correlation will be denoted as $r_{i,j|k,\ldots,n}$.

Partial correlations are also of interest in this article. These can be defined in terms of partial regression coefficients. Consider variables X_i with mean zero and standard deviation σ_i for $i = 1, \ldots, n$ and let the numbers $b_{1,2;3,\ldots,n}, \ldots, b_{1,n;2,\ldots,n-1}$ minimise

$$E[(X_1 - b_{1,2;3,\ldots,n}X_2 - \ldots - b_{1,n;2,\ldots,n-1}X_n)^2].$$

The partial correlation of X_1 and X_2 based on $X_3,\ldots,X_n$ is

$$\rho_{1,2;3,\ldots,n} = \mathrm{sgn}\,(b_{1,2;3,\ldots,n})(b_{1,2;3,\ldots,n}b_{2,1;3,\ldots,n})^{1/2},$$

where sgn is the signum function of $b_{1,2,3,\ldots,n}$. Partial correlations can be computed recursively from correlations (see Yule & Kendall, 1965):

$$\rho_{1,2;3,\ldots,n} = \frac{\rho_{1,2;3,\ldots,n-1} - \rho_{1,n,3,\ldots,n-1}\rho_{2,n;3,\ldots,n-1}}{((1-\rho^2_{1,n;3,\ldots,n-1})(1-\rho^2_{2,n;3,\ldots,n-1}))^{1/2}}, \quad (1)$$

One of the most common bivariate copulas is the normal copula. Denote by Φ_ρ the bivariate standard normal cumulative distribution function with correlation ρ, and Φ^{-1} the inverse of the univariate standard normal distribution function, then

$$C_\rho(u,v) = \Phi_\rho(\Phi^{-1}(u), \Phi^{-1}(v)), \quad (u,v) \in [0,1]^2,$$

is called the normal copula. Notice that ρ is a parameter of the normal copula. In case of a conditional bivariate copula the parameter $\rho_{1,2;3,\ldots,n}$ is used instead. The relationship between the correlation of the normal copula r (the rank correlation of the normal variables) and the parameters ρ (or $\rho_{1,2;3,\ldots,n}$ for conditional copulas) is known and given by the following formula (Pearson, 1907, p. 12):

$$\rho = 2\sin\left(\frac{\pi}{6}r\right). \tag{2}$$

Bayesian networks

The influences between adjacent nodes in a BN may be probabilistic or deterministic.[2] The parent set of variable X_i will be denoted as $\mathrm{Pa}(X_i)$. To specify a joint distribution through a BN the graph must be specified together with conditional probability functions of each variable given its parents:

$$f_{X_1,\ldots,X_n} = \prod_{i=1}^{n} f_{X_i|\mathrm{Pa}(X_i)}. \tag{3}$$

If $\mathrm{Pa}(X_i) = \emptyset$ then $f_{X_i|\mathrm{Pa}(X_i)} = f_{X_i}$. In the case that all nodes in the BN are discrete, the functions to be specified are the conditional probability tables (CPT) of each node given its parents. When variables are continuous, one possibility is to discretise them into a large enough number of states and use discrete BNs. In general, the number of probabilities to be assessed K for a discrete BN on n nodes with k_i states for each X_i for $i = 1, \ldots, n$ is

$$K = \sum_{j\in S} k_j - |S| + \sum_{l\in C}(k_l - 1)\prod_{m\in M(l)} k_m \tag{4}$$

where $S = \{X_j{:}\mathrm{Pa}(X_j) = \emptyset\}$ and $C = \{X_l{:}\mathrm{Pa}(X_l) \neq \emptyset\}$, $|S| + |C| = n$ and $M(l) = \{m{:}\ X_m \in \mathrm{Pa}(X_l)\}$. It is clear from Equation (4) that K grows rather quickly as the number of states of each X_i grows. This is one of the main drawbacks of discrete BNs (Cowell, Dawid, Lauritzen, & Spiegelhalter, 1999; Hanea et al., 2006).

Non-parametric continuous BNs

Another way to deal with continuous nodes in a BN is with the use of normal (Shachter & Kenley, 1989) or discrete-normal BNs. For discrete-normal BNs (Cowell et al., 1999), unconditional means and conditional variances must be assessed for every normal variable. For each arc, partial regression coefficients must be assessed. In the absence of data, the assessment of partial regression coefficients and conditional variances by experts is difficult if the normality assumption does not hold. Continuous-discrete non-parametric BNs (Hanea et al., 2006; Kurowicka & Cooke, 2005) have been developed to cope with some of the drawbacks that a discrete (and a discrete-normal) model imposes.

Essentially, a *non-parametric continuous* (*or continuous-discrete*) BN (*NPCDBN*) is a directed acyclic graph whose nodes are associated with arbitrary univariate random variables (that has an invertible distribution function) and whose arcs are associated with parent–child (un)conditional rank correlations. For each variable X_i in the network with m parents $X_j = \mathrm{Pa}_1(X_i), \ldots, X_k = Pa_m(X_i)$, associate the arc $\mathrm{Pa}_j(X_i) \rightarrow X_i$ with the conditional rank correlation:

$$\begin{cases} r\left(X_i, \mathrm{Pa}_j(X_i)\right), & j = 1 \\ r\left(X_i, \mathrm{Pa}_j(X_i)|\mathrm{Pa}_1(X_i), \ldots, \mathrm{Pa}_{j-1}(X_i)\right), & j \geq 2 \end{cases} \tag{5}$$

The assignment is vacuous if $\mathrm{Pa}(X_i) = \emptyset$. Notice that the ordering $1, \ldots, m$ is not unique and need not correspond to the original labelling of the variables $j, \ldots, k$. These assignments together with a copula family for which zero correlation implies independence and with the conditional independence statements embedded in the graph structure of a BN are sufficient to construct a unique joint distribution. Moreover, the conditional rank correlations in Equation (5) are algebraically independent, hence any number in $(-1, 1)$ can be attached to the arcs of a NPCDBN (Hanea et al., 2006).

Choosing the normal copula presents advantages with respect to other copulas for building the joint distribution. Observe that for the normal copula relation Equation (2) holds, and since conditional correlations are equal to partial correlations then Equation (1) may be used to compute the correlation matrix corresponding to the graph. Moreover, since for the joint normal distribution, conditional distributions are also normal, then analytical updating is possible by this choice (Hanea et al., 2006, p. 724).

