Delft University of Technology
Faculty of Electrical Engineering, Mathematics and Computer Science
Delft Institute of Applied Mathematics

Probabilistic Inversion in Priority Setting of Emerging Zöonoses

A thesis submitted to the
Delft Institute of Applied Mathematics
in partial fulfillment of the requirements

for the degree

MASTER OF SCIENCE
in
APPLIED MATHEMATICS

by

CĂTĂLIN ALEXANDRU BUCURA

Delft, the Netherlands
June 2008

MSc THESIS APPLIED MATHEMATICS

"Probabilistic Inversion in Priority Setting of Emerging Zöonoses"

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June 2008

Delft, the Netherlands

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Acknowledgements

This thesis is the result of work of not only one person, but much more. I would like thank Prof. Dr. Arie Havelaar for all his support, encouragement and patience he has shown during this project. I would like to thank my two supervisors, Dr. Dorota Kurowicka and Prof. Dr. Roger Cooke for their confidence, guidance and feed-back throughout the project. Many thanks to Rabin Neslo, for his prompt support and for the software used, which made my life easier. Many thanks to Anca and Dana Hanea, who have shown lots of patience for reading all my drafts, and giving me constantly support. I would like to thank to all the people involved in the project, for their time and effort.

The decision to come to Delft for studying has improved my life significantly. A very big influence in taking this decision belongs to Remus Hanea. For this, and for all that he has done for me I would like to sincerely thank him. In the same time, sincerely thanks to Dana and Anca Hanea, and to Dan Ababei for their help, support and friendship. Many thanks for Prof. Dr. Ir. Arnold Heemink for his great support and prompt help. I would also like to thank Kasia J., Kasia K., Lukasz, Mihai, Iulia, Bogdan and Catalin for being my close friends. Last, but not least, special thanks to Asia, for supporting and encouraging me every day, no matter what. I hope I can return some day her help. I would also like to thank my mom for trusting in me and helping me during all this time. The list is long, and I apologise if I omitted anybody.

Chapter 1

Introduction

"Complex and partially yet unknown risk factors will lead to the introduction of new infections in the human population. Although we do not know which disease will emerge next, recent emerging infections came predominantly from animal reservoirs. Therefore, animal populations are considered the main reservoir for emerging infectious diseases".[1]

"In Europe, zoonoses* originating from wildlife sources transmitted by arthropods are considered to become more important in the future. Climate and ecological changes may favour already existing arthropods to expand to other regions and thus to introduce new pathogens to areas in Europe." [2]

Emerging zoonoses

In 1959, the World Health Organisation (WHO) defined an emerging disease as "a disease that has appeared in a human population for the first time, or has occurred previously but is increasing in incidence or expanding into areas where it has not previously been reported".[3] At the WHO Geneva conference in 2004, a new definition for emerging zoonoses was formulated: "An emerging zoonoses is a zoonosis that is newly recognised or newly evolved, or that has occurred previously but shows an increase in incidence or expansion in geographic, host, or vector range. It is noted that some of this diseases may further evolve and become effectively and essentially transmissible from human to human." In the current research the last definition is used.

Because mankind is more and more threatened by zoonoses, in 2007, The Dutch National Institute of Public Health and Environment (RIVM), as a result of its research, published the 'Zoonoses and Zoonotic Agents in Humans, Food, Animals and Feed in The Netherlands 2003 - 2006' report, which contains data that is reported annually to the European Commission, in accordance with the Directive 2003/99/EC on the monitoring of zoonoses and zoonotic agents. After the existing pathogens have been identified, a natural step to follow, with respect to public health, is to prioritise these pathogens based on their severity. A second aspect is to ensure a good prevention of the new (emerging) zoonoses.

*zoonoses represent pathogens that are transmitted from animals to humans, e.g. bird flu

In 2007 the consortium consisting of institutes involved in veterinary medicine and infectious disease control in the Netherlands started the project of Emerging Zoonoses. This project aims to build a mathematical model that helps Dutch decision makers to establish the priority of emerging zoonoses.

In order to use the model to prioritise the pathogens, we first must choose different characteristics that will be used to describe the pathogen. We call this different characteristics, *attributes*. After a series of discussions between institutions participating in the project, nine attributes defining the most relevant aspects of risk of a pathogen were selected. Each attribute has four or five levels, and to each level corresponds a value. The convention is that the lowest level (level 1) corresponds to the least threatening case and the highest level (level 4 or 5) signifies the most threatening situation. The nine criteria used in this project are briefly described below. More details about the attributes can be found in Appendix A.

1. Probability of introduction the pathogen in the Netherlands
    a) level 1 corresponds to 0% chances of introduction;
    b) level 2 corresponds to 0.5% chances of introduction;
    c) level 3 corresponds to 50% chances of introduction;
    d) level 4 corresponds to 50% chances of introduction;
    e) level 5 corresponds to 100% chances of introduction.
2. Speed of spread of the pathogen between animals
    a) level 1 corresponds to 10,000 days (it takes 10,000 days for the pathogen to spread; 10,000 days was chosen by analyst to keep values of this attribute monotonic. This basically it means that it does not spread);
    b) level 2 corresponds to 30 days;
    c) level 3 corresponds to 10 days;
    d) level 4 corresponds to 1 day.
3. Economic damage within animals
    a) level 1 corresponds to 5 M€[†] damage;
    b) level 2 corresponds to 50 M€;
    c) level 3 corresponds to 500 M€;
    d) level 4 corresponds to 5000 M€.
4. Probability of transmission of the pathogen from animals to humans
    a) level 1 corresponds to 1:10,000 (one human must get in contact with 10,000 infested animals to catch the virus);
    b) level 2 corresponds to 1:1,000;

[†] M€denotes million euros

c) level 3 corresponds to 1:100;

d) level 4 corresponds to 1:10.

5. Speed of spread of the pathogen between humans

   a) level 1 corresponds to 10,000 days (it takes 10,000 days for the pathogen to spread);

   b) level 2 corresponds to 30 days;

   c) level 3 corresponds to 10 days;

   d) level 4 corresponds to 1 day.

6. Gravity of illness, morbidity

   a) level 1 corresponds to 0.02 gravity;

   b) level 2 corresponds to 0.06;

   c) level 3 corresponds to 0.2;

   d) level 4 corresponds to 0.6.

7. Chances of dying, mortality of human population

   a) level 1 corresponds to 0% chances of dying;

   b) level 2 corresponds to 0.5%;

   c) level 3 corresponds to 5%

   d) level 4 corresponds to 50%

   e) level 5 corresponds to 100%.

8. Economic damage within humans

   a) level 1 corresponds to 5 M€damage;

   b) level 2 corresponds to 50 M€;

   c) level 3 corresponds to 500 M€;

   d) level 4 corresponds to 5000 M€.

9. Risk perception

   a) level 1 corresponds to 0;

   b) level 2 corresponds to 2;

   c) level 3 corresponds to 4;

   d) level 4 corresponds to 6.

   This last criterion describes the level in which subjective risk attributes influence the perception of the Dutch society. The following consequences are possible. Depending of how many out of possible aspects apply, the pathogen is considered not threatening, moderately threatening, etc:

   - Involuntary exposure

- Inequity (who profits)
- Cannot be avoided through personal behaviour
- Unknown or new and unnatural risk
- Hidden, postponed and irreversible damage
- Possibility of identification with victims (e.g. children or pregnant women)

The pathogen is considered:

- Not threatening if 0 of 6 subjective aspects apply;
- Moderately threatening if 2 of 6 subjective aspects apply;
- Threatening if 4 of 6 subjective aspects apply;
- Very Threatening if 6 of 6 subjective aspects apply.

Looking on the above presented attributes, we notice that they are expressed in different units. We need to transform the scale of attributes such that we can represent all of them in a increasing scale from 0 to 1. More information about transformations can be found in Chapter 4.

We want to compare pathogens in terms of severity using these nine criteria. A solution for this, is to create random combinations of one the levels of each criteria, which we call *scenarios*. For our problem we randomly generate 30 different scenarios. The scenarios reflect hypothetical zoonoses. Note that scenarios have been generated such that none of them is "majorising" the others, which means there is no scenario for which all attributes have higher or equal value than any other scenario. The advantage of using randomly chosen scenarios, rather than designing them otherwise, is that the bias is not introduced.

The total number of scenarios is divided into 6 groups, each group consisting of 7 scenarios. Scenarios are overlapping within the groups. In the first five groups the last two scenarios of one group are repeated as being the first ones in the consecutive group. In the sixth group, the first four scenarios are the last ones from group 5. This way experts' consistency when ordering the same scenarios in different groups can be tested. In the Section 1.1 we will discuss experts' assessments, and in Chapter 5 we discuss their consistency. Scenarios in first groups are in general more severe then in last groups. This means that the attributes' values from these scenarios are in general higher.

Table 1.1 contains scenarios from the first group.

The first column in Table 1.1 represents the scenarios numbering, e.g. S_1. Columns two and three contain information about the first attribute: column two shows the levels of the first attribute, whereas column three shows the value corresponding to this level.

We see that in scenario S_1, the first attribute, the chance of introduction is at level 4 (50%), speed of spreading between animals is 3 (it takes 10 days for the virus to spread), the economical damage within animals is 3 (5000 million euros), probability of transmission of the pathogen from animal to human is 3 (one human must have contact with 100 animals to get the virus), speed of spread between humans is 3 (it takes 10 days for

Table 1.1: Group I - first seven scenarios

	Attributes																	
	1		2		3		4		5		6		7		8		9	
S_1	4	50	3	10	3	500	3	100	3	10	3	0.2	4	50	3	500	3	6
S_2	3	5	2	30	3	500	4	10	4	3	3	0.2	3	5	2	50	4	8
S_3	4	50	3	10	1	5	3	100	4	3	3	0.2	4	50	1	5	4	8
S_4	4	50	4	3	4	5000	4	10	3	10	2	0.06	3	5	2	50	1	2
S_5	4	50	1	0	3	500	2	1000	4	3	4	0.6	3	5	3	500	2	4
S_6	2	0.5	2	30	4	5000	3	100	3	10	3	0.2	4	50	2	50	4	8
S_7	3	5	3	10	2	50	3	100	4	3	3	0.2	4	50	2	50	3	6

the virus to spread), the gravity of illness produced by the pathogen is 3 (average, 0.2), chances of dying once the pathogen has been caught are 4 (50%), and the risk perception is 3 (4).

The randomly chosen scenarios do not describe any particular pathogen, they are going to be ordered be experts, increasingly, in terms of severity. From experts assessments the model for scoring scenarios' severity will be recovered using *probabilistic inversion (PI)* technique. Before explaining PI, we need more information about experts, and how we obtain information from them.

1.1 Expert Judgement

In general, statistical data are an important base to build forecast, calculate estimates or support decisions. Unfortunately, it is common to find real life examples where data are not always available and/or complete. One possible solution to this situation is expert judgement.[11]

In September 2007 the elicitation[‡] took place, at RIVM headquarters. In this procedure 11 experts (9 male and 2 female) have participated. Due to confidentiality reasons, the names of experts are not revealed.

Prior to the elicitation, experts have been explained the procedure, the attributes and the scenarios, using a training set. The elicitation was organised as follows: each scenario from each group was written down on a cardboard. Experts were asked to arrange cards with scenarios in increasing order of severity. The cardboards corresponding to each of the six groups were coloured differently.

The experts were divided into two groups. The first group of experts started to order the cardboards with scenarios from the first three groups, whereas the second group of experts started ordering the last three groups of scenarios. This was done to avoid the case that expert might be tired when analysing the last groups.

Two weeks after the elicitation, another panel sesion was organised. Two out of six

[‡]Expert elicitation is the synthesis of opinions of experts of a subject where there is uncertainty due to insufficient data, when such data is unattainable because of physical constraints or lack of resources. Expert elicitation is essentially a scientific consensus methodology.

groups have been randomly chosen (and these two are group 2 and group 5), and they were sent by post mail to the eleven experts. They were asked to order again the seven scenarios, based on their severity. From 11 experts, only 9 have returned their assessments. Table 1.1 presents the ordering of scenarios from group 2, of experts 2 and 3. First column, denoted *e2_set2_1* represents the ordering of expert two, of group 2, after the first elicitation. Next column, *e2_set2_2* contains the ordering of the same expert, of the same group, but from the second panel session.

	e2_set2_1	e2_set2_2	e3_set2_1	e3_set2_2
1	1	2	7	6
2	5	1	5	4
3	3	4	3	2
4	6	7	4	7
5	7	6	1	1
6	4	5	2	3
7	2	3	6	5

Figure 1.1: Experts assessments for Group I

To examine how experts agree with their own answers, during the panel session (measuring 1) and the panel session (measuring 2), we calculate the rank correlation of each expert, shown in Table 1.2.

Table 1.2: Rank correlation coefficient for group 2 and 5, for each expert

	Rank correlation	
expert	**group 2**	**group 5**
2	0.61	0.71
3	0.75	0.86
4	0.64	0.39
5	0.64	0.32
8	0.32	0.25
9	0.82	0.82
1	0.86	0.32
6	0.75	0.50
7	0.46	0.76

It is visible that some of the experts obtained a very low correlation, which means their assessments for the same group, but at different time period was different.

Experts assessments for the first group are presented in Figure 1.2. We explain this first group, and all the other groups are presented in Appendix B.

The first column in Figure 1.2 is the numbering of scenarios. The second column contains the scenarios codification. In this thesis we replace this codification of scenarios by S_i, where $i = 1 \ldots\ 30$. The top row shows the 11 experts. The rest of the columns represent orderings provided by experts. For instance, if we follow scenario number 1, QJ, we observe that it is ranked by expert number 1 on the fourth place, by expert 2 on the sixth position, by the third expert on the last position, and so on. We consider the

SCENARIOS		EXPERTS 1	2	3	4	5	6	7	8	9	10	11
1	QJ	JR	WL	ZC	ZC	WL	ZC	VG	VG	JR	ZC	ZC
2	VG	VG	PX	VG	JR	VG	PX	ZC	ZC	VG	PX	VG
3	GF	PX	ZC	PX	VG	ZC	JR	JR	JR	ZC	JR	PX
4	JR	QJ	VG	GF	WL	JR	GF	GF	WL	WL	VG	JR
5	ZC	ZC	JR	JR	PX	PX	VG	PX	PX	PX	WL	WL
6	WL	GF	QJ	WL	QJ	QJ	QJ	WL	QJ	QJ	GF	GF
7	PX	WL	GF	QJ	GF	GF	WL	QJ	GF	GF	QJ	QJ

Figure 1.2: Experts assessments for Group I

last seventh place as the most severe state, whereas the first position denotes the least severe situation. Table 1.2 shows that experts considered scenario QJ relatively severe. We could compare it with the scenario PX which is considered slightly less severe than QJ.

Experts orderings for other groups have been obtained in a similar way as for group 1. This information has been summarised for further analysis in Tables 1.3 and 1.4.

Table 1.3: Experts assessments for the first three groups

	scores			1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}
G	S_1	68	S_1				1		6	4
R	S_2	28	S_3	2	5	1	2	1		
O	S_3	65	S_6				3		3	5
U	S_4	34	S_7	2	1	4	2	2		
P	S_5	23	S_4	5	2	3		1		
	S_6	50	S_2	2			3	2	2	2
I	S_7	40	S_5		3	3		5		
G	S_6	34	S_{10}	2	4	2		1	1	1
R	S_7	40	S_9	1	3	1		6		
O	S_8	32	S_{11}	3		5	2		1	
U	S_9	62	S_7		1		1	1	5	3
P	S_{10}	64	S_6	1			2		1	7
	S_{11}	42	S_{12}	1	1	1	5	2	1	
II	S_{12}	34	S_8	3	2	2	1	1	2	
G	S_{11}	53	S_{16}		1	1	2	3	3	1
R	S_{12}	17	S_{13}	10						1
O	S_{13}	55	S_{11}		2	1	1	2	1	4
U	S_{14}	46	S_{14}			4	3	2	2	
P	S_{15}	38	S_{17}	1	2	2	4	1	1	
	S_{16}	56	S_{15}		2	1	1	2		5
III	S_{17}	43	S_{12}		4	2		1	4	

Tables 1.3 and 1.4 contain the following information:

1. first column defines the six groups;
2. second column defines the scenarios, from S_1 to S_{30};
3. third column shows the rank scores of each scenario obtained from experts. The rank ordering technique gives an indication of the ordering of scenarios within each

Table 1.4: Experts assessments for the last three groups

	scores			1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}
G	S_{16}	33	S_{20}	3	2	3		1	2	
R	S_{17}	40	S_{21}			5	5	1		
O	S_{18}	49	S_{18}		3		1	4	2	1
U	S_{19}	47	S_{19}	1		3	2	3		2
P	S_{20}	67	S_{17}				2	2		7
	S_{21}	53	S_{16}	1	2			1	6	1
IV	S_{22}	20	S_{22}	6	4				1	
G	S_{21}	60	S_{21}	2				2	1	6
R	S_{22}	33	S_{23}	3	3	1	2		1	1
O	S_{23}	55	S_{26}	1	1		1	1	6	1
U	S_{24}	38	S_{27}		2	5	2	1	1	
P	S_{25}	32	S_{24}	4	2	1	1	2		1
	S_{26}	45	S_{22}		3	2	2	1	1	2
V	S_{27}	45	S_{25}	1		2	3	4	1	
G	S_{24}	36	S_{29}	2	2	3	2		1	1
R	S_{25}	35	S_{26}	4		2	1	3	1	
O	S_{26}	49	S_{27}	1	3		2		1	4
U	S_{27}	47	S_{28}	2	1		2	3	1	2
P	S_{28}	45	S_{30}		2	2	3	2	1	1
	S_{29}	56	S_{24}			2	1	2	6	
VI	S_{30}	40	S_{25}	2	3	2		1		3

group. The scores are obtained by multiplying the number of experts who ranked scenario i as j^{th} by its rank order, thus j, $j = 1, ...7$ and summing over j;

4. fourth column shows scenarios ordered from most to least severe within the group based on rank order technique;

5. fifth column contains the number of experts that ranked a given scenario as first in the ordering hence the least severe, sixth column shows the number of experts that considered this scenario second in the ordering etc. and finally the eleventh column shows number of experts that considered a given scenario as the most severe.

Looking at scenario S_1 from Table 1.3, we can read that out of eleven experts **one** expert ranked the scenario S_1 as fourth, **six** experts ranked it as sixth and **four** experts ranked it on the seventh place. The rank score was calculated as:

$$1 \times 4 + 6 \times 6 + 4 \times 7 = 68 \tag{1.1}$$

Using the information that we obtained from experts we want to build the model which recovers their preferences. This is done using probabilistic inversion technique, which is presented in the next section.

1.2 Probabilistic Inversion

Expert judgement can be applied whenever the variables under consideration can be theoretically measured or observed. However, there are some complex situations where the variables of interest can neither be measured, nor observed. Therefore experts are not able to give either quantiles, or any approximation for the variables of interest. Instead trying to quantify an un-observable quantities, an analyst can find an observable variable that is related to the variable of interest through a function.

For example, the analyst might be interested in a parameter of a physical model that is not observable in the field. If the function relating the parameter and an observable variable is known, then it is possible to find information about the specific parameter. Based on this information probabilistic inversion, (PI), can be applied to obtain the parameter's distribution.

Before providing the mathematical definition of probabilistic inversion, we give a short intuitive explanation. Consider a given model M with input data, A and output, B, hence $B = M(A)$. Assume that we can observe and measure the output B. Then probabilistic inversion inverts the information that we have about the output B, and this way we can obtain information about the input, A.

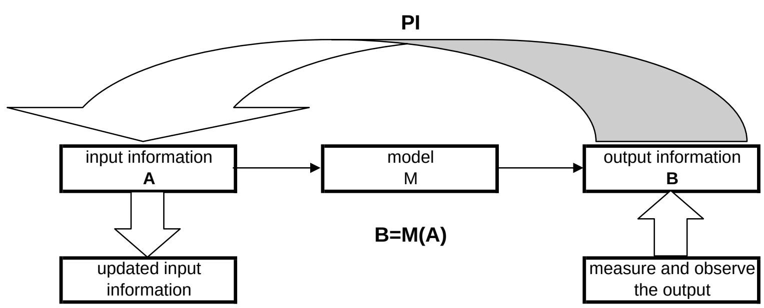


Figure 1.3: Intuitive scheme of probabilistic inversion

In our case, the output information (see Figure1.3) is represented by the experts' assessments. Using PI, we invert information from experts, which in this case represent the input. Next we want to find a model such that using the obtained input information, we recover experts' preferences.

The mathematical definition of probabilistic inversion method is as follows: let X and Y be two random vectors in $\mathbb{R}^n$ and $\mathbb{R}^m$ respectively; and F a measurable function from $\mathbb{R}^n$ to $\mathbb{R}^m$. If $F(x) = y$, then $x \in \mathbb{R}^n$ is the inverse of $y \in \mathbb{R}^m$ under F. Correspondingly, if $F(X)$ (the function F does not have to be the same used before) shares the same distribution as Y (we say $F(X) \sim Y$), then X is the probabilistic inverse of Y under F.

The probabilistic inversion problem can be defined as follows: assume that vector

Y is the vector of the observable variables, and that the physical model relating the variables and the parameters is given by the set of functions F. Then the problem consists in finding the joint distribution of random vector X such that $F(X)$ given by:

$$F(X) = [F_1(x), F_2(x), \dots F_n(x)]$$

has the same distribution as a random vector Y.

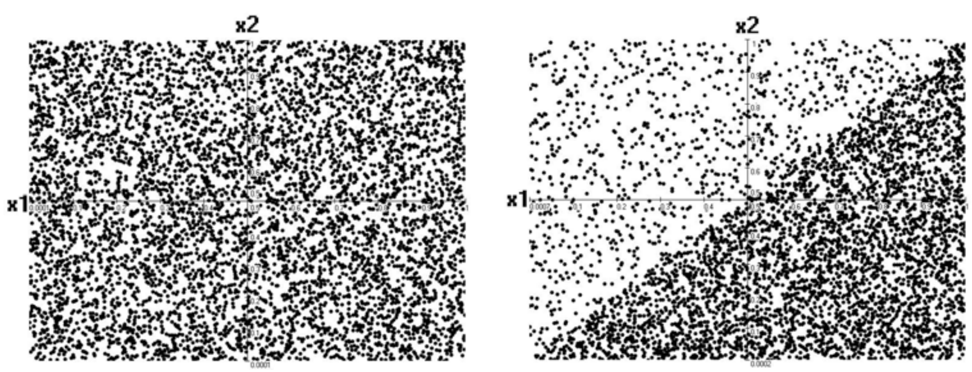
A solution to probabilistic inversion does not always have to exist, and if there is a solution then this solution does not have to be unique. If the problem is feasible it may have many solutions and we require a preferred solution [13]. In case of infeasibility we seek a random vector X such that such that the $F(X)$ is *as close as possible* to Y distribution-wise[12]. Usually for measuring the differences between these two distributions the relative information is used.[14]

We explain probabilistic inversion on a simple example:

Suppose we have two independent uniformly distributed random variables: X_1 and X_2 such that $X_1 \perp X_2$, $(X_1, X_2) \sim U[0,1]^2$. Since these two variables are independent then the probability of X_1 being bigger than X_2 (or X_2 being bigger than X_1) is 0.5, i.e.:

$$P(X_1 > X_2) = 0.5.$$

By sampling and plotting 10,000 samples of X_1 and X_2 we expect to have a uniform spread of mass on the unit square. Figure 1.4(a) confirms our expectation.



(a) with probability 0.5 (b) with probability 0.8

Figure 1.4: Scatter plot of 10,000 samples

Consider for $P(X_1 > X_2)$ a different value than 0.5, say 0.8. We solve probabilistic inversion method using the *sampling re-weighting technique*. An iterative algorithm called *iterative proportional fitting (IPF)*[19] is applied at this point to find weights for these samples such that after re-sampling the new imposed probability will be satisfied. This means that samples satisfying the constraint will get bigger weights, and after re-sampling we will see more mass concentrated in the bottom right corner of the unit square, see Figure 1.4(b) where X_1 is bigger than X_2. This change can also be noticed on the plot of the cumulative distribution functions. In Figure 1.5 X_2 is represented with a dotted line, and X_1 with a solid one. If we look at the marked points in the same figure we see that with probability 0.5, X_2 is approximately equal to 0.32 whereas X_1 with probability 0.5 is almost 0.7. Hence $X_1 > X_2$.

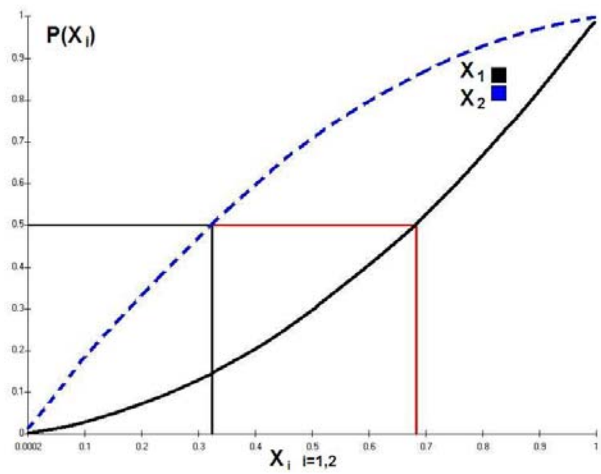


Figure 1.5: Cumulative distribution functions of variables X_1 and X_2

1.3 Iterative Proportional Fitting (IPF) algorithm

Probabilistic inversion problems are solved using different types of algorithms. In the literature there are available few algorithms for this problems, namely: iterative proportional fitting algorithm (IPF)[19], parameter fitting for uncertain models (PARFUM)[15]. We use in our analysis iterative algorithms for numerically solving probabilistic inversion problems, because these methods do not require model inversion. They are based on sample re-weighting techniques. Their advantage is that they do not require special knowledge about the problem at hand, or complicated heuristic steering on the part of the user. Moreover, operations on the sample are performed one-at-a-time, so the entire sample does not need to be kept in memory.

The iterative methods are re-sampling methods that will start with a large set of samples of X and $F(X)$ and re-weight the samples in the set such that $F(X)$ is *as close as possible* to Y. The starting distribution for X can be any distribution such that the range of $F(X)$ covers the domain of Y.

If the probabilistic inversion problem is feasible, then IPF[19] method is preferred over the other iterative methods, because it converges faster. In case of infeasibility PARFUM and PARFUM-like algorithms will converge to minimally infeasible solution.

Further we continue explaining IPF algorithm. Looking at the previous example, before applying the algorithm the weight for each sample was equal to $\frac{1}{10000}$. After the inversion, the weights corresponding to samples which satisfy the constraint should change from $\frac{1}{10000}$ to $\frac{0.8}{0.5} \cdot \frac{1}{10000}$. For only one constraint, namely $P(X_1 > X_2) = 0.8$ it is easy to find how samples should be weighted. If more constraints are added, more sophisticated method have to be used.

In the previous example, we add one more constraint: $P(X_1 > 1 - X_2) = 0.8$. Next, we run IPF on these two constraints, and plot the scatter plot, in Figure 1.3. We notice now that samples have been re-weighted such that they satisfy also the second constraint, and hence we see mass concentrated in the top right corner as well.

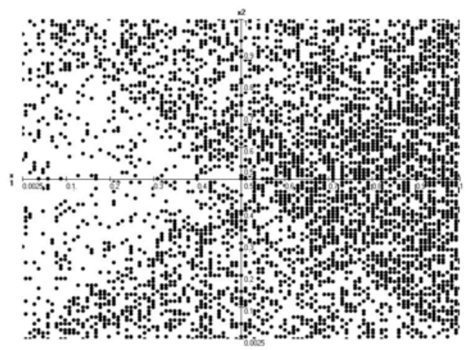


Figure 1.6: Scatter plot of 10,000 samples, with two constraints

Consider now, in the previous example, another random variable, X_3, also uniformly distributed. Our example is now as follows: we have three independent uniformly distributed random variables: X_1, X_2 and X_3. We impose three constraints, which we denote s_1, s_2 and s_3. We also denote value taken by the probabilities by quantiles (Q).

$$\begin{aligned} s_1 = & \quad P(X_1 > X_2) & = 0.8 \\ s_2 = & \quad P(X_1 > 1 - X_2) & = 0.8 \\ s_3 = & \quad P(X_2 > X_3) & = 0.2 \end{aligned}$$

Because of the software tool used, in our program we have to impose instead of 0.8 for instance, 1-0.8. Below we present the IPF algorithm for this small example. We present the probabilities we want to impose, see Table 1.5, first column, titled "imposed Q". Next, we will follow the evolution of IPF after several number of iterations.

Because there are more than one constraint and more than two variables, the probabilities are not recovered from the first iteration. However, it is visible that after 4 iterations, the obtained probabilities are relatively close to the ones which we imposed. IPF algorithm hence is re-weighting each sample such that they satisfy the imposed probabilities. Because IPF is an iterative procedure, errors do occur.

Table 1.5: IPF example

imposed Q	obtained Q			
	1^{st}iteration	2^{nd}iteration	3^{rd}iteration	4^{th}iteration
0.2	0.13280	0.18902	0.19862	0.19982
0.2	0.26530	0.20556	0.20065	0.20008
0.8	0.80000	0.80000	0.79999	0.79999
error	0.000183	$3.344{\cdot}10^{-5}$	$4.148{\cdot}10^{-5}$	$5.172{\cdot}10^{-5}$

Last row from the tables presents the obtained error after each iteration. We say that the problem is feasible, or that IPF converges, when the obtained probabilities are as close as possible to the imposed ones. In this small example it is visible that IPF converges after 3 iterations. However, we show up to 20 iterations, where the differences between obtained and imposed probabilities are of order of 10^{-17}. (see Table 1.7 last row, last column).

Table 1.6: IPF example

imposed Q	obtained Q			
	5^{th}iteration	6^{th}iteration	7^{th}iteration	8^{th}iteration
0.2	0.19997	0.19999	0.19999	0.19999
0.2	0.20001	0.20000	0.20000	0.20000
0.8	0.79999	0.79999	0.80000	0.79999
error	$6.436{\cdot}10^{-8}$	$8.011{\cdot}10^{-10}$	$9.967{\cdot}10^{-10}$	$1.240{\cdot}10^{-11}$

Table 1.7: IPF example

imposed Q	obtained Q			
	9^{th}iteration	10^{th}iteration	⋯	20^{th}iteration
0.2	0.19999	0.19999	⋯	0.20000
0.2	0.20000	0.20000	⋯	0.20000
0.8	0.80000	0.79000	⋯	0.80000
error	$1.543{\cdot}10^{-11}$	$1.920{\cdot}10^{-12}$	⋯	$1.561{\cdot}10^{-17}$

The number of iterations needed for IPF to converge differs from case to case. This can be seen in Figure 1.7. The PI software provides us with the plot which contains the number of iterations against the error. On the X-axis we plot the number of iterations performed, and on the Y-axis the value of error obtained. Because the example is very simple, it is visible from the plot that the number of iterations necessary for convergence is approximately 4. For the example presented above, we performed in the end 100 iterations, although after 20 iterations the error obtained is very small. The same number of iterations (100) will be used further on in the analysis.

We have presented the approach that we are going to use in solving our problem. Briefly we recall our goal in this thesis: we want to build a model that can be used in prioritising pathogens based on their severity. We do not want to make any apriori assumptions about the model type, as linearity for instance.

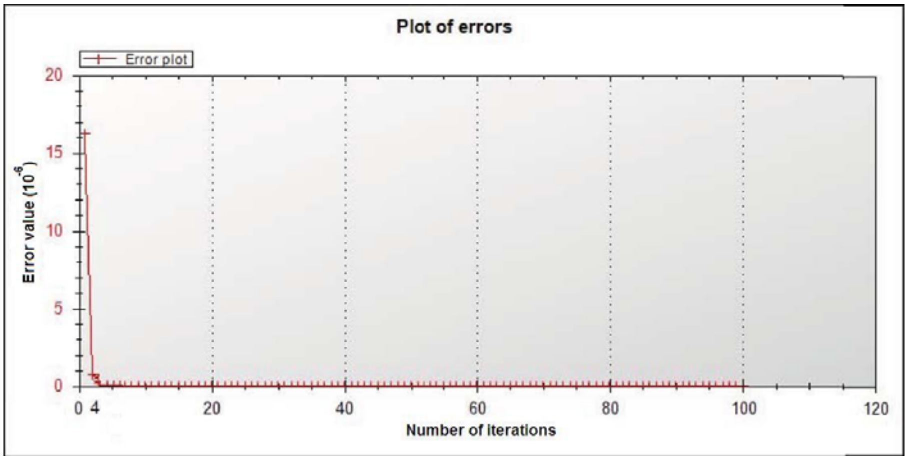


Figure 1.7: An example of number of iterations versus the error

In literature other existing approaches for this problem can be found. In the next chapter we present one alternative and discuss its drawbacks very briefly.

1.4 Multi-criteria decision making

Another approach to solve our problem would be to use the multi-criteria decision making methods (MCDM). [5] Using MCDM, it is possible to find a linear model for scores. MCDM require assigning weights of importance to attributes. They are usually chosen by analyst or by discussion with experts. [6][9] Besides the intuitive way of choosing weights, and the assumptions about the model, there are more assumptions that have to be done.[4] We do not want to start make any assumptions, therefore we want to let mathematical procedures recover the model from experts' assessments.

1.5 Outline

The present document is organised as follows:

In *Chapter 2*, we introduce a simple model which we analyse in a similar way as the main analysis. This model is called *toy model*. Firstly, we want to explain our methodology using a simpler example, for a better understanding, and secondly we want to test if the procedure that we propose for analysis really works. Based on this research, in *Chapter 3* we proceed with the analysis on the real data obtained from experts. *Chapter 4* contains the justification of our decisions, taken while analysing the real data. *Chapter 5* contains extra analysis that we have performed, and the corresponding results. The chapter ends with conclusions after the analysis. Finally, this thesis ends with the conclusions and future work, presented in *Chapter 6*.

Chapter 2

Toy model

In this chapter we study a simple problem to explain and test the procedure that will be applied to the real data. We first construct a set of artificial scenarios containing three attributes and we compare these scenarios based on our preferences. We call this set of artificial scenarios "Toy Model". The chapter ends with our conclusions after analysis of this toy model.

2.1 Toy model description

We start with creating an artificial set of four scenarios which contain three attributes and we compare these scenarios based on our preferences. In this example the probabilities of preferences are chosen by the author for illustrative purpose only. In the real zoonoses project we obtain them from experts.