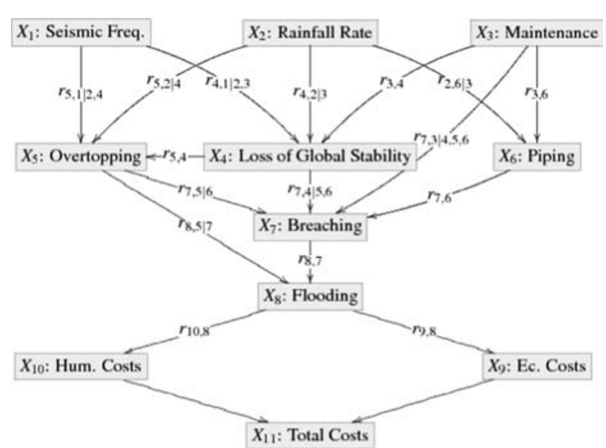

Figure 2. Example of NPCDBN on 11 nodes.

Figure 2 illustrates an example of a NPCDBN on 11 nodes. To illustrate the assignment of rank and conditional rank correlations consider variable X_4 with parents $\mathrm{Pa}(X_4) = \{\mathrm{Pa}_1(X_4) = X_3,\ \mathrm{Pa}_2(X_4) = X_2,\ \mathrm{Pa}_3(X_4) = X_1\}$. Notice that the indexing in the parent set is not the same as the original index of variables and is not unique. According to the assignment in Equation (5), the unconditional rank correlation $r_{4,3}$ is used first. Then the conditional rank correlations $r_{4,2|3}$ and $r_{4,1|3,2}$ are assigned, and so on with the other nodes.

The structure of the BN gives information about the dependence structure in the joint distribution. For example, variables X_1, X_2 and X_3 are independent of each other and their dependence with other variables is described in terms of (conditional) rank correlations. In general every variable in the graph is conditionally independent of its ancestors given its parents. For a more complete description of the semantics of a BN see Pearl (1998, Chapter 3).

Non-parametric continuous BNs in earth dam safety in central Mexico

Variables involved in the analysis of contributing factors, failure modes and consequences of dam failure are mostly continuous. For example, rainfall rate is measured in millimetres, the piping is measured in 1 feet per second or some similar unit and consequences of flooding are measured in monetary or some utility unit such as US$, € or quality adjusted life years (qalys). Often, these continuous quantities are discretised to facilitate the calculus of probabilities. Then, variables as the ones described previously are given states such as *low*, *medium* and *high*.

The model represents a case study for the State of Mexico in the central region of the country. Seven dams were identified, their names being: Embajomuy (E), San Joaquín (SJ), José Trinidad Fabela (JTF), Dolores (D), José Antonio Alzate or San Bernabé (JAA), Ignacio Ramírez or La Gavia (IR) and El Guarda (EG). Their heights range from 15 to 24 m, their ages from 36 to 66 years and their capacities from 0.55 to 52.5 hm^3. They are used mainly for purposes of irrigation, flooding prevention and hydroelectric power generation.

Variables of interest

The BN presented in Figure 2 represents the model for earth dams' safety to be discussed in this article. The nodes of the graph represent univariate random variables with invertible distribution functions and the arcs represent rank and conditional rank correlations between these nodes. Ten variables were identified as most relevant for the study. Their description, units and source for the univariate marginal distributions is presented in Table 1. The quantification of rank correlations will be discussed later.

Table 1. Variables of interest (SEJ = Structured Expert Judgement).

X_1: *Seismic frequency*. It refers to the distribution of earthquakes >5.5 per year, in Richter magnitude scale, between 2000 and 2008 for the locations of interest. Data is available from the Mexican National Seismographic System. Approximated by a Gamma distribution with shape parameter 9.79 and scale parameter 0.68	Field data
X_2: *Rainfall rate*. Average value over the seven-basins (i.e. the area of influence of the 7 dams of interest) of the five-days moving averages in [mm/day]. Data is available from 'ERIC', a Mexican database from 1961 to 1998. A short overview of ERIC may be found in Carrera-Hernández and Gaskin (2008). The empirical distribution for this variable was used	Field data
X_3: *Maintenance*. Is the number of years between maintenance activities which would lead the dam to an 'as good as new' condition	SEJ
X_4: *Overtopping*. Water level from the crest during an event in which such a level may increase beyond the total embankment height (mm)	SEJ
X_5: *Loss of global stability*. Distribution of the safety factors (resisting moment/causing moment), for each of the seven dams based on their design geometrical features. This variable helps to quantify sliding probabilities. The so called 'Swedish method' is used for calculating such factors (SRH, 1976). Approximated by a Weibull distribution with scale parameter 1.76 and shape parameter 7.57 (unitless)	Field data
X_6: *Piping*. Distribution of water flowing through the embankment that causes its internal erosion apart from the spillway and outlet pipe torrents (l/s)	SEJ
X_7: *Breaching*. Refers to the average breach width, i.e. the mean of both superior and inferior breach widths, due to embankment's crest erosion (m). Calculated with the methods reported in Wahl (1998) with data from SRH (1976). An empirical distribution was used	Field data
X_8: *Flooding*. Average water level per day in the downstream flooded area during a dam failure event (mm/day)	SEJ
X_9: *Economic cost*. Both public and private total costs, due to all possible damages in infrastructures (e.g. schools, hospitals, bridges, roads, transport systems), fields (e.g. farms, crops), housing, supply, commercial and entertainment centres, caused by a flooding, consequence of a dam failure. It is measured in current US$	SEJ
X_{10}: *Human costs*. Both public and private total costs over a time period equivalent to the maximum human remaining life span, due to all possible damages, health and life losses, caused by a flooding, consequence of a dam failure. It is measured in current US$	SEJ

Note: SEJ = Structured Expert Judgement.

Variables are broadly grouped into three categories: contributing factors (seismic frequency, rainfall rate and maintenance), failure modes (loss of global stability, piping, overtopping and breaching) and consequences (flooding, human and economic costs). The model has been kept small in order to make the quantification feasible through structured expert judgement (SEJ) which is a necessary step due to the lack of field data. This restriction leads to simplifications which, nevertheless it is strongly believed, do not inhibit the ability of the model to provide decision-makers with valuable information. Moreover, the model as presented here should serve as benchmark for a larger-scale model that would encompass larger structures across the country.

The elicitation of marginal distributions from experts has been discussed extensively in the past. In 17 years, approximately 67,000 experts' subjective probability distributions have been elicited from 521 domain specialists with the classical model (Cooke, 1991; Cooke & Goossens, 2008). The classical model aims at rational consensus. Basically, it is a performance-based linear pooling (weighted average) model. Loosely speaking, in addition to the variables of interest, experts are queried about seed or calibration variables. The latter are variables whose value is known to the analyst but not to the expert at the moment of the elicitation. The objective of seed variables is to evaluate experts as uncertainty assessors and not directly on their field of expertise. Experts' performance as uncertainty assessors is measured by the calibration and information scores from seed variables. The calibration score, ranging from 0 to 1, is the probability that the divergence between the experts' assessments and the observed values on seed variables might have arisen by chance. A score higher than a significant level α (for instance 0.05) means that the experts' assessments are statistically supported by the set of seed variables.

On the other hand, the information score measures the degree to which a distribution is concentrated relative to a background measure. The uniform and log uniform are the most common choices for the background measures. The overall information score is the mean of information scores for each variable. The weights in the classical model are proportional to the product of calibration and information and satisfy a proper scoring rule constraint. These are used to derive the weights entered in the linear pooling.

A total of four experts participated in the quantification of the model in Figure 2. The elicitation of variables of interest followed the same lines as the elicitation of calibration variables. One example of a calibration variable for this elicitation is: *Consider the 7-day moving average of the daily average precipitation (mm) from the two stations related to the Embajomuy Dam from January 1961 to August 1999 in ERIC II of CONAGUA (Carrera-Hernández, & Gaskin, 2008). What is the maximum moving average for the time period of reference? (please state the 5th, 50th and 95th percentiles of your uncertainty distribution)*. The format for variables of interest is similar.

In total three questions about seismicity, four over general characteristics of the sample of seven dams in the State of Mexico, nine over precipitation and two about water discharge were used as calibration variables. Notice that experts could give an exact answer to the question above based on data. However, the objective is to measure whether experts' assessments are accurate in a statistical sense. For example, for a well-calibrated expert, one would expect that approximately 1 true value from the set of calibration variables would fall below the 5th percentile of the experts' assessments, 1 above the 95th, 12 above the median and 12 below the median and so on. The elicitation of all rank and conditional rank correlations required in the model was carried out through SEJ.

Quantification of dependence

The literature available to guide researchers in the elicitation of a joint distribution is much less than that available for the elicitation of univariate distributions (O'Hagan, 2005). Methods for eliciting rank correlations from experts have been proposed and used in the past (Clemen et al., 2000). One of the options is directly asking experts for an estimate of the rank correlation between pairs of variables. Another one is asking experts for estimates of some other quantity, for example a conditional probability of exceedence, or probabilities of concordance or discordance prior to estimating, under certain assumptions (an underlying copula, for example), the rank correlation of interest. In Morales et al. (2008), the elicitation of rank correlations from domain experts is carried out through conditional probabilities of exceedence, the approach is also applied in Hanea and Morales-Napoles (2012). When the (conditional) rank correlations of one child with many parents should be elicited from experts, this approach would require the elicitation of conditional probabilities on a large number of conditioning events, making it unintuitive to some specialists.

Additionally, while not conclusive, previous results indicate that the most accurate way to obtain a subjective measure of bivariate dependence is simply to ask the expert to estimate the correlation between the two variables in question (Clemen et al., 2000); a total of 20 questions were asked to every expert. For each child node, experts were asked to rank parent variables according to some criteria of their preference. For example, the largest unconditional rank correlation with the child in absolute value was used. These questions were meant to help experts in their next assessments.

Then for unconditional rank correlations experts would assess $P(X_{\text{child}} > median|X_{\text{parent}} > median)$. This question is translated in the application for expert *A* and the rank correlation $r_{4,3}$ as: *Consider a situation in which the number of years to undertake maintenance action* (X_3) *is above its median (30 years). What is the probability that the loss of global stability* (X_4) *is also above its median (1.66)*? Observe that, for this question, the median value for the loss of global stability is equal for all experts since it comes from data. The number of years to undertake maintenance actions, however, comes from expert judgement and is different across experts. When combining experts' assessments it is necessary to take these kinds of differences into account.

The relationship between $P(X_4 > median|X_3 > median)$ and $r_{4,3}$ under the normal copula assumption is shown in Figure 3. Once the expert has given an assessment for the probability of exceedence, the analyst then finds the *r* which satisfies the expert's conditional probability assessment and transforms this into the corresponding rank correlation using the inverse function of Equation (2).

If an expert believes that X_4 and X_3 are independent, then because of the zero independence property, zero correlation would entail that the experts' answer for $P(X_4 > median|X_3 > median) = 0.5$. A conditional probability value in the interval [0, 0.5) corresponds to negative correlation. Positive correlation is attained when $P(X_4 > median|X_3 > median) > 0.5$. In the example presented, expert A believes that when the number of years between rebuild maintenance are more than 30 for earth dams, then the probability that the loss of global stability is above 1.66 is as low as 0.3. This assessment corresponds to a value of $r_{4,3} = -0.57$.

Once the experts have provided an assessment for the unconditional rank correlation of a given child node to its first parent, ratios of rank correlations of the child with the rest of the parents to the first rank correlation elicited are asked to them. In the example, the question for *expert A* would be: *Given your previous estimates, what is the ratio of* $r_{4,2}/r_{4,3}$? The answer to this question depends on a number of points. First, the statement 'given your previous estimates' in this case refers to a value of $r_{4,3} = -0.57$, which is computed during the elicitation, and then given to the expert. Observe that $r_{4,3}$ is negative. Hence, if the expert believes that the loss of global stability is positively correlated with rainfall rate, then the ratio $r_{4,2}/r_{4,3} < 0$, negative correlation between the loss of global stability and rainfall rate corresponds to $r_{4,2}/r_{4,3} > 0$. Notice that $|r_{4,2}/r_{4,3}| > 1$ corresponds to the expert's belief that $|r_{4,2}| > |r_{4,3}|$ and an analogous situation is observed when $|r_{4,2}/r_{4,3}| < 1$.

Figures 4 and 5 depict the relationship of $r_{4,2}/r_{4,3}$ to $r_{4,2|3}$ and $r_{4,2}$, respectively. Observe the ratio $r_{4,2}/r_{4,3} \in (-1.38, 1.38)$. Because X_2 and X_3 are independent, $r_{4,2}/r_{4,3}$ is in a symmetric interval around zero. However, if X_3 and X_2 were correlated, additional constraints would be present for the assessment of $r_{4,2}/r_{4,3}$ (see Morales et al., 2008). The assignment of rank correlations in relation (5) entails that $r_{4,2|3}$ can be chosen freely and hence $r_{4,2|3} \in (-1, 1)$. This is shown in Figure 4. However, $r_{4,2}$ will be restricted by the expert's previous answer. In this case, given the expert's answer that $P(X_4 > median|X_3 > median) = 0.3 \rightarrow r_{4,3} = -0.57$, $r_{4,2} \in (-0.78, 0.78)$ as observed in Figure 5.