The scenarios are defined as follows:

$$\text{Scenario 1} : \{0\ 1\ 2\}; \tag{2.1}$$

$$\text{Scenario 2} : \{0\ 2\ 1\}; \tag{2.2}$$

$$\text{Scenario 3} : \{1\ 2\ 0\}; \tag{2.3}$$

$$\text{Scenario 4} : \{2\ 1\ 0\}. \tag{2.4}$$

The score of each scenario is defined as a linear combination of values attributes levels as follows:

$$S_i = B_1 X_{1i} + B_2 X_{2i} + B_3 X_{3i}, \quad i = 1, \ldots, 4 \tag{2.5}$$

where

- X_{ki} is a value of k^{th} attribute in i^{th} scenario. The possible values are $\{0, 1, 2\}$. The attributes have three levels, 0, 1 and 2, where 0 corresponds to the least severe and 2 corresponds to the most severe consequence (e.g 0-nobody dies, 1 -100 people die, 2- 1000 people die).

- B_i's are the starting uniformly distributed and independent coefficients of attributes in the linear model (2.5).

Below we briefly synthesise the steps of analysis. We analyse the real data which contains the 30 scenarios, in the same way.

1. we sample $(B_1, B_2, B_3) \sim U[0,1]^3$, and compute the scores $S_1,\ S_2,\ S_3,\ S_4$ with (2.5);

2. in the next step we need the probabilities which we want to impose. Since in this artificial example we are playing the experts role, we specify what is our probability that a given scenario is more sever than another. In the real data we take these constraints from experts. The probabilities will be calculated as the number of experts that prefer scenario S_i to S_j, divided by the total number of experts.

3. we take the sample file obtained in step 1 and run probabilistic inversion algorithm with probabilities obtained in step 2 .

This way we obtain a new distribution for (B_1, B_2, B_3) which satisfies constraints in the form of probabilities of preferences.

2.2 Analysis of the toy model

In this section we analyse the toy model.

Scenarios defined in relations (2.1-2.4) are chosen such that two consecutive scenarios differ on the values of only two attributes. When we know which scenario is more severe according to experts, we can deduce which attribute influences the severity of the given scenario the most. The coefficient of the more influential attribute in the final model should be bigger than the coefficient of the less influential one.

We start with the coefficients from model (2.5) being uniformly distributed and independent:

$$B_i \sim U[0,1], \quad i = 1, 2, 3$$

and we define the score of scenario S_i, $i = 1 \ldots 4$ as follows:

$$S_1 = 0 \times B_1 + 1 \times B_2 + 2 \times B_3; \tag{2.6}$$
$$S_2 = 0 \times B_1 + 2 \times B_2 + 1 \times B_3; \tag{2.7}$$
$$S_3 = 1 \times B_1 + 2 \times B_2 + 0 \times B_3; \tag{2.8}$$
$$S_4 = 2 \times B_1 + 1 \times B_2 + 0 \times B_3. \tag{2.9}$$

$$P(S_2 > S_3) = 0.8; \quad (2.10)$$
$$P(S_2 > S_1) = 0.8; \quad (2.11)$$
$$P(S_3 > S_4) = 0.8; \quad (2.12)$$
$$P(S_1 > S_4) = 0.8. \quad (2.13)$$

The interpretation of relation 2.10 is that scenario S_2 was seen as more severe than the scenario S_3 ($S_2 > S_3$) by 80% of the experts. Scenario S_2 differs from S_3 on the values of the first and the third attribute. This means that bigger severity of the second scenario with respect to the third one is caused by bigger influence of the third attribute as compared to the first attribute. Similarly $S_2 > S_1$ leads to the conclusion that the second attribute is more important than the third one etc. Hence this example was constructed such that the most influential attribute is the second one and the least influential is the first attribute.

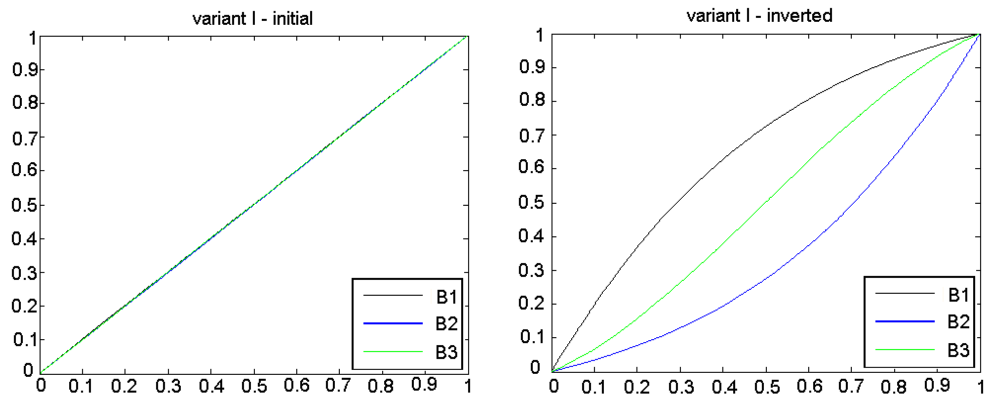
It is worth mentioning again that to make $P(S_2 > S_3) = 0.8$ the probabilistic inversion method would have to "reward" samples for which B_3 is bigger than B_1(by giving them a larger weight). Similarly the coefficient of the second attribute would have to be bigger than B_3. Table 2.1 presents the means and variances of B_i's obtained after running the probabilistic inversion algorithm using constraints (2.10-2.13).

Table 2.1: Means and variances of B_i with 0.8

	mean	**variance**
B_1	0.3496	0.0683
B_2	0.6501	0.0680
B_3	0.5004	0.0684

Figure 2.1(a) represents the empirical cumulative distribution functions of B_i's before re-weighting. In Figure 2.1(b), the new empirical cumulative distribution functions of B_i's after re-weighting are showed. We see that B_i's change significantly to accommodate preference information for the scenarios. In Figure 2.1(b) one can see that the curve of B_3 did not change too much, but according to the construction of this example, it does not mean that the third attribute is not influential. This means that even if the distribution function of the third attribute does not differ too much from the uniform distribution, we still cannot exclude this third attribute.

Probabilistic inversion made B_i's slightly dependent. Table 2.2 contains the correlation matrix of B's obtained in Table 2.1. All correlations are rather small.



(a) CDF for uniform B_i's (b) CDF for new distribution of B_i's

Figure 2.1: Cumulative distribution functions of B_i

Table 2.2: Correlation matrix of B_i with 0.8

	B_1	B_2	B_3
B_1	1.00	0.01	0.17
B_2	0.01	1.00	0.17
B_3	0.17	0.17	1.00

2.3 Application of the toy model

In this section we want to show how to apply the model (2.1) in order to compute the score of each scenario. We also show how the ordering is done.

In applying the model to score different scenarios we can either use the joint distribution of (B_1, B_2, B_3), or we can simplify the model by using only means of B_i's, given in Table 2.1. Taking the means of B_i's, give us the means of S's. With means from Table 2.1, the values of X_i from relations (2.1, 2.4), based on the model (2.1) we compute the sore of a scenario as follows:

$$S = 0.3496 \times X_1 + 0.6501 \times X_2 + 0.5005 \times X_3 \tag{2.14}$$

where X_i denotes the value of i^{th} attribute.

Scenario 2 is the most severe and it gets the highest score, as expected equal to 1.8007. Using the same procedure we compute the score of the other scenarios. Scenario 1 gets score equals to 1.6511, scenario 3 equals to 1.6498 and scenario 4 equals to 1.3493. Based on this values for scores, the ordering of scenarios is: 2,1,3,4.

We can now use the model (2.14) to compare some new scenarios. For example, if we consider a new scenario, $\{1, 1, 1\}$, its corresponding score is 1.5002. The score of this new scenario is therefore smaller than the score for the scenario 3 (1.6498) but bigger than the score for the scenario 4 (1.3493).

2.4 Sensitivity of results to the probabilities of preference

In the previous section we took the imposed probabilities all equal to 0.8. We want to check how sensitive is our model to the change of these probabilities. For instance, instead of 0.8 we take 0.9.

$$P(S_2 > S_3) = 0.9; \tag{2.15}$$
$$P(S_2 > S_1) = 0.9; \tag{2.16}$$
$$P(S_3 > S_4) = 0.9; \tag{2.17}$$
$$P(S_1 > S_4) = 0.9. \tag{2.18}$$

Table 2.3 contains the results obtained when using constraints (2.15 - 2.18). If we compare them with values from Table 2.1, the mean of B_1 becomes slightly smaller whereas B_2 increases a little. We notice no big difference in B_i's means. Figure 2.2 shows the graph of cumulative distribution functions.

Table 2.3: Means and variances of B_i with 0.9

	mean	**variance**
B_1	0.3020	0.0555
B_2	0.6995	0.0554
B_3	0.5001	0.0597

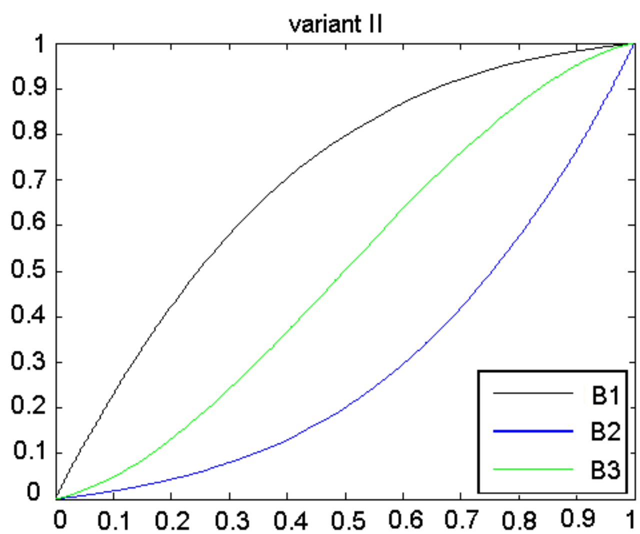


Figure 2.2: CDF of re-weighted B_i's with 0.9 probability

Naturally if all probabilities were taken to be 0.5, then probabilistic inversion would not have to adjust distribution of (B_1, B_2, B_3) at all. They would stay uniform and independent.
Based on the above investigation we conclude that the model is sensitive to the choice of probability.

2.5 Can the toy model be simplified?

We investigate whether it is possible to observe if an attribute can be removed from the model as being not important. We showed already in the previous section that even if the coefficient of the third attribute, after the probabilistic inversion does not change significantly the third attribute is the second in terms of importance. If the third attribute is removed from the model, the scores become:

$$S_1 = 0 \times B_1 + 1 \times B_2;$$
$$S_2 = 0 \times B_1 + 2 \times B_2;$$
$$S_3 = 1 \times B_1 + 2 \times B_2;$$
$$S_4 = 2 \times B_1 + 1 \times B_2.$$

This would lead to the situation where the score of the second scenario is always bigger than the score of the first scenario. We do not want any scenario to "majorise" any other scenarios. Similarly S_3 is always bigger than S_2 etc.

2.6 Conclusions

We have presented a technique to recover coefficients of attributes in a given model from preference assessments that can be obtained from experts. In the next part of this thesis we use this technique to analyse real data in the zoonosis project.

In this chapter we have used a simple example to give an intuition how the method works, to help the reader understand better the results obtained in the real application. We have showed that the method gives as results according to our intuition and allows us to build a model that can be later on, used to score and compare other scenarios.

Nevertheless the linear model which we built does work properly, in the sense that the ordering of our scenarios is satisfied and the results support our intuition. We conclude that this method provides a traceable and defendable way of quantifying the model for scores, using experts assessments.

Chapter 3

Main analysis

In this chapter we analyse the real data obtained from experts and presented in Section 1.1.

Before we present the model that we plan to use in our analysis, we refer to Tables 1.3 and 1.4 from Section 1.1 and discuss the constraints that we include in the analysis. Columns 5 to 11 represent the constraints that we are taking into consideration in our analysis. For example scenario S_1 was ranked on the last place by 4 experts. Then we consider the probability that scenario S_1 is ranked on the last place, to be equal to $\frac{4}{11}$. In a similar way, for instance, the probability that the same scenario is ranked on the sixth place (6 experts ranked S_1 on the sixth place), equals to $\frac{6}{11}$.

The total number of constraints needed to combine all scenarios using all information provided by experts is 200 (all nonempty cells in columns 5 to 11 from Tables 1.3 and 1.4), would have to be imposed. Such analysis is impossible, because probabilistic inversion method will not work due to such a large number of constraints. More about the strategies that we use to reduce the number of constraints we discuss after we present the model and the transformations of attributes that we will use.

In Chapter 1 we presented the nine attributes that we use. Remember that they are expressed in different units. We need therefore to transform their scale such that we can represent all of them in a monotonic scale from 0 to 1. Transformations that we used can be found in Chapter 4.

We start our analysis by considering the simple linear model for scores. The score of each scenario is defined as a linear combination of values attributes levels as follows:

$$S_i = B_1X_{1i} + B_2X_{2i} + B_3X_{3i} + B_4X_{4i} + B_5X_{5i} \\ + B_6X_{6i} + B_7X_{7i} + B_8X_{8i} + B_9X_{9i}, \quad i = 1, \ldots, 30 \tag{3.1}$$

where

- X_{ki} is a value of k^{th} attribute in i^{th} scenario.
- B_i's are uniformly distributed and independent coefficients of attributes in the linear model (3.1).

Our goal is, after analysing all groups together, to recover the coefficients of attributes (B_i's) from linear model (3.1), such that, after computing the scores of each scenario using the same model, we obtain the ordering obtained when using all constraints. Because it is impossible to use in the analysis all constraints, we will try to choose a variant that would allow good reconstruction of ranking order technique with minimum number of constraints. In our analysis we use 100 iterations of IPF algorithm, and 100,000 samples.

It is worth reminding that IPF is an iterative procedure, therefore the differences between probabilities that we imposed and the ones we obtained are acceptable.

Next we start presenting the variants we choose with their corresponding results.

3.1 Group 2

We first provide a detailed discussion of GROUP 2 and then we show results obtained in a similar way, for other groups.

There are 35 nonempty cells in group 2, see columns 3 to 9 from Table 3.1. Table 3.1 is a part from Table 1.3 containing summary of experts ordering. This means that we have 35 constraints to impose on the joint distribution of scores. They are of the following type: for the sixth scenario we have that the chance that S_6 is the smallest within the second group is equal to $\frac{2}{11}$, the chance that S_6 is second smallest is equal to $\frac{4}{11}$ etc., and finally the chance that S_6 is the most severe is $\frac{1}{11}$. (see Appendix **??**)

We first impose all 35 constraints and check the ordering of scenarios obtained from this constraints. Then we consider few variants with smaller number of constraints and compare their performance. We start with the variant containing all constraints (variant I, $1 \div 7$), presented in Table 3.1.

Table 3.1: Variant I of Group II - constraints used

	scenario	**1**st	**2**nd	**3**rd	**4**th	**5**th	**6**th	**7**th
VAR I	S_6	*2*	*4*	*2*		*1*	*1*	*1*
	S_7	*1*	*3*	*1*		*6*		
	S_8	*3*		*5*	*2*		*1*	
	S_9		*1*		*1*	*1*	*5*	*3*
	S_{10}	*1*			*2*		*1*	*7*
	S_{11}	*1*	*1*	*1*	*5*	*2*	*1*	
	S_{12}	*3*	*2*	*2*	*1*	*1*	*2*	

Table 3.2: Variant I of Group II - results obtained

VARIANT I: 1÷7

	mean	**variance**	**scores**	**ordering rank**	**ordering PI**	**#**	**EQ**	**OQ**	**QD**
B_1	0.7208	0.0643	S_6=1.390	S_{10}	S_{10}	‖	0.818	0.82720	0.009
B_2	0.5865	0.0761	S_7=1.326	S_9	S_9	‖	0.909	0.89764	0.011
B_3	0.2211	0.0637	S_8=1.366	S_{12}	S_{11}	‖	0.727	0.79806	0.071
B_4	0.2885	0.0549	S_9=1.503	S_{11}	S_7	‖	0.909	0.91177	0.003
B_5	0.2300	0.0443	S_{10}=1.529	S_6	S_6	‖	0.909	0.89769	0.011
B_6	0.3225	0.0453	S_{11}=1.434	S_8	S_{12}	‖	0.727	0.69834	0.029
B_7	0.5955	0.0626	S_{12}=1.435	S_7	S_8	‖	0.909	0.94041	0.031
B_8	0.5040	0.0971				‖	0.727	0.75773	0.030
B_9	0.6055	0.0472				‖	0.364	0.42983	0.066
						‖	0.636	0.64914	0.013
						‖	0.727	0.73881	0.012
						‖	0.909	0.76784	0.041
						‖	0.909	0.90126	0.008
						‖	0.818	0.84296	0.025
						‖	0.909	0.87689	0.032
							0.909	0.88544	0.024
						3	0.545	0.59949	0.054
						5	0.909	0.90746	0.002
							0.909	0.89937	0.010
						‖	0.818	0.83225	0.014
						‖	0.818	0.83887	0.021
						‖	0.909	0.90560	0.003
						‖	0.545	0.61963	0.074
						‖	0.909	0.87249	0.037
						‖	0.818	0.82738	0.009
						‖	0.909	0.92824	0.019
						‖	0.455	0.54305	0.088
						‖	0.909	0.90555	0.004
						‖	0.818	0.79495	0.023
						‖	0.909	0.91589	0.007
						‖	0.818	0.81016	0.008
						‖	0.909	0.92782	0.019
						‖	0.818	0.85331	0.035
						‖	0.545	0.54229	0.003
						‖	0.909	0.90900	0.000

 # - the number of constraints used
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ-OQ)

Table 3.2 provides us results of the probabilistic inversion (PI) analysis for 35 constraints.

Column 8 corresponds to "expert quantiles", hence the probabilities that we have we imposed. These values are computed as explained at the beginning of this section. Next column presents the "obtained quantiles". This represents the probabilities that we obtained after running probabilistic inversion. The last column provides the differences between the probabilities that we imposed and the ones that we obtained. We see that the differences for each quantile are of order 10^{-3}, which means that the problem is feasible.

After PI we compute the score of each scenario, corresponding to each sample. Next we take the mean of each distribution of B_i, and compute the mean scores. The second and third columns of Table 3.2 contain the means and variances of coefficients of attributes in the linear model for scenario scores. The fourth column shows the scores of each scenario computed with the linear model. Based on these scores we find an ordering from the most to the least sever scenario within the second group. These are given in the fifth column, titled PI. The sixth column shows the ordering of the scenarios in the second group based on rank ordering technique, titled RANK. Notice that these orderings are not the same but the most severe scenarios 10 and 9 are ranked as top ones for both methods.

We can also analyse the obtained sample file. We check for each score the frequency of occurrence, and then we compute the probability of each scenario to be ranked on each position. In other words, we imposed the probability of scenario S_6 to be ranked on the 1^{st} place, to be $\frac{2}{11}$. We check from the sample file, the number of times when the score of S_6 had the smallest number. By dividing this number to the total number of samples, we obtain the probability that scenario S_1 was placed on the first place. We expect that this probability to be as close as possible to $\frac{2}{11}$. We call this procedure *in samples validation*. We perform the same analysis for the rest of the variants, and for each group. The complete results are presented in Appendix B. Table 3.3 contains the probabilities obtained for each scenario, before and after the inversion technique.

Based on this validation of samples we compute two root mean square errors:

- first, RMSE of "fitting", with which we check how good we fit the model to our data. This error represents the square root of the means of squared differences between the imposed probabilities and obtained ones. (i.e. we imposed for S_6 to be ranked on the first place probability 2/11, and we obtained 1.9059/11. We check all these squared differences, and take the square root of their mean)

- second, RMSE of samples validation. This error is computed as the square root of the means of squared differences between the obtained probability from each variant and obtained probabilities from variant I. (i.e. we subtract from

Table 3.3: Out of sample validation for Variant I of Group II

	scenario	1^{st} obt pbty	2^{nd} obt pbty	3^{rd} obt pbty	4^{th} obt pbty	5^{th} obt pbty	6^{th} obt pbty	7^{th} obt pbty
VAR I	S_6	*2* 1.9059	*4* 4.0464	*2* 1.8715	0.0605	*1* 0.8468	*1* 1.1464	*1* 1.1225
	S_7	*1* 1.1697	*3* 2.7366	*1* 0.8804	0.4342	*6* 5.7790	0.0001	0
	S_8	*3* 2.9187	0	*5* 4.8386	*2* 1.8168	0.3682	*1* 0.9668	0.0910
	S_9	0.2680	*1* 0.8712	0.1118	*1* 0.9197	*1* 1.0177	*5* 4.8181	*3* 2.9935
	S_{10}	*1* 1.0276	0.4194	0.2111	*2* 1.8166	0.0001	*1* 1.1578	*7* 6.3674
	S_{11}	*1* 0.9140	*1* 1.0035	*1* 0.9231	*5* 4.9357	*2* 2.0093	*1* 0.9044	0.3100
	S_{12}	*3* 2.7961	*2* 1.9229	*2* 2.1635	*1* 1.0165	*1* 0.9790	*2* 2.0064	0.1156
RMSE of fitting = 0.2226								
RMSE of validation = 0.0000								

1.9059 (the obtained probability that S_6 is ranked on the first place, when considering all constraints) the probability that S_6 is ranked on the first place, but obtained in all other variants. For instance, in Table 3.5 this probability equals to 1.7595) Using this error we check how far we are from the validation obtained in the first variant. Due to lack of space, we will present here only the values of the two errors, and the complete results can be found in Appendix B.

We see in Table 3.3 that the probabilities computed from the sample file obtained after PI are close to the ones imposed. We will present the results for each variant of this second group.

We are interested in another variant with a smaller number of constraints and yet small differences between scores obtained in variant 1, and the scores obtained with the new variant. In the same time, we want the error of fitting and validation to be as small as possible.

Let us consider the second variant, in which we take constraints corresponding to the first three and the last three columns of Table 3.4. Hence we are taking into account all constraints except the ones that give percentages of experts that considered a given scenario as forth in terms of severity. This choice reduces the number of constraints from 35 to 30. We denote it as variant II (1,2,3,5,6,7). Table 3.4 contains cells used as constraints in PI procedure, and Table 3.6 provides the results after probabilistic inversion. We notice that both errors (validation and fitting) have increased in this case. We continue investigating the problem, by removing constraints and observing the evolution of these two errors.

Table 3.4: Variant II of Group II - constraints used

	scenario	$\mathbf{1}^{st}$	$\mathbf{2}^{nd}$	$\mathbf{3}^{rd}$	$\mathbf{4}^{th}$	$\mathbf{5}^{th}$	$\mathbf{6}^{th}$	$\mathbf{7}^{th}$
V A R I I	S_6	*2*	*4*	*2*		*1*	*1*	*1*
	S_7	*1*	*3*	*1*		*6*		
	S_8	*3*		*5*			*1*	
	S_9		*1*			*1*	*5*	*3*
	S_{10}	*1*					*1*	*7*
	S_{11}	*1*	*1*	*1*		*2*	*1*	
	S_{12}	*3*	*2*	*2*		*1*	*2*	

Table 3.5: Out of sample validation for Variant II of Group II

	scenario	$\mathbf{1}^{st}$ **obt pbty**	$\mathbf{2}^{nd}$ **obt pbty**	$\mathbf{3}^{rd}$ **obt pbty**	$\mathbf{4}^{th}$ **obt pbty**	$\mathbf{5}^{th}$ **obt pbty**	$\mathbf{6}^{th}$ **obt pbty**	$\mathbf{7}^{th}$ **obt pbty**
V A R I I	S_6	*2* 1.7595	*4* 4.0912	*2* 1.7219	0.5778	*1* 0.8000	*1* 1.1208	*1* 0.9287
	S_7	*1* 0.9833	*3* 2.6356	*1* 0.8007	0.6698	*6* 5.9106	0	0
	S_8	*3* 2.9827	0	*5* 5.2307	(2)1.5581	0.2746	*1* 0.9536	0.0003
	S_9	0	*1* 0.9975	0.0007	(1)1.8285	*1* 1.0099	*5* 4.4587	*3* 2.7047
	S_{10}	*1* 1.0047	0.0590	0.0006	(2)2.5877	0	*1* 1.1378	*7* 6.2102
	S_{11}	*1* 1.0695	*1* 1.2263	*1* 3.0609	(5)1.6032	*2* 2.0050	*1* 1.1148	0.9203
	S_{12}	*3* 3.2003	*2* 1.9905	*2* 0.1846	(1)2.1747	*1* 0.9999	*2* 2.2142	0.2358
RMSE of fitting = 0.9154								
RMSE of validation = 0.9298								

We notice in columns 8, 9 and 10 from Table 3.6 the same type of information as for the first variant. The differences between the imposed probabilities and the obtained ones are relatively small, which means that the problem is feasible. We notice a new column in this table, the differences of scores. This represents the difference of the scores obtained in the first variant, and the scores obtained in this second variant. Because in variant II we include less constraints this means we omit some of the experts' assessments, which creates an error of the obtained scores. We measure this error with the Root Mean Square Error (RMSE). We want this error to be as small as possible. Considering variant with 30 constraints the RMSE of scores equals to 0.484768.

Table 3.5 presents the probabilities of each scenario computed from the sample file.

These errors can be compared for different variants, and the variant having the smallest number of constraints and in the same time the small error will be preferred for further analysis.

Our goal is to find a variant such that:

- our problem is feasible;
- we recover the ordering of scenarios given by experts;
- the scores differences with variant I to be as small as possible.

Table 3.6: Variant II of Group II - results obtained

VARIANT II: 1,2,3 5,6,7

	mean	variance	scores	ordering rank	ordering PI	#	EQ	OQ	QD	SD
B_1	0.7398	0.0568	S_6=1.370	S_{10}	S_{10}		0.818	0.83811	0.020	0.020
B_2	0.6092	0.0846	S_7=1.318	S_9	S_9		0.909	0.91807	0.009	0.008
B_3	0.1887	0.0301	S_8=1.298	S_{11}	S_{11}		0.727	0.72152	0.006	0.068
B_4	0.2636	0.0412	S_9=1.538	S_6	S_7		0.909	0.91657	0.007	0.035
B_5	0.2354	0.0571	S_{10}=1.541	S_{12}	S_6		0.909	0.90069	0.008	0.012
B_6	0.2388	0.0366	S_{11}=1.436	S_7	S_{12}		0.727	0.70504	0.022	0.002
B_7	0.6417	0.0510	S_{12}=1.345	S_8	S_8		0.909	0.92436	0.015	0.090
B_8	0.5485	0.0908					0.727	0.76561	0.038	
B_9	0.5989	0.0472					0.364	0.36151	0.002	
							0.636	0.65520	0.019	
							0.727	0.73355	0.006	
							0.909	0.91457	0.005	
							0.909	0.90652	0.003	
							0.818	0.80345	0.015	
							0.909	0.91904	0.010	
							0.909	0.91227	0.003	
						3	0.545	0.56320	0.018	
						0	0.909	0.91356	0.004	
							0.909	0.89817	0.011	
							0.818	0.80706	0.011	
							0.818	0.82560	0.007	
							0.909	0.91289	0.004	
							0.545	0.55328	0.008	
							0.909	0.90433	0.005	
							0.818	0.80938	0.009	
							0.909	0.91830	0.009	
							0.455	0.46307	0.009	
							0.909	0.90900	0.000	
							0.818	0.81800	0.000	
							0.909	0.90900	0.000	

 # - the number of constraints used RMSE=0.484767986
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ − OQ)
SD - scores differences (scores from variant I − scores from variant II)
RMSE - square root of the sum of scores differences

30 constraints for each group would still be to much to use in combining all groups. Let us consider variant III in which only constraints corresponding to two least severe and two most severe rankings are included (1,2,6,7). Table 3.7 shows the included constraints and Table 3.9 presents results obtained for variant III.

In this case we reduced the number of constraints to 20. Looking in column 10 from Table 3.7 we find that the problem is feasible. The differences between scores obtained in this variant and the ones from variant I, are reflected in RMSE. We observe a slight increase of this value as compared to the previous variant.

Table 3.7: Variant III of Group II - constraints used

	scenario	**1st**	**2nd**	**3rd**	**4th**	**5th**	**6th**	**7th**
VARIII	S_6	*2*	*4*				*1*	*1*
	S_7	*1*	*3*					
	S_8	*3*					*1*	
	S_9		*1*				*5*	*3*
	S_{10}	*1*					*1*	*7*
	S_{11}	*1*	*1*				*1*	
	S_{12}	*3*	*2*				*2*	

Table 3.8: Out of sample validation for Variant III of Group II

	scenario	**1st obt pbty**	**2nd obt pbty**	**3rd obt pbty**	**4th obt pbty**	**5th obt pbty**	**6th obt pbty**	**7th obt pbty**
VARIII	S_6	*2* 1.9766	*4* 4.0070	(2)0.7264	1.1607	(1)1.1681	*1* 0.9890	*1* 0.9722
	S_7	*1* 1.0081	*3* 2.9864	(1)4.9960	1.3396	(6)0.6409	0.0290	0
	S_8	*3* 2.9817	0.0299	(5)1.3602	(2)2.3099	3.3080	*1* 1.0028	0.0076
	S_9	0.0228	*1* 1.0025	0.5900	(1)0.8228	(1)0.5963	*5* 4.9721	*3* 2.9934
	S_{10}	*1* 1.0050	0.0053	0.6008	(2)0.7726	0.6275	*1* 0.9839	*7* 7.0048
	S_{11}	*1* 1.0079	*1* 0.9827	(1)1.9866	(5)3.2245	(2)2.7718	*1* 1.0232	0.0032
	S_{12}	*3* 2.9979	*2* 1.9862	(2)0.7400	(1)1.3698	(1)1.8874	*2* 1.9999	0.0188

RMSE of fitting = 2.0323

RMSE of validation = 2.1003

Table 3.9: Variant III of Group II - results obtained

VARIANT III: 1,2 6,7

	mean	variance	scores	ordering rank	ordering PI	#	EQ	OQ	QD	SD
B_1	0.7301	0.0601	S_6=1.356	S_{10}	S_{10}	||	0.818	0.81839	0.000210	0.034
B_2	0.5568	0.0797	S_7=1.304	S_9	S_9	||	0.909	0.90917	0.000083	0.022
B_3	0.2120	0.0232	S_8=1.312	S_{11}	S_{11}	||	0.727	0.72733	0.000054	0.054
B_4	0.2312	0.0321	S_9=1.523	S_6	S_7	||	0.909	0.90909	0.000004	0.020
B_5	0.2253	0.0557	S_{10}=1.524	S_{12}	S_6	||	0.909	0.90907	0.000025	0.005
B_6	0.2520	0.0335	S_{11}=1.409	S_8	S_{12}	||	0.727	0.72700	0.000273	0.025
B_7	0.6346	0.0474	S_{12}=1.350	S_7	S_8	||	0.909	0.90909	0.000002	0.085
B_8	0.5418	0.0828					0.727	0.72719	0.000086	
B_9	0.5477	0.0674				2	0.364	0.36400	0.000363	
						0	0.636	0.63665	0.000289	
							0.727	0.72728	0.000007	
						||	0.909	0.90907	0.000021	
						||	0.909	0.90905	0.000044	
						||	0.818	0.81800	0.000182	
						||	0.909	0.90924	0.000152	
						||	0.909	0.90922	0.000128	
						||	0.545	0.54549	0.000031	
						||	0.909	0.90907	0.000018	
						||	0.909	0.90905	0.000042	
						||	0.818	0.81800	0.000182	

 # - the number of constraints used RMSE=0.494974747
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ – OQ)
SD - scores differences (scores from variant I – scores from variant III)
RMSE - square root of the sum of scores differences

Next we take out more constraints: the second and the sixth column, and this way we obtain variant IV, denoted (1,7). We only take into consideration experts opinions regarding the most sever and the least sever scenarios. As shown in the Table 3.10 the constraints' number reduces significantly from 20 in the previous variant to 9. The next important aspect is to check the RMSE which increases but not noticeable. Its value is now 0.558091845.

In variant II the error of scores equals to 0.484767986, and in variant IV to 0.558091845. The error in variant IV is larger than the one in Variant II, but only slightly. However, the gain in reduction of number of constraints is significant (we use 9 instead of 30).

Table 3.10: Variant IV of Group II - constraints used

	scenario	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}
VARIANT IV	S_6	*2*						*1*
	S_7	*1*						
	S_8	*3*						
	S_9							*3*
	S_{10}	*1*						*7*
	S_{11}	*1*						
	S_{12}	*3*						

Table 3.11: Out of sample validation for Variant IV of Group II

	scenario	1^{st} obt pbty	2^{nd} obt pbty	3^{rd} obt pbty	4^{th} obt pbty	5^{th} obt pbty	6^{th} obt pbty	7^{th} obt pbty
VARIANT IV	S_6	*2* 1.9546	(4) 0.5807	(2) 1.2015	2.4484	(1) 2.3003	(1) 1.5365	*1* 0.9780
	S_7	*1* 0.9898	(3) 4.2375	(1) 3.5070	1.2429	(6) 0.7414	0.2813	0.0001
	S_8	*3* 2.9849	2.4874	(5) 1.5934	(2) 1.3692	1.4083	(1) 1.1510	0.0058
	S_9	0.0331	(1) 1.0551	1.5291	(1) 2.1561	(1) 1.8277	(5) 1.4160	*3* 2.9829
	S_{10}	*1* 1.0068	0.7738	0.6006	(2) 0.5357	0.5970	(1) 0.4777	*7* 7.0085
	S_{11}	*1* 0.9861	(1) 0.6787	(1) 1.4266	(5) 1.9330	(2) 2.3995	(1) 3.5733	0.0029
	S_{12}	*3* 3.0447	(2) 1.1869	(2) 1.1419	(1) 1.3147	(1) 1.7259	(2) 2.5642	0.0217

RMSE of fitting = 3.5302

RMSE of validation = 3.6110

Table 3.12: Variant IV of Group II - results obtained

VARIANT IV: 1 7

	mean	variance	scores	ordering rank	ordering PI	#	EQ	OQ	QD	SD
B_1	0.6757	0.0654	S_6=1.448	S_{10}	S_{10}	‖	0.818	0.8139	0.00071	0.058
B_2	0.5294	0.0848	S_7=1.372	S_9	S_9	‖	0.909	0.9094	0.00031	0.046
B_3	0.2208	0.0371	S_8=1.301	S_{11}	S_{11}	‖	0.727	0.7277	0.00047	0.065
B_4	0.2531	0.0402	S_9=1.536	S_6	S_7		0.909	0.9092	0.00011	0.033
B_5	0.2658	0.0713	S_{10}=1.556	S_{12}	S_6	9	0.909	0.9092	0.00007	0.027
B_6	0.2529	0.0393	S_{11}=1.473	S_7	S_{12}		0.727	0.7270	0.00028	0.039
B_7	0.6356	0.0513	S_{12}=1.391	S_8	S_8	‖	0.909	0.9098	0.00076	0.044
B_8	0.5324	0.0804				‖	0.727	0.7288	0.00151	
B_9	0.6355	0.0716				‖	0.364	0.3640	0.00036	

 # - the number of constraints used RMSE=0.558091845

EQ - experts quantiles (experts assessments)

OQ - obtained quantiles (after PI)

QD - quantiles differences (EQ − OQ)

SD - scores differences (scores from variant I − scores from variant IV)

RMSE - square root of the sum of scores differences

At the moment there is no statistical method to decide when we should stop removing constraints. Moreover, we do not have a structural way of deciding which constraints we should choose.

Next we present one more variant. It is called variant V ($\geq$3). The idea here is that we take only those constraints for which at least 3 experts agreed that the scenario should have a certain ranking. For instance, looking at the sixth scenario from Table 3.13, this means that only the constraint that the chance that S_6 is second smallest is 4/11 is included. In the Table 3.13 the cells included in PI are presented.

Table 3.13: Variant V of Group II - constraints used

	scenario	**1st**	**2nd**	**3rd**	**4th**	**5th**	**6th**	**7th**
VAR V	S_6		*4*					
	S_7		*3*			*6*		
	S_8	*3*		*5*				
	S_9						*5*	*3*
	S_{10}							*7*
	S_{11}				*5*			
	S_{12}	*3*						

Table 3.14: Out of sample validation for Variant V of Group II

	scenario	**1st obt pbty**	**2nd obt pbty**	**3rd obt pbty**	**4th obt pbty**	**5th obt pbty**	**6th obt pbty**	**7th obt pbty**
VAR V	S_6	(2)0.0278	*4* 3.9963	(2)1.1543	2.3857	(1)0.9133	(1)2.4702	(1)0.0524
	S_7	(1)1.5705	*3* 3.0278	(1)0.3896	0.0334	*6* 5.9773	0.0014	0
	S_8	*3* 3.0138	0.0773	*5* 5.0181	(2)1.0442	0.6866	(1)0.5027	0.0273
	S_9	0.0979	(1)0.5331	0.9481	(1)1.1331	(1)0.2735	*5* 5.0030	*3* 3.0114
	S_{10}	(1)2.9853	0.3935	0.2949	(2)0.2950	0.0066	(1)0.0252	*7* 6.9995
	S_{11}	(1)0.2914	(1)1.0193	(1)1.5893	*5* 4.9936	(2)1.6948	(1)1.3362	0.0756
	S_{12}	*3* 3.0133	(2)1.3229	(2)1.6057	(1)1.1150	(1)1.4479	(2)1.6613	0.8339

RMSE of fitting = 1.6301
RMSE of validation = 1.6914

Table 3.15: Variant V of Group II - results obtained

VARIANT V: ≥3

	mean	variance	scores	ordering rank	ordering PI	#	EQ	OQ	QD	SD
B_1	0.6804	0.0556	S_6=1.381	S_9	S_{10}	‖	0.727	0.7270	0.00027	0.009
B_2	0.5133	0.0790	S_7=1.314	S_{10}	S_9	‖	0.727	0.7270	0.00027	0.012
B_3	0.2250	0.0399	S_8=1.256	S_{11}	S_{11}	‖	0.727	0.7270	0.00027	0.110
B_4	0.3183	0.0643	S_9=1.474	S_{12}	S_7		0.364	0.3640	0.00036	0.029
B_5	0.2950	0.0928	S_{10}=1.464	S_6	S_6	10	0.636	0.6360	0.00036	0.065
B_6	0.2546	0.0444	S_{11}=1.409	S_7	S_{12}		0.727	0.7270	0.00027	0.025
B_7	0.6232	0.0490	S_{12}=1.397	S_8	S_8	‖	0.909	0.9090	0.00009	0.038
B_8	0.4811	0.0850				‖	0.545	0.5450	0.00145	
B_9	0.6056	0.0754				‖	0.455	0.4550	0.00045	
						‖	0.545	0.5450	0.00045	

 # - the number of constraints used RMSE=0.53705847
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ − OQ)
SD - scores differences (scores from variant I − scores from variant V)
RMSE - square root of the sum of scores differences

When we compare variant V with variant IV we notice an increase in the number of constraints by 1. However the RMSE of scores is now smaller and equals to 0.53705847. This last variant also provides promising results. Moreover, the two errors (fitting and validation) have decreased significantly in this case (they are now approximately 1.69, see Table 3.14. In this group we conclude that the best variant to be considered for further analysis is the last variant.

For convenience we present results for all discussed variants for group 2 together in Table 3.16.

Table 3.16: Group II - all variants

GROUP II		**mean**	**variance**	**scores**	**ordering rank**	**PI**	**#**	
Variant I	B_1	0.7208	0.0643	S_6=1.326	S_{10}	S_{10}	||	
	B_2	0.5865	0.0761	S_7=1.366	S_9	S_9	||	
	B_3	0.2211	0.0637	S_8=1.503	S_{12}	S_{11}		
	B_4	0.2885	0.0549	S_9=1.529	S_{11}	S_7	3	
	B_5	0.2300	0.0443	S_{10}=1.434	S_6	S_6	5	
	B_6	0.3225	0.0453	S_{10}=1.435	S_8	S_{12}		
	B_7	0.5955	0.0626		S_7	S_8	||	
	B_8	0.5040	0.0971				||	
	B_9	0.6055	0.0472				||	**SD**
Variant II	B_1	0.7398	0.0568	S_6=1.370	S_{10}	S_{10}	||	0.020
	B_2	0.6092	0.0846	S_7=1.318	S_9	S_9	||	0.008
	B_3	0.1887	0.0301	S_8=1.298	S_{11}	S_{11}		0.068
	B_4	0.2636	0.0412	S_9=1.538	S_6	S_7	3	0.035
	B_5	0.2354	0.0571	S_{10}=1.541	S_{12}	S_6	0	0.012
	B_6	0.2388	0.0366	S_{11}=1.436	S_7	S_{12}		0.002
	B_7	0.6417	0.0510	S_{12}=1.345	S_8	S_8	||	0.090
	B_8	0.5485	0.0908				||	RMSE
	B_9	0.5989	0.0472				||	0.48476799
Variant III	B_1	0.7301	0.0601	S_6=1.356	S_{10}	S_{10}	||	0.034
	B_2	0.5568	0.0797	S_7=1.304	S_9	S_9	||	0.022
	B_3	0.2120	0.0232	S_8=1.312	S_{11}	S_{11}		0.054
	B_4	0.2312	0.0321	S_9=1.523	S_6	S_7	2	0.020
	B_5	0.2253	0.0557	S_{10}=1.524	S_{12}	S_6	0	0.005
	B_6	0.2520	0.0335	S_{11}=1.409	S_8	S_{12}		0.025
	B_7	0.6346	0.0474	S_{12}=1.350	S_7	S_8	||	0.085
	B_8	0.5418	0.0828				||	RMSE
	9	0.5477	0.0674				||	0.49497475
Variant IV	B_1	0.6757	0.0654	S_6=1.448	S_{10}	S_{10}	||	0.058
	B_2	0.5294	0.0848	S_7=1.372	S_9	S_9	||	0.046
	B_3	0.2208	0.0371	S_8=1.301	S_{11}	S_{11}	||	0.065
	B_4	0.2531	0.0402	S_9=1.536	S_6	S_7		0.033
	B_5	0.2658	0.0713	S_{10}=1.556	S_{12}	S_6	9	0.027
	B_6	0.2529	0.0393	S_{11}=1.473	S_7	S_{12}		0.039
	B_7	0.6356	0.0513	S_{12}=1.391	S_8	S_8	||	0.044
	B_8	0.5324	0.0804				||	RMSE
	B_9	0.6355	0.0716				||	0.55809184
Variant V	B_1	0.6807	0.0566	S_6=1.381	S_9	S_{10}	||	
	B_2	0.5133	0.0790	S_7=1.314	S_{10}	S_9	||	
	B_3	0.2250	0.0399	S_8=1.256	S_{11}	S_{11}	||	
	B_4	0.3183	0.0643	S_9=1.474	S_{12}	S_7		
	B_5	0.2950	0.0928	S_{10}=1.464	S_6	S_6	10	
	B_6	0.2546	0.0444	S_{11}=1.409	S_7	S_{12}		
	B_7	0.6232	0.0490	S_{12}=1.397	S_8	S_8	||	
	B_8	0.4811	0.0850				||	RMSE
	B_9	0.6056	0.0754				||	0.53705847

 # - the number of constraints used
SD - scores differences with variant I

Table 3.16 gives the relationship between the number of constraints and the root mean square error of scores. For instance, we see that using 30 constraints out of 35 the error is approximately 0.48. If we take 20 constraints, the error increases a little, and equals almost to 0.49, and so on. Another important aspect is that in all five variants we recover the ordering of scenarios, given by experts. Finally, the problem is feasible for all five variants.

Keeping in mind that the ordering of scenarios is recovered for all variants, we decide that "best variant" for group II is the last variant. We will use this last variant for further analysis.

Next we present the plots of the imposed and obtained probability, corresponding to each rank (position). Figure 3.1 shows the imposed probabilities, and the obtained ones in all of the 5 variants. We can see from this plot that the line corresponding to variant 5 follows the most accurate the line corresponding to imposed probabilities. In the same manner all the ranks are treated, and it is visible that for all ranks the variant V has performed the best. If we combine this knowledge with the obtained results for the two errors (fitting and validation), and with the error for scores, we conclude that the last variant is suitable for the further analysis.

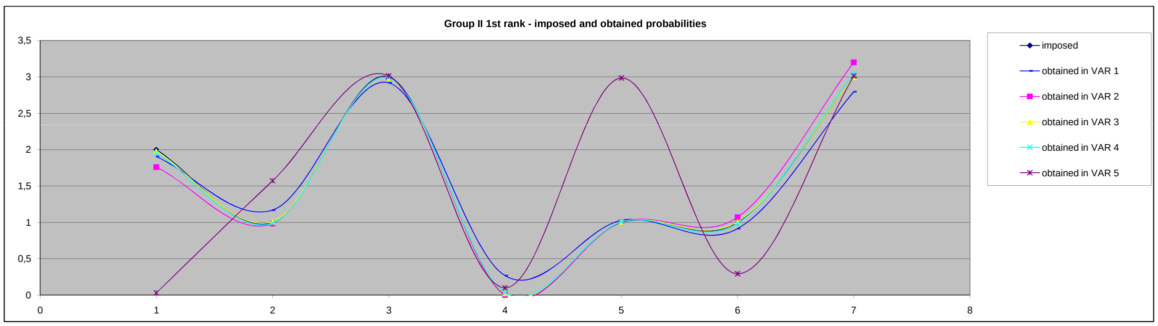


Figure 3.1: Group II - first rank plot

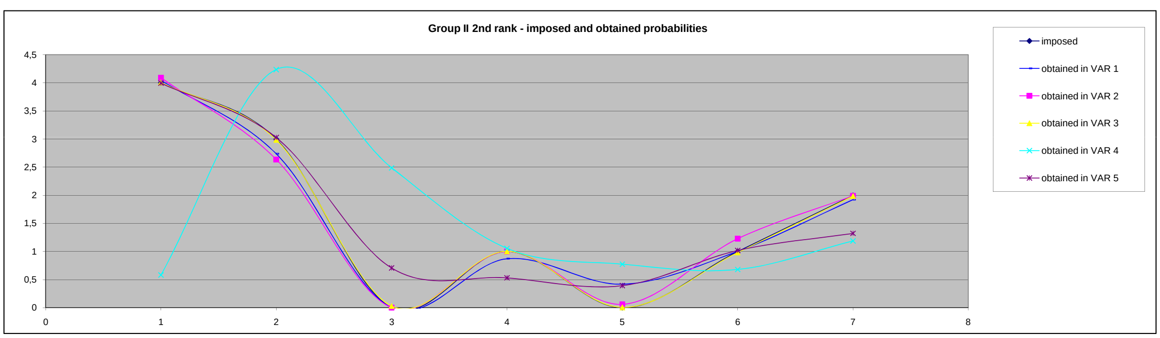


Figure 3.2: Group II - second rank plot

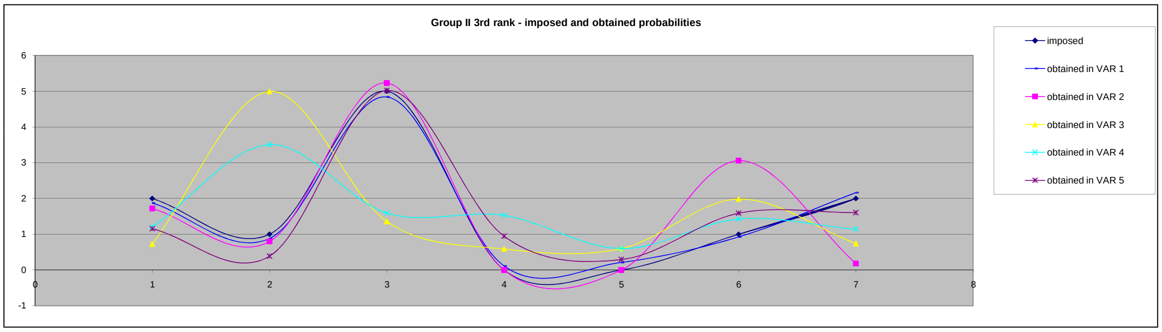


Figure 3.3: Group II - third rank plot

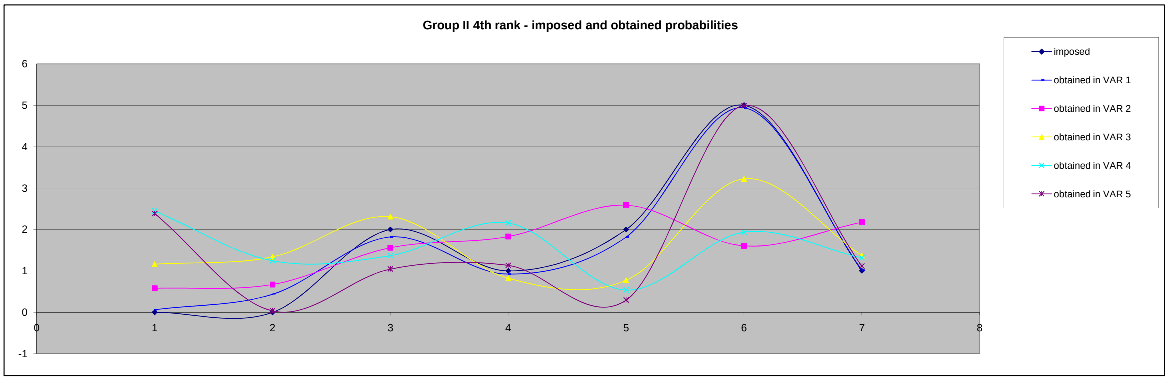


Figure 3.4: Group II - fourth rank plot

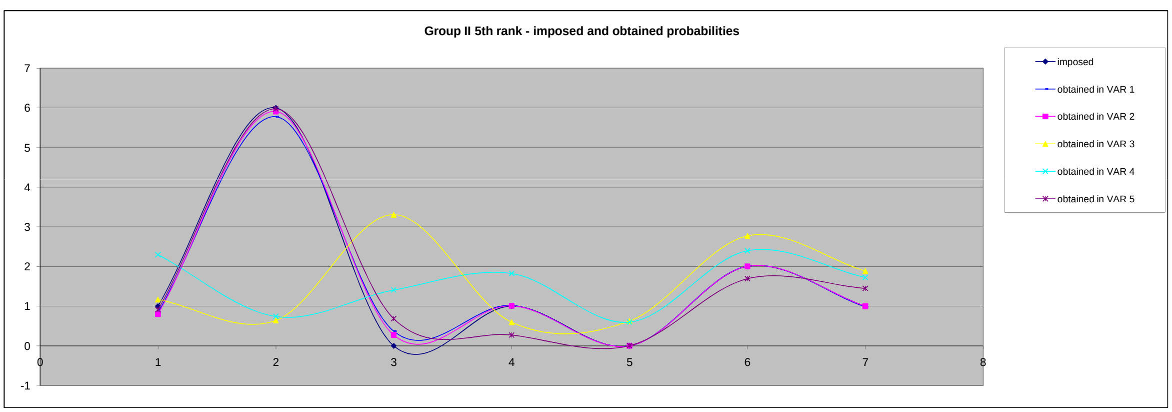


Figure 3.5: Group II - fifth rank plot

We analyse similarly the other groups. The same variants as for group 2 are considered further.

3.2 Group 1

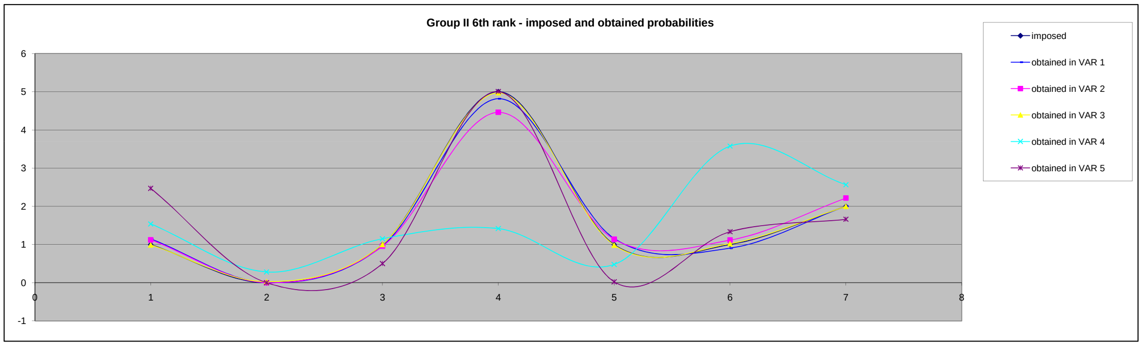


Figure 3.6: Group II - sixth rank plot

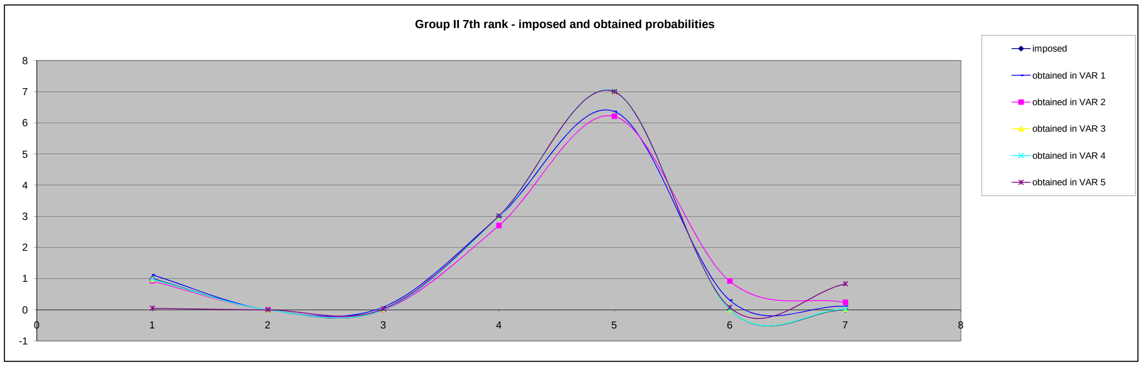


Figure 3.7: Group II - seventh rank plot

Taking first three and last three columns of constraints results in a small value of RMSE, but in a large number of constraints. Very good results are obtained in variant (III), where the value of RMSE is 0.50149932 and the number of constraints 14. In variant (IV) the number of constraints is small but RMSE increased significantly. This should be further investigated. The last variant is variant V with the number of constraints equal to 13 and RMSE equal to 0.54665841.

As a conclusion for this group, we state that variant III (1,2,6,7) and V ($\geq$3) are suitable for the analysis, but we consider in our analysis variant V.

3.3 Group 3

In this group due to an error, scenario number 12 is not the same as scenario number 12 from group 2. The difference is in the value of the first attribute: "chance of introduction". This value was taken equal to zero by mistake ((1) corresponds to 0% chances of introduction, whereas (5) corresponds to 100% chances of introduction).

Because of this error 10 out of 11 experts considered this scenario the least sever, and one out of 11 experts considered this scenario as the most severe. This is a strange result, and this scenario might have been misunderstood. After the elicitation, the discussion with experts revealed that they were very confused about

Table 3.17: Group I - all variants

GROUP I					**ordering**			
		mean	**variance**	**scores**	**rank**	**PI**	**#**	
Variant I	B_1	0.4849	0.1136	S_1=1.584	S_1	S_1	‖	
	B_2	0.5460	0.0751	S_2=1.263	S_3	S_3	‖	
	B_3	0.5060	0.0940	S_3=1.550	S_6	S_6		
	B_4	0.1794	0.0211	S_4=1.233	S_7	S_7	2	
	B_5	0.4746	0.0880	S_5=1.112	S_4	S_2	8	
	B_6	0.4399	0.0803	S_6=1.380	S_2	S_4		
	B_7	0.6913	0.0551	S_7=1.334	S_5	S_5	‖	
	B_8	0.4021	0.0776				‖	
	B_9	0.3629	0.0662				‖	**SD**
Variant II	B_1	0.4847	0.1136	S_1=1.580	S_1	S_1	‖	0.004
	B_2	0.5479	0.0751	S_2=1.258	S_3	S_3	‖	0.005
	B_3	0.5034	0.0945	S_3=1.546	S_6	S_6		0.004
	B_4	0.1784	0.0211	S_4=1.229	S_7	S_7	2	0.004
	B_5	0.4747	0.0881	S_5=1.109	S_4	S_2	3	0.003
	B_6	0.4409	0.0807	S_6=1.376	S_2	S_4		0.004
	B_7	0.6895	0.0557	S_7=1.331	S_5	S_5	‖	0.003
	B_8	0.4028	0.0779				‖	RMSE
	B_9	0.3616	0.0662				‖	0.16785829
Variant III	B_1	0.5321	0.1020	S_1=1.626	S_1	S_1	‖	0.042
	B_2	0.5533	0.0766	S_2=1.301	S_3	S_3	‖	0.038
	B_3	0.5006	0.0938	S_3=1.596	S_6	S_6		0.046
	B_4	0.1864	0.0227	S_4=1.274	S_7	S_7	1	0.041
	B_5	0.4673	0.0855	S_5=1.153	S_4	S_2	4	0.041
	B_6	0.4460	0.0797	S_6=1.403	S_2	S_4		0.023
	B_7	0.6726	0.0587	S_7=1.354	S_5	S_5	‖	0.020
	B_8	0.4140	0.0785				‖	RMSE
	B_9	0.3886	0.0734				‖	0.50149932
Variant IV	B_1	0.5427	0.0946	S_1=1.688	S_1	S_1	‖	0.104
	B_2	0.5532	0.0740	S_2=1.451	S_3	S_3	‖	0.188
	B_3	0.4520	0.0886	S_3=1.683	S_6	S_6	‖	0.133
	B_4	0.2445	0.0317	S_4=1.346	S_7	S_2		0.113
	B_5	0.5064	0.0847	S_5=1.233	S_4	S_7	7	0.121
	B_6	0.4768	0.0803	S_6=1.471	S_2	S_4		0.091
	B_7	0.6302	0.0671	S_7=1.415	S_5	S_5	‖	0.081
	B_8	0.4692	0.0823				‖	RMSE
	B_9	0.4758	0.0865				‖	0.91119084
Variant V	B_1	0.5074	0.0825	S_1=1.612	S_1	S_1	‖	0.029
	B_2	0.5666	0.0760	S_2=1.351	S_3	S_3	‖	0.088
	B_3	0.4154	0.0844	S_3=1.595	S_6	S_6		0.045
	B_4	0.2385	0.0439	S_4=1.278	S_7	S_7	1	0.045
	B_5	0.5302	0.0826	S_5=1.174	S_4	S_2	3	0.062
	B_6	0.5058	0.0796	S_6=1.388	S_2	S_4		0.005
	B_7	0.6618	0.0645	S_7=1.360	S_5	S_5	‖	0.026
	B_8	0.4875	0.0829				‖	RMSE
	B_9	0.3988	0.0809				‖	0.54665841

 # - the number of constraints used
SD - scores differences with variant I

this scenario, as costs and spread can be high only if the pathogen have been introduced. We had few alternatives that we could have chosen in order to solve this problem. One of them was to redo the elicitation process, with corrected scenario

12. Another alternative was to exclude the whole group 3 from our analysis. We could have also changed the model, or, the fourth alternative was that we found the most attractive to remove scenario 12 from our analysis.

Redoing the elicitation would probably be the best alternative to take. Unfortunately, the time constraints do not allow us to choose this alternative. To change the model, due to a mistake was not considered the best option. Moreover, if we have removed scenario 12 from the group, the linear model seems to fit. Because of this, we decided to remove scenario 12 from the group, in all considered variants.

Table 3.18: Group III - all variants

GROUP III					**ordering**			
		mean	**variance**	**scores**	**rank**	**PI**	**#**	
Variant I	B_1	0.3617	0.0558	$S_{11}=1.302$	S_{16}	S_{16}	‖	
	B_2	0.2121	0.0716		S_{13}	S_{15}	‖	
	B_3	0.4194	0.0631	$S_{13}=1.239$	S_{11}	S_{14}		
	B_4	0.2215	0.0588	$S_{14}=1.395$	S_{14}	S_{11}	3	
	B_5	0.5048	0.1225	$S_{15}=1.401$	S_{17}	S_{17}	1	
	B_6	0.3664	0.0964	$S_{16}=1.608$	S_{15}	S_{13}		
	B_7	0.5719	0.0842	$S_{17}=1.254$			‖	
	B_8	0.6336	0.0894				‖	
	B_9	0.3126	0.0968				‖	**SD**
Variant II	B_1	0.4083	0.0729	$S_{11}=1.299$	S_{16}	S_{16}	‖	0.003
	B_2	0.1990	0.0456		S_{13}	S_{15}	‖	
	B_3	0.4489	0.0759	$S_{13}=1.268$	S_{11}	S_{14}		0.029
	B_4	0.2658	0.1044	$S_{14}=1.444$	S_{14}	S_{11}	2	0.050
	B_5	0.4710	0.1050	$S_{15}=1.468$	S_{17}	S_{13}	6	0.066
	B_6	0.3509	0.0974	$S_{16}=1.566$	S_{15}	S_{17}		0.042
	B_7	0.5772	0.0698	$S_{17}=1.261$			‖	0.007
	B_8	0.6678	0.0712				‖	RMSE
	B_9	0.2253	0.0553				‖	0.44410977
Variant III	B_1	0.4687	0.1005	$S_{11}=1.402$	S_{16}	S_{16}	‖	0.099
	B_2	0.2503	0.0769		S_{13}	S_{14}	‖	
	B_3	0.4315	0.0844	$S_{13}=1.285$	S_{11}	S_{15}		0.046
	B_4	0.1698	0.0417	$S_{14}=1.441$	S_{14}	S_{11}	1	0.047
	B_5	0.4913	0.0888	$S_{15}=1.439$	S_{17}	S_{13}	4	0.038
	B_6	0.3616	0.0870	$S_{16}=1.645$	S_{15}	S_{17}		0.037
	B_7	0.5507	0.0928	$S_{17}=1.273$			‖	0.020
	B_8	0.6216	0.0812				‖	RMSE
	B_9	0.3668	0.0833				‖	0.53491265
Variant IV	B_1	0.5689	0.0829	$S_{11}=1.454$	S_{16}	S_{16}	‖	0.0151
	B_2	0.3505	0.1060		S_{13}	S_{14}	‖	
	B_3	0.3825	0.0793	$S_{13}=1.256$	S_{11}	S_{15}	‖	0.016
	B_4	0.1972	0.0405	$S_{14}=1.485$	S_{14}	S_{11}		0.090
	B_5	0.6004	0.0789	$S_{15}=1.482$	S_{17}	S_{13}	4	0.081
	B_6	0.4161	0.0810	$S_{16}=1.674$	S_{15}	S_{17}		0.066
	B_7	0.4739	0.0864	$S_{17}=1.202$			‖	0.052
	B_8	0.5872	0.0813				‖	RMSE
	B_9	0.4797	0.0839				‖	0.67568840
Variant V	B_1	0.4582	0.0969	$S_{11}=1.361$	S_{16}	S_{16}	‖	0.059
	B_2	0.3540	0.1009		S_{13}	S_{15}	‖	
	B_3	0.4541	0.0811	$S_{13}=1.173$	S_{11}	S_{14}	‖	0.066
	B_4	0.2016	0.0531	$S_{14}=1.419$	S_{14}	S_{11}		0.024
	B_5	0.5064	0.0866	$S_{15}=1.440$	S_{17}	S_{17}	9	0.039
	B_6	0.4043	0.0867	$S_{16}=1.660$	S_{15}	S_{13}		0.052
	B_7	0.5566	0.0924	$S_{17}=1.239$			‖	0.015
	B_8	0.5909	0.0884				‖	RMSE
	B_9	0.3613	0.0802				‖	0.50543491

 # - the number of constraints used
SD - scores differences with variant I

3.4 Group 4

In an analogous way we present the results for group 4 in Table 3.19.

Table 3.19: Group IV - all variants

GROUP IV					**ordering**			
		mean	**variance**	**scores**	**rank**	**PI**	**#**	
Variant I	B_1	0.5333	0.0966	S_{16}=1.235	S_{20}	S_{20}	‖	
	B_2	0.2234	0.0665	S_{17}=1.204	S_{21}	S_{21}	‖	
	B_3	0.6953	0.0429	S_{18}=1.187	S_{16}	S_{16}		
	B_4	0.6013	0.0619	S_{19}=1.188	S_{17}	S_{17}	2	
	B_5	0.1229	0.0385	S_{20}=1.468	S_{19}	S_{19}	9	
	B_6	0.1023	0.0351	S_{21}=1.275	S_{18}	S_{18}		
	B_7	0.3754	0.0895	S_{22}=1.084	S_{22}	S_{22}	‖	
	B_8	0.4140	0.0859				‖	
	B_9	0.1906	0.0295				‖	**SD**
Variant II	B_1	0.5096	0.0902	S_{16}=1.252	S_{20}	S_{20}	‖	0.017
	B_2	0.2182	0.0698	S_{17}=1.221	S_{21}	S_{21}	‖	0.017
	B_3	0.7196	0.0406	S_{18}=1.206	S_{16}	S_{16}		0.019
	B_4	0.5977	0.0601	S_{19}=1.202	S_{17}	S_{17}	2	0.014
	B_5	0.1256	0.0369	S_{20}=1.473	S_{19}	S_{18}	5	0.005
	B_6	0.1057	0.0335	S_{21}=1.287	S_{18}	S_{19}		0.012
	B_7	0.3747	0.0881	S_{22}=1.099	S_{22}	S_{22}	‖	0.016
	B_8	0.4098	0.0823				‖	RMSE
	B_9	0.1912	0.0302				‖	0.31584207
Variant III	B_1	0.5716	0.1011	S_{16}=1.359	S_{20}	S_{20}	‖	0.124
	B_2	0.2467	0.0590	S_{17}=1.289	S_{21}	S_{21}	‖	0.085
	B_3	0.7369	0.0469	S_{18}=1.258	S_{16}	S_{16}		0.071
	B_4	0.6176	0.0631	S_{19}=1.272	S_{17}	S_{17}	1	0.085
	B_5	0.1434	0.0436	S_{20}=1.568	S_{19}	S_{19}	6	0.100
	B_6	0.1472	0.0420	S_{21}=1.389	S_{18}	S_{18}		0.114
	B_7	0.3935	0.0825	S_{22}=1.157	S_{22}	S_{22}	‖	0.074
	B_8	0.4315	0.1007				‖	RMSE
	B_9	0.2258	0.0443				‖	0.80744958
Variant IV	B_1	0.5492	0.0963	S_{16}=1.416	S_{20}	S_{20}	‖	0.183
	B_2	0.3152	0.0599	S_{17}=1.297	S_{21}	S_{16}	‖	0.093
	B_3	0.7203	0.0505	S_{18}=1.239	S_{16}	S_{21}	‖	0.052
	B_4	0.5798	0.0818	S_{19}=1.279	S_{17}	S_{17}		0.091
	B_5	0.1898	0.0486	S_{20}=1.574	S_{19}	S_{19}	8	0.106
	B_6	0.1627	0.0373	S_{21}=1.357	S_{18}	S_{18}		0.082
	B_7	0.4569	0.1077	S_{22}=1.154	S_{22}	S_{22}	‖	0.070
	B_8	0.4302	0.0965				‖	RMSE
	B_9	0.2695	0.0550				‖	0.82279450
Variant V	B_1	0.6357	0.0624	S_{16}=1.495	S_{20}	S_{20}	‖	0.259
	B_2	0.2897	0.0715	S_{17}=1.312	S_{21}	S_{21}	‖	0.107
	B_3	0.6866	0.0601	S_{18}=1.242	S_{16}	S_{16}		0.055
	B_4	0.5513	0.0748	S_{19}=1.293	S_{17}	S_{17}	1	0.106
	B_5	0.1701	0.0343	S_{20}=1.598	S_{19}	S_{19}	2	0.130
	B_6	0.2713	0.0886	S_{21}=1.499	S_{18}	S_{18}		0.224
	B_7	0.4081	0.0821	S_{22}=1.158	S_{22}	S_{22}	‖	0.074
	B_8	0.4587	0.0922				‖	RMSE
	B_9	0.2545	0.0671				‖	0.97731014

 # - the number of constraints used
SD - scores differences with variant I

Notice that in this group variant IV (1,7) performs better than variant V ($\geq$3). In the former we have 8 constraints and we obtain an RMSE of 0.804259, whereas in the latter we have 12 constraints and obtain an RMSE of 0.977310. Both variant IV and V provide good results, but variant IV is better.