In the example, expert *A* has stated that $r_{4,2}/r_{4,3} = 0.25$. This entails that he believes that the rank correlation between

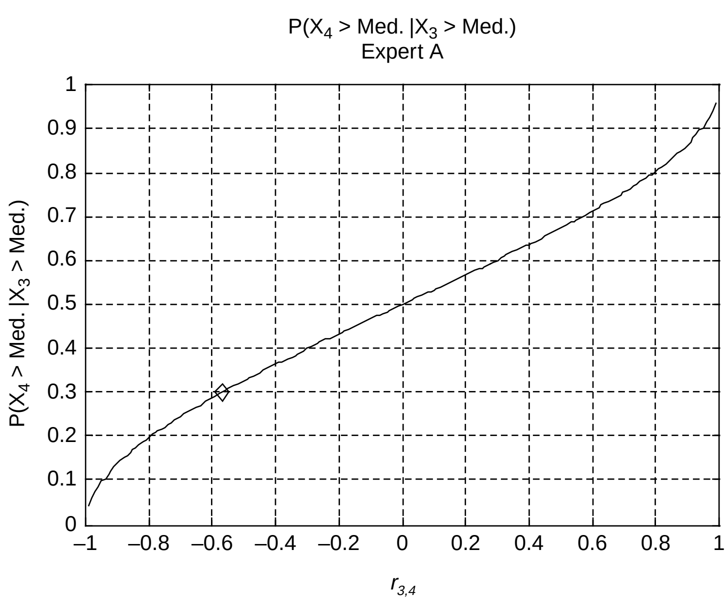

Figure 3. $P(X_4 > median|X_3 > median)$ versus $r_{4,3}$ for expert *A* under the normal copula assumption.

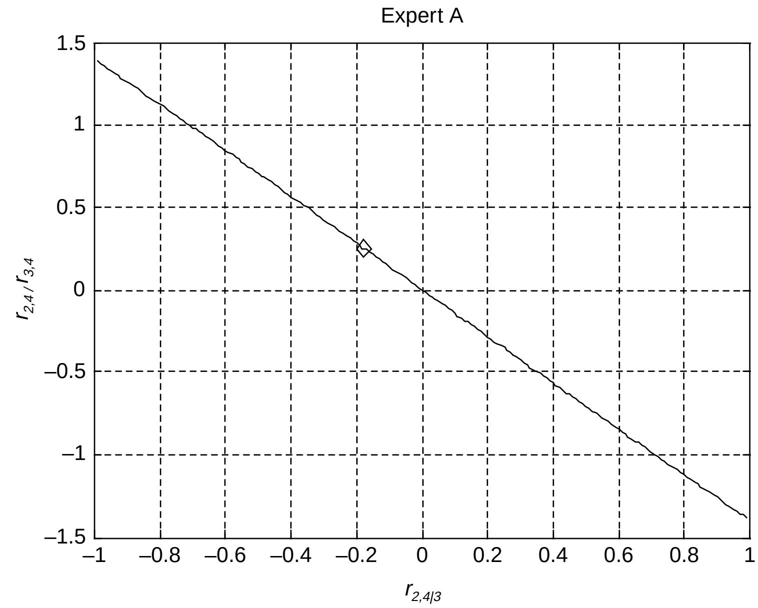

Figure 4. $r_{4,2}/r_{4,3}$ versus $r_{4,2|3}$ for *expert A* under the normal copula assumption given $P(X_4 > median|X_3 > median) = 0.3$ and X_2 and X_3 are independent.

the loss of global stability and rainfall rate is negative and smaller in absolute value than $r_{4,3}$. In particular, $r_{4,2} = -0.14$ for him. Using the dependence statements embedded in the graph and Equations (2) and (1), the analyst may compute the value of $r_{4,2|3} = -0.17$ for this expert. According to Figure 2, loss of global stability has one last parent node for which the conditional rank correlation $r_{4,1|2,3}$ needs to be specified. Thus, the expert would be confronted with the following question: *Given your previous estimates, what is the ratio of* $r_{4,2}/r_{4,3}$? In this case, the statement 'given your previous estimates' refers to $r_{4,3} = -0.57$ and $r_{4,2} = -0.14$. Again, the possible answers to this question are restricted by the expert's previous answers. The relationship of $r_{4,1}/r_{4,3}$ with $r_{4,1|2,3}$ and $r_{4,1}$ is shown in Figures 6 and 7, respectively; in this case, given the expert's previous answers $r_{4,1} \in (-0.77, 0.77)$.

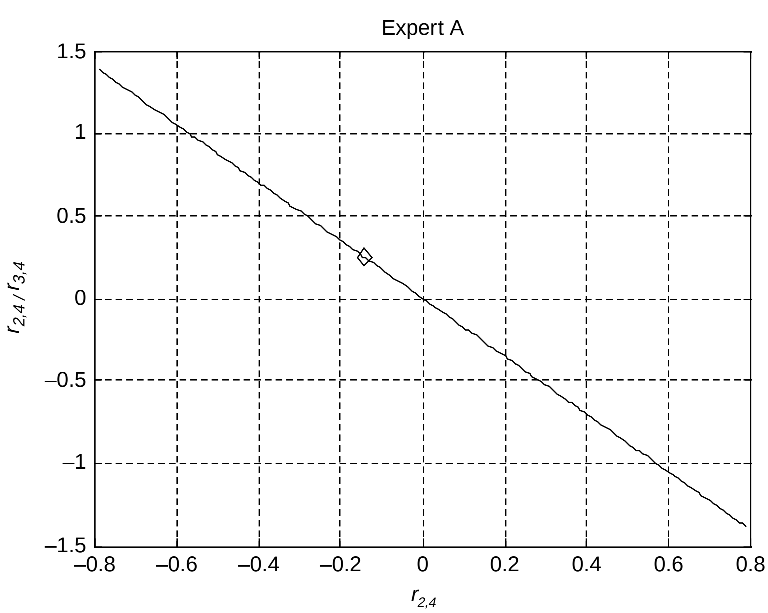

Figure 5. $r_{4,2}/r_{4,3}$ versus $r_{4,2}$ for *expert A* under the normal copula assumption given $P(X_4 > median|X_3 > median) = 0.3$ and X_2 and X_3 are independent.