3.5 Group 5

Table 3.20: Group V - all variants

GROUP V		**mean**	**variance**	**scores**	**ordering rank**	**ordering PI**	**#**	
Variant I	B_1	0.5255	0.0818	S_{21}=1.192	S_{21}	S_{21}	||	
	B_2	0.2938	0.0615	S_{22}=0.901	S_{23}	S_{23}	||	
	B_3	0.2926	0.0609	S_{23}=1.129	S_{26}	S_{26}		
	B_4	0.3821	0.0747	S_{24}=0.923	S_{27}	S_{24}	3	
	B_5	0.4996	0.0874	S_{25}=0.916	S_{24}	S_{27}	8	
	B_6	0.2936	0.0757	S_{26}=0.953	S_{22}	S_{25}		
	B_7	0.5416	0.0703	S_{27}=0.923	S_{25}	S_{22}	||	
	B_8	0.7221	0.0684				||	
	B_9	0.1443	0.0288				||	**SD**
Variant II	B_1	0.5266	0.0822	S_{21}=1.190	S_{21}	S_{21}	||	0.002
	B_2	0.2939	0.0614	S_{22}=0.900	S_{23}	S_{23}	||	0.001
	B_3	0.2929	0.0606	S_{23}=1.126	S_{26}	S_{26}		0.003
	B_4	0.3826	0.0749	S_{24}=0.922	S_{27}	S_{24}	3	0.001
	B_5	0.4998	0.0875	S_{25}=0.915	S_{24}	S_{27}	2	0.002
	B_6	0.2914	0.0745	S_{26}=0.952	S_{22}	S_{25}		0.002
	B_7	0.5416	0.0704	S_{27}=0.920	S_{25}	S_{22}	||	0.002
	B_8	0.7199	0.0691				||	RMSE
	B_9	0.1456	0.0294				||	0.11259413
Variant III	B_1	0.5106	0.0847	S_{21}=1.259	S_{21}	S_{21}	||	0.067
	B_2	0.2947	0.0662	S_{22}=0.965	S_{23}	S_{23}	||	0.063
	B_3	0.3448	0.0719	S_{23}=1.190	S_{26}	S_{26}		0.061
	B_4	0.4129	0.0793	S_{24}=0.995	S_{27}	S_{24}	2	0.072
	B_5	0.5287	0.0811	S_{25}=0.976	S_{24}	S_{25}	1	0.060
	B_6	0.2872	0.0746	S_{26}=1.025	S_{22}	S_{22}		0.071
	B_7	0.5857	0.0659	S_{27}=0.955	S_{25}	S_{27}	||	0.032
	B_8	0.7129	0.0672				||	RMSE
	B_9	0.1910	0.0380				||	0.65403060
Variant IV	B_1	0.4824	0.0861	S_{21}=1.303	S_{21}	S_{21}	||	0.111
	B_2	0.2952	0.0704	S_{22}=0.994	S_{23}	S_{23}	||	0.093
	B_3	0.4204	0.0871	S_{23}=1.220	S_{26}	S_{26}		0.091
	B_4	0.4584	0.0837	S_{24}=1.032	S_{27}	S_{24}	1	0.109
	B_5	0.5226	0.0850	S_{25}=1.015	S_{24}	S_{25}	0	0.099
	B_6	0.2936	0.0821	S_{26}=1.066	S_{22}	S_{22}		0.113
	B_7	0.6142	0.0689	S_{27}=0.955	S_{25}	S_{27}	||	0.032
	B_8	0.6537	0.0778				||	RMSE
	B_9	0.2124	0.0370				||	0.80573760
Variant V	B_1	0.5483	0.0809	S_{21}=1.428	S_{21}	S_{21}	||	0.236
	B_2	0.2920	0.0635	S_{22}=0.988	S_{23}	S_{23}	||	0.087
	B_3	0.3176	0.0705	S_{23}=1.391	S_{26}	S_{25}	||	0.262
	B_4	0.3933	0.0748	S_{24}=1.012	S_{27}	S_{24}		0.089
	B_5	0.4894	0.0822	S_{25}=1.021	S_{24}	S_{27}	9	0.105
	B_6	0.4609	0.0935	S_{26}=1.958	S_{22}	S_{22}		0.005
	B_7	0.4967	0.0739	S_{27}=0.990	S_{25}	S_{26}	||	0.067
	B_8	0.6870	0.0715				||	RMSE
	B_9	0.2484	0.0560				||	0.92172131

 # - the number of constraints used
SD - scores differences with variant I

Table 3.20 presents results for group 5. We notice that when we take the first and the last column of constraints we obtain better results than in other cases. Variant IV has 10 constraints and a satisfactory RMSE of 0.769146 whereas the last one provides us with a lower number of constraints, 9, but a higher value of RMSE, 0.921721. For this group we would suggest variant IV to be chosen.

3.6 Group 6

Finally the last group is the 6^{th} one. We discuss this last group in more details. In group 6 four scenarios from group 5 are repeated. Hence last group provides just three new scenarios: 28, 29 and 30. When we run probabilistic inversion with all constraints, the problem is not feasible. This can be seen in Table 3.21. The error in this case equals to 1.23695431, whereas for teach group, when analysing with all constraints, this error is on the order 10^{-3}. The linear model is not appropriate for this last group.

Table 3.21: Variant I of Group VI - results obtained

VARIANT I: 1÷7

	mean	variance	scores	ordering rank	ordering PI	#	EQ	OQ	QD
B_1	0.4148	0.0531	S_{24}=1.073	S_{29}	S_{30}	‖	0.818	0.94337	0.1252
B_2	0.3192	0.0478	S_{25}=1.018	S_{26}	S_{24}	‖	0.636	0.81995	0.1836
B_3	0.3874	0.1086	S_{26}=1.027	S_{27}	S_{26}	‖	0.909	0.87676	0.0323
B_4	0.3396	0.1000	S_{27}=0.913	S_{28}	S_{25}	‖	0.818	0.69024	0.1279
B_5	0.3207	0.0883	S_{28}=1.003	S_{30}	S_{28}	‖	0.818	0.90444	0.0863
B_6	0.2721	0.0315	S_{29}=0.861	S_{24}	S_{27}	‖	0.909	0.88508	0.0240
B_7	0.6198	0.0848	S_{30}=1.090	S_{25}	S_{29}	‖	0.636	0.64227	0.0059
B_8	0.5880	0.0775				‖	0.818	0.88006	0.0619
B_9	0.3838	0.0131				‖	0.909	0.90807	0.0010
						‖	0.727	0.70230	0.0250
						‖	0.818	0.85672	0.0385
						‖	0.727	0.80746	0.0802
						‖	0.909	0.93611	0.0270
						‖	0.818	0.85234	0.0342
						‖	0.727	0.75167	0.0244
							0.909	0.92236	0.0133
						3	0.909	0.86796	0.0411
						7	0.909	0.86682	0.0423
							0.909	0.95580	0.0467
						‖	0.909	0.91511	0.0060
						‖	0.455	0.63805	0.1835
						‖	0.727	0.72874	0.0015
						‖	0.818	0.79951	0.0187
						‖	0.818	0.81207	0.0061
						‖	0.818	0.79123	0.0270
						‖	0.818	0.90856	0.0904
						‖	0.727	0.78409	0.0568
						‖	0.727	0.74012	0.0128
						‖	0.818	0.87588	0.0577
						‖	0.818	0.83603	0.0178
						‖	0.909	0.91390	0.0048
						‖	0.818	0.81258	0.0056
						‖	0.909	0.91346	0.0044
						‖	0.818	0.82679	0.0086
						‖	0.818	0.81882	0.0006
						‖	0.727	0.73411	0.0068
						‖	0.909	0.90900	0.0001
								RMSE	**1.23695431**

 # - the number of constraints used
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ-OQ)

We tried to increase the number of samples, but we were not successful. We investigated whether the change of the starting distributions of B_i's would make the linear model feasible. Unfortunately, all these alternatives we tried led us to the same answer: not feasible.

There can be many reasons why the linear model does not fit properly the data from the last group. It can be due to experts fatigue that they gave preferences of scenarios differently than in other groups. It may be that for the attributes which have low values (the scenarios do not differ too much of each other), experts scored scenarios with different than linear model in mind, or just give their ordering randomly. This problem will be further investigated in Chapter 5. We will motivate there our decision of removing the sixth group from the analysis.

Our analysis is divided into two parts. In the first part, we make the analysis of all combined groups together using variant V for all of them. Taking this variant for all groups, leads to the reduction of number of constraints from 200 to 53, which is manageable for PI. In the second part, we combine all groups together but we consider variant V for the first three groups, and variant IV for the last two. This way we choose the variant which has performed the best within each group. In this case we reduce the number of constraints from 200 to 50.

In the following sections we present the results corresponding to both analyses.

3.7 Results obtained under variant V

This section presents results of analysis with variant V firstly for each group taken separately, and then with all scenarios considered together. For fair comparison we have used the same samples for all groups. Table 3.22 contains the mean and variances of B's, scores obtained, orderings obtained with probabilistic inversion and ordering with rank ordering technique.

In Table 3.22 the constraints included are shown in first column. For instance, in the second group, sixth scenario, has only one constraint included, denoted as $S_{6.2<2}$ (Table 3.23 first column, row 5). $S_{6.2<2}$ means that scenario 6, in group 2, must be second smallest in the ranking. This convention is used for all constraints. In column 2, row 5 of the same table we see that the chance of the sixth scenario to be second smallest within the second group should be $\frac{4}{11}$ (the value 1-$\frac{4}{11}$=0.636 in the software). Next to 0.363 we see 0.3630 and 0.00036. The first one, 0.3630 is the value recovered by probabilistic inversion procedure for this constraint whereas the last one, 0.00036 represents the error obtained by subtracting the value obtained by probabilistic inversion from the value imposed by experts. RMSE$_q$ expresses the error of values for quantiles imposed and obtained in this second group, and RMSE$_s$

represents the error of scores computed by subtracting the scores obtained in variant V from scores obtained in variant I.

Table 3.22: Final results for GROUP I when Variant V used

VARIANT V: $\geq$3				**GROUP I**				**ordering**		
	EQ	**OQ**	**QD**		**mean**	**variance**	**scores**	**rank**	**PI**	**#**
$S_{5.1<}$	0.545	0.5450	0.00044	b_1	0.5074	0.0825	S_1=1.612	S_1	S_1	‖
$S_{1.1>}$	0.636	0.6360	0.00032	b_2	0.5666	0.0760	S_2=1.351	S_3	S_3	‖
$S_{3.1>}$	0.545	0.5475	0.00208	b_3	0.4154	0.0844	S_3=1.595	S_6	S_6	‖
$S_{2.1<2}$	0.545	0.5449	0.00059	b_4	0.2385	0.0439	S_4=1.278	S_7	S_7	
$S_{7.1<2}$	0.727	0.7292	0.00197	b_5	0.5302	0.0826	S_5=1.174	S_4	S_2	1
$S_{1.1>2}$	0.455	0.4571	0.00254	b_6	0.5058	0.0796	S_6=1.385	S_2	S_4	3
$S_{3.1>2}$	0.727	0.7278	0.00048	b_7	0.6618	0.0645	S_7=1.360	S_5	S_5	
$S_{4.1<3}$	0.636	0.6361	0.00028	b_8	0.4875	0.0829				‖
$S_{5.1<3}$	0.727	0.7274	0.00014	b_9	0.3998	0.0809				‖
$S_{7.1<3}$	0.727	0.7287	0.00142							‖
$S_{7.1>3}$	0.545	0.5449	0.00057							‖
$S_{3.1<4}$	0.727	0.7270	0.00028							‖
$S_{6.1<4}$	0.727	0.7270	0.00027							‖
		RMSE$_q$	**0.1064**			**RMSE$_s$**	**0.5561**			

 # - the number of constraints used
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ − OQ)
RMSE$_q$ - error of quantiles
RMSE$_s$ - error of scores

Tables 3.27 and 3.28 provide the results obtained when running the analysis with all groups together. Due to alignment in the document we split the table into two parts. It is visible that the RMSE is bigger than the ones we obtained for each group separately, and is now equal to 2.33273.

We have now many constraints and not all of them can be fitted properly. Nevertheless the ordering of scenarios is still quite good. When analysing the groups separately we have noticed that variant V is not always the best option to take. We investigate now the results obtained when we take for each group the best performing variant. These results are presented in the next section.

Table 3.23: Final results for GROUP II when Variant V used

VARIANT V: ≥3				**GROUP II**				**ordering**		
	EQ	**OQ**	**QD**		**mean**	**variance**	**scores**	**rank**	**PI**	**#**
$S_{8.2<}$	0.727	0.7270	0.00027	b_1	0.6807	0.0566	S_6=1.381	S_{10}	S_{10}	‖
$S_{12.2<}$	0.727	0.7270	0.00027	b_2	0.5133	0.0790	S_7=1.314	S_9	S_9	‖
$S_{9.2>}$	0.727	0.7270	0.00027	b_3	0.2250	0.0299	S_8=1.256	S_{11}	S_{11}	‖
$S_{10.2>}$	0.364	0.3640	0.00036	b_4	0.3183	0.0643	S_9=1.474	S_{12}	S_7	
$S_{6.2<2}$	0.636	0.6360	0.00036	b_5	0.2950	0.0928	S_{10}=1.465	S_6	S_6	1
$S_{7.2<2}$	0.727	0.7270	0.00027	b_6	0.2546	0.0444	S_{11}=1.409	S_7	S_{12}	0
$S_{9.2<2}$	0.909	0.9090	0.00009	b_7	0.6232	0.0490	S_{12}=1.397	S_8	S_8	
$S_{8.2<3}$	0.545	0.5450	0.00145	b_8	0.4811	0.0850				‖
$S_{7.2>3}$	0.455	0.4550	0.00045	b_9	0.6056	0.0754				‖
$S_{11.2<4}$	0.545	0.5450	0.00045							‖
		RMSE$_q$	**0.0572**			**RMSE$_s$**	**0.5500**			

 # - the number of constraints used
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ − OQ)
RMSE$_q$ - error of quantiles
RMSE$_s$ - error of scores

Table 3.24: Final results for GROUP III when Variant V used

VARIANT V: ≥3				**GROUP III**				**ordering**		
	EQ	**OQ**	**QD**		**mean**	**variance**	**scores**	**rank**	**PI**	**#**
$S_{13.3>}$	0.636	0.6360	0.00036	b_1	0.4582	0.0969	S_{11}=1.361	S_{16}	S_{16}	‖
$S_{16.3>}$	0.545	0.5450	0.00045	b_2	0.3540	0.1009		S_{13}	S_{15}	‖
$S_{17.3<2}$	0.636	0.6360	0.00036	b_3	0.4541	0.0811	S_{13}=1.173	S_{11}	S_{14}	‖
$S_{11.2>2}$	0.727	0.7270	0.00027	b_4	0.2016	0.0531	S_{14}=1.419	S_{14}	S_{11}	
$S_{17.3>2}$	0.636	0.6360	0.00036	b_5	0.5064	0.0866	S_{15}=1.440	S_{17}	S_{17}	9
$S_{14.3<3}$	0.636	0.6360	0.00036	b_6	0.4043	0.0867	S_{16}=1.660	S_{15}	S_{13}	
$S_{11.3>3}$	0.727	0.7270	0.00027	b_7	0.5566	0.0924	S_{17}=1.239			‖
$S_{14.3<4}$	0.727	0.7270	0.00027	b_8	0.5909	0.0884				‖
$S_{15.3<4}$	0.636	0.6360	0.00036	b_9	0.3613	0.0802				‖
		RMSE$_q$	**0.0556**			**RMSE$_s$**	**0.5039**			

 # - the number of constraints used
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ − OQ)
RMSE$_q$ - error of quantiles
RMSE$_s$ - error of scores

Table 3.25: Final results for GROUP IV when Variant V used

VARIANT V: ≥3				**GROUP IV**				**ordering**		
	EQ	**OQ**	**QD**	**mean**		**variance**	**scores**	**rank**	**PI**	**#**
$S_{16.4<}$	0.727	0.7271	0.00021	b_1	0.6357	0.0624	S_{16}=1.495	S_{20}	S_{20}	‖
$S_{22.4<}$	0.455	0.4549	0.00040	b_2	0.2897	0.0715	S_{17}=1.312	S_{21}	S_{21}	‖
$S_{20.4>}$	0.364	0.3640	0.00034	b_3	0.6966	0.0601	S_{18}=1.242	S_{18}	S_{16}	‖
$S_{18.4<2}$	0.727	0.7270	0.00028	b_4	0.5513	0.0748	S_{19}=1.293	S_{19}	S_{17}	
$S_{22.4<2}$	0.636	0.6360	0.00032	b_5	0.1701	0.0343	S_{20}=1.598	S_{17}	S_{19}	1
$S_{21.4>2}$	0.455	0.4549	0.00040	b_6	0.2713	0.0886	S_{21}=1.499	S_{16}	S_{18}	2
$S_{16.4<3}$	0.727	0.7282	0.00094	b_7	0.4081	0.0821	S_{22}=1.158	S_{22}	S_{22}	
$S_{17.4<3}$	0.545	0.5458	0.00033	b_8	0.4587	0.0922				‖
$S_{19.4<3}$	0.727	0.7270	0.00030	b_9	0.2545	0.0671				‖
$S_{18.4>3}$	0.636	0.6360	0.00039							‖
$S_{19.4>3}$	0.727	0.7270	0.00026							‖
$S_{17.4<4}$	0.545	0.5450	0.00045							‖
		RMSE$_q$	**0.0680**			**RMSE$_s$**	**0.9773**			

 # - the number of constraints used
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ − OQ)
RMSE$_q$ - error of quantiles
RMSE$_s$ - error of scores

Table 3.26: Final results for GROUP V when Variant V used

VARIANT V: ≥3				**GROUP V**				**ordering**		
	EQ	**OQ**	**QD**	**mean**		**variance**	**scores**	**rank**	**PI**	**#**
$S_{22.5<}$	0.727	0.7270	0.00027	b_1	0.5483	0.0809	S_{21}=1.428	S_{21}	S_{21}	‖
$S_{25.5<}$	0.636	0.6360	0.00036	b_2	0.2920	0.0635	S_{22}=0.988	S_{23}	S_{23}	‖
$S_{21.5>}$	0.455	0.4550	0.00045	b_3	0.3176	0.0705	S_{23}=1.391	S_{26}	S_{25}	‖
$S_{22.5<2}$	0.727	0.7270	0.00027	b_4	0.3933	0.0748	S_{24}=1.012	S_{27}	S_{24}	
$S_{26.5<2}$	0.727	0.7270	0.00027	b_5	0.4894	0.0822	S_{25}=1.021	S_{24}	S_{27}	9
$S_{23.5>2}$	0.455	0.4550	0.00045	b_6	0.4609	0.0935	S_{26}=0.958	S_{22}	S_{22}	
$S_{24.5<3}$	0.545	0.5450	0.00045	b_7	0.4967	0.0739	S_{27}=0.990	S_{25}	S_{26}	‖
$S_{27.5>3}$	0.636	0.6360	0.00036	b_8	0.6870	0.0715				‖
$S_{27.5<4}$	0.727	0.7270	0.00027	b_9	0.2484	0.0560				‖
		RMSE$_q$	**0.0680**			**RMSE$_s$**	**0.9217**			

 # - the number of constraints used
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ − OQ)
RMSE$_q$ - error of quantiles
RMSE$_s$ - error of scores

Table 3.27: Final results for ALL GROUPS when Variant V used (PART I)

		EQ	**OQ**	**QD**	**mean**		**variance**	**scores**	**ordering rank**	**PI**	**#**
GROUP 1 ≥ 3	$S_{5.1<}$	0.545	0.7095	0.16407	b_1	0.5570	0.0930	S_1=1.452	S_1	S_1	‖
	$S_{1.1>}$	0.636	0.4890	0.14740	b_2	0.2210	0.0295	S_2=1.240	S_3	S_3	‖
	$S_{3.1>}$	0.545	0.7927	0.24722	b_3	0.2988	0.0474	S_3=1.357	S_6	S_4	‖
	$S_{2.1<2}$	0.545	0.4942	0.05128	b_4	0.2965	0.0825	S_4=1.346	S_7	S_2	
	$S_{7.1<2}$	0.727	0.7027	0.02456	b_5	0.3394	0.0608	S_5=1.123	S_4	S_6	1
	$S_{1.1>2}$	0.455	0.7783	0.32372	b_6	0.1521	0.0261	S_6=1.167	S_2	S_7	3
	$S_{3.1>2}$	0.727	0.6257	0.10154	b_7	0.5159	0.1027	S_7=1.151	S_5	S_5	
	$S_{4.1<3}$	0.636	0.8631	0.22673	b_8	0.6207	0.0883				‖
	$S_{5.1<3}$	0.727	0.5861	0.14115	b_9	0.2004	0.0546				‖
	$S_{7.1<3}$	0.727	0.7425	0.01523							‖
	$S_{7.1>3}$	0.545	0.6734	0.12793							‖
	$S_{3.1<4}$	0.727	0.6282	0.09907							‖
	$S_{6.1<4}$	0.727	0.7487	0.02142							‖
GROUP 2 ≥ 3	$S_{8.2<}$	0.727	0.9102	0.18293				S_6= 1.167	S_{10}	S_{12}	‖
	$S_{12.2<}$	0.727	0.9250	0.19769				S_7= 1.151	S_9	S_9	‖
	$S_{9.2>}$	0.727	0.7162	0.01105				S_8= 1.196	S_{11}	S_8	‖
	$S_{10.2>}$	0.364	0.8090	0.44540				S_9= 1.351	S_7	S_{10}	
	$S_{6.2<2}$	0.636	0.7545	0.11813				S_{10}=1.185	S_6	S_{11}	1
	$S_{7.2<2}$	0.727	0.8390	0.11174				S_{11}=1.178	S_{12}	S_6	0
	$S_{9.2<2}$	0.909	0.8938	0.01524				S_{12}=1.408	S_8	S_7	
	$S_{8.2<3}$	0.545	0.6556	0.12012							‖
	$S_{7.2>3}$	0.455	0.6655	0.21092							‖
	$S_{11.2<4}$	0.545	0.6224	0.07695							‖
GROUP 3 ≥ 3	$S_{13.3>}$	0.636	0.7656	0.12928				S_{11}= 1.178	S_{16}	S_{14}	‖
	$S_{16.3>}$	0.545	0.5879	0.04248				S_{13}= 1.101	S_{13}	S_{15}	‖
	$S_{17.3<2}$	0.636	0.7895	0.15309				S_{14}= 1.370	S_{11}	S_{16}	‖
	$S_{11.3>2}$	0.727	0.8021	0.07483				S_{15}= 1.359	S_{14}	S_{11}	
	$S_{17.3>2}$	0.636	0.6648	0.02846				S_{16}= 1.201	S_{17}	S_{13}	9
	$S_{14.3<3}$	0.636	0.7614	0.12501				S_{17}= 1.039	S_{15}	S_{17}	
	$S_{11.3>3}$	0.727	0.6494	0.07783							‖
	$S_{14.3<4}$	0.727	0.7835	0.05618							‖
	$S_{15.3<4}$	0.636	0.6973	0.06098							‖

Table 3.28: Final results for ALL GROUPS when Variant V used (PART II)

		EQ	**OQ**	**QD**	**mean**	**variance**	**scores**	**ordering rank**	**PI**	**#**
GROUP 4 ≥ 3	$S_{16.4<}$	0.727	0.9163	0.18901			S_{16}= 1.201	S_{20}	S_{20}	‖
	$S_{22.4<}$	0.455	0.5278	0.07330			S_{17}= 1.039	S_{21}	S_{16}	‖
	$S_{20.4>}$	0.364	0.3605	0.00309			S_{18}= 0.930	S_{18}	S_{21}	‖
	$S_{18.4<2}$	0.727	0.6395	0.08774			S_{19}= 0.867	S_{19}	S_{17}	
	$S_{22.4<2}$	0.636	0.7187	0.08231			S_{20}= 1.305	S_{17}	S_{18}	1
	$S_{21.4>2}$	0.455	0.4332	0.02135			S_{21}= 1.161	S_{16}	S_{22}	2
	$S_{16.4<3}$	0.727	0.7972	0.06997			S_{22}= 0.883	S_{22}	S_{19}	
	$S_{17.4<3}$	0.545	0.7037	0.15827						‖
	$S_{19.4<3}$	0.727	0.7060	0.02123						‖
	$S_{18.4>3}$	0.636	0.8288	0.19239						‖
	$S_{19.4>3}$	0.727	0.8555	0.12820						‖
	$S_{17.4<4}$	0.545	0.6398	0.09436						‖
GROUP 5 ≥ 3	$S_{22.5<}$	0.727	0.8281	0.10081			S_{21}= 1.161	S_{21}	S_{21}	‖
	$S_{25.5<}$	0.636	0.6045	0.03190			S_{22}= 0.883	S_{23}	S_{23}	‖
	$S_{21.5>}$	0.455	0.4569	0.00232			S_{23}= 1.034	S_{26}	S_{26}	‖
	$S_{22.5<2}$	0.727	0.6740	0.05323			S_{24}= 0.903	S_{27}	S_{24}	
	$S_{26.5<2}$	0.727	0.8317	0.10439			S_{25}= 0.851	S_{24}	S_{22}	9
	$S_{23.5>2}$	0.455	0.4236	0.03094			S_{26}= 0.959	S_{22}	S_{25}	
	$S_{24.5<3}$	0.545	0.5850	0.0.958			S_{27}= 0.839	S_{25}	S_{27}	‖
	$S_{27.5>3}$	0.636	0.6637	0.02733						‖
	$S_{27.5<4}$	0.727	0.7270	0.00027						‖
			RMSE$_q$	**2.33273**						

 # - the number of constraints used
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ − OQ)
RMSE$_s$ - error of scores

3.8 Results obtained under the combination of variants

As mentioned previously, in this section we present the results obtained when we combine variants with the best performance in terms of the number of constraints and RMSE, in the group. For groups I, II and III the best results are obtained with Variant V, while for the group IV and V the best results are obtained under variant IV.

Just like in the previous section, means and variances of B's, scores obtained, orderings obtained with probabilistic inversion and ordering obtained with rank ordering technique, and constraints that we have included in the analysis of the five groups are shown in Tables 3.29, 3.30, 3.31, 3.32 and 3.33 respectively.

For the first three groups nothing changes relative to results obtained in the previous section. In the last two we notice a decrease of RMSE. Noticeable is that in group 4 we have 8 constraints instead of 12 and in group 5, 10 instead of 9.

Table 3.29: Final results for GROUP I when Combined Variant used

VARIANT V: $\geq$3	**EQ**	**OQ**	**QD**	**GROUP I**	**mean**	**variance**	**scores**	**ordering rank**	**PI**	**#**
$S_{5.1<}$	0.545	0.5450	0.00044	b_1	0.5074	0.0825	S_1=1.612	S_1	S_1	$\|$
$S_{1.1>}$	0.636	0.6360	0.00032	b_2	0.5666	0.0760	S_2=1.351	S_3	S_3	$\|$
$S_{3.1>}$	0.545	0.5475	0.00208	b_3	0.4154	0.0844	S_3=1.595	S_6	S_6	$\|$
$S_{2.1<2}$	0.545	0.5449	0.00059	b_4	0.2385	0.0439	S_4=1.278	S_7	S_7	
$S_{7.1<2}$	0.727	0.7292	0.00197	b_5	0.5302	0.0826	S_5=1.174	S_4	S_2	1
$S_{1.1>2}$	0.455	0.4571	0.00254	b_6	0.5058	0.0796	S_6=1.385	S_2	S_4	3
$S_{3.1>2}$	0.727	0.7278	0.00048	b_7	0.6618	0.0645	S_7=1.360	S_5	S_5	
$S_{4.1<3}$	0.636	0.6361	0.00028	b_8	0.4875	0.0829				$\|$
$S_{5.1<3}$	0.727	0.7274	0.00014	b_9	0.3998	0.0809				$\|$
$S_{7.1<3}$	0.727	0.7287	0.00142							$\|$
$S_{7.1>3}$	0.545	0.5449	0.00057							$\|$
$S_{3.1<4}$	0.727	0.7270	0.00028							$\|$
$S_{6.1<4}$	0.727	0.7270	0.00027							$\|$
		$RMSE_q$	**0.1064**			**$RMSE_s$**	**0.5561**			

 # - the number of constraints used
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ − OQ)
$RMSE_q$ - error of quantiles
$RMSE_s$ - error of scores

Table 3.30: Final results for GROUP II when Combined Variant used

VARIANT V: $\geq$3	**EQ**	**OQ**	**QD**	**GROUP II**	**mean**	**variance**	**scores**	**ordering rank**	**PI**	**#**
$S_{8.2<}$	0.727	0.7270	0.00027	b_1	0.6807	0.0566	S_6=1.381	S_{10}	S_{10}	$\|$
$S_{12.2<}$	0.727	0.7270	0.00027	b_2	0.5133	0.0790	S_7=1.314	S_9	S_9	$\|$
$S_{9.2>}$	0.727	0.7270	0.00027	b_3	0.2250	0.0299	S_8=1.256	S_{11}	S_{11}	$\|$
$S_{10.2>}$	0.364	0.3640	0.00036	b_4	0.3183	0.0643	S_9=1.474	S_{12}	S_7	
$S_{6.2<2}$	0.636	0.6360	0.00036	b_5	0.2950	0.0928	S_{10}=1.465	S_6	S_6	1
$S_{7.2<2}$	0.727	0.7270	0.00027	b_6	0.2546	0.0444	S_{11}=1.409	S_7	S_{12}	0
$S_{9.2<2}$	0.909	0.9090	0.00009	b_7	0.6232	0.0490	S_{12}=1.397	S_8	S_8	
$S_{8.2<3}$	0.545	0.5450	0.00145	b_8	0.4811	0.0850				$\|$
$S_{7.2>3}$	0.455	0.4550	0.00045	b_9	0.6056	0.0754				$\|$
$S_{11.2<4}$	0.545	0.5450	0.00045							$\|$
		$RMSE_q$	**0.0572**			**$RMSE_s$**	**0.5500**			

 # - the number of constraints used
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ − OQ)
$RMSE_q$ - error of quantiles
$RMSE_s$ - error of scores

Table 3.31: Final results for GROUP III when Combined Variant used

VARIANT V: ≥3				**GROUP III**				**ordering**		
	EQ	**OQ**	**QD**		**mean**	**variance**	**scores**	**rank**	**PI**	**#**
$S_{13.3>}$	0.636	0.6360	0.00036	b_1	0.4582	0.0969	S_{11}=1.361	S_{16}	S_{16}	‖
$S_{16.3>}$	0.545	0.5450	0.00045	b_2	0.3540	0.1009		S_{13}	S_{15}	‖
$S_{17.3<2}$	0.636	0.6360	0.00036	b_3	0.4541	0.0811	S_{13}=1.173	S_{11}	S_{14}	‖
$S_{11.2>2}$	0.727	0.7270	0.00027	b_4	0.2016	0.0531	S_{14}=1.419	S_{14}	S_{11}	
$S_{17.3>2}$	0.636	0.6360	0.00036	b_5	0.5064	0.0866	S_{15}=1.440	S_{17}	S_{17}	9
$S_{14.3<3}$	0.636	0.6360	0.00036	b_6	0.4043	0.0867	S_{16}=1.660	S_{15}	S_{13}	
$S_{11.3>3}$	0.727	0.7270	0.00027	b_7	0.5566	0.0924	S_{17}=1.239			‖
$S_{14.3<4}$	0.727	0.7270	0.00027	b_8	0.5909	0.0884				‖
$S_{15.3<4}$	0.636	0.6360	0.00036	b_9	0.3613	0.0802				‖
		RMSE$_q$	**0.0556**			**RMSE$_s$**	**0.5039**			

 # - the number of constraints used
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ − OQ)
RMSE$_q$ - error of quantiles
RMSE$_s$ - error of scores

Table 3.32: Final results for GROUP IV when Combined Variant used

VARIANT IV: (1,7)				**GROUP IV**				**ordering**		
	EQ	**OQ**	**QD**		**mean**	**variance**	**scores**	**rank**	**PI**	**#**
$S_{16.4<}$	0.727	0.7289	0.00162	b_1	0.5492	0.0963	S_{16}=1.418	S_{20}	S_{20}	‖
$S_{19.4<}$	0.909	0.9096	0.00046	b_2	0.3152	0.0599	S_{17}=1.297	S_{21}	S_{16}	‖
$S_{21.4<}$	0.909	0.9095	0.00038	b_3	0.7203	0.0505	S_{18}=1.239	S_{18}	S_{21}	‖
$S_{22.4<}$	0.455	0.4550	0.00046	b_4	0.5798	0.0818	S_{19}=1.279	S_{19}	S_{17}	‖
$S_{18.4>}$	0.909	0.9098	0.00072	b_5	0.1898	0.0486	S_{20}=1.574	S_{17}	S_{19}	
$S_{19.4>}$	0.818	0.8193	0.00111	b_6	0.1627	0.0373	S_{21}=1.357	S_{16}	S_{18}	8
$S_{20.4>}$	0.364	0.3646	0.00093	b_7	0.4569	0.1077	S_{22}=1.154	S_{22}	S_{22}	
$S_{21.4>}$	0.909	0.9090	0.00009	b_8	0.4302	0.0965				‖
				b_9	0.2695	0.0550				‖
		RMSE$_q$	**0.0058**			**RMSE$_s$**	**1.0931**			

 # - the number of constraints used
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ − OQ)
RMSE$_q$ - error of quantiles
RMSE$_s$ - error of scores

Finally Tables 3.34 and 3.35 present the results obtained by taking all groups together. RMSE shows that indeed, taking for each group the variant which has performed the best is a better approach. From Table 3.35 we read he error equal to 1.91919. When we took variant V for all five groups we obtained an error equal to 2.33273.

Table 3.33: Final results for GROUP V when Combined Variant used

VARIANT IV: (1,7)				**GROUP V**				**ordering**		
	EQ	**OQ**	**QD**		**mean**	**variance**	**scores**	**rank**	**PI**	**#**
$S_{21.5<}$	0.818	0.8198	0.00159	b_1	0.4824	0.0861	S_{21}=1.303	S_{21}	S_{21}	‖
$S_{22.5<}$	0.727	0.7287	0.00139	b_2	0.2952	0.0704	S_{22}=0.994	S_{23}	S_{23}	‖
$S_{23.5<}$	0.909	0.9095	0.00039	b_3	0.4204	0.0871	S_{23}=1.220	S_{26}	S_{26}	‖
$S_{25.5<}$	0.636	0.6363	0.00002	b_4	0.4584	0.0837	S_{24}=1.032	S_{27}	S_{24}	
$S_{27.5<}$	0.909	0.9090	0.00010	b_5	0.5226	0.0850	S_{25}=1.015	S_{24}	S_{25}	1
$S_{21.5>}$	0.455	0.4574	0.00282	b_6	0.2936	0.0821	S_{26}=1.066	S_{22}	S_{22}	0
$S_{22.5>}$	0.909	0.9093	0.00023	b_7	0.6142	0.0689	S_{27}=0.955	S_{25}	S_{27}	
$S_{23.5>}$	0.909	0.9092	0.00015	b_8	0.6537	0.0778				‖
$S_{25.5>}$	0.909	0.9092	0.00007	b_9	0.2124	0.0370				‖
$S_{26.5>}$	0.818	0.8180	0.00018							‖
		$RMSE_q$	**0.0832**			**$RMSE_s$**	**0.86557**			

 # - the number of constraints used
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ − OQ)
$RMSE_q$ - error of quantiles
$RMSE_s$ - error of scores

Table 3.34: Final results for ALL GROUPS when Combined Variant used (PART I)

									ordering		
		EQ	**OQ**	**QD**	**mean**		**variance**	**scores**	**rank**	**PI**	**#**
GROUP 1 ≥ 3	$S_{5.1<}$	0.545	0.4946	0.05090	b_1	0.4763	0.0959	S_1=1.491	S_1	S_1	‖
	$S_{1.1>}$	0.636	0.4510	0.18535	b_2	0.3235	0.0688	S_2=1.240	S_3	S_3	‖
	$S_{3.1>}$	0.545	0.7783	0.23283	b_3	0.3395	0.0921	S_3=1.397	S_6	S_6	‖
	$S_{2.1<2}$	0.545	0.5919	0.04642	b_4	0.1808	0.0249	S_4=1.241	S_7	S_4	
	$S_{7.1<2}$	0.727	0.7836	0.05637	b_5	0.2894	0.0788	S_5=1.072	S_4	S_7	1
	$S_{1.1>2}$	0.455	0.6568	0.20221	b_6	0.1839	0.0415	S_6=1.253	S_2	S_2	3
	$S_{3.1>2}$	0.727	0.7109	0.01633	b_7	0.5692	0.1043	S_7=1.217	S_5	S_5	
	$S_{4.1<3}$	0.636	0.7777	0.14129	b_8	0.6084	0.1290				‖
	$S_{5.1<3}$	0.727	0.7490	0.02174	b_9	0.2560	0.0508				‖
	$S_{7.1<3}$	0.727	0.7349	0.00764							‖
	$S_{7.1>3}$	0.545	0.6414	0.09596							‖
	$S_{3.1<4}$	0.727	0.7253	0.00196							‖
	$S_{6.1<4}$	0.727	0.7827	0.05546							‖
GROUP 2 ≥ 3	$S_{8.2<}$	0.727	0.7831	0.05585				S_6= 1.253	S_{10}	S_9	‖
	$S_{12.2<}$	0.727	0.8661	0.13882				S_7= 1.217	S_9	S_{12}	‖
	$S_{9.2>}$	0.727	0.7047	0.02260				S_8= 1.211	S_{11}	S_{10}	‖
	$S_{10.2>}$	0.364	0.7417	0.37804				S_9= 1.378	S_7	S_6	
	$S_{6.2<2}$	0.636	0.6791	0.04271				S_{10}=1.257	S_6	S_7	1
	$S_{7.2<2}$	0.727	0.9203	0.19305				S_{11}=1.203	S_{12}	S_8	0
	$S_{9.2<2}$	0.909	0.8972	0.01186				S_{12}=1.375	S_8	S_{11}	
	$S_{8.2<3}$	0.545	0.7322	0.18678							‖
	$S_{7.2>3}$	0.455	0.6150	0.16042							‖
	$S_{11.2<4}$	0.545	0.5798	0.03439							‖
GROUP 3 ≥ 3	$S_{13.3>}$	0.636	0.7997	0.16334				S_{11}= 1.203	S_{16}	S_{16}	‖
	$S_{16.3>}$	0.545	0.5468	0.00137				S_{13}= 1.023	S_{13}	S_{14}	‖
	$S_{17.3<2}$	0.636	0.6278	0.00861				S_{14}= 1.283	S_{11}	S_{15}	‖
	$S_{11.3>2}$	0.727	0.8535	0.12618				S_{15}= 1.275	S_{14}	S_{11}	
	$S_{17.3>2}$	0.636	0.7201	0.08377				S_{16}= 1.334	S_{17}	S_{17}	9
	$S_{14.3<3}$	0.636	0.6623	0.02592				S_{17}= 1.106	S_{15}	S_{13}	
	$S_{11.3>3}$	0.727	0.7045	0.02276							‖
	$S_{14.3<4}$	0.727	0.8096	0.08229							‖
	$S_{15.3<4}$	0.636	0.6193	0.01708							‖

Table 3.35: Final results for ALL GROUPS when Combined Variant used (PART II)

		EQ	**OQ**	**QD**	**mean**	**variance**	**scores**	**ordering rank**	**PI #**
GROUP 4 1, 7	$S_{16.4<}$	0.727	0.9393	0.21205			S_{16}= 1.334	S_{20}	S_{20} ‖
	$S_{19.4<}$	0.909	0.8391	0.07001			S_{17}= 1.106	S_{21}	S_{16} ‖
	$S_{21.4<}$	0.909	0.8187	0.09038			S_{18}= 0.991	S_{18}	S_{21} ‖
	$S_{22.4<}$	0.455	0.4559	0.00132			S_{19}= 1.006	S_{19}	S_{17}
	$S_{18.4>}$	0.909	0.8757	0.03343			S_{20}= 1.341	S_{17}	S_{19} 8
	$S_{19.4>}$	0.818	0.9262	0.10806			S_{21}= 1.160	S_{16}	S_{18}
	$S_{20.4>}$	0.364	0.4722	0.10861			S_{22}= 0.938	S_{22}	S_{22} ‖
	$S_{21.4>}$	0.909	0.8956	0.01353					‖
									‖
GROUP 5 1, 7	$S_{21.5<}$	0.818	0.8571	0.03892			S_{21}= 1.160	S_{21}	S_{21} ‖
	$S_{22.5<}$	0.727	0.8105	0.08318			S_{22}= 0.938	S_{23}	S_{23} ‖
	$S_{23.5<}$	0.909	0.9412	0.03210			S_{23}= 1.117	S_{26}	S_{22} ‖
	$S_{25.5<}$	0.636	0.6449	0.00857			S_{24}= 0.930	S_{27}	S_{24}
	$S_{27.5<}$	0.909	0.9196	0.01052			S_{25}= 0.885	S_{24}	S_{26} 9
	$S_{21.5>}$	0.455	0.4550	0.00048			S_{26}= 0.927	S_{22}	S_{25}
	$S_{22.5>}$	0.909	0.9092	0.00009			S_{27}= 0.843	S_{25}	S_{27} ‖
	$S_{23.5>}$	0.909	0.9100	0.00092					‖
	$S_{25.5>}$	0.909	0.9097	0.00062					‖
	$S_{26.5>}$	0.818	0.8180	0.00018					‖
			RMSE$_q$	**1.91919**					

 # - the number of constraints used
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ − OQ)
RMSE$_s$ - error of scores

We started with $B'i$'s independently distributed. Probabilistic inversion made B_i's dependent. Table 3.36 contains the correlation matrix of B's obtained in Table 3.34.

Table 3.36: Correlation matrix of B_i

	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9
B_1	1.0000	-0.1806	-0.3992	-0.0172	-0.0730	-0.2817	-0.1100	-0.1316	0.2597
B_2	-0.1806	1.0000	0.4664	-0.1096	-0.2143	-0.2089	-0.3034	-0.4633	0.2486
B_3	-0.3992	0.4664	1.0000	-0.0915	0.0387	0.0256	-0.4929	-0.2265	-0.0322
B_4	-0.0172	-0.1096	-0.0915	1.0000	-0.1256	-0.2352	0.0481	0.1229	-0.0118
B_5	-0.0730	-0.2143	0.0387	-0.1256	1.0000	-0.0525	0.2031	0.4128	-0.3785
B_6	-0.2817	-0.2089	0.0256	-0.2352	-0.0525	1.0000	0.2415	0.1135	-0.0201
B_7	-0.1100	-0.3034	-0.4929	0.0481	0.2031	0.2415	1.0000	0.3864	0.0849
B_8	-0.1316	-0.4633	-0.2265	0.1229	0.4128	0.1135	0.3864	1.0000	-0.0721
B_9	0.2597	0.2486	-0.0322	-0.0118	-0.3785	-0.0201	0.0849	-0.0721	1.0000

Figure 3.8 shows the cumulative distribution functions of B_i's obtained in Table 3.34.

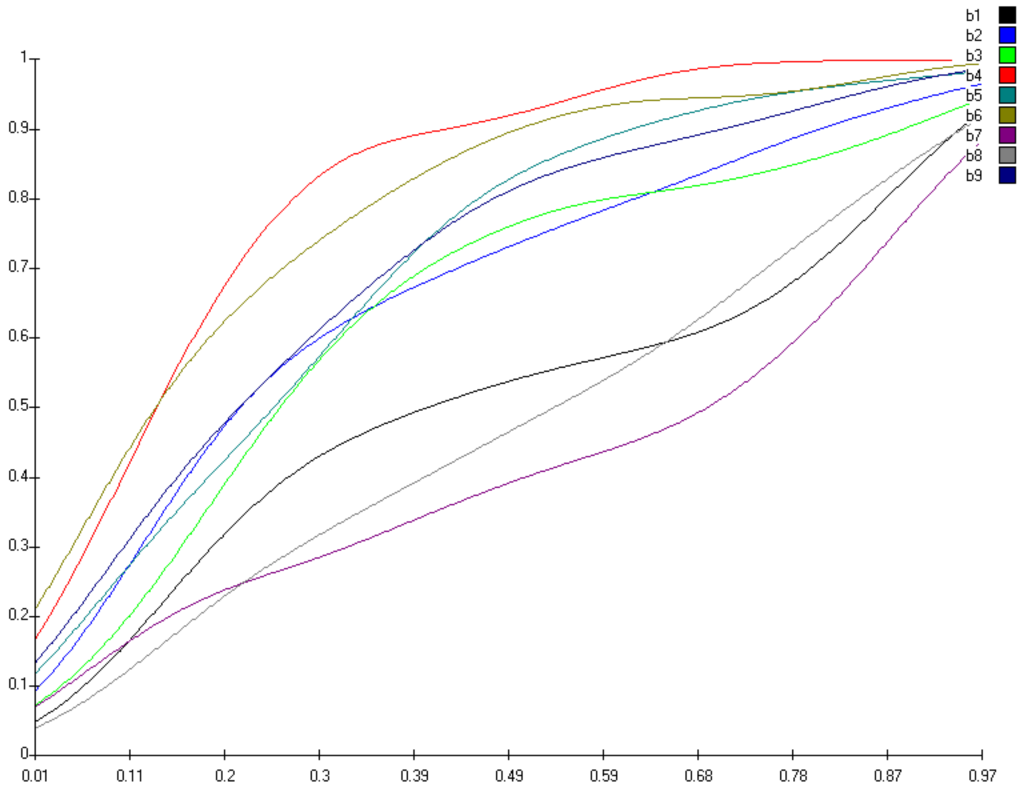


Figure 3.8: Cumulative distribution functions of B_i's

3.9 Summary and conclusions

Our goal in this thesis as part of Emerging Zoonoses project is to find out the model based on which the pathogens transmitted from animals to humans can be prioritizied.

We had available for analysis 30 scenarios representing hypothetical pathogens, which have been ordered based on their severity by experts. These scenarios were divided into 6 groups, each group consisting of 7 scenarios. Due to several reasons, which have been explained during the chapters, 5 groups out of 6 have been analysed. Our purpose was to combine all groups together, and due to software constraints, it was not possible to include all constraints. We investigated therefore each group, to find out a way of removing the constraints, without a too big loss of information.

The linear model that we used was feasible for these five groups.

Chapter 4

Analysis of the rankings given by experts

In Chapter 3 we obtained a linear model of scores from experts' ordering, summarised in Tables 1.3 and 1.4 (see Section 1.1). We have encountered problems with fitting the linear model to the data from the last set. Many reasons which could cause this problem were proposed. In this chapter we investigate experts' ordering with the help of the statistical method that checks if orderings were given at random or not. For this purpose we use the coefficient of concordance (W). We test the null hypothesis that the preferences are at random. [11][22].

We first explain the coefficient of concordance method and then apply it to experts' orderings.

We use the following notations:

n	the number of experts
$A(1),\ldots,A(t)$	objects to be compared
t	the number of objects to be ranked
$R(i,e)$	the rank of $A(i)$ obtained from the responses of expert e the value of $R(i,e)$ ranges from 1 to t

We denote the sum over all experts from their assessments for each scenario, by $R(i)$[11]:

$$R(i) = \sum_{e} R(i,e)$$

The sum of squares of the observed deviations from the mean of $R(i)$, is denoted by S and equals to:

$$S = \sum_{i} \left[R(i) - \frac{1}{t} \sum_{j} R(j) \right]^2$$

Siegel [22] defines W:

$$W = \frac{S}{\frac{1}{12}n^2(t^3 - t)}$$

In case of complete agreement, W equals to 1,[11] and gets smaller as the experts agreement diminishes.

For the null hypothesis that experts gave their preferences at random, in [22] we find a table which contains the critical values * of S, for t between 3 and 7 and n between 3 and 20. [11]

Table 4.1: Critical values of S at .05 level of significance

n			t		
	3	4	5	6	7
3			64.4	103.9	157.3
4		49.5	88.4	143.3	217.0
5		62.6	112.3	182.4	276.2
6		75.7	136.1	221.4	335.2
8	48.1	101.7	183.7	299.0	453.1
10	60.0	127.8	231.2	376.7	571.0
15	89.8	192.9	349.8	570.5	864.9
20	119.7	258.0	468.5	764.4	1158.7

In Table 4.1, n represents the number of experts (in our case 11), and t the number of objects to be ranked (in our case 7). In case n is larger than 20, the corresponding values should be computed like described in [24].

We compute the values of S and W obtained for each group:

Table 4.2: Values of S and W for each group

	GROUP 1	GROUP 2	GROUP 3	GROUP 4	GROUP 5	GROUP 6
S	1866	1088	1116	1356	680	340
W	0.5508	0.3211	0.3294	0.4005	0.2007	0.1004

In our problem experts had to order 7 scenarios in each group. We used 11 experts in our analysis. For each group we computed the values of S and W, shown in Table 4.2. These values have to be compared with the values in the 7^{th} column of Table 4.1. However, Table 4.1 does not contain the case of n=11 experts, hence we can check our results against values for n=10 or n=15. If the hypothesis is rejected on level n=15, it is surely rejected for n=11. On the other hand, if we accept the hypothesis on level n=10, than it must surely be accepted for n=11. For the first 4 groups, the obtained values of S are significantly bigger than 864.9 (critical value for 15 experts).

*Critical value is the values which corresponds to a given *significance level*. This value determines the boundary between those samples resulting in a test statistic that lead to rejecting the null hypothesis, and those which lead to a decision not to reject the null hypothesis. The corresponding values for .01 level of significance are larger than the ones for .05, hence we choose .01

We can see that for the last group, the hypothesis that experts gave their orderings at random is accepted, as the value of S is much smaller than 571.0 (critical value for 10 experts).

For group 5, S equals to 680 which is smaller than significance level for 15 experts (864.9), but larger than for 11 experts (571.0). Hence this group is on a verge of acceptance. In this case we would need to find the exact critical value for n=11. We have not done this, and, as a simple observation, we show in Figure 4.1 the relationship between the number of experts, n and the critical value of S for 7 objects to be rank, from Table 4.1.

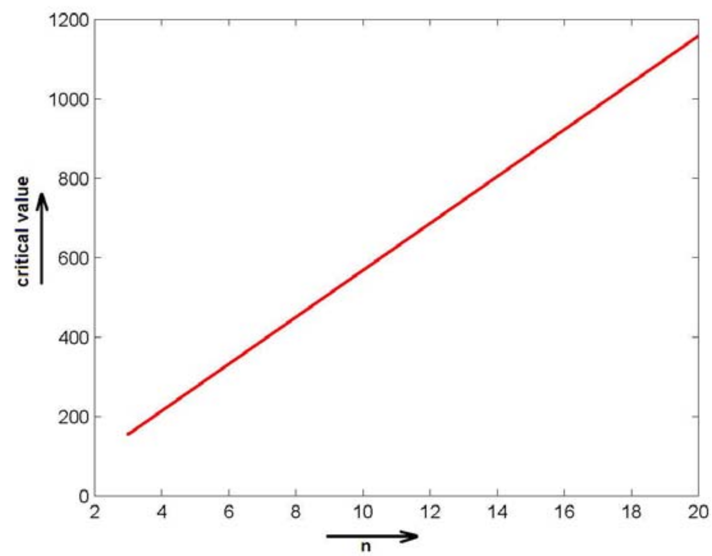


Figure 4.1:

The following line equation:

$$y = ax + b \tag{4.1}$$

is roughly satisfied by each pair of points which form the above plot, where a ∼59 and $b \sim 19.7$. Using this equation, we find out the approximate value of S for 11 experts, which is 629,78.

For the first five groups, based on the values obtained for S, we reject the null hypothesis. Moreover, coefficient of concordance, W,[11] shows the same facts: for the first group its value equals to 0.5508 whereas for the sixth group decreases up to 0.1004. This means that experts agreed the most in the first group, and their agreement diminishes while advancing in the groups.

In the first group, for instance, there are bigger differences between scenarios (least severe - more severe), hence the experts could differentiate them easier. In the following groups these differences become smaller and smaller.

In Chapter 1 we mentioned that in the first five groups the last two scenarios of one group are repeated as being the first ones in the consecutive group. This was done to see if experts are consistent when ordering the same scenarios in different groups. Looking at the table with experts' assessments from Appendix B, we can follow expert number 1. Scenario PX from the first group is identical with NA from the second group, and WL from the first group with SK from the second group. Expert was consistent if he kept his preference while ordering these two scenarios in each group. This means, that in group 1, expert number 1 considered PX more severe than WL. In the second group, the same expert considered NA more severe than SK. (he ordered the same these two identical scenarios from different groups). However, there are cases in which experts were not consistent. The same expert, in group 3 ordered GU as more severe than BE, and in fourth group he ordered BY more severe than AG, where GU=AG and BE=BY. In this case he reversed the ordering.

Table 5.1 presents each expert's consistency within each group. We notice that expert 7 was consistent during all analysis, whereas expert 11 was not consistent at all.

Table 4.3: Experts' consistency within each group

	Experts										
Groups	1	2	3	4	5	6	7	8	9	10	11
gr.1-gr.2	√	√	√	√	√	√	√	√	√	√	
gr.2-gr.3	√	√		√	√		√				
gr.3-gr.4		√		√	√		√				
gr.4-gr.5		√			√		√				
gr.5-gr.6							√				

Based on the consistency of experts we can assign to each expert a weight. Next, with this weight, we perform the analysis of one group, and check the results. This approach is presented in the next chapter.

Chapter 5

Extra analysis

In this chapter we perform extra analysis to check sensitivities of the procedure that we have used in previous chapter to find the model to score pathogens. Firstly we test if our procedure is sensitive to different choices of transformations for attributes. THen we test how the results change with different choice of starting distributions of B_i's. Moreover, we analyse Group 2 by considering weights for experts, as stated in the previous chapter.

5.1 Weights for experts

We recall from the previous chapter the table presenting experts' consistency. Based on this results, we assign to each expert a weight. These weights are shown in Table 5.2. In this section we present the results obtained by considering weights for experts, for Group 2.

Table 5.1: Experts' consistency within each group

	Experts										
Groups	1	2	3	4	5	6	7	8	9	10	11
gr.1-gr.2	√	√	√	√	√	√	√	√	√	√	
gr.2-gr.3	√	√		√	√		√				
gr.3-gr.4		√		√	√		√				
gr.4-gr.5		√			√		√				
gr.5-gr.6							√				

Table 5.2: Experts' weights

experts	1	2	3	4	5	6	7	8	9	10	11
weights	0.0869	0.1739	0.0435	0.1304	0.1739	0.0435	0.2174	0.0435	0.0435	0.0345	0

The weights from the previous table have obtained as follows: we sum the number of times that experts have been consistent, and this number equals to 23. Next, for each expert, we divide the number of times that he/she was consistent, to the total number of times that all expert have been consistent (23). For instance, for the first expert: 2/23=0.0869, and this number represent the weight of the first expert. In a similar way we obtained all the other experts' weights. Because we assign a weight to each expert, the constraints from the second group, which we have to impose are changed. Table 5.3 shows the constraints obtained for each scenario, considering weights for experts. We explain briefly how these new constraints have been obtained. For instance, we know that S_6 was ranked on the first place by two experts. We check in the table which contains experts' assessments from Appendix B, which experts ranked S_6 on the first place, and then sum their weights. S_6 was ranked on the first place by expert 2 and expert 11. We look in the table which contains the weights for experts, and see that expert 2 obtained weight 0.1739, and expert 11 obtained weight 0. By summing these two weights, we obtain the probability of scenario S_6 to be ranked on the first place. In a similar way we obtained all the constraints from Table 5.3.

Table 5.3: Group II - updated constraints

	scenario	**1**st	**2**nd	**3**rd	**4**th	**5**th	**6**th	**7**th
V A R I	S_6	*0.1739*	*0.4783*	*0.2173*		*0.0435*	*0.0435*	*0.0435*
	S_7	*0.2174*	*0.1304*	*0.0435*		*0.6087*		
	S_8	*0.2174*		*0.4783*	*0.3043*			
	S_9		*0.0435*		*0.0435*	*0.0869*	*0.6087*	*0.2174*
	S_{10}	*0.0435*			*0.0435*		*0.1739*	*0.7391*
	S_{11}	*0.0345*	*0.0435*	*0.0435*	*0.5652*	*0.2174*	*0.0869*	
	S_{12}	*0.3043*	*0.3043*	*0.2174*	*0.0435*	*0.0435*	*0.0870*	

We analyse this second group in a similar way as we did before, by considering the 5 variants. In the first variant we performed the analysis considering all constraints. Table 5.4 contains the means, variances and scores obtained in this first variant. The out of samples validation has also been performed for this group, and due to space constraints we do not provide this table here, they are presented in Appendix B.

Table 5.4: Variant I of Group II - results obtained with weights for experts

VARIANT I: 1÷7

	mean	variance	scores	ordering rank	ordering PI	#	EQ	OQ	QD
B_1	0.6504	0.0811	S_6=1.216	S_{10}	S_9	||	0.826	0.87353	0.047
B_2	0.6113	0.0683	S_7=1.180	S_9	S_{10}	||	0.783	0.69382	0.089
B_3	0.2371	0.0582	S_8=1.147	S_{12}	S_{11}	||	0.783	0.75237	0.030
B_4	0.2217	0.0449	S_9=1.397	S_{11}	S_6	||	0.957	0.93833	0.018
B_5	0.2353	0.0561	S_{10}=1.346	S_6	S_{12}	||	0.957	0.96491	0.008
B_6	0.1773	0.0201	S_{11}=1.268	S_8	S_7	||	0.696	0.79467	0.099
B_7	0.6127	0.0469	S_{12}=1.196	S_7	S_8	||	0.957	0.90505	0.051
B_8	0.4940	0.0926				||	0.783	0.72892	0.054
B_9	0.4800	0.0707				||	0.261	0.50813	0.247
						||	0.522	0.53898	0.017
						||	0.870	0.92860	0.059
						||	0.957	0.94287	0.014
						||	0.957	0.91605	0.040
						||	0.696	0.75942	0.064
						||	0.957	0.93010	0.026
							0.391	0.55255	0.161
						3	0.826	0.69664	0.129
						4	0.913	0.96461	0.052
							0.913	0.91279	0.001
						||	0.783	0.85812	0.075
						||	0.957	0.97325	0.017
						||	0.522	0.64177	0.120
						||	0.957	0.96017	0.004
						||	0.783	0.69768	0.085
						||	0.957	0.98237	0.026
						||	0.391	0.42196	0.031
						||	0.913	0.90527	0.008
						||	0.783	0.81384	0.031
						||	0.957	0.94936	0.007
						||	0.696	0.77233	0.077
						||	0.957	0.96747	0.011
						||	0.957	0.96792	0.011
						||	0.435	0.44104	0.006
						||	0.957	0.95700	0.001

 # - the number of constraints used
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ-OQ)

The following tables contain the constraints considered in each variant and the results obtained.

Table 5.5: Variant II Group II - constraints used

	scenario	$\mathbf{1}^{st}$	$\mathbf{2}^{nd}$	$\mathbf{3}^{rd}$	$\mathbf{4}^{th}$	$\mathbf{5}^{th}$	$\mathbf{6}^{th}$	$\mathbf{7}^{th}$
VAR II	S_6	*0.1739*	*0.4783*	*0.2173*		*0.0435*	*0.0435*	*0.0435*
	S_7	*0.2174*	*0.1304*	*0.0435*		*0.6087*		
	S_8	*0.2174*		*0.4783*				
	S_9		*0.0435*			*0.0869*	*0.6087*	*0.2174*
	S_{10}	*0.0435*					*0.1739*	*0.7391*
	S_{11}	*0.0345*	*0.0435*	*0.0435*		*0.2174*	*0.0869*	
	S_{12}	*0.3043*	*0.3043*	*0.2174*		*0.0435*	*0.0870*	

Table 5.6: Variant III Group II - constraints used

	scenario	$\mathbf{1}^{st}$	$\mathbf{2}^{nd}$	$\mathbf{3}^{rd}$	$\mathbf{4}^{th}$	$\mathbf{5}^{th}$	$\mathbf{6}^{th}$	$\mathbf{7}^{th}$
VAR III	S_6	*0.1739*	*0.4783*				*0.0435*	*0.0435*
	S_7	*0.2174*	*0.1304*					
	S_8	*0.2174*						
	S_9		*0.0435*				*0.6087*	*0.2174*
	S_{10}	*0.0435*					*0.1739*	*0.7391*
	S_{11}	*0.0345*	*0.0435*				*0.0869*	
	S_{12}	*0.3043*	*0.3043*				*0.0870*	

Table 5.7: Variant IV Group II - constraints used

	scenario	$\mathbf{1}^{st}$	$\mathbf{2}^{nd}$	$\mathbf{3}^{rd}$	$\mathbf{4}^{th}$	$\mathbf{5}^{th}$	$\mathbf{6}^{th}$	$\mathbf{7}^{th}$
VAR IV	S_6	*0.1739*						*0.0435*
	S_7	*0.2174*						
	S_8	*0.2174*						
	S_9							*0.2174*
	S_{10}	*0.0435*						*0.7391*
	S_{11}	*0.0345*						
	S_{12}	*0.3043*						

Table 5.8: Variant V Group II - constraints used

	scenario	$\mathbf{1}^{st}$	$\mathbf{2}^{nd}$	$\mathbf{3}^{rd}$	$\mathbf{4}^{th}$	$\mathbf{5}^{th}$	$\mathbf{6}^{th}$	$\mathbf{7}^{th}$
VAR V	S_6		*0.4783*					
	S_7		*0.1304*			*0.6087*		
	S_8	*0.2174*		*0.4783*				
	S_9						*0.6087*	*0.2174*
	S_{10}							*0.7391*
	S_{11}				*0.5652*			
	S_{12}	*0.3043*						

Table 5.9: Variant II of Group II - results obtained with weights for experts

VARIANT II: 1,2,3   5,6,7

	mean	variance	scores	ordering rank	ordering PI	#	EQ	OQ	QD	SD
B_1	0.6752	0.0741	S_6=1.085	S_{10}	S_9	||	0.836	0.88599	0.060	0.130
B_2	0.6558	0.0652	S_7=1.063	S_9	S_{11}	||	0.783	0.72996	0.053	0.116
B_3	0.1606	0.0485	S_8=1.048	S_{11}	S_{10}	||	0.783	0.79675	0.014	0.100
B_4	0.2434	0.0350	S_9=1.303	S_6	S_6	||	0.957	0.96270	0.006	0.094
B_5	0.2621	0.0608	S_{10}=1.227	S_6	S_7	||	0.957	0.95294	0.004	0.119
B_6	0.1290	0.0232	S_{11}=1.233	S_{12}	S_8	||	0.696	0.67166	0.024	0.035
B_7	0.4939	0.0538	S_{12}=1.015	S_8	S_{12}	||	0.957	0.95156	0.005	0.181
B_8	0.4892	0.0908				||	0.783	0.80032	0.018	
B_9	0.5264	0.0472				||	0.261	0.54394	0.283	
						||	0.522	0.53099	0.009	
						||	0.870	0.92010	0.050	
						||	0.957	0.96393	0.007	
						||	0.957	0.91464	0.042	
						||	0.696	0.67035	0.025	
						||	0.957	0.94782	0.009	
							0.391	0.49200	0.101	
						2	0.826	0.75188	0.074	
						9	0.913	0.92877	0.016	
							0.913	0.93689	0.024	
						||	0.783	0.83151	0.049	
						||	0.957	0.97288	0.016	
						||	0.522	0.51288	0.009	
						||	0.957	0.97134	0.015	
						||	0.783	0.80015	0.018	
						||	0.957	0.97319	0.017	
						||	0.391	0.40388	0.013	
						||	0.913	0.91324	0.000	
						||	0.783	0.78305	0.000	
						||	0.957	0.95700	0.000	

 # - the number of constraints used RMSE=0.881088997
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ – OQ)
SD - scores differences (scores from variant I – scores from variant II)
RMSE - square root of the sum of scores differences

Table 5.10: Variant III of Group II - results obtained with weights for experts

VARIANT III: 1,2 6,7				**ordering**						
	mean	**variance**	**scores**	**rank**	**PI**	**#**	**EQ**	**OQ**	**QD**	**SD**
B_1	0.7810	0.0400	S_6=1.257	S_{10}	S_{10}	||	0.826	0.82605	0.00005	0.041
B_2	0.5859	0.0741	S_7=1.209	S_9	S_9	||	0.783	0.78305	0.00045	0.030
B_3	0.2783	0.0642	S_8=1.251	S_{11}	S_{11}	||	0.783	0.78302	0.00042	0.104
B_4	0.1764	0.0256	S_9=1.475	S_6	S_6	||	0.957	0.95701	0.00050	0.078
B_5	0.1495	0.0312	S_{10}=1.499	S_7	S_8	||	0.957	0.95700	0.00050	0.153
B_6	0.2114	0.0241	S_{11}=1.331	S_{12}	S_{12}	||	0.696	0.69600	0.00030	0.064
B_7	0.6546	0.0442	S_{12}=1.238	S_8	S_7	||	0.957	0.95734	0.00084	0.041
B_8	0.4458	0.0995				||	0.783	0.78438	0.00178	
B_9	0.4627	0.0651				||	0.261	0.26088	0.00002	
						||	0.522	0.52381	0.00210	
						||	0.870	0.87037	0.00076	
						||	0.957	0.95712	0.00061	
						||	0.957	0.95710	0.00060	
						||	0.696	0.69603	0.00033	
						||	0.957	0.95741	0.00091	
							0.391	0.39304	0.00173	
						2	0.826	0.82630	0.00019	
						9	0.913	0.91307	0.00002	
							0.913	0.91300	0.00000	

 # - the number of constraints used RMSE=0.71446919
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ − OQ)
SD - scores differences (scores from variant I − scores from variant III)
RMSE - square root of the sum of scores differences

After analysing Group 2 considering weights for experts we observed some interesting results. Variant I (when we considered all constraints) did not perform too well, and the differences between imposed and obtained probabilities are bigger than the ones obtained for the same variant, but without weights for experts. However, interesting is that the more constraints we take out (see variant II, III, IV and V), the differences between imposed and obtained probabilities become smaller than for the same variants, but without weights of experts. We believe that better results could be obtained if all groups are analysed together using variant V, and considering weights for experts. However we leave this as an open question, regarding future work.

Table 5.11: Variant IV of Group II - results obtained with weights for experts

VARIANT IV: 1,7

	mean	variance	scores	ordering rank	ordering PI	#	EQ	OQ	QD	SD
B_1	0.7366	0.0487	S_6=1.588	S_{10}	S_{10}	‖	0.826	0.8268	0.00065	0.372
B_2	0.5387	0.0845	S_7=1.474	S_9	S_9	‖	0.783	0.7837	0.00108	0.294
B_3	0.4028	0.0781	S_8=1.485	S_{11}	S_{11}		0.783	0.7834	0.00083	0.338
B_4	0.2363	0.0379	S_9=1.657	S_6	S_6	9	0.957	0.9571	0.00058	0.260
B_5	0.1978	0.0401	S_{10}=1.761	S_7	S_{12}		0.957	0.9571	0.00057	0.415
B_6	0.2463	0.0365	S_{11}=1.655	S_{12}	S_8	‖	0.696	0.6960	0.00030	0.387
B_7	0.5584	0.0624	S_{12}=1.534	S_8	S_7	‖	0.957	0.9573	0.00083	0.338
B_8	0.5384	0.0824				‖	0.783	0.7843	0.00169	
B_9	0.6166	0.0808				‖	0.261	0.2610	0.00010	

 # - the number of constraints used RMSE=1.550230825
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ – OQ)
SD - scores differences (scores from variant I – scores from variant IV)
RMSE - square root of the sum of scores differences

Table 5.12: Variant V of Group II - results obtained with weights for experts

VARIANT V: ≥3

	mean	variance	scores	ordering rank	ordering PI	#	EQ	OQ	QD	SD
B_1	0.6015	0.0884	S_6=1.391	S_{10}	S_9	‖	0.783	0.7830	0.00040	0.176
B_2	0.6063	0.0869	S_7=1.338	S_9	S_{10}	‖	0.696	0.6960	0.00030	0.159
B_3	0.2800	0.0576	S_8=1.253	S_{11}	S_6	‖	0.783	0.8446	0.06203	0.105
B_4	0.2415	0.0620	S_9=1.510	S_6	S_{12}	‖	0.261	0.2959	0.03503	0.113
B_5	0.2868	0.1025	S_{10}=1.450	S_7	S_{11}	‖	0.522	0.5074	0.01433	0.104
B_6	0.2207	0.0320	S_{11}=1.375	S_{12}	S_7	‖	0.870	0.8761	0.00654	0.108
B_7	0.6813	0.0334	S_{12}=1.381	S_8	S_8	‖	0.391	0.4684	0.07715	0.185
B_8	0.5498	0.0889				‖	0.522	0.5294	0.00765	
B_9	0.5290	0.0958				‖	0.391	0.3910	0.00030	
						‖	0.435	0.4350	0.00020	

 # - the number of constraints used RMSE=0.974351082
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ – OQ)
SD - scores differences (scores from variant I – scores from variant II)
RMSE - square root of the sum of scores differences

5.2 Transformations

In this section we first present the transformations that we have used in the project, and afterwards we discuss their interpretation. Next, we investigate the sensitivity of results with respect to these transformations, and present the results obtained. Table 5.13 shows the transformations that we have used for our analysis.

We need to transform the scale of attributes such that we can represent all of them in a monotonic increasing scale from 0 to 1.

Because some attributes values are very large (for instance the economic damage - five thousand million €) we transform these numbers using a logarithmic scale. This way we deal with more convenient values. These attributes are: ”animal spreading speed”, ”animal economic damage”, ”human spreading speed” and ”human economic damage”. The log-transformed values corresponding to these criteria are shown in column 4 of Table 5.13. Note that for the rest of the attributes column 4 contains the same values as the third column (no transformation was applied).

First column from Table 5.13 expresses the levels of each attributes. Second column contains the point estimates of each level of the attributes. And, finally, third column contains the numerical expression of the point estimates.

It is also worth mentioning that all attributes, except second and fifth, have after transformation an increasing monotonic scale. The convention is the higher the value of the attribute, the higher the threat. The second and fifth attributes have decreasing monotonic scale. The convention here is the less the value, the higher the threat. This is explained by the nature of the attribute. For instance, a high value, say 30 means that it takes 30 days for the pathogen to spread, where as for a low value, say 3, it takes only 3 days. For this reason we used minus sign in computing the scores.

The first value of the second attribute is null. This means that the pathogen does not spread. In our mathematical model we took this value equal to 10,000 days, therefore we approximate zero by a very small probability of occurrence. We took 10,000 days because we want to have monotonicity. 10,000 days is our choice, and is equivalent to almost 27 years and we consider that if a pathogen does not spread in this period, then it does not spread at all. Also we used a logarithmic scale for this attribute.

These transformations were chosen by analysts. Many other transformations could have been used. Few questions can be posted at this point:

1. Do these transformations influence the results (which would be ordering of scenarios)? If yes,
2. Can we propose transformations in some sense, optimal for the analysts?