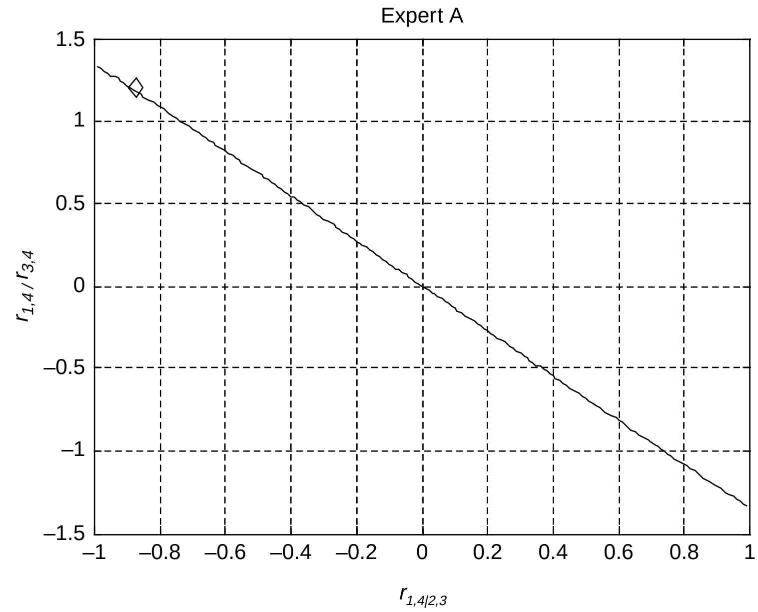

Figure 6. $r_{4,1}/r_{4,3}$ versus $r_{4,1|2,3}$ for *expert A* under the normal copula assumption given $P(X_4 > median|X_3 > median) = 0.3$, $r_{4,2}/r_{4,3} = 0.25$ X_1 and X_2 and X_3 are independent.

In the example, expert *A* believes that there is a negative correlation between seismic frequency and the loss of global stability. The expert also believes that the correlation between the loss of global stability and the years until rebuild maintenance is smaller than the correlation between the loss of global stability and the seismic frequency. The value for the ratio stated by expert *A* is $r_{4,1}/r_{4,3} = 1.2$. This value corresponds to $r_{4,1} = -0.68$. Again, by using the dependence statements embedded in the graph and Equations (2) and (1), $r_{4,1} = -0.68$ is translated into a conditional rank correlation of $r_{4,1|2,3} = -0.87$.

The full quantification of rank correlations in the BN in Figure 2 follows the same procedure as the one described for the elicitation of $r_{4,3}$, $r_{4,2|3}$ and $r_{4,1|2,3}$. For each expert a BN is obtained. Once a joint distribution is available for each expert these may be combined via the weight derived from the calibration and information scores in the classical model, as described in Morales-Nápoles (2010).

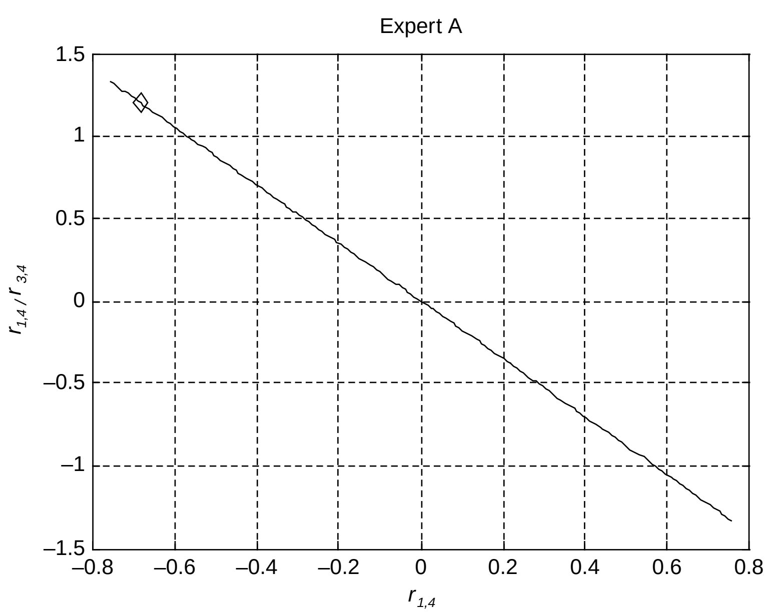

Figure 7. $r_{4,1}/r_{4,3}$ versus $r_{4,1}$ for *expert A* under the normal copula assumption given $P(X_4 > median|X_3 > median) = 0.3$, $r_{4,2}/r_{4,3} = 0.25$ and X_2 and X_3 are independent.

Elicitation results for the BN for earth dam safety in the State of Mexico

In total four experts (*A, B, C* and *D*) participated in the elicitation exercise. Three of them hold positions at the National Water Commission (CONAGUA) in the State of Mexico. The other holds a position in the Municipality of Zinacantepec as water manager. Two of the experts are lecturers in the civil engineering programme at the Autonomous University of the State of Mexico (UAEM). Having invited them to take part in the research, individual interviews were agreed with each expert and the questionnaire included six questions to elicit marginal distributions, 20 to elicit the rank and conditional rank correlations from Figure 2 and 20 calibration variables.

After processing the results of the elicitation, no individual expert had a calibration score corresponding to a *p*-value above 5%. Information scores were within a 2.5 factor for the four experts. *Expert B* had the lowest calibration score, however was also the most informative. In contrast, *Expert A* had the largest calibration score and is the least informative. This is a recurrent pattern; however, low informativeness does not translate automatically into better calibration (Cooke & Goossens, 2008, p. 669).

In the classical model, experts' scores are the product of calibration and information scores. Experts with a calibration score lower than the significance level are weighted with zero. The global weight decision-maker (GWDM) uses the information score per variable, while the item weight decision-maker (IWDM) uses the average information score. In the example, the GWDM and IWDM were equal, hence in the future the latter exclusively will be referred. In this elicitation, the GWDM is better calibrated than each expert individually; however, its information scores are lower than the information scores of each expert individually. The calibration score of the GWDM is still lower than 5% which fails to confer the requisite level of confidence for the study.

On the other hand, the equal weight decision-maker (EWDM) does not take into account experts' performance on calibration variables and gives equal weight to all experts. However, this was the only expert with a *p*-value above 5%. For this reason, the EWDM is the recommended choice and further analysis will be conducted with this combination. The cost of this choice is in the information scores (about three times smaller than the GWDM). The results of the combination scheme are presented in Table 2.