We notice that the transformations used, presented in Table 5.13, do not lead to values of the attributes that are uniformly spread. For PI procedure it would be advantageous to have them "nicely" spread (it would be easier to get samples in all the intervals). This is why we now investigate what the result would be if instead of the values of levels (e.g. 50%, 5,000 million €, etc.) we take the levels themselves (e.g. 1, 5, 4, 3 etc.) and normalise them. By doing this, we imposed a uniform spread of each level attribute. This means, for instance, that for first attribute we used for level 1, the value 0.25, for level 2, 0.5, for level 3, 0.75, and for level 4, 0.75, instead of 0%, 0.5%, 50% and 100% (attribute one has four levels). Obviously it would be very difficult to find transformations for all attributes that would give us similar result. But if this would really help improving feasibility of the problem in PI procedure, than it is worth investigating further.

Next we present the results obtained using this uniform spread of attributes values. We skip presenting the results for each group taken separately, and provide the results obtained when all the groups are placed together. Tables 5.14 and 5.15 show the obtained scores of each scenario and the means of B_i's. We used these changed transformations in "combined variant" (i.e. we take groups I, II and III with variant V , ($\geq$3), and groups IV and V with variant IV, (1,7)).

Table 5.13: Transformations of values of attributes

	Attributes	Range		f(x)
	I chances of introduction			
1	0%	0	0	0.000
2	0.5%	0.005	0.005	0.005
3	5%	0.05	0.05	0.050
4	50%	5	5	0.500
5	100%	1	1	1.000
	II animal spreading speed			
1	0	10,000	4	1.000
2	30	30	1.47712125	0.369
3	10	10	1	0.250
4	3	3	0.47712125	0.119
	III animal economic damage			
1	5M€per year	5×10^6	6.69897	0.691
2	50M€per year	5×10^7	7.69897	0.794
3	500M€per year	5×10^8	8.69897	0.897
4	5000M€per year	5×10^9	9.69897	1.000
	IV animal to human transmitting chance			
1	1:10,000	0.0001	0.0001	0.001
2	1:1,000	0.001	0.001	0.010
3	1:100	0.01	0.01	0.100
4	1:10	0.1	0.1	1.000
	V human spreading speed			
1	0	10,000	4	1.000
2	30	30	1.47712125	0.369
3	10	10	1	0.250
4	3	3	0.47712125	0.119
	VI gravity of illness			
1	0.02	0.02	.204	1.000
2	0.06	0.06	1.47712125	0.369
3	0.20	0.20	1	0.250
4	0.60	0.60	0.47712125	0.119
	VII chances of dying			
1	0%	0	0	0.000
2	0.5%	0.005	0.005	0.005
3	5%	0.05	0.05	0.050
4	50%	5	5	0.500
5	100%	1	1	1.000
	VIII human economic damage			
1	5M€per year	5×10^6	6.69897	0.691
2	50M€per year	5×10^7	7.69897	0.794
3	500M€per year	5×10^8	8.69897	0.897
4	5000M€per year	5×10^9	9.69897	1.000
	IX perception			
1	0	0	0	0
2	2	2	2	0.333
3	4	4	4	0.667
4	6	6	6	1.000

Table 5.14: Results for ALL GROUPS with transformations (PART I)

		EQ	**OQ**	**QD**	**mean**		**scores**	**rank**	**ordering PI**	**#**
GROUP 1 ≥ 3	$S_{5.1<}$	0.545	0.5690	0.02359	B_1	0.5807	S_1=1.9859	S_1	S_1	‖
	$S_{1.1>}$	0.636	0.4350	0.20139	B_2	0.3397	S_2=1.7953	S_3	S_3	‖
	$S_{3.1>}$	0.545	0.7801	0.23463	B_3	0.2303	S_3=1.8175	S_6	S_2	‖
	$S_{2.1<2}$	0.545	0.5756	0.03014	B_4	0.2749	S_4=1.6872	S_7	S_6	
	$S_{7.1<2}$	0.727	0.6965	0.03077	B_5	0.3031	S_5=1.5304	S_4	S_7	1
	$S_{1.1>2}$	0.455	0.5816	0.12706	B_6	0.2992	S_6=1.7762	S_2	S_4	3
	$S_{3.1>2}$	0.727	0.7482	0.02088	B_7	0.6929	S_7=1.7600	S_5	S_5	
	$S_{4.1<3}$	0.636	0.6974	0.06103	B_8	0.3302				‖
	$S_{5.1<3}$	0.727	0.7341	0.00678	B_9	0.3262				‖
	$S_{7.1<3}$	0.727	0.7461	0.01886						‖
	$S_{7.1>3}$	0.545	0.6354	0.08994						‖
	$S_{3.1<4}$	0.727	0.6969	0.03036						‖
	$S_{6.1<4}$	0.727	0.7475	0.02023						‖
GROUP 2 ≥ 3	$S_{8.2<}$	0.727	0.7775	0.05019			S_6=1.7762	S_{10}	S_6	‖
	$S_{12.2<}$	0.727	0.8512	0.12394			S_7=1.7600	S_9	S_7	‖
	$S_{9.2>}$	0.727	0.8369	0.10958			S_8=1.5969	S_{11}	S_{12}	‖
	$S_{10.2>}$	0.364	0.5022	0.13853			S_9=1.7100	S_7	S_{10}	
	$S_{6.2<2}$	0.636	0.7667	0.13033			S_{10}=1.7162	S_6	S_9	1
	$S_{7.2<2}$	0.727	0.8102	0.08296			S_{11}=1.6583	S_{12}	S_{11}	0
	$S_{9.2<2}$	0.909	0.9096	0.00047			S_{12}=1.7274	S_8	S_8	
	$S_{8.2<3}$	0.545	0.4794	0.06602						‖
	$S_{7.2>3}$	0.455	0.4998	0.04522						‖
	$S_{11.2<4}$	0.545	0.6826	0.13719						‖
GROUP 3 ≥ 3	$S_{13.3>}$	0.636	0.7176	0.08126			S_{11}=1.6583	S_{16}	S_{14}	‖
	$S_{16.3>}$	0.545	0.5250	0.02049			S_{13}=1.5733	S_{13}	S_{17}	‖
	$S_{17.3<2}$	0.636	0.7474	0.11100			S_{14}=1.7760	S_{11}	S_{15}	‖
	$S_{11.3>2}$	0.727	0.7883	0.06107			S_{15}=1.6646	S_{14}	S_{11}	
	$S_{17.3>2}$	0.636	0.6168	0.01959			S_{16}=1.6088	S_{17}	S_{16}	9
	$S_{14.3<3}$	0.636	0.7266	0.09026			S_{17}=1.6654	S_{15}	S_{13}	
	$S_{11.3>3}$	0.727	0.7721	0.04480						‖
	$S_{14.3<4}$	0.727	0.7763	0.04903						‖
	$S_{15.3<4}$	0.636	0.5976	0.03875						‖
		$RMSE_q$	**0.1064**							
$RMSE_s$	**0.5561**									

 # - the number of constraints used
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ − OQ)
$RMSE_s$ - error of scores

Table 5.15: Results for ALL GROUPS with transformations (PART II)

		EQ	**OQ**	**QD**	**scores**	**rank**	**ordering PI**	**#**
GROUP 4 1, 7	$S_{16.4<}$	0.727	0.7472	0.01997	S_{16}=1.6088	S_{20}	S_{20}	‖
	$S_{19.4<}$	0.909	0.9019	0.00714	S_{17}=1.6654	S_{21}	S_{17}	‖
	$S_{21.4<}$	0.909	0.8842	0.02493	S_{18}=1.5751	S_{18}	S_{16}	‖
	$S_{22.4<}$	0.455	0.4798	0.02524	S_{19}=1.5201	S_{19}	S_{18}	
	$S_{18.4>}$	0.909	0.8846	0.02445	S_{20}=1.7883	S_{17}	S_{19}	8
	$S_{19.4>}$	0.818	0.8845	0.06633	S_{21}=1.4673	S_{16}	S_{21}	
	$S_{20.4>}$	0.364	0.4604	0.09681	S_{22}=1.3781	S_{22}	S_{22}	‖
	$S_{21.4>}$	0.909	0.8981	0.01095				‖
								‖
GROUP 5 1, 7	$S_{21.5<}$	0.818	0.8354	0.01726	S_{21}= 1.4673	S_{21}	S_{23}	‖
	$S_{22.5<}$	0.727	0.7635	0.03625	S_{22}= 1.3781	S_{23}	S_{21}	‖
	$S_{23.5<}$	0.909	0.9174	0.00832	S_{23}= 1.5024	S_{26}	S_{24}	‖
	$S_{25.5<}$	0.636	0.7168	0.08047	S_{24}= 1.3980	S_{27}	S_{22}	
	$S_{27.5<}$	0.909	0.9007	0.00838	S_{25}= 1.3427	S_{24}	S_{25}	9
	$S_{21.5>}$	0.455	0.4563	0.00174	S_{26}= 1.3093	S_{22}	S_{27}	
	$S_{22.5>}$	0.909	0.9061	0.00302	S_{27}= 1.3245	S_{25}	S_{26}	‖
	$S_{23.5>}$	0.909	0.9111	0.00198				‖
	$S_{25.5>}$	0.909	0.9088	0.00025				‖
	$S_{26.5>}$	0.818	0.8180	0.00018				‖
			RMSE$_q$	**1.65227**				
RMSE$_s$	**0.5561**							

 # - the number of constraints used
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ − OQ)
RMSE$_s$ - error of scores

The results that we presented in this section show that indeed, the choice of transformations does influence the results. More explicitly, when we look at RMSE in this case, it equals to 1.65227. When we used previous transformations, we obtained for RMSE a larger value, 1.91919. This result shows that because we transformed the values of attributes such that they are uniformly spread, PI performed much better. however, even if we obtained a smaller value for RMSE, the ordering obtained is not so accurate, because we do not follow the reality for those values. We emphasise that we wanted to show that the more uniformly spread are the values of attributes after transformations, the less error we obtain.

Further we try to answer to the second question: can we propose any transformations? It is difficult to assume what it should be done. However, our proposal is that, first of all the attributes values should be chosen such that they express as accurate as possible the reality. Secondly, the transformation can be done using any kind of relation, function, etc, such that they lead to a uniform spread of the attributes values.

5.3 Weights

In this section we present another approach. Instead of starting with uniform distribution for B_i's, we start with Dirichlet distribution, to get weights for B_i's. Let B be a random vector, where each of the elements are independent and have Gamma distribution with scale equal to 1.

$$B \sim Gamma(shape = \alpha_i, scale = 1),$$

for $i = 1, .., 9$. Then, the random vector $V = (B_1/T, B_2/T, B_3/T, ..., B_9/T)$, where

$$T = \sum_{i=1}^{9} B_i$$

has a Dirichlet distribution with parameters α_i, $i = 1, .., 9$

We skip presenting the results for each group taken separately, and provide just the results obtained when all the groups are placed together. Tables 5.16 and 5.17 show the obtained scores of each scenario and the means of B_i's.

We specify that we started from "combined variant" (i.e. we take groups I, II and III with variant V , (≥3), and groups IV and V with variant IV, (1,7)), and we took B_i's as weights.

In this approach we investigated the sensitivity of the results with respect to starting distribution. The results are shown in Tables 5.16 and 5.17.

Table 5.16: Results for ALL GROUPS with weights (PART I)

		EQ	OQ	QD	mean	scores	rank	ordering PI	#
GROUP 1 ≥ 3	$S_{5.1<}$	0.545	0.5849	0.03943	B_1 0.2193	S_1=0.463	S_1	S_1	‖
	$S_{1.1>}$	0.636	0.6518	0.01542	B_2 0.0786	S_2=0.329	S_3	S_3	‖
	$S_{3.1>}$	0.545	0.6647	0.11928	B_3 0.0763	S_3=0.439	S_6	S_4	‖
	$S_{2.1<2}$	0.545	0.5925	0.04700	B_4 0.0782	S_4=0.394	S_7	S_6	
	$S_{7.1<2}$	0.727	0.7374	0.01018	B_5 0.0685	S_5=0.303	S_4	S_7	1
	$S_{1.1>2}$	0.455	0.5666	0.11208	B_6 0.0257	S_6=0.352	S_2	S_2	3
	$S_{3.1>2}$	0.727	0.7330	0.00572	B_7 0.2322	S_7=0.349	S_5	S_5	
	$S_{4.1<3}$	0.636	0.6829	0.04650	B_8 0.1598				‖
	$S_{5.1<3}$	0.727	0.8037	0.07646	B_9 0.0614				‖
	$S_{7.1<3}$	0.727	0.7421	0.01482					‖
	$S_{7.1>3}$	0.545	0.5867	0.04122					‖
	$S_{3.1<4}$	0.727	0.6767	0.05062					‖
	$S_{6.1<4}$	0.727	0.7968	0.06954					‖
GROUP 2 ≥ 3	$S_{8.2<}$	0.727	0.7515	0.02422		S_6= 0.352	S_{10}	S_9	‖
	$S_{12.2<}$	0.727	0.7374	0.01015		S_7= 0.349	S_9	S_{10}	‖
	$S_{9.2>}$	0.727	0.7233	0.00400		S_8= 0.337	S_{11}	S_{12}	‖
	$S_{10.2>}$	0.364	0.4787	0.11505		S_9= 0.438	S_7	S_6	
	$S_{6.2<2}$	0.636	0.5735	0.06285		S_{10}=0.412	S_6	S_1	1
	$S_{7.2<2}$	0.727	0.8043	0.07706		S_{11}=0.352	S_{12}	S_7	0
	$S_{9.2<2}$	0.909	0.8835	0.02556		S_{12}=0.402	S_8	S_8	
	$S_{8.2<3}$	0.545	0.5162	0.02923					‖
	$S_{7.2>3}$	0.455	0.5402	0.08564					‖
	$S_{11.2<4}$	0.545	0.5715	0.02607					‖
GROUP 3 ≥ 3	$S_{13.3>}$	0.636	0.7450	0.10865		S_{11}=0.352	S_{16}	S_{14}	‖
	$S_{16.3>}$	0.545	0.5024	0.04305		S_{13}=0.309	S_{13}	S_{15}	‖
	$S_{17.3<2}$	0.636	0.6408	0.00439		S_{14}=0.404	S_{11}	S_{16}	‖
	$S_{11.3>2}$	0.727	0.7668	0.03949		S_{15}=0.394	S_{14}	S_{11}	
	$S_{17.3>2}$	0.636	0.6471	0.01073		S_{16}=0.357	S_{17}	S_{17}	9
	$S_{14.3<3}$	0.636	0.6246	0.01172		S_{17}=0.323	S_{15}	S_{13}	
	$S_{11.3>3}$	0.727	0.7321	0.00485					‖
	$S_{14.3<4}$	0.727	0.7780	0.05070					‖
	$S_{15.3<4}$	0.636	0.6646	0.02820					‖

Table 5.17: Results for ALL GROUPS with weights (PART II)

		EQ	**OQ**	**QD**	**scores**	**rank**	**ordering PI**	**#**
GROUP 4 1, 7	$S_{16.4<}$	0.727	0.7895	0.06218	S_{16}= 0.357	S_{20}	S_{20}	$\|$
	$S_{19.4<}$	0.909	0.8845	0.02457	S_{17}= 0.323	S_{21}	S_{21}	$\|$
	$S_{21.4<}$	0.909	0.8958	0.01328	S_{18}= 0.256	S_{18}	S_{16}	$\|$
	$S_{22.4<}$	0.455	0.4683	0.01374	S_{19}= 0.298	S_{19}	S_{17}	
	$S_{18.4>}$	0.909	0.9077	0.00138	S_{20}= 0.431	S_{17}	S_{19}	8
	$S_{19.4>}$	0.818	0.8739	0.05569	S_{21}= 0.365	S_{16}	S_{18}	
	$S_{20.4>}$	0.364	0.4100	0.04632	S_{22}= 0.236	S_{22}	S_{22}	$\|$
	$S_{21.4>}$	0.909	0.9094	0.00028				$\|$
								$\|$
GROUP 5 1, 7	$S_{21.5<}$	0.818	0.8222	0.00403	S_{21}= 0.365	S_{21}	S_{21}	$\|$
	$S_{22.5<}$	0.727	0.7946	0.06729	S_{22}= 0.236	S_{23}	S_{26}	$\|$
	$S_{23.5<}$	0.909	0.9129	0.00379	S_{23}= 0.265	S_{26}	S_{23}	$\|$
	$S_{25.5<}$	0.636	0.6366	0.00024	S_{24}= 0.233	S_{27}	S_{22}	
	$S_{27.5<}$	0.909	0.9120	0.00287	S_{25}= 0.217	S_{24}	S_{24}	9
	$S_{21.5>}$	0.455	0.4600	0.00548	S_{26}= 0.282	S_{22}	S_{25}	
	$S_{22.5>}$	0.909	0.9108	0.00170	S_{27}= 0.212	S_{25}	S_{27}	$\|$
	$S_{23.5>}$	0.909	0.9124	0.00330				$\|$
	$S_{25.5>}$	0.909	0.9094	0.00033				$\|$
	$S_{26.5>}$	0.818	0.8180	0.00018				$\|$
			RMSE$_q$	**1.31016**				

 # - the number of constraints used
EQ - experts quantiles (experts assessments)
OQ - obtained quantiles (after PI)
QD - quantiles differences (EQ − OQ)
RMSE$_q$ - error of quantiles

Looking at Table 5.17 we notice that error obtained using weights equals to 1.31016. This value is the lowest error that we have obtained in in this analysis. However, there are cases when recovery of ordering that we obtain is poor (i.e. group 3). This problem should be further investigated.

5.4 Summary of obtained results

We present in this section four tables containing the results obtained under the four strategies that we performed in this thesis. Table 5.18 contains scores obtained by each scenarios, the ordering obtained from rank ordering technique, and the one obtained with PI. For a fair comparison, we normalise the values of means and scores of each scenario. In Table 5.19 we present the scores obtained by each scenario in the four strategies we used. For each strategy we define a minimum and maximum score, and we normalised the scores with respect to this minimum and maximum, to make the comparison possible. Minimum score is obtained with means from each variant, and all attributes having the smallest values. Similarly, the maximum score is computed using the maximum values for the attributes.

We denote by A the strategy in which we used for all groups together variant V ($\geq$3). With B we denote the strategy in which we considered ”combined variant” (for groups 1,2,3 - variant V, $\geq$3, and for groups 4, 5 - variant IV, 1,7). With C we denote the strategy in which we used weights, and D presents results obtained when we changed the transformations.

We notice that scores obtained for strategy A and B are close to each other. This is because the two strategies are pretty similar (in A we use for all groups variant V, whereas in B we use for group 1 to 4, variant V, and for groups 5 and 6 variant IV). For the other two strategies, we notice a slightly difference of scores. This difference appears to be higher for D, because, as shown in the previous section, the results are sensitive to the choice of transformations. Different transformations, better developed and quantified, may lead to better results. However, the scores obtained in the four strategies do not differ from each other too much.

Table 5.18: Results of scores for ALL GROUPS in four strategies

A		B		C		D		
scores	**PI**	**scores**	**PI**	**scores**	**PI**	**scores**	**PI**	**rank**
S_1=1.452	S_1	S_1=1.491	S_1	S_1=0.463	S_1	S_1=1.986	S_1	S_1
S_2=1.240	S_3	S_2=1.240	S_3	S_2=0.329	S_3	S_2=1.795	S_3	S_3
S_3=1.357	S_4	S_3=1.397	S_6	S_3=0.439	S_4	S_3=1.818	S_2	S_6
S_4=1.346	S_2	S_4=1.241	S_4	S_4=0.394	S_6	S_4=1.687	S_6	S_7
S_5=1.123	S_6	S_5=1.072	S_7	S_5=0.303	S_7	S_5=1.503	S_7	S_4
S_6=1.167	S_7	S_6=1.253	S_2	S_6=0.352	S_2	S_6=1.776	S_4	S_2
S_7=1.151	S_5	S_7=1.217	S_5	S_7=0.349	S_5	S_7=1.760	S_5	S_5
S_6=1.167	S_{12}	S_6=1.253	S_9	S_6=0.352	S_9	S_6=1.776	S_6	S_{10}
S_7=1.151	S_9	S_7=1.217	S_{12}	S_7=0.349	S_{10}	S_7=1.760	S_7	S_9
S_8=1.196	S_8	S_8=1.211	S_{10}	S_8=0.337	S_{12}	S_8=1.597	S_{12}	S_{11}
S_9=1.351	S_{10}	S_9=1.378	S_6	S_9=0.438	S_6	S_9=1.710	S_{10}	S_7
S_{10}=1.185	S_{11}	S_{10}=1.257	S_7	S_{10}=0412.	S_{11}	S_{10}=1.716	S_9	S_6
S_{11}=1.178	S_6	S_{11}=1.203	S_8	S_{11}=0352.	S_7	S_{11}=1.658	S_{11}	S_{12}
S_{12}=1.408	S_7	S_{12}=1.375	S_{11}	S_{12}=0402.	S_8	S_{12}=1.727	S_8	S_8
S_{11}=1.178	S_{14}	S_{11}=1.203	S_{16}	S_{11}=0.352	S_{14}	S_{11}=1.658	S_{14}	S_{16}
S_{13}=1.101	S_{15}	S_{13}=1.023	S_{14}	S_{13}=0.309	S_{15}	S_{13}=1.573	S_{17}	S_{13}
S_{14}=1.370	S_{16}	S_{14}=1.283	S_{15}	S_{14}=0.404	S_{16}	S_{14}=1.776	S_{15}	S_{11}
S_{15}=1.359	S_{11}	S_{15}=1.275	S_{11}	S_{15}=0.394	S_{11}	S_{15}=1.665	S_{11}	S_{14}
S_{16}=1.201	S_{13}	S_{16}=1.334	S_{17}	S_{16}=0.357	S_{17}	S_{16}=1.609	S_{16}	S_{17}
S_{17}=1.039	S_{17}	S_{17}=1.106	S_{13}	S_{17}=0.323	S_{13}	S_{17}=1.665	S_{13}	S_{15}
S_{16}=1.201	S_{20}	S_{16}=1.334	S_{20}	S_{16}=0.357	S_{20}	S_{16}=1.609	S_{20}	S_{20}
S_{17}=1.039	S_{16}	S_{17}=1.106	S_{16}	S_{17}=0.323	S_{21}	S_{17}=1.665	S_{17}	S_{21}
S_{18}=0.930	S_{21}	S_{18}=0.991	S_{21}	S_{18}=0.256	S_{16}	S_{18}=1.575	S_{16}	S_{18}
S_{19}=0.867	S_{17}	S_{19}=1.006	S_{17}	S_{19}=0.298	S_{17}	S_{19}=1.502	S_{18}	S_{19}
S_{20}=1.305	S_{18}	S_{20}=1.641	S_{19}	S_{20}=0.431	S_{19}	S_{20}=1.788	S_{19}	S_{17}
S_{21}=1.161	S_{22}	S_{21}=1.160	S_{18}	S_{21}=0.365	S_{18}	S_{21}=1.467	S_{21}	S_{16}
S_{22}=0.883	S_{19}	S_{22}=0.938	S_{22}	S_{22}=0.236	S_{22}	S_{22}=1.378	S_{22}	S_{22}
S_{21}=1.161	S_{21}	S_{21}=1.160	S_{21}	S_{21}=0.365	S_{21}	S_{21}=1.467	S_{23}	S_{21}
S_{22}=0.883	S_{23}	S_{22}=0.938	S_{23}	S_{22}=0.236	S_{26}	S_{22}=1.378	S_{21}	S_{23}
S_{23}=1.034	S_{26}	S_{23}=0.117	S_{22}	S_{23}=0.265	S_{23}	S_{23}=1.502	S_{24}	S_{26}
S_{24}=0.903	S_{24}	S_{24}=0.930	S_{24}	S_{24}=0.233	S_{22}	S_{24}=1.398	S_{22}	S_{27}
S_{25}=0.851	S_{22}	S_{25}=0.885	S_{26}	S_{25}=0.217	S_{24}	S_{25}=1.343	S_{25}	S_{24}
S_{26}=0.959	S_{25}	S_{26}=0.927	S_{25}	S_{26}=0.282	S_{25}	S_{26}=1.309	S_{27}	S_{22}
S_{27}=0.839	S_{27}	S_{27}=0.843	S_{27}	S_{27}=0.212	S_{27}	S_{27}=1.325	S_{26}	S_{25}
RMSE: RMSE:2.23273		**RMSE: 1.91919**		**RMSE: 1.91919**		**RMSE: 1.65227**		

Table 5.19: Results of scores for ALL GROUPS in four strategies - normalised values

A		B		C		D		
scores	**PI**	**scores**	**PI**	**scores**	**PI**	**scores**	**PI**	**rank**
S_{min}=0.2473		S_{min}=0.2366		S_{min}=0.1969		S_{min}=0.1645		
S_1=0.4534	S_1	S_1=0.4621	S_1	S_1=0.4632	S_1	S_1=0.5880	S_1	S_1
S_2=0.3872	S_3	S_2=0.3669	S_3	S_2=0.3288	S_3	S_2=0.5316	S_3	S_3
S_3=0.4237	S_4	S_3=0.4330	S_6	S_3=0.4388	S_4	S_3=0.5382	S_2	S_6
S_4=0.4205	S_2	S_4=0.3847	S_4	S_4=0.3940	S_6	S_4=0.4996	S_6	S_7
S_5=0.3507	S_6	S_5=0.3323	S_7	S_5=0.3034	S_7	S_5=0.4532	S_7	S_4
S_6=0.3643	S_7	S_6=0.3883	S_2	S_6=0.3520	S_2	S_6=0.5260	S_4	S_2
S_7=0.3594	S_5	S_7=0.3771	S_5	S_7=0.3491	S_5	S_7=0.5211	S_5	S_5
S_6=0.3643	S_{12}	S_6=0.3883	S_9	S_6=0.3520	S_9	S_6=0.5260	S_6	S_{10}
S_7=0.3594	S_9	S_7=0.3771	S_{12}	S_7=0.3491	S_{10}	S_7=0.2155	S_7	S_9
S_8=0.3734	S_8	S_8=0.3753	S_{10}	S_8=0.3371	S_{12}	S_8=0.4728	S_{12}	S_{11}
S_9=0.4220	S_{10}	S_9=0.4270	S_6	S_9=0.4383	S_6	S_9=0.5063	S_{10}	S_7
S_{10}=0.3702	S_{11}	S_{10}=0.3894	S_7	S_{10}=0.4118	S_{11}	S_{10}=0.5082	S_9	S_6
S_{11}=0.3678	S_6	S_{11}=0.3727	S_8	S_{11}=0.3518	S_7	S_{11}=0.4910	S_{11}	S_{12}
S_{12}=0.4398	S_7	S_{12}=0.4259	S_{11}	S_{12}=0.4017	S_8	S_{12}=0.5115	S_8	S_8
S_{11}=0.3678	S_{14}	S_{11}=0.3727	S_{16}	S_{11}=0.3518	S_{14}	S_{11}=0.4910	S_{14}	S_{16}
S_{13}=0.3437	S_{15}	S_{13}=0.3170	S_{14}	S_{13}=0.3094	S_{15}	S_{13}=0.4659	S_{17}	S_{13}
S_{14}=0.4279	S_{16}	S_{14}=0.3976	S_{15}	S_{14}=0.4044	S_{16}	S_{14}=0.5259	S_{15}	S_{11}
S_{15}=0.4244	S_{11}	S_{15}=0.3950	S_{11}	S_{15}=0.3940	S_{11}	S_{15}=0.4929	S_{11}	S_{14}
S_{16}=0.3752	S_{13}	S_{16}=0.4132	S_{17}	S_{16}=0.3574	S_{17}	S_{16}=0.4764	S_{16}	S_{17}
S_{17}=0.3246	S_{17}	S_{17}=0.3427	S_{13}	S_{17}=0.3226	S_{13}	S_{17}=0.4931	S_{13}	S_{15}
S_{16}=0.3752	S_{20}	S_{16}=0.4132	S_{20}	S_{16}=0.3574	S_{20}	S_{16}=0.4764	S_{20}	S_{20}
S_{17}=0.3426	S_{16}	S_{17}=0.3427	S_{16}	S_{17}=0.3226	S_{21}	S_{17}=0.4931	S_{17}	S_{21}
S_{18}=0.2904	S_{21}	S_{18}=0.3070	S_{21}	S_{18}=0.2563	S_{16}	S_{18}=0.4664	S_{16}	S_{18}
S_{19}=0.2707	S_{17}	S_{19}=0.3116	S_{17}	S_{19}=0.2979	S_{17}	S_{19}=0.4501	S_{18}	S_{19}
S_{20}=0.4075	S_{18}	S_{20}=0.4157	S_{19}	S_{20}=0.4306	S_{19}	S_{20}=0.5295	S_{19}	S_{17}
S_{21}=0.3627	S_{22}	S_{21}=0.3594	S_{18}	S_{21}=0.3653	S_{18}	S_{21}=0.4345	S_{21}	S_{16}
S_{22}=0.2758	S_{19}	S_{22}=0.2906	S_{22}	S_{22}=0.2361	S_{22}	S_{22}=0.4081	S_{22}	S_{22}
S_{21}=0.3627	S_{21}	S_{21}=0.3594	S_{21}	S_{21}=0.3563	S_{21}	S_{21}=0.4345	S_{23}	S_{21}
S_{22}=0.2758	S_{23}	S_{22}=0.2906	S_{23}	S_{22}=0.2361	S_{26}	S_{22}=0.4081	S_{21}	S_{23}
S_{23}=0.3230	S_{26}	S_{23}=0.3462	S_{22}	S_{23}=0.2649	S_{23}	S_{23}=0.4449	S_{24}	S_{26}
S_{24}=0.2821	S_{24}	S_{24}=0.2882	S_{24}	S_{24}=0.2333	S_{22}	S_{24}=0.4140	S_{22}	S_{27}
S_{25}=0.2658	S_{22}	S_{25}=0.2741	S_{26}	S_{25}=0.2168	S_{24}	S_{25}=0.3976	S_{25}	S_{24}
S_{26}=0.2996	S_{25}	S_{26}=0.2873	S_{25}	S_{26}=0.2816	S_{25}	S_{26}=0.3877	S_{27}	S_{22}
S_{27}=0.2621	S_{27}	S_{27}=0.2611	S_{27}	S_{27}=0.2121	S_{27}	S_{27}=0.3922	S_{26}	S_{25}
S_{max}=3.2018		S_{max}=3.2270		S_{max}=1		S_{max}=3.3777		

Chapter 6

Conclusions and future work

6.1 Application of results - Priority of Zoonoses

Effective surveillance, prevention and control of zoonoses require focusing on the most relevant ones. To establish a list in which all relevant zoonoses are ranked, a priority setting procedure must be followed. Several priority setting procedures can be used to acquire a final prioritized list including discussions, voting, and group consensus or with an online survey (Public Health Foundation, 2006). Another method of prioritizing is to build a model used to score each pathogen, and based on this score, the ordering of pathogens from most to least severe is obtained.

Our goal in this thesis was to build a model such that it can be used for prioritising the zoonoses based on their severity. In this project, 92 emerging zoonotic agents are considered for their importance for The Netherlands. Technical experts (e.g. scientists of the Central Veterinary Institute[21]) scored these 92 pathogens on nine criteria. We use this information to recover the coefficients of the attributes, using PI technique. We proposed as a starting model, a linear one. After testing this model, we found out that this linear model can be used in analysis of the real data. Because of software constraints we could not use all constraints in our analysis. We have investigated few strategies for removing constraints, and we obtained the best one to combine all groups. This variant performed the best with respect to error obtained and number of constraints used. Table 6.1 shows the means used in the prioritising of pathogens.

For each pathogen, information necessary to score the pathogens on the nine criteria was acquired from websites of the organizations such as (governmental) organisations that are concerned with animal or human health and welfare like WHO (World Health Organisation), OIE (World Organisation for Animal Health), ECDC (European Centre for Disease Prevention and Control), CDC (Centres for Disease Control and Prevention in the USA), RIVM (National Institute For Public Health and Environment in The Netherlands), HPA (Health Protection Agency in the UK)

Table 6.1: Means of B_i used for prioritising pathogens

B_1	0.4763
B_2	0.3235
B_3	0.3395
B_4	0.1808
B_5	0.2894
B_6	0.1839
B_7	0.5692
B_8	0.6084
B_9	0.2560

and VLA (Veterinary Laboratory Agency in the UK)[21]. In some instances, recent articles and books were used to acquire information. The information that was missing was obtained by more specific sources.

For some of the criteria, not enough or even no information was available. These criteria have an uncertainty; the exact score of the criteria is somewhere between the lowest and the highest score. For example, the costs involved with a human infection with a particular pathogen are not precisely known. However, around 5-15% of the patients will visit their physician and the duration of the illness varies between one and two weeks. In this case, the costs are estimated to be between 5 and 50 M Euro a week. The scores of the criteria were added to a database in which general information (taxonomy, disease, reservoir, transmission routes and distribution) of each pathogen was already gathered. For each criterion, the exact or estimated ranges of the scores were filled-in and information used from the source was added. The references were added to be able to retrace the information.

Monte Carlo simulation is a technique that involves using random numbers and probabilities to solve statistical problems. The goal of a Monte Carlo simulation is to determine how random variation (lack of information, or error) affects the sensitivity, performance, or reliability of the system that is being modelled. The data generated from the simulation can be represented as probability distributions, converted to error bars, reliability predictions, tolerance zones, or confidence intervals. Any given sample may fall anywhere within the range of the input distribution. The simulation can involve over 10000 evaluations[21]. This is the first time that a Monte Carlo simulation is used for prioritising of the emerging zoonotic pathogens in this project. The estimated range of the scores were included in the prioritising process by randomly choosing a number out of the range (10000 times) with help of the Monte Carlo simulation (software tools[21], using 10000 simulations). The output of the Monte Carlo simulation is multiplied by the weight for each criterion (which was received from the panel sessions with the policy makers). The scores are normalised to the maximum high threat that was set at 1 and the minimum threat that was set as 0. The scores of criteria 1, 4, 6, 7, and 9 were linear-transformed and the scores of criteria 2, 3, 5 and 8 were log-transformed.

The virus Crimean-Congo Haemorrhagic Fever Virus (CCHFV), an emerging pathogen, is discussed in more detail and serves as an illustrative example of the scoring process using the nine criteria. In short, CCHFV is not present in The Netherlands, but the chance of introduction is high because the agent is already present in other parts of Europe (criterion 1 = 50%). Arthropod borne zoonoses, like CCHFV, have an average rate of spread within the animal population (criterion 2 = 10 days). Economic damage for spread in animals is smaller than 10 M Euro per year as control can be performed at farm level (criterion 3 = 5 M. Euro). The probability of transmission of pathogen from a vertebrate animal to human is not found in any source. This criterion is therefore scored from the lowest (1:10000) till the highest (1:10) (criterion 4 = 1:10000 till 1:10). Humans who become infected with CCHFV acquire the virus from direct contact with blood or other infected tissues from livestock during this time, or they may become infected from a tick bite (criterion 5 = 10 days). CCHF is a hemorrhagic and a toxic syndrome disease (criterion 6 = 0.6) and has a case fatality rate of 30% (criterion 7= 50%). According to the decision rules, the costs of hospital admission, which is required with CCHFV, infections are high (criterion 8 = 500 M. Euro). In the perception criterion 4 out of 6 risk attributes appear to be valid (criterion 9 = 4). After normalisation, weighing and aggregation of the scores of all criteria, CCHFV ranked 7th on the preliminary prioritised list of emerging zoonoses.