Model use

One of the advantages of BNs is that once the model has been fully quantified, whenever evidence becomes available the joint distribution may be updated accordingly. In the case of NPBNs, one may exploit the fact that the normal copula is used to build the joint distribution. This process is referred to as conditionalisation or inference. Figure 8 shows the model from Figure 2 in UNINET, a stand-alone application developed at the Technical University of Delft for NPBNs (Cooke et al., 2007). Marginal distributions are represented by histograms. Means and standard deviations (after the ± sign) are shown. Figure 8 will be referred to as baseline or unconditional case. Figure 9 presents the model when 18 earthquakes per year, 16 mm/day of rain in a five-day moving average period and 60 years without rebuilt maintenance are observed.

Table 2. EWDM dependence estimates for the dam safety model from Figure 2.

(Un)Conditional rank correlation		(Un)Conditional rank correlation	
$r_{3,6}$	0.1799	$r_{7,6}$	0.5025
$r_{2,6\|3}$	0.1067	$r_{7,5\|6}$	0.5793
$r_{4,3}$	− 0.3996	$r_{7,4\|5,6}$	− 0.4647
$r_{4,2\|3}$	− 0.3164	$r_{7,3\|4,5,6}$	0.2212
$r_{4,1\|2,3}$	− 0.4307	$r_{8,7}$	0.1135
$r_{5,4}$	− 0.1278	$r_{8,5\|7}$	0.0669
$r_{5,2\|4}$	0.1711	$r_{10,8}$	0.1384
$r_{5,1\|2,4}$	0.3025	$r_{9,8}$	0.2281

The original baseline case histograms are shown in Figure 9 in grey, whereas the updated belief (conditional distribution) is shown in black. Conditional expectations and standard deviations are also visible. The reader may observe that the possible failure modes (*overtopping*, *loss of global stability*, *piping* and *breaching*) present larger changes than the possible consequences (*flooding* and *costs*). This is because the experts' combined opinion says that given a dam failure in the State of Mexico, the consequences are almost the same regardless of the size of the failure. Similar type of analysis may be used in risk and reliability analysis. This is illustrated with the two examples presented in Figures 10 and 11.

Maintenance decisions

To illustrate one possible use of the model in decision-making under uncertainty, an example is given. The focus will be on the variables maintenance, loss of global stability and rainfall rate. It is worth to be mentioned that since no records of maintenance activities were available for all seven structures, it was treated as a random variable (there are about 56 earth dams in the State of Mexico with no maintenance records). Such a decision also helped to express the variation across the ages of the dams. Note, however, that maintenance could have been a decision variable rather than a random variable or a deterministic value provided that there were records of the conservation activities.

Conditionalisation needs not be done in the direction of the arcs. For instance, to make inference with respect to

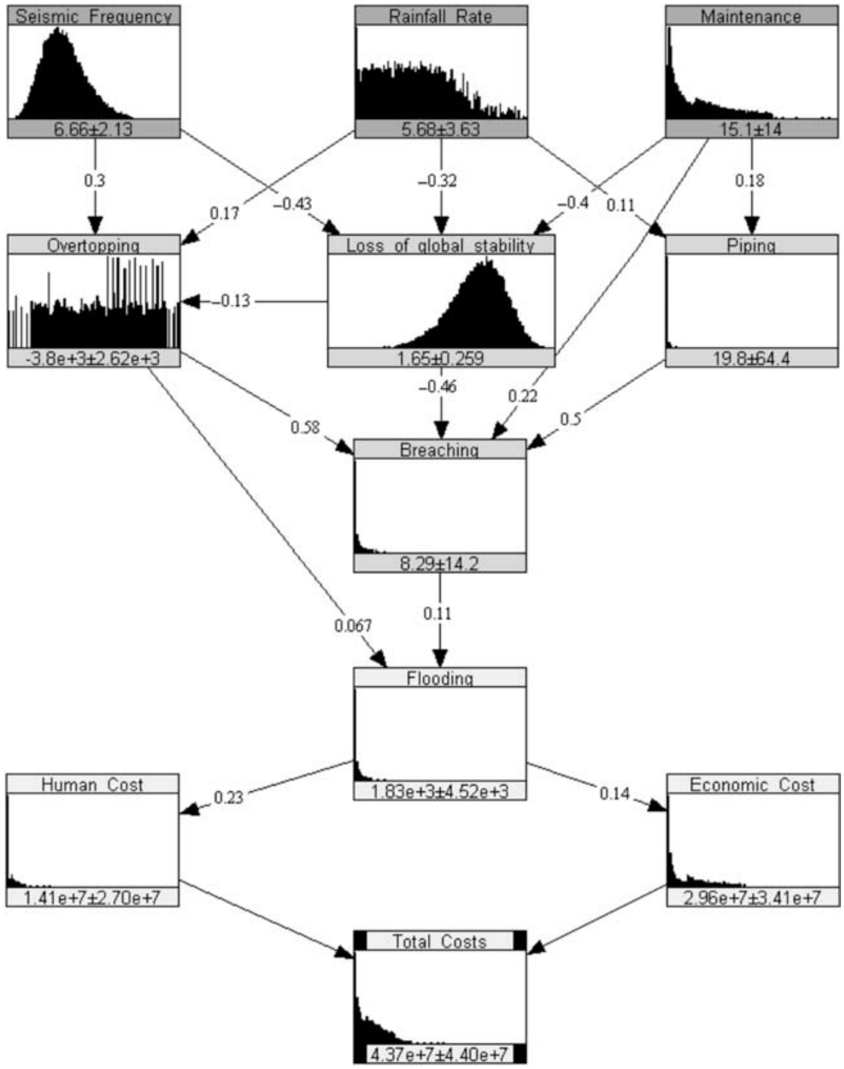

Figure 8. Unconditional dam safety model.

the amount of maintenance required to achieve a particular loss of global stability of, say 2.3, Figure 10 would be employed. The interest is in the conditional distribution of maintenance. Its conditional distribution would reflect the distribution of *recommended* next maintenance times for a certain subset of dams in the population. In this case, the expected maintenance should be every 3.3 years (with percentiles 5th = 0.7 and 95th = 13 years). The decision for future maintenance across the hypothetical subpopulation included in the 90% uncertainty interval could then be based on criteria other than loss of global stability. If the assumption that the 95th percentile gives the required level of confidence for the next maintenance programme for the whole subpopulation without any additional evidence, then the recommended time for the next intervention would be 13 years (not shown in Figure 10). Notice that this value is lower than the expectation of maintenance in the unconditional case shown in Figure 8.