The results of the priority setting process are shown in Figure 6.1. For comparison, the scores of two additional scenarios are also included, i.e. high and low threat, respectively. These represent (hypothetical) zoonoses that would have all variables set to the maximum (1.00) or minimal (0.00) threat level. Scores for all 92 zoonotic pathogens have been normalised to this range.

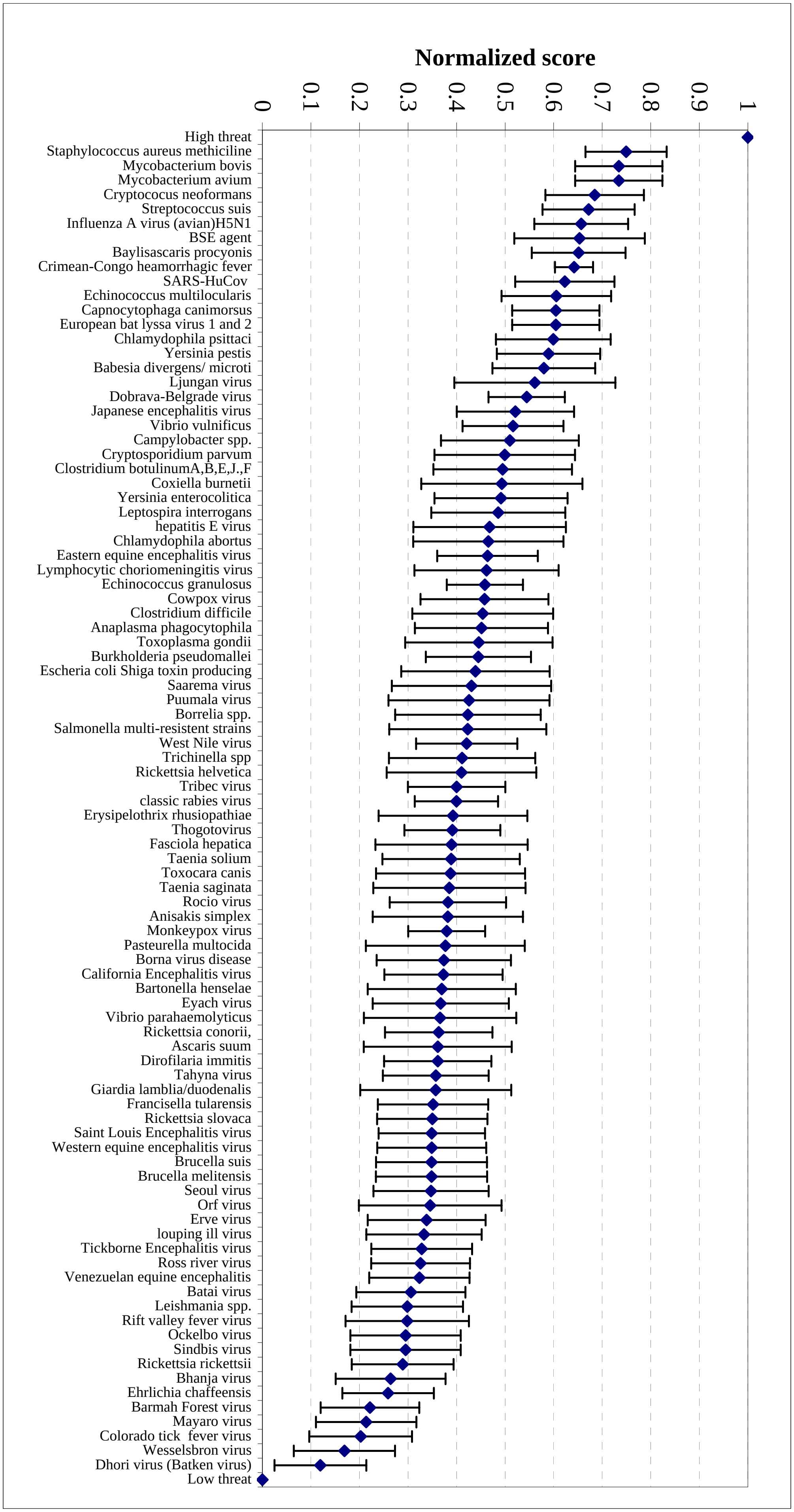


Figure 6.1: Final results of prioritising of Zoonoses

The uncertainty of some of the information resulted in large confidence intervals for the normalized score of almost all zoonotic pathogens as shown by the error bars in Figure 6.1. This overall uncertainty was mainly due to the fact that hardly any information was available to score criterion four. As a result for nearly all zoonotic pathogens, criterion four was scored as an interval between the lowest and highest possible score (from 1:10000 till 1:10), which gives rise to a high uncertainty. However, for a few new discovered pathogens, information on any of the criteria was hard to find, which left us with very high uncertainty in the scores. Criterion four was changed from 'number of infected animals needed to infect one person' into 'transmission route from animal to human' and information has been processed for this new criterion. The score for the new criterion four is calculated differently. The score is now log-transformed (instead of linear-transformed), and a high value for the scores for this new criterion indicate now low thread (same as for criteria 2 and 5). From now on all results include the modified criterion four.

With Monte Carlo simulation the 43 variable weight factors were included in the scoring process (instead of the mean weight scores[21]), which resulted in an additional uncertainty (see Figure 6.1). To obtain normalised scores including the variable weight factors, new estimations were made. For each pathogen the score for each criterion is multiplied by the sample of weights which is the unique combination for each criterion linked to the number of occurrences (with use of a software tool[21]). For more details about the unique combinations and the number of occurrences see [21].

6.2 Conclusions and future work

Conclusions The world that we are living in, is changing constantly. Human mankind is also evolving constantly, while the time passes by. Unfortunately for us, we are not the only organisms that are evolving. During the history, man had been the victim of different influential factors, which caused sometimes severe consequences, other times less severe consequences. Diseases represent one of the category that put human mankind under danger.

There exist, nowadays, many types of diseases, some of them lethal, some of them less dangerous. Unfortunately not even the modern medicine research is not able to provide medicines and treatments for all existing diseases. Thus it is very important for us to give a lot of interest in rather prevention, than treatment of diseases.

Diseases, in general, are provoked by viruses, or pathogens. One category of diseases is represented by the ones that come from the animal reign. These pathogens, which are transmitted from animals to humans are called *zoonoses*. The National

Institute for Public Health and the Environment (RIVM) has been allocating for many years, a lot of resources in this direction. A first step was to identify the existing zoonoses, all over the world, and more important the ones from Europe and The Netherlands. At the moment there are many institutions and organizations which are constantly updating this list of pathogens.

Once the pathogens have been identified, a natural step would be to develop a method such that their severity can be quantified. This way the prevention of infecting with these pathogens would be easier.

Within this thesis, RIVM in collaboration with Technical University of Delft, has performed a research, having as result, building a model that can be used in prioritising the existing pathogens, and, moreover, that can be used in prioritising the new (emerging) pathogens that may occur. The method used for ranking the zoonotic pathogens has many advantages. The used quantitative method is transparent, repeatable and more objective. The normalised scores for each zoonotic pathogen can help in the effective policy making, control and surveillance. Surveillance and control systems can be improved or developed for those pathogens having the highest normalised score. And also, human and animal medication and vaccines, for those pathogens, can be improved or developed. Making decisions based on the normalised score would be better than using the ranking. This is because the difference between normalised score of the disease ranked number 20 and the one ranked number 30 is very small. Therefore, it would be better to focus on the zoonotic pathogens above a certain normalised score instead of focusing on, for example, the top 20. The methods used for quantifying the information corresponding to each attribute need some improvement and also the weight values need more attention. After improving the method, the final normalised scores can be used for policy making. However, the model must than be kept up to date, newly available or updated information about the pathogens have to be included in the model. Only then the model is reliable and can be used.

Future work One proposal for future work is that another eilicitation procedure should be organised. In further research more people from different backgrounds (e.g. students, doctors or civilians) can take part in these sessions, which may give a more universal outcome. It is interesting to include in the model which criterion or criteria the Dutch citizens find more or less important. Next it would be interesting to find a statistical test to check whether the assessments of the two types of experts differ or not. During the elicitation procedure, we suggest that everything should be checked very well, to avoid any mistakes in formulating the scenarios, for example (as we did with scenario 12). It is worth investigating if using a different number of groups, and maybe less than 7 scenarios in each group would make a difference. We also suggest that scenarios should be constructed such that it would be easier to differentiate them (i.e. if two scenarios have attributes with similar values, for instance, low, then it is difficult to choose between these two scenarios). In the same time, it should be avoided the situation in which there are scenarios with high values for all attributes.

Another suggestion is related to the software which we used. A favourable case would be that the software allows using more than 100,000 samples for big number of constraints which we used.

Transformations of the attributes values consist another research direction. As we have seen in the previous chapters, transformations do influence the accuracy of results. We believe that they should be chosen such that the values of attributes, after transformation, have a uniform spread. This would be very advantageous for PI program, as samples would be distributed uniformly.

As mentioned in the end of the previous section, the decision under uncertainty about the prioritisation of the pathogens should be further investigated.
There may exist the possibility of building a integrated system available on the Internet, where information about the attributes can be updated in real time, by anybody who has knowledge and access the web page. The program will automatically include the new information and generate the updated list with the prioritised pathogens. For the moment however, the list provided contains the latest information.

Appendices

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Appendix A

Criteria definitions

This chapter is published with the permission of the authors. We briefly present the nine criteria used in this project. For the full description of criteria along with the decision rules on which the estimation of criteria relies on please refer to [21].

A.1 Probability of introduction of a pathogen in the Netherlands

Significance: Probability of introduction of pathogen, percentage [%]

Definition: This criterion describes the probability that a zoonotic pathogen will be introduced in the Netherlands in the following year. This probability depends on the introduction of an infected entity. Moreover, it depends on the prevalence of an infection in such an entity and the intensity in which those entities enter the Netherlands. The result depends on the type of entity in question.

Point estimates: The probability of introduction of a pathogen will be estimated using the decision rules described below appointing it to one of five probability intervals:

- 0%;
- $<$ 1%, point estimate 0,5%;
- 1-9, point estimate 5%;
- 10-99%, point estimate 50%;
- 100%.

The explanation and translation between the probability intervals is described in [21]. We just present the possible values of each coefficient.

A.2 Rate of transmission of pathogen in animal reservoirs

Significance: Time between new infections in animals [days]

Definition: This criterion describes the rate by which an infection spreads in a sensitive animal population. This rate depends on many factors including the infectiousness of the disease and duration of the infectious period. The rate is expressed as the time that passes between a primary and secondary infection. The estimate is based on the level of section of the pathogen or its transmission route.

Point estimates: The rate of spread of a pathogen will be estimated using the decision rules described below appointing it to one of four intervals:

- null (in the mathematical model we use 10.000 days);
- 30 days;
- 10 days;
- 3 days.

A.3 Economic damage (animal)

Significance: Criterion: Costs ([MEuro/ year])

Definition: This criterion describes the costs for the Dutch society given the discovery of an infection in the Dutch animal reservoir, and transmission between animals has occurred. The costs relate to the agricultural sector (production animal farms, suppliers, slaughter houses, and food industry) and the government. The costs include costs associated with control of the disease (culling, vaccination, compensation etc) and the costs of lack of occupancy of stables, loss of breeding animals, lost returns and the damage to the market though the loss of a share in the market for long period of time and loss in the tourist industry. These costs depend on preceding criteria, because a zoonotic agent that also causes animal diseases and spreads quickly will demand more intense and expensive control measures.

Point estimates: The costs of the emerging pathogen will be estimated using the decision rules described below appointing it to one of four intervals:

- < 10 MEuro per year, point estimate 5 M.Euro per year;
- 10 - 100 MEuro per year, point estimate 50 M.Euro per year;
- 100 - 1,000 MEuro per year, point estimate 500 M.Euro per year;
- 1,000 MEuro per year, point estimate 5000 M.Euro per year.

A.4 Probability of transmission of pathogen from animal to human

Significance: The number of human cases due to one infected animal

Definition: This criterion describes the probability that an infection is transmitted from a vertebrate animal to a human, given that infected animals are present. For example, with a probability of 1:1000, one human gets ill for every 1000 infected animals. For this, the current hygienic practises, level of contact between human and animals and the level of infectiousness for humans are taken into account. The probability on transmission is the result of a complex relationship between the different factors. This phenomenon is difficult to describe with simple decision rules. This criterion is scored on ground of observations/estimates in countries where the infection is endemic.

Point estimates:

- 1:10,000;
- 1:1,000;
- 1:100;
- 1:10.

A.5 Rate of spread of pathogen within human population

Significance: Time between new infections in humans ([days])

Definition: This criterion describes the rate in which an infection spreads in a sensitive human population. This rate depends on many factors including the infectiousness of the disease and duration of the infective period. The rate is expressed in the time that passes between a primary and secondary infection. The estimate is based on the level of section of the pathogen or its transmission route.

Point estimates: The rate of spread of a pathogen will be estimated using the decision rules described below appointing it to one of following four intervals):

- null (in mathematical model we use 10.000 days);
- 30 days;
- 10 days;
- 3 days.

A.6 Morbidity (human) - gravity of illness

Significance: Loss of health related quality of life

Definition: This criterion reflects the effect of the disease on the health related quality of life and it is expressed in the number of years in which the disease appears. The value of the criterion is anchored between 0 (full health) and 1 (worst possible health state) and depends on both the severity and the duration of the disease.

Point estimates: Four intervals for the morbidity are used:

- disability weight ¡ 0.03; point estimate 0.02;
- 0.03 ¡ disability weight ¡ 0.1; point estimate 0.06;
- 0.1 ¡ disability weight ¡ 0.3; point estimate 0.2;
- disability weight ¿ 0.3; point estimate 0.6.

A.7 Mortality (human) - chances of dying

Significance: Case fatality rate (percentage [%])

Definition: This criterion describes the case-fatality rate of the illness, which depends on the nature of the infection and the health status of the infected person.

Point estimates: Four intervals for the mortality are used:

- 0%;
- $<$ 1%, point estimate 0,5%;
- 1-10%, point estimate 5%;
- 10-100%, point estimate 50%;
- 100%.

A.8 Economic damage (human)

Significance: Costs ([MEuro/ year])

Definition: This criterion describes the costs for the Dutch society involving with the presence of the infection within the Dutch human population. The costs relate to the health care sector (physicians, hospitals, drugs etc) and not-health care costs including lost of productivity due to illness or untimely death, but also costs to control an epidemic like closing schools or industries, closing airports etc. These costs depend on preceding criteria. A zoonotic agent that spreads quickly between humans will demand more intense and expensive control measures. Similarly, a more serious illness will result in more costs than less severe ones.

Point estimates: The costs of the emerging pathogen will be estimated using the decision rules described below appointing it to one of following four intervals):

- < 10 M.Euro per year, point estimate 5 M.Euro per year;
- 10 - 100 M.Euro per year, point estimate 50 M.Euro per year;
- 100 - 1000 M.Euro per year, point estimate 500 M.Euro per year;
- 1000 M.Euro per year, point estimate 5000 M.Euro per year.

A.9 Perception

Significance: Number of applicable risk attributes

Definition: This criterion described the level in which subjective risk attributes influence the perception of the Dutch society.

Point estimates:

- Involuntary exposure;
- Unknown or new and unnatural risk;
- Hidden, postponed and irreversible damage;
- Possibility of identification with victims (e.g. children or pregnant women).

Appendix B

Scenarios list

ORDER	1	2	3	4	5	6	7	8	9	10	11
1	JR	WL	ZC	ZC	WL	ZC	VG	VG	JR	ZC	ZC
2	VG	PX	VG	JR	VG	PX	ZC	ZC	VG	PX	VG
3	PX	ZC	PX	VG	ZC	JR	JR	JR	ZC	JR	PX
4	QJ	VG	GF	WL	JR	GF	GF	WL	WL	VG	JR
5	ZC	JR	JR	PX	PX	VG	PX	PX	PX	WL	WL
6	GF	QJ	WL	QJ	QJ	QJ	WL	QJ	QJ	GF	GF
7	WL	GF	QJ	GF	GF	WL	QJ	GF	GF	QJ	QJ
1	LQ	SK	AS	FZ	LQ	FZ	NA	FZ	OS	LQ	SK
2	NA	LQ	OS	LQ	SK	EV	SK	SK	SK	NA	NA
3	SK	FZ	FZ	SK	FZ	NA	LQ	OS	FZ	FZ	LQ
4	FZ	OS	EV	OS	OS	OS	FZ	LQ	AS	OS	AS
5	EV	NA	NA	NA	NA	LQ	OS	NA	NA	SK	OS
6	OS	EV	LQ	EV	AS	SK	EV	EV	LQ	EV	FZ
7	AS	AS	SK	AS	EV	AS	AS	AS	EV	AS	EV
1	RZ	RZ	RZ	RZ	RZ	RZ	RZ	RZ	RZ	KC	RZ
2	GU	BE	YU	GU	CI	GU	BE	KC	KC	GU	YU
3	BE	GU	KC	YU	DP	DP	KC	DP	CI	DP	GU
4	KC	KC	DP	CI	KC	KC	DP	YU	DP	CI	BE
5	CI	DP	GU	KC	BE	YU	CI	CI	YU	BE	DP
6	DP	CI	CI	DP	GU	CI	GU	GU	GU	YU	KC
7	YU	YU	BE	BE	YU	BE	YU	BE	BE	RZ	CI
1	FV	BY	MF	FV	FV	EW	FV	FV	FV	BY	BY
2	BY	FV	EW	EW	JD	FV	BY	JD	JD	FV	FV
3	AG	MF	BY	AG	BY	AG	AG	BY	MF	MF	AG
4	MF	AG	AG	KD	AG	KD	JD	MF	AG	AG	EW
5	JD	JD	KD	JD	MF	BY	MF	AG	KD	JD	MF
6	EW	EW	FV	BY	EW	JD	EW	EW	BY	EW	JD
7	KD	KD	JD	MF	KD	MF	KD	KD	EW	KD	KD
1	PT	PT	LO	TB	DI	PT	DI	RY	DI	PT	LO
2	XN	DI	PT	XN	UB	RY	PT	DI	UB	DI	XN
3	TB	UB	XN	UB	TB	DI	UB	UB	PT	XN	UB
4	DI	TB	DI	RY	XN	UB	TB	TB	XN	UB	PT
5	UB	XN	TB	LO	PT	LO	RY	PT	TB	TB	TB
6	RY	RY	UB	DI	RY	TB	XN	LO	RY	RY	RY
7	LO	LO	RY	PT	LO	XN	LO	XN	LO	LO	DI
1	IA	IA	CM	CM	NW	QX	IA	NW	QX	IA	MJ
2	MJ	CM	QX	MJ	CM	CM	QX	MJ	NW	OE	OE
3	QX	OE	OE	IT	QX	IA	CM	CM	IA	QX	IT
4	OE	QX	MJ	OE	IA	NW	NW	IT	OE	MJ	QX
5	NW	IT	IA	NW	OE	OE	IT	IA	CM	NW	IA
6	IT	MJ	IT	IA	IT	IT	OE	QX	IT	IT	NW
7	CM	NW	NW	QX	MJ	MJ	MJ	OE	MJ	CM	CM

1		ATTRIBUTES																	
Scenario		1		2		3		4		5		6		7		8		9	
		Intro Kans		Vrspr.Dier		$Dier		OvrdMens		Vrspr.Mens		ZkErnst		KansDood		$Mens		Perceptie	
		IK(1-5)		VD(1-4)		ESD(1-4)		Ok(1-4)		VM(1-4)		ZE(1-4)		OVK(1-5)		ESM(1-4)		P(1-5)	
1	QJ	4	50	3	10	3	500	3	100	3	10	3	0.2	4	50	3	500	3	6
2	VG	3	5	2	30	3	500	4	10	4	3	3	0.2	3	5	2	50	4	8
3	GF	4	50	3	10	1	5	3	100	4	3	3	0.2	4	50	1	5	4	8
4	JR	4	50	4	3	4	5000	4	10	3	10	2	0.06	3	5	2	50	1	2
5	ZC	4	50	1	niet	3	500	2	1000	4	3	4	0.6	3	5	3	500	2	4
6	WL	2	0.5	2	30	4	5000	3	100	3	10	3	0.2	4	50	2	50	4	8
7	PX	3	5	3	10	2	50	3	100	4	3	3	0.2	4	50	2	50	3	6

2		ATTRIBUTES																	
Scenario		1		2		3		4		5		6		7		8		9	
		Intro Kans		Vrspr.Dier		$Dier		OvrdMens		Vrspr.Mens		ZkErnst		KansDood		$Mens		Perceptie	
		IK(1-5)		VD(1-4)		ESD(1-4)		Ok(1-4)		VM(1-4)		ZE(1-4)		OVK(1-5)		ESM(1-4)		P(1-5)	
6	SK	2	0.5	2	30	4	5000	3	100	3	10	3	0.2	4	50	2	50	4	8
7	NA	3	5	3	10	2	50	3	100	4	3	3	0.2	4	50	2	50	3	6
8	FZ	4	50	4	3	2	50	3	100	3	10	4	0.6	2	0.5	3	500	1	2
9	EV	4	50	4	3	1	5	2	1000	4	3	2	0.06	4	50	3	500	2	4
10	AS	4	50	2	30	3	500	3	100	1	niet	2	0.06	4	50	3	500	4	8
11	OS	4	50	3	10	3	500	2	1000	4	3	1	0.02	3	5	2	50	4	8
12	LQ	2	0.5	2	30	4	5000	4	10	2	30	4	0.6	4	50	3	500	1	2

3		ATTRIBUTES																	
Scenario		1		2		3		4		5		6		7		8		9	
		Intro Kans		Vrspr.Dier		$Dier		OvrdMens		Vrspr.Mens		ZkErnst		KansDood		$Mens		Perceptie	
		IK(1-5)		VD(1-4)		ESD(1-4)		Ok(1-4)		VM(1-4)		ZE(1-4)		OVK(1-5)		ESM(1-4)		P(1-5)	
11	CI	4	50	3	10	3	500	2	1000	4	3	1	0.02	3	5	2	50	4	8
12	RZ	1	0	2	30	4	5000	4	10	2	30	4	0.6	4	50	3	500	1	2
13	YU	4	50	1	niet	1	5	3	100	4	3	3	0.2	3	5	4	5000	3	6
14	DP	4	50	2	30	3	500	4	1000	2	30	2	0.06	3	5	2	50	4	8
15	KC	4	50	3	10	3	500	4	1000	2	30	3	0.2	2	0.5	2	50	3	6
16	BE	2	0.5	4	3	1	5	1	10	3	10	4	0.6	4	50	2	50	4	8
17	GU	3	5	2	10	2	50	3	100	2	30	3	0.2	4	50	2	50	3	6

4 Scenario		1 Intro Kans IK(1-5)		2 Vrspr.Dier VD(1-4)		3 $Dier ESD(1-4)		4 OvrdMens Ok(1-4)		5 Vrspr.Mens VM(1-4)		6 ZkErnst ZE(1-4)		7 KansDood OVK(1-5)		8 $Mens ESM(1-4)		9 Perceptie P(1-5)	
16	BY	**2**	0.5	**4**	3	**1**	5	**1**	10000	**3**	10	**4**	0.6	**4**	50	**2**	50	**4**	8
17	AG	**3**	5	**2**	30	**2**	50	**3**	100	**2**	30	**3**	0.2	**4**	50	**2**	50	**3**	6
18	JD	**3**	5	**4**	3	**4**	5000	**2**	1000	**2**	30	**1**	0.02	**3**	5	**4**	5000	**2**	4
19	MF	**3**	5	**4**	3	**2**	50	**3**	100	**1**	niet	**3**	0.2	**4**	50	**1**	5	**4**	8
20	KD	**4**	50	**2**	30	**2**	50	**3**	100	**2**	30	**2**	0.06	**4**	50	**2**	50	**4**	8
21	EW	**4**	50	**1**	niet	**2**	50	**2**	1000	**2**	30	**4**	0.6	**4**	50	**1**	5	**3**	6
22	FV	**2**	0.5	**4**	3	**2**	50	**3**	100	**3**	10	**2**	0.06	**2**	0.5	**3**	500	**3**	6

5 Scenario		1 Intro Kans IK(1-5)		2 Vrspr.Dier VD(1-4)		3 $Dier ESD(1-4)		4 OvrdMens Ok(1-4)		5 Vrspr.Mens VM(1-4)		6 ZkErnst ZE(1-4)		7 KansDood OVK(1-5)		8 $Mens ESM(1-4)		9 Perceptie P(1-5)	
21	LO	**4**	50	**1**	niet	**2**	50	**2**	1000	**2**	30	**4**	0.6	**4**	50	**1**	5	**3**	6
22	DI	**2**	0.5	**4**	3	**2**	50	**3**	100	**3**	10	**2**	0.06	**2**	0.5	**3**	500	**3**	6
23	RY	**3**	5	**2**	30	**2**	50	**2**	1000	**2**	30	**4**	0.6	**1**	0	**4**	5000	**4**	8
24	UB	**2**	0.5	**2**	30	**3**	500	**3**	100	**4**	3	**2**	0.06	**2**	0.5	**3**	500	**3**	6
25	PT	**2**	0.5	**3**	10	**3**	500	**3**	100	**4**	3	**3**	0.2	**2**	0.5	**2**	50	**2**	4
26	XN	**2**	0.5	**1**	niet	**4**	5000	**3**	100	**3**	10	**1**	0.02	**4**	50	**4**	5000	**1**	2
27	TB	**3**	5	**2**	30	**1**	5	**2**	1000	**4**	3	**3**	0.2	**2**	0.5	**4**	5000	**1**	2

6 Scenario		1 Intro Kans IK(1-5)		2 Vrspr.Dier VD(1-4)		3 $Dier ESD(1-4)		4 OvrdMens Ok(1-4)		5 Vrspr.Mens VM(1-4)		6 ZkErnst ZE(1-4)		7 KansDood OVK(1-5)		8 $Mens ESM(1-4)		9 Perceptie P(1-5)	
24	QX	**2**	0.5	**2**	30	**3**	500	**3**	100	**4**	3	**2**	0.06	**2**	0.5	**3**	500	**3**	6
25	IA	**2**	0.5	**3**	10	**3**	500	**3**	100	**4**	3	**3**	0.2	**2**	0.5	**2**	50	**2**	4
26	MJ	**2**	0.5	**1**	niet	**4**	5000	**3**	100	**3**	10	**1**	0.02	**4**	50	**4**	5000	**1**	2
27	NW	**3**	5	**2**	30	**1**	5	**2**	1000	**4**	3	**3**	0.2	**2**	0.5	**4**	5000	**1**	2
28	OE	**2**	0.5	**3**	10	**3**	500	**2**	1000	**2**	30	**2**	0.06	**3**	5	**1**	5	**4**	8
29	IT	**3**	5	**2**	30	**4**	5000	**1**	10000	**1**	niet	**1**	0.02	**3**	5	**2**	50	**4**	8
30	CM	**4**	50	**4**	3	**2**	50	**1**	10000	**1**	niet	**4**	0.6	**2**	0.5	**2**	50	**2**	4

GROUP 1 - samples validation, errors of samples validation and errors od fitting

VARIANT I

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
QJ	1		0	0	0		0.0012	0	1.4E-06		0.0098	0	9.6E-05	1	0.9988	0	1.4E-06		0.0513	0	0.00263	6	5.9495	0	0.00255	4	3.9895	0	0.00011
VG	2	2	2.0148	0	0.00022	5	4.9623	0	0.00142	1	0.9816	0	0.00034	2	1.9757	0	0.00059	1	1.0154	0	0.00024		0.0241	0	0.00058		0.0261	0	0.00068
GF	3		0.0099	0	9.8E-05		0.01	0	0.0001		0.0149	0	0.00022	3	2.9810	0	0.00036		0.0049	0	2.4E-05	3	3.013	0	0.00017	5	4.9663	0	0.00114
JR	4	2	1.9983	0	2.9E-06	1	0.9837	0	0.00027	4	4.0094	0	8.8E-05	2	2.0040	0	1.6E-05	2	1.9721	0	0.00078		0.0181	0	0.00033		0.0144	0	0.00021
ZC	5	5	4.9875	0	0.00016	2	2.0217	0	0.00047	3	2.9882	0	0.00014		0.0287	0	0.00082	1	0.966	0	0.00116		0.004	0	1.6E-05		0.004	0	1.6E-05
WL	6	2	1.9848	0	0.00023		0.0183	0	0.00033		0.0098	0	9.6E-05	3	3.0056	0	3.1E-05	2	1.9906	0	8.8E-05	2	1.9911	0	7.9E-05	2	1.9998	0	4E-08
PX	7		0.0047	0	2.2E-05	3	3.0028	0	7.8E-06	3	2.9864	0	0.00018		0.0062	0	3.8E-05	5	4.9997	0	9E-08		0.0002	0	4E-08		0	0	0

		constraints
sum error validation	0.0000	34
RMSE validation	0.0000	
sum error fitting	0.0171	
RMSE fitting	0.0225	

VARIANT II

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
QJ	1		0	0	0		0.0001	1.2E-06	1E-08		0.0032	4.4E-05	1E-05	1	1.0155	0.000279	0.00024		0.036	0.00023	0.0013	6	5.933	0.00027	0.00449	4	4.0122	0.00052	0.00015
VG	2	2	2.0029	0.00014	8.4E-06	5	4.9662	1.5E-05	0.00114	1	0.9947	0.00017	2.8E-05	2	2.0257	0.0025	0.00066	1	0.9679	0.00226	0.00103		0.021	9.6E-06	0.00044		0.0217	1.9E-05	0.00047
GF	3		0.0294	0.00038	0.00086		0.0125	6.3E-06	0.00016		0.0239	8.1E-05	0.00057	3	2.9039	0.005944	0.00924		0.0266	0.00047	0.00071	3	3.0155	6.2E-06	0.00024	5	4.9882	0.00048	0.00014
JR	4	2	1.9697	0.00082	0.00092	1	0.9986	0.00022	2E-06	4	3.9801	0.00086	0.0004	2	2.0622	0.003387	0.00387	2	1.9803	6.7E-05	0.00039		0.0057	0.00015	3.2E-05		0.0034	0.00012	1.2E-05
ZC	5	5	4.9973	9.6E-05	7.3E-06	2	2.0104	0.00013	0.00011	3	2.9487	0.00156	0.00263		0.0698	0.001689	0.00487	1	0.9736	5.8E-05	0.0007		0.0002	1.4E-05	4E-08		0	1.6E-05	0
WL	6	2	2.0006	0.00025	3.6E-07		0.0543	0.0013	0.00295		0.0455	0.00127	0.00207	3	2.9005	0.011046	0.0099	2	2	8.8E-05	0	2	2.0246	0.00112	0.00061	2	1.9745	0.00064	0.00065
PX	7		0.0002	2E-05	4E-08	3	2.9579	0.00202	0.00177	3	3.0039	0.00031	1.5E-05		0.0224	0.000262	0.0005	5	5.0156	0.00025	0.00024		0	4E-08	0		0	0	0

		constraints
sum error validation	0.0416	29
RMSE validation	0.0379	
sum error fitting	0.0545	
RMSE fitting	0.0434	

VARIANT III

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
QJ	1		0	0	0		0	1.4E-06	0		0.0449	0.00123	0.00202	1	0.2600	0.545825	0.5476		0.7488	0.48651	0.5607	6	5.9628	0.00018	0.00138	4	3.9835	3.6E-05	0.00027
VG	2	2	2.0114	1.2E-05	0.00013	5	4.9739	0.00013	0.00068	1	1.1749	0.03736	0.03059	2	1.2535	0.521573	0.55726	1	1.5567	0.29301	0.30991		0.0124	0.00014	0.00015		0.0173	7.7E-05	0.0003
GF	3		0.0068	9.6E-06	4.6E-05		0.0044	3.1E-05	1.9E-05		0.4734	0.21022	0.22411	3	1.1292	3.429163	3.49989		1.3773	1.88348	1.89696	3	3.0142	1.4E-06	0.0002	5	4.9947	0.00081	2.8E-05
JR	4	2	1.9764	0.00048	0.00056	1	0.9903	4.4E-05	9.4E-05	4	3.2216	0.62063	0.60591	2	2.3068	0.091688	0.09413	2	2.4922	0.2705	0.24226		0.009	8.3E-05	8.1E-05		0.0037	0.00011	1.4E-05
ZC	5	5	4.9772	0.00011	0.00052	2	1.9978	0.00057	4.8E-06	3	1.3852	2.56961	2.60758		1.1274	1.207142	1.27103	1	1.5008	0.28601	0.2508		0.0068	7.8E-06	4.6E-05		0.0047	4.9E-07	2.2E-05
WL	6	2	2.0078	0.00053	6.1E-05		0.0301	0.00014	0.00091		1.9632	3.81577	3.85415	3	1.8049	1.44168	1.42826	2	1.2043	0.61827	0.63314	2	1.9936	6.2E-06	4.1E-05	2	1.9961	1.4E-05	1.5E-05
PX	7		0.0205	0.00025	0.00042	3	3.0034	3.6E-07	1.2E-05	3	2.7368	0.0623	0.06927		3.1183	9.685166	9.72379	5	2.1199	8.29325	8.29498		0.0011	8.1E-07	1.2E-06		0	0	0

		constraints
sum error validation	36.3742	29
RMSE validation	1.1199	
sum error fitting	36.7104	
RMSE fitting	1.1251	