Suppose, additionally, that for a particular sub-region from the ones represented in the sample, a long dry season is expected. On the basis of a rainfall rate of maximum 0.5 mm/day for 5 days moving average, the next maintenance would be planned. This situation is exemplified in Figure 11. The conditional expected maintenance is now 3.6 years, though in this case, the 95th percentile of the distribution would indicate that the next maintenance may be performed after 14.3 years. In this section, the maintenance time is presented as a random variable, but when specific conditions regarding particular information for each dam under investigation are available, the maintenance time becomes a deterministic value as shown in the accompanying paper (Delgado-Hernandez et al., 2012) and in the next section.

The use of decision nodes is a very common procedure used in discrete BNs. Only a little amount of evidence has been documented regarding the implementation of decision

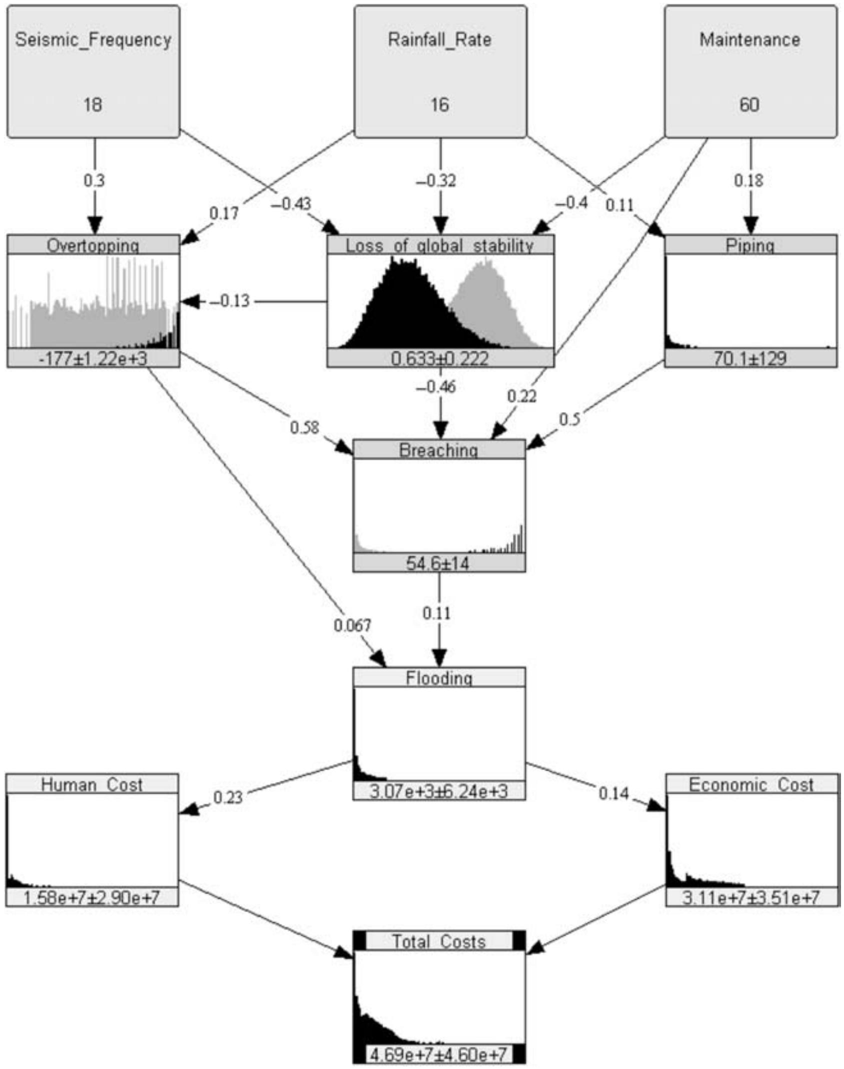

Figure 9. Dam safety model given that the seismic frequency is 18 earthquakes above 5.5 Richter degrees per year, rainfall rate is 16 mm/day and maintenance is 60 years between conservation activities.

nodes in continuous and hybrid BNs, e.g. are some of the works published by Cobb (2008, 2009, 2010); the decision nodes were implemented with BNs using the mixtures of truncated exponentials framework. Finally, the treatment of the variable *maintenance* in this model is perhaps not the most adequate. A disadvantage of the modelling shown in this section is that the maintenance variable has not been treated as a decision node; an alternative approach, not shown in this article, would be to modify the BN in Figure 2 to consider node maintenance as a decision node. Decision nodes have not yet been investigated in the NPBN framework. The implementation of decision nodes is the subject of future research within the NPBN community.

Design discharges

Extreme value analysis is often required in reliability assessment for the computation of design values. Some would say that the flood problem is the oldest problem connected with extreme values (Gumbel, 1958, p. 4). For example, in the probabilistic design of river dikes, the design discharges are defined as those with an average return period of 1250 years. It is not unusual to look at probabilities in the order of 10^{-6} or smaller for this kind of return periods (van Noortwijk, 2003).

In order to compute design values for the earth dams in the State of Mexico, one may use the model here presented. Samples may be retrieved from either the unconditional distribution (Figure 8) or any conditional distribution as the one shown in Figure 9. Then, extrapolation may be useful on the basis of these samples. Figure 12 shows this procedure.

The case denoted *Conditional* in Figure 12 refers to the conditional distribution of flooding given the following values: *seismic frequency* $= 18$, *rainfall rate* $= 16$ and *maintenance* $= 60$. Note that, in this case, maintenance

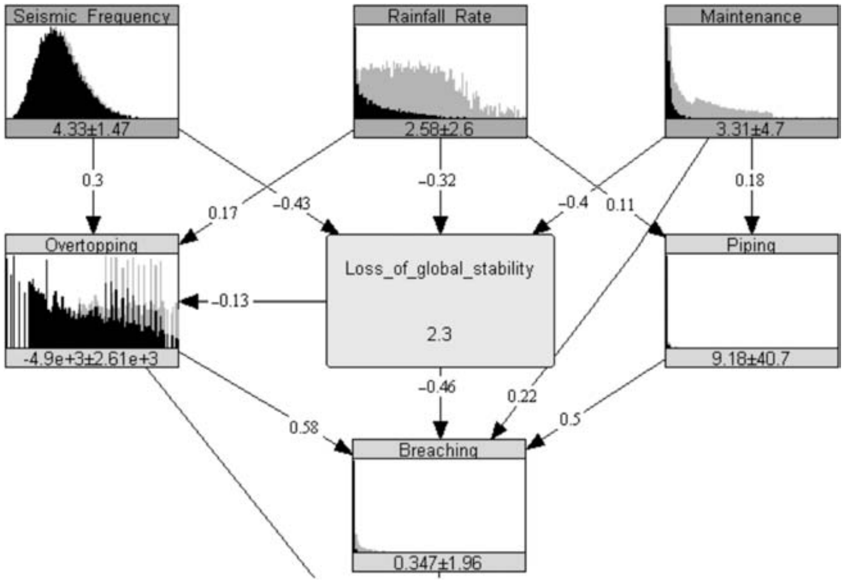

Figure 10. Dam safety model given a loss of global stability = 2.3.

becomes a deterministic value in relation to observed previous maintenance. Figure 12 shows the empirical distributions of both, the baseline case and the conditional case plus the function fitted to them for the extrapolation. The function considered for extrapolation corresponds to a mixture of Gaussians:

$$\hat{f}(y_j) = \sum_{i=1}^{c} \hat{\pi}_i \phi_i(y_j; \hat{\mu}_i, \hat{\sigma}_i), \tag{6}$$

where ϕ_i represent a normal density with mean $\hat{\mu}_i$ and standard deviation $\hat{\sigma}_i$, and each $\hat{\pi}_i$ is a non-negative quantity and all sum to one. The $\hat{\pi}_i$'s are called *mixing proportions* and the ϕ_i's *component densities* of the mixture. The expectation-maximisation (EM) algorithm (McLachlan & Peel, 2000) is used in the fitting; however, other techniques might also be used (see, e.g. van Noortwijk, 2003).

The conditional case corresponds to the case when the dams of interest require design values with high levels of safety. If the design probabilities are in both cases 10^{-8} then the design discharge values for earth dams in the State of Mexico would correspond to 4×10^4 and 4.5×10^4 l/day for the baseline and conditional cases, respectively.

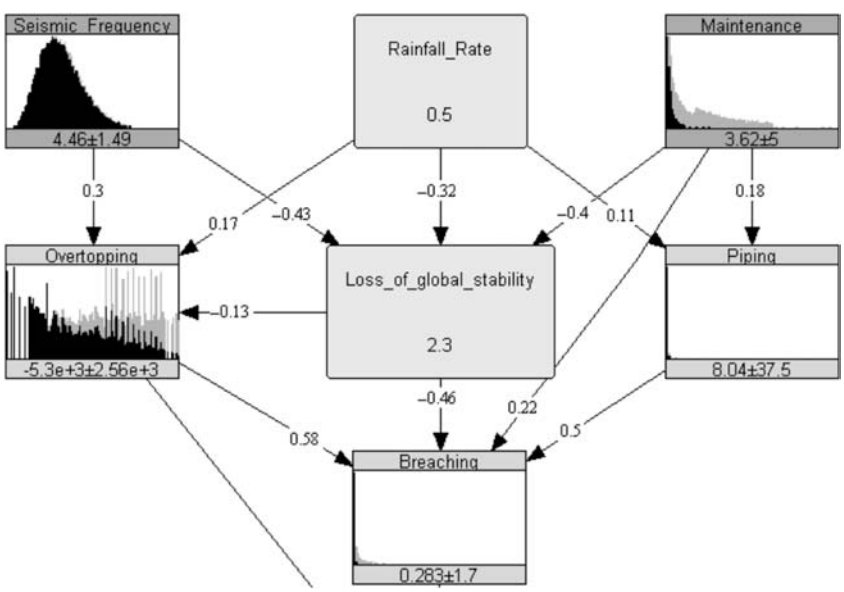

Figure 11. Dam safety model given a loss of global stability = 2.3 and rainfall rate = 0.5 mm/day.

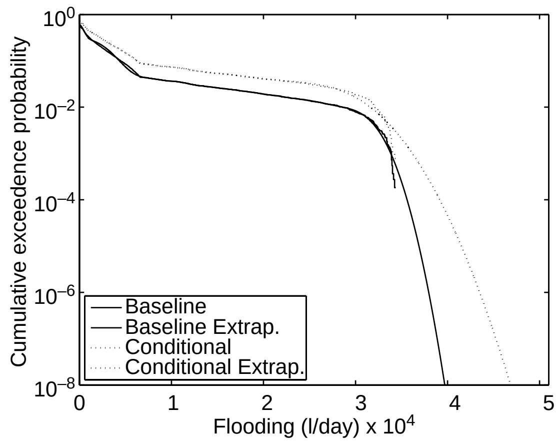

Figure 12. Extrapolation of discharge values for computation of design values for earth dams in the State of Mexico.

The economic impact of one or the other choice could be evaluated by the decision-maker. A similar analysis can be carried out with the other variables.

Conclusions and final comments

This article shows a BN for causes and possible consequences of earth dam failure and their failure modes. The main focus has been on the description of the type of BN used and its quantification through both SEJ and field data. In particular, this document introduces an elicitation technique for conditional rank correlations on the basis of ratios of unconditional rank correlations. In an accompanying paper, emphasis is made on risk assessment in dams within the State of Mexico. The combination of BNs and expert judgement recognises that dam managers need intuitive, useful and practical tools for carrying out quantitative risk assessment based on a solid theoretical foundation and often with the absence of field data.

The availability of experts was one of the main difficulties encountered during the quantification process. This step is critical in future refinements of the model. The inclusion of more variables should be considered. This is particularly relevant if some local cases have shown that other variables are important in the risk evaluation apart from those reported in international statistics. The equal weight combination was proposed as the preferred choice for the decision-maker. The training of experts in probabilistic assessments and a representative set of seed variables is fundamental for the classical methodology for SEJ. These are aspects to consider when refining the model.

In spite of these observations, it is strongly believed that the methodology utilised to build the model can be applied to carry out similar exercises in different locations. Overall, this research has demonstrated that the use of NPBN in Mexican dams' risk assessment is not only feasible but also beneficial. This research is the starting point of a bigger project aimed at developing a more comprehensive model applicable to different types of dams.

Acknowledgements

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Notes

1. We use the name non-parametric Bayesian networks in order to be consistent with previous literature. Perhaps a more appropriate name would be semi-parametric Bayesian networks, since generally a parametric family of copulas (the normal copula) is used in applications. The original motivation to use the non-parametric qualification is to stress that one-dimensional margins may be chosen freely.
2. When an influence is deterministic, nodes will be called functional. The discussion presented next refers to probabilistic influences unless otherwise specified.

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