VARIANT IV

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
QJ	1		0	0	0		0.0032	4E-06	1E-05		0.0944	0.00716	0.00891	1	0.6160	0.146536	0.14746		1.7575	2.91112	3.08881	6	4.5747	1.89008	2.03148	4	3.9543	0.00124	0.00209
VG	2	2	2.0084	4.1E-05	7.1E-05	5	1.1202	14.7617	15.0528	1	2.2706	1.66152	1.61442	2	2.1894	0.045668	0.03587	1	1.9478	0.86937	0.89832		1.4333	1.98584	2.05435		0.0302	1.7E-05	0.00091
GF	3		0.0019	6.4E-05	3.6E-06		0.1058	0.00918	0.01119		0.2541	0.05722	0.06457	3	0.8493	4.544145	4.62551		1.6896	2.83821	2.85475	3	3.1044	0.00835	0.0109	5	4.9949	0.00082	2.6E-05
JR	4	2	1.9889	8.8E-05	0.00012	1	3.5514	6.59308	6.50964	4	1.2883	7.40439	7.35332	2	1.6599	0.118405	0.11567	2	1.7287	0.05924	0.0736		0.7612	0.5522	0.57943		0.0217	5.3E-05	0.00047
ZC	5	5	4.9849	6.8E-06	0.00023	2	1.8584	0.02667	0.02005	3	1.2679	2.95943	3.00017		1.0607	1.065024	1.12508	1	1.2514	0.08145	0.0632		0.5695	0.31979	0.32433		0.0073	1.1E-05	5.3E-05
WL	6	2	1.9842	3.6E-07	0.00025		1.0436	1.05124	1.0891		1.7478	3.02064	3.0548	3	2.0636	0.887364	0.87684	2	1.6974	0.08597	0.09157	2	0.4718	2.30827	2.3354	2	1.9917	6.6E-05	6.9E-05
PX	7		0.0318	0.00073	0.00101	3	3.3174	0.09897	0.10074	3	4.0769	1.18919	1.15971		2.5610	6.527003	6.55872	5	0.9277	16.5812	16.5836		0.0851	0.00721	0.00724		0	0	0

		constraints
sum error validation	82.6760	29
RMSE validation	1.6885	
sum error fitting	83.5270	
RMSE fitting	1.6971	

VARIANT V

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
QJ	1	0	0.0018	3.2E-06	3.2E-06	0	0.0098	7.4E-05	9.6E-05	0	0.0218	0.00014	0.00048	1	0.3368	0.438244	0.43983	0	0.662	0.37295	0.43824	6	5.9737	0.00059	0.00069	4	3.9942	2.2E-05	3.4E-05
VG	2	2	0.8014	1.47234	1.43664	5	4.9911	0.00083	7.9E-05	1	0.2578	0.52389	0.55086	2	2.3549	0.143793	0.12595	1	1.4257	0.16835	0.18122		0.4129	0.15117	0.17049		0.7561	0.5329	0.57169
GF	3		0.0019	6.4E-05	3.6E-06		0.0018	6.7E-05	3.2E-06		0.001	0.00019	1E-06	3	2.9832	4.84E-06	0.00028		0.0207	0.00025	0.00043	3	2.9872	0.00067	0.00016	5	5.0043	0.00144	1.8E-05
JR	4	2	3.5583	2.4336	2.4283	1	0.8199	0.02683	0.03244	4	4.0151	3.2E-05	0.00023	2	1.0930	0.829921	0.82265	2	0.729	1.5453	1.61544		0.1475	0.01674	0.02176		0.6372	0.38788	0.40602
ZC	5	5	5.0106	0.00053	0.00011	2	0.8699	1.32664	1.27713	3	2.9808	5.5E-05	0.00037		1.2167	1.411344	1.48036	1	0.628	0.11424	0.13838		0.0903	0.00745	0.00815		0.2037	0.03988	0.04149
WL	6	2	1.6158	0.13616	0.14761		1.3111	1.67133	1.71898		0.7509	0.54923	0.56385	3	2.9970	7.4E-05	9E-06	2	2.5333	0.29452	0.28441	2	1.3877	0.36409	0.37491	2	0.4044	2.5453	2.54594
PX	7		0.0103	3.1E-05	0.00011	3	2.9964	4.1E-05	1.3E-05	3	2.9726	0.00019	0.00075		0.0185	0.000151	0.00034	5	5.0014	2.9E-06	2E-06		0.0008	3.6E-07	6.4E-07		0	0	0

		constraints
sum error validation	16.6975	29
RMSE validation	0.7588	
sum error fitting	16.9476	
RMSE fitting	0.7645	

GROUP 2 - samples validation, errors of samples validation and errors od fitting

VARIANT I

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
SK	6	2	1.9059	0	0.00885	4	4.0464	0	0.00215	2	1.8715	0	0.01651	0	0.0605	0	0.00366	1	0.8468	0	0.02347	1	1.1464	0	0.02143	1	1.1225	0	0.01501
NA	7	1	1.1697	0	0.0288	3	2.7366	0	0.06938	1	0.8804	0	0.0143	0	0.4342	0	0.18853	6	5.779	0	0.04884	0	0.0001	0	1E-08	0	0	0	0
FZ	8	3	2.9187	0	0.00661	0	0	0	0	5	4.8386	0	0.02605	2	1.8168	0	0.03356	0	0.3682	0	0.13557	1	0.9668	0	0.0011	0	0.091	0	0.00828
EV	9	0	0.268	0	0.07182	1	0.8712	0	0.01659	0	0.1118	0	0.0125	1	0.9197	0	0.00645	1	1.0177	0	0.00031	5	4.8181	0	0.03309	3	2.9935	0	4.2E-05
AS	10	1	1.0276	0	0.00076	0	0.4194	0	0.1759	0	0.2111	0	0.04456	2	1.8166	0	0.03364	0	0.0001	0	1E-08	1	1.1578	0	0.0249	7	6.3674	0	0.40018
OS	11	1	0.914	0	0.0074	1	1.0035	0	1.2E-05	1	0.9231	0	0.00591	5	4.9357	0	0.00413	2	2.0093	0	8.6E-05	1	0.9044	0	0.00914	0	0.31	0	0.0961
LQ	12	3	2.7961	0	0.04158	2	1.9229	0	0.00594	2	2.1635	0	0.02673	1	1.0165	0	0.00027	1	0.979	0	0.00044	2	2.0064	0	4.1E-05	0	0.1156	0	0.01336

sum error validation	0.0000	constraints
RMSE validation	0.0000	34
sum error fitting	1.6840	
RMSE fitting	0.2226	

VARIANT II

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
SK	6	2	1.7595	0.021433	0.05784	4	4.0912	0.002007	0.00832	2	1.7219	0.02238	0.07734		0.5778	0.267599	0.33385	1	0.8	0.00219	0.04	1	1.1208	0.00066	0.01459	1	0.9287	0.03756	0.00508
NA	7	1	0.9833	0.034745	0.00028	3	2.6356	0.010201	0.13279	1	0.8007	0.006352	0.03972		0.6698	0.055507	0.44863	6	5.9106	0.01732	0.00799		0	1E-08	0		0	0	0
FZ	8	3	2.9827	0.004096	0.0003		0	0	0	5	5.2307	0.153742	0.05322	2	1.5581	0.066926	0.19528		0.2746	0.00876	0.07541	1	0.9536	0.00017	0.00215		0.0003	0.00823	9E-08
EV	9		0	0.071824	0	1	0.9975	0.0159517	6.2E-06		0.0007	0.012343	4.9E-07	1	1.8285	0.825917	0.68641	1	1.0099	6.1E-05	9.8E-05	5	4.4587	0.12917	0.29301	3	2.7047	0.08341	0.0872
AS	10	1	1.0047	0.000524	2.2E-05		0.059	0.1298882	0.00348		0.0006	0.04431	3.6E-07	2	2.5877	0.594595	0.34539		0	1E-08	0	1	1.1378	0.0004	0.01899	7	6.2102	0.02471	0.62378
OS	11	1	1.0695	0.02418	0.00483	1	1.2263	0.0496398	0.05121	1	3.0609	4.570189	4.24731	5	1.6032	11.10556	11.5383	2	2.005	1.8E-05	2.5E-05	1	1.1148	0.04427	0.01318		0.9203	0.37247	0.84695
LQ	12	3	3.2003	0.163378	0.04012	2	1.9905	0.0045698	9E-05	2	0.1846	3.916045	3.29568	1	2.1747	1.341427	1.37992	1	0.9999	0.00044	1E-08	2	2.2142	0.04318	0.04588		0.2358	0.01445	0.0556

sum error validation	24.3028	constraints
RMSE validation	0.9154	29
sum error fitting	25.0702	
RMSE fitting	0.9298	

VARIANT III

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
SK	6	2	1.9766	0.004998	0.00055	4	4.007	0.0015524	4.9E-05	2	0.7264	1.311254	1.62206		1.1607	1.21044	1.34722	1	1.1681	0.10323	0.02826	1	0.989	0.02477	0.00012	1	0.9722	0.02259	0.00077
NA	7	1	1.0081	0.026115	6.6E-05	3	2.9864	0.0624	0.00018	1	4.996	16.93816	15.968		1.3396	0.819749	1.79453	6	0.6409	26.4001	28.72		0.029	0.00084	0.00084		0	0	0
FZ	8	3	2.9817	0.003969	0.00033		0.0299	0.000894	0.00089	5	1.3602	12.09927	13.2481	2	2.3099	0.243148	0.09604		3.308	8.64242	10.9429	1	1.0028	0.0013	7.8E-06		0.0076	0.00696	5.8E-05
EV	9		0.0228	0.060123	0.00052	1	1.0025	0.0172397	6.2E-06		0.59	0.228675	0.3481	1	0.8228	0.00939	0.0314	1	0.5963	0.17758	0.16297	5	4.9721	0.02372	0.00078	3	2.9934	1E-08	4.4E-05
AS	10	1	1.005	0.000511	2.5E-05		0.0053	0.1714788	2.8E-05		0.6008	0.151866	0.36096	2	0.7726	1.089936	1.50651		0.6275	0.39363	0.39376	1	0.9839	0.03024	0.00026	7	7.0048	0.40628	2.3E-05
OS	11	1	1.0079	0.008817	6.2E-05	1	0.9827	0.0004326	0.0003	1	1.9866	1.131032	0.97338	5	3.2245	2.928205	3.1524	2	2.7718	0.58141	0.59568	1	1.0232	0.01411	0.00054		0.0032	0.09413	1E-05
LQ	12	3	2.9979	0.040723	4.4E-06	2	1.9862	0.0040069	0.00019	2	0.74	2.026352	1.5876	1	1.3698	0.124821	0.13675	1	1.8874	0.82519	0.78748	2	1.9999	4.2E-05	1E-08		0.0188	0.00937	0.00035

sum error validation	78.4734	constraints
RMSE validation	2.0323	19
sum error fitting	83.8111	
RMSE fitting	2.1003	

VARIANT IV

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
SK	6	2	1.9546	0.002372	0.00206	4	0.5807	12.011076	11.6916	2	1.2015	0.4489	0.6376		2.4484	5.702066	5.99466	1	2.3003	2.11266	1.69078	1	1.5365	0.15218	0.28783	1	0.978	0.02088	0.00048
NA	7	1	0.9898	0.032364	0.0001	3	4.2375	2.2527008	1.53141	1	3.507	6.899028	6.28505		1.2429	0.653996	1.5448	6	0.7414	25.3774	27.6529		0.2813	0.07907	0.07913		0.0001	1E-08	1E-08
FZ	8	3	2.9849	0.004382	0.00023		2.4874	6.1871588	6.18716	5	1.5934	10.53132	11.6049	2	1.3692	0.200346	0.39791		1.4083	1.08181	1.98331	1	1.151	0.03393	0.0228		0.0058	0.00726	3.4E-05
EV	9		0.0331	0.055178	0.0011	1	1.0551	0.0338192	0.00304		1.5291	2.008739	2.33815	1	2.1561	1.528685	1.33657	1	1.8277	0.6561	0.68509	5	1.416	11.5743	12.8451	3	2.9829	0.00011	0.00029
AS	10	1	1.0068	0.000433	4.6E-05		0.7738	0.1255994	0.59877		0.6006	0.15171	0.36072	2	0.5357	1.640705	2.14417		0.597	0.35629	0.35641	1	0.4777	0.46254	0.2728	7	7.0085	0.41101	7.2E-05
OS	11	1	0.9861	0.005198	0.00019	1	0.6787	0.105495	0.10323	1	1.4266	0.253512	0.18199	5	1.933	9.016207	9.40649	2	2.3995	0.15226	0.1596	1	3.5733	7.12303	6.62187		0.0029	0.09431	8.4E-06
LQ	12	3	3.0447	0.061802	0.002	2	1.1869	0.541696	0.66113	2	1.1419	1.043667	0.73634	1	1.3147	0.088923	0.09904	1	1.7259	0.55786	0.52693	2	2.5642	0.31114	0.31832		0.0217	0.00882	0.00047

sum error validation	112.1600	constraints
RMSE validation	3.5302	9
sum error fitting	117.3546	
RMSE fitting	3.6110	

VARIANT V

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
SK	6	2	0.0278	3.52726	3.88957	4	3.9963	0.00251	1.4E-05	2	1.1543	0.514376	0.71521		2.3857	5.406555	5.69156	1	0.9133	0.00442	0.00752	1	2.4702	1.75245	2.16149	1	0.0524	1.14511	0.89795
NA	7	1	1.5705	0.160641	0.32547	3	3.0278	0.0847974	0.00077	1	0.3896	0.240885	0.37259		0.0334	0.160641	0.00112	6	5.9773	0.03932	0.00052		0.0014	1.7E-06	2E-06		0	0	0
FZ	8	3	3.0138	0.009044	0.00019		0.7073	0.5002733	0.50027	5	5.0181	0.03222	0.00033	2	1.0442	0.596911	0.91355		0.6866	0.10138	0.47142	1	0.5027	0.21539	0.24731		0.0273	0.00406	0.00075
EV	9		0.0979	0.028934	0.00958	1	0.5331	0.1143116	0.218		0.9481	0.699398	0.89889	1	1.1331	0.04554	0.01772	1	0.2735	0.55383	0.5278	5	5.003	0.03419	9E-06	3	3.0114	0.00032	0.00013
AS	10	1	2.9853	3.832589	3.94142		0.3935	0.0006708	0.15484		0.2949	0.007022	0.08697	2	0.295	2.315267	2.90703		0.0066	4.2E-05	4.4E-05	1	0.0252	1.28278	0.95024	7	6.9995	0.39955	2.5E-07
OS	11	1	0.2914	0.387631	0.50211	1	1.0193	0.0002496	0.00037	1	1.5893	0.443822	0.34727	5	4.9936	0.003352	4.1E-05	2	1.6948	0.09891	0.09315	1	1.3362	0.18645	0.11303		0.0756	0.05494	0.00572
LQ	12	3	3.0133	0.047176	0.00018	2	1.3229	0.36	0.45846	2	1.6057	0.311141	0.15547	1	1.115	0.009702	0.01323	1	1.4479	0.21987	0.20061	2	1.6613	0.11909	0.11472		0.8339	0.51595	0.69539

sum error validation	14.2183	constraints
RMSE validation	1.1924	10
sum error fitting	15.2467	
RMSE fitting	1.2348	

GROUP 3 - samples validation, errors of samples validation and errors od fitting

		scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
VARIANT I	CI	11		0	0	0	1	3.9853	0	8.91202	1	0.9865	0	0.00018	2	0.9645	0	1.07226	3	2.6957	0	0.0926	3	1.5195	0	2.19188	1	0.8485	0	0.02295
	RZ	12	10	10.9918	0	0.98367		0.0083	0	6.9E-05		0	0	0		0	0	0		0	0	0		0	0	0	1	0	0	1
	YU	13		0.0079	0	6.2E-05	2	2.9247	0	0.85507	1	1.9554	0	0.91279	1	1.0086	0	7.4E-05	2	0.7775	0	1.49451	1	1.0262	0	0.00069	4	3.2998	0	0.49028
	DP	14		0.0003	0	9E-08		1.4065	0	1.97824	4	0.7285	0	10.7027	3	4.1145	0	1.24211	2	2.4099	0	0.16802	2	2.0345	0	0.00119		0.3058	0	0.09351
	KC	15	1	0	0	1	2	1.158	0	0.70896	2	2.1789	0	0.03201	4	1.9578	0	4.17058	1	3.1098	0	4.45126	1	1.267	0	0.07129		1.3286	0	1.76518
	BE	16		0	0	0	2	0.5944	0	1.97571	1	1.6898	0	0.47582	1	0.9627	0	0.00139	2	0.99	0	1.0201		1.5457	0	2.38919	5	5.2173	0	0.04722
	GU	17		0	0	0	4	0.9229	0	9.46854	2	3.4609	0	2.13423		1.9919	0	3.96767	1	1.0172	0	0.0003	4	3.6071	0	0.15437		0	0	0

sum error validation	0.0000		constraints
RMSE validation	0.0000		34
sum error fitting	66.0487		
RMSE fitting	1.3938		

		scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
VARIANT II	CI	11		0	0	0	1	3.9853	0	8.91202	1	0.9865	0	0.00018	2	0.9645	0	1.07226	3	2.6957	0	0.0926	3	1.5195	0	2.19188	1	0.8485	0	0.02295
	RZ	12	10	10.9918	0	0.98367		0.0083	0	6.9E-05		0	0	0		0	0	0		0	0	0		0	0	0	1	0	0	1
	YU	13		0.0079	0	6.2E-05	2	2.9247	0	0.85507	1	1.9554	0	0.91279	1	1.0086	0	7.4E-05	2	0.7775	0	1.49451	1	1.0262	0	0.00069	4	3.2998	0	0.49028
	DP	14		0.0003	0	9E-08		1.4065	0	1.97824	4	0.7285	0	10.7027	3	4.1145	0	1.24211	2	2.4099	0	0.16802	2	2.0345	0	0.00119		0.3058	0	0.09351
	KC	15	1	0	0	1	2	1.158	0	0.70896	2	2.1789	0	0.03201	4	1.9578	0	4.17058	1	3.1098	0	4.45126	1	1.267	0	0.07129		1.3286	0	1.76518
	BE	16		0	0	0	2	0.5944	0	1.97571	1	1.6898	0	0.47582	1	0.9627	0	0.00139	2	0.99	0	1.0201		1.5457	0	2.38919	5	5.2173	0	0.04722
	GU	17		0	0	0	4	0.9229	0	9.46854	2	3.4609	0	2.13423		1.9919	0	3.96767	1	1.0172	0	0.0003	4	3.6071	0	0.15437		0	0	0

sum error validation	0.0000		constraints
RMSE validation	0.0000		29
sum error fitting	66.0487		
RMSE fitting	1.5092		

		scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
VARIANT III	CI	11		0	0	0	1	1.1835	7.8500832	0.03367	1	0.9867	4E-08	0.00018	2	2.5659	2.564482	0.32024	3	2.2717	0.17978	0.53042	3	2.9757	2.12052	0.00059	1	1.0165	0.02822	0.00027
	RZ	12	10	10.9344	0.003295	0.8731		0.0595	0.0026214	0.00354		0.0052	2.7E-05	2.7E-05		0.0009	8.1E-07	8.1E-07		0	0	0		0	0	0	1	0	0	1
	YU	13		0.0602	0.002735	0.00362	2	3.4858	0.3148332	2.2076	1	1.9756	0.000408	0.9518	1	0.2127	0.633457	0.61984	2	0.2427	0.28601	3.0881	1	1.006	0.00041	3.6E-05	4	4.0171	0.51452	0.00029
	DP	14		0.0051	2.3E-05	2.6E-05		0.9054	0.2511012	0.81975	4	0.0409	0.472794	15.6745	3	4.7755	0.436921	3.1524	2	2.8987	0.23893	0.80766	2	2.0015	0.00109	2.3E-06		0.3729	0.0045	0.13905
	KC	15	1	0	0	1	2	1.0001	0.0249324	0.9998	2	1.9733	0.042271	0.00071	4	2.5065	0.301072	2.23054	1	3.8782	0.59044	8.28404	1	1.0053	0.06849	2.8E-05		0.6367	0.47873	0.40539
	BE	16		0	0	0	2	1.3797	0.6166961	0.38477	1	1.999	0.095605	0.998	1	0.916	0.002181	0.00706	2	1.6998	0.50382	0.09012		0.0486	2.24131	0.00236	5	4.9568	0.06786	0.00187
	GU	17		0.0003	9E-08	9E-08	4	2.986	4.2563816	1.0282	2	4.0193	0.311811	4.07757		0.0226	3.878142	0.00051	1	0.0089	1.01667	0.98228	4	3.963	0.12666	0.00137		0	0	0

sum error validation	30.5298		constraints
RMSE validation	1.2676		19
sum error fitting	50.7213		
RMSE fitting	1.6339		

		scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
VARIANT IV	CI	11		0	0	0	1	0.1682	14.570252	0.69189	1	1.0255	0.001521	0.00065	2	3.056	4.374372	1.11514	3	2.4925	0.04129	0.25756	3	3.2599	3.02899	0.06755	1	0.9979	0.02232	4.4E-06
	RZ	12	10	10.8933	0.009702	0.79798		0.0947	0.007465	0.00897		0.0088	7.74E-05	7.7E-05		0.0026	6.76E-06	6.8E-06		0.0006	3.6E-07	3.6E-07		0	0	0	1	0	0	1
	YU	13		0.105	0.009428	0.01103	2	4.3965	2.1661952	5.74321	1	1.4594	0.246016	0.21105	1	0.5295	0.229537	0.22137	2	0.2907	0.23697	2.92171	1	0.22	0.64996	0.6084	4	3.9988	0.4886	1.4E-06
	DP	14		0	9E-08	0		0.1046	1.6949436	0.01094	4	1.7217	0.986446	5.19065	3	2.8037	1.718197	0.03853	2	3.4441	1.06957	2.08542	2	2.5891	0.30758	0.34704		0.3368	0.00096	0.11343
	KC	15	1	0	0	1	2	1.0101	0.0218744	0.9799	2	1.3564	0.676506	0.41422	4	2.4804	0.273111	2.30918	1	2.8775	0.05396	3.52501	1	2.8821	2.60855	3.5423		0.3935	0.87441	0.15484
	BE	16		0	0	0	2	0.033	0.31517	3.86909	1	1.736	0.002134	0.5417	1	0.9629	4E-08	0.00138	2	1.4608	0.22165	0.29074		1.5343	0.00013	2.35408	5	5.273	0.0031	0.07453
	GU	17		0.0016	2.56E-06	2.6E-06	4	5.1929	18.2329	1.42301	2	3.6921	0.053453	2.8632		1.1648	0.684094	1.35676	1	0.4338	0.34036	0.32058	4	0.5147	9.56294	12.1473		0	0	0

sum error validation	65.7848		constraints
RMSE validation	2.7036		9
sum error fitting	58.6104		
RMSE fitting	2.5519		

		scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
VARIANT V	CI	11		0	0	0	1	0.5578	11.747756	0.19554	1	1.7908	0.646898	0.62536	2	2.5125	2.396304	0.26266	3	2.9922	0.08791	6.1E-05	3	3.013	2.23054	0.00017	1	0.1336	0.51108	0.75065
	RZ	12	10	10.845	0.02155	0.71403		0.1393	0.017161	0.0194		0.0143	0.000204	0.0002		0.0006	3.6E-07	3.6E-07		0.0001	1E-08	1E-08		0.0008	6.4E-07	6.4E-07	1	0	0	1
	YU	13		0.1541	0.021374	0.02375	2	5.7059	7.7350734	13.7337	1	0.6215	1.779289	0.14326	1	0.4026	0.367236	0.35689	2	0.0922	0.46964	3.6397	1	0.0091	1.03449	0.98188	4	4.0146	0.51094	0.00021
	DP	14		0.0004	1E-08	1.6E-07		0.4588	0.8981353	0.2105	4	1.7027	0.949066	5.27759	3	4.0124	0.010424	1.02495	2	2.9988	0.3468	0.9976	2	0.9837	1.10418	1.03287		0.8431	0.28869	0.71082
	KC	15	1	0	0	1	2	0.4118	0.5568144	2.52238	2	1.4101	0.591053	0.34798	4	2.7232	0.585837	1.63022	1	4.0242	0.83613	9.14579	1	1.4385	0.02941	0.19228		0.9922	0.11316	0.98446
	BE	16		0	0	0	2	0.8127	0.0476549	1.40968	1	1.4839	0.042395	0.23416	1	1.2452	0.079806	0.06012	2	0.8882	0.01036	1.2361		1.5535	6.1E-05	2.41336	5	5.0164	0.04036	0.00027
	GU	17		0.0004	1.6E-07	1.6E-07	4	2.9137	3.9632846	1.18005	2	3.9767	0.26605	3.90734		0.1036	3.565677	0.01073	1	0.0042	1.02617	0.99162	4	4.0014	0.15547	2E-06		0	0	0

sum error validation	27.4640		constraints
RMSE validation	1.6572		10
sum error fitting	57.1339		
RMSE fitting	2.3903		

GROUP 4 - samples validation, errors of samples validation and errors od fitting

VARIANT I

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
BY	16	3	2.6595	0	0.11594	2	1.9558	0	0.00195	3	2.737	0	0.06917		0.9378	0	0.87947	1	0.8808	0	0.01421	2	1.6472	0	0.12447		0.1819	0	0.03309
AG	17		0.2373	0	0.05631		0.0369	0	0.00136	5	4.867	0	0.01769	5	4.9669	0	0.0011	1	0.513	0	0.23717		0.379	0	0.14364		0	0	0
JD	18		0.0287	0	0.00082	3	2.8881	0	0.01252		0.5213	0	0.27175	1	1.0018	0	3.2E-06	4	3.7453	0	0.06487	2	1.8742	0	0.01583	1	0.9407	0	0.00352
MF	19	1	1.0326	0	0.00106		0.3656	0	0.13366	3	2.8747	0	0.0157	2	2.1274	0	0.01623	3	2.8998	0	0.01004		0.0097	0	9.4E-05	2	1.6902	0	0.09598
KD	20		0.029	0	0.00084		0	0	0		0	0	0	2	1.625	0	0.14063	2	1.9747	0	0.00064		0.0614	0	0.00377	7	7.3098	0	0.09598
EW	21	1	1.1739	0	0.03024	2	1.7785	0	0.04906		0	0	0		0.0723	0	0.00523	1	0.9864	0	0.00018	6	6.1116	0	0.01245	1	0.8774	0	0.01503
FV	22	6	5.839	0	0.02592	4	3.9752	0	0.00062		0	0	0		0.2688	0	0.07225		0	0	0	1	0.917	0	0.00689		0	0	0

		constraints
sum error validation	0.0000	34
RMSE validation	0.0000	
sum error fitting	2.7974	
RMSE fitting	0.2868	

VARIANT II

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
BY	16	3	2.6934	0.001149	0.094	2	1.8601	0.0091585	0.01957	3	2.7812	0.001954	0.04787		0.7865	0.022892	0.61858	1	0.935	0.00294	0.00422	2	1.7063	0.00349	0.08626		0.2375	0.00309	0.05641
AG	17		0.0027	0.055037	7.3E-06		0	0.0013616	0	5	4.789	0.006084	0.04452	5	5.8583	0.794594	0.73668	1	0.2956	0.04726	0.49618		0.0545	0.1053	0.00297		0	0	0
JD	18		0.1316	0.010588	0.01732	3	2.8698	0.0003349	0.01695		0.4499	0.005098	0.20241	1	0.7996	0.040885	0.04016	4	3.8698	0.0155	0.01695	2	1.9159	0.00174	0.00707	1	0.9635	0.00052	0.00133
MF	19	1	1.0541	0.000462	0.00293		0.5181	0.0232563	0.26843	3	2.9799	0.011067	0.0004	2	1.6827	0.197758	0.10068	3	2.9539	0.00293	0.00213		0.1005	0.00824	0.0101	2	1.7107	0.00042	0.08369
KD	20		0.1173	0.007797	0.01376		0	0	0		0	0	0	2	1.4632	0.026179	0.28815	2	1.9462	0.00081	0.00289		0.2881	0.05139	0.083	7	7.1852	0.01553	0.0343
EW	21	1	1.1293	0.001989	0.01672	2	1.8158	0.0013913	0.03393		0	0	0		0.1729	0.01012	0.02989	1	0.9995	0.00017	2.5E-07	6	5.9795	0.01745	0.00042	1	0.9031	0.00066	0.00939
FV	22	6	5.8717	0.001069	0.01646	4	3.9362	0.001521	0.00407		0	0	0		0.2368	0.001024	0.05607		0	0	0	1	0.9552	0.00146	0.00201		0	0	0

		constraints
sum error validation	1.5117	29
RMSE validation	0.2283	
sum error fitting	3.5689	
RMSE fitting	0.3508	

VARIANT III

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
BY	16	3	2.9679	0.095111	0.00103	2	1.9834	0.0007618	0.00028	3	0.1057	6.92374	8.37697		0.6227	0.099288	0.38776	1	3.2908	5.8081	5.24776	2	1.9868	0.11533	0.00017		0.0427	0.01938	0.00182
AG	17		0.0001	0.056264	1E-08		0.0096	0.0007453	9.2E-05	5	4.876	8.1E-05	0.01538	5	4.849	0.0139	0.0228	1	1.2097	0.48539	0.04397		0.0557	0.10452	0.0031		0	0	0
JD	18		0.0306	3.61E-06	0.00094	3	2.9659	0.0060528	0.00116		1.4588	0.878906	2.1281	1	1.7824	0.609336	0.61215	4	1.7794	3.86476	4.93106	2	1.9868	0.01268	0.00017	1	0.9961	0.00307	1.5E-05
MF	19	1	1.0109	0.000471	0.00012		0.033	0.1106228	0.00109	3	4.1492	1.62435	1.32066	2	1.5179	0.37149	0.23242	3	2.2855	0.37736	0.51051		0.0002	9E-05	4E-08	2	2.0033	0.09803	1.1E-05
KD	20		0	0.000841	0		0	0	0		0.0833	0.006939	0.00694	2	1.5058	0.014209	0.24423	2	2.4256	0.20331	0.18114		0.0228	0.00149	0.00052	7	6.9626	0.12055	0.0014
EW	21	1	1.0065	0.028023	4.2E-05	2	2.0034	0.05058	1.2E-05		0.3229	0.104264	0.10426		0.7173	0.416025	0.51452	1	0.0037	0.9657	0.99261	6	5.9508	0.02586	0.00242	1	0.9954	0.01392	2.1E-05
FV	22	6	5.984	0.021025	0.00026	4	4.0047	0.0008702	2.2E-05		0.0042	1.76E-05	1.8E-05		0.0048	0.069696	2.3E-05		0.0054	2.9E-05	2.9E-05	1	0.9969	0.00638	9.6E-06		0	0	0

		constraints
sum error validation	23.7296	19
RMSE validation	1.1176	
sum error fitting	25.8880	
RMSE fitting	1.1673	

VARIANT IV

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
BY	16	3	2.9921	0.110623	6.2E-05	2	0.0461	3.6469541	3.81773	3	0.0903	7.005021	8.46635		0.9871	0.00243	0.97437	1	2.4256	2.38641	2.03234	2	4.3683	7.40439	5.60884		0.0904	0.00837	0.00817
AG	17		0.0015	0.055602	2.3E-06		1.2042	1.3625893	1.4501	5	3.6445	1.494506	1.83738	5	5.0839	0.013689	0.00704	1	0.5621	0.00241	0.19176		0.5038	0.01558	0.25381		0	0	0
JD	18		0.0319	1.02E-05	0.00102	3	4.2193	1.7720934	1.48669		2.0947	2.475588	4.38777	1	0.97	0.001011	0.0009	4	1.386	5.5663	6.833	2	1.3908	0.23368	0.37112	1	0.9073	0.00112	0.00859
MF	19	1	0.9997	0.001082	9E-08		2.1227	3.0874004	4.50586	3	2.5912	0.080372	0.16712	2	1.1318	0.991219	0.75377	3	1.3824	2.3025	2.61663		0.7674	0.57411	0.5889	2	2.005	0.0991	2.5E-05
KD	20		0.0001	0.000835	1E-08		0.0073	5.329E-05	5.3E-05		0.0833	0.006939	0.00694	2	0.3591	1.602503	2.69255	2	2.3178	0.11772	0.101		1.2346	1.3764	1.52424	7	6.9978	0.09734	4.8E-06
EW	21	1	0.9826	0.036596	0.0003	2	1.7911	0.0001588	0.04364		1.4366	2.06382	2.06382		1.167	1.198368	1.36189	1	2.0748	1.18461	1.1552	6	2.5483	12.6971	11.9142	1	0.9996	0.01493	1.6E-07
FV	22	6	5.992	0.023409	6.4E-05	4	1.6094	5.5970096	5.71497		1.0594	1.122328	1.12233		1.3011	1.065643	1.69286		0.8513	0.72471	0.72471	1	0.1868	0.53319	0.66129		0	0	0

		constraints
sum error validation	70.1578	9
RMSE validation	2.7920	
sum error fitting	77.1494	
RMSE fitting	2.9278	

VARIANT V

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
BY	16	3	2.9868	0.107125	0.00017	2	0.0228	3.736489	3.90932	3	2.9993	0.068801	4.9E-07		0.1047	0.694056	0.01096	1	0.3479	0.28398	0.42523	2	1.3432	0.09242	0.43139		3.1953	9.08058	10.2099
AG	17		0.0244	0.045326	0.0006		0.3295	0.0856148	0.10857	5	4.9854	0.014019	0.00021	5	5.0226	0.003102	0.00051	1	0.5704	0.00329	0.18456		0.0678	0.09685	0.0046		0	0	0
JD	18		1.2828	1.572767	1.64558	3	3.0113	0.0151782	0.00013		0.0139	0.257455	0.00019	1	1.1691	0.027989	0.02859	4	4.0121	0.07118	0.00015	2	1.4761	0.15848	0.27447	1	0.0348	0.82065	0.93161
MF	19	1	0.408	0.390125	0.35046		2.9587	6.7241676	8.75391	3	2.9757	0.010201	0.00059	2	1.371	0.572141	0.39564	3	2.9973	0.00951	7.3E-06		0.2883	0.07762	0.08312	2	0.001	2.8534	3.996
KD	20		0.0052	0.000566	2.7E-05		0.0738	0.0054464	0.00545		0.0026	6.76E-06	6.8E-06	2	0.5811	1.089727	2.01328	2	1.5892	0.14861	0.16876		1.7596	2.88388	3.09619	7	6.9885	0.10323	0.00013
EW	21	1	0.2814	0.796556	0.51639	2	0.6213	1.3391118	1.90081		0.0186	0.000346	0.00035		1.9917	3.684096	3.96687	1	1.3096	0.10446	0.09585	6	5.9983	0.01284	2.9E-06	1	0.7792	0.00964	0.04875
FV	22	6	6.0114	0.029722	0.00013	4	3.9828	5.776E-05	0.0003		0.0045	2.03E-05	2E-05		0.7598	0.241081	0.5773		0.1736	0.03014	0.03014	1	0.0668	0.72284	0.87086		0.0012	1.4E-06	1.4E-06

		constraints
sum error validation	25.0114	10
RMSE validation	1.5815	
sum error fitting	30.0511	
RMSE fitting	1.7335	

GROUP 5 - samples validation, errors of samples validation and errors od fitting

VARIANT I

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
LO	21	2	1.9901	0	9.8E-05		0.0149	0	0.00022		0.0224	0	0.0005		0.0352	0	0.00124	2	1.9695	0	0.00093	1	1.0051	0	2.6E-05	6	5.9628	0	0.00138
DI	22	3	3.023	0	0.00053	3	2.988	0	0.00014	1	0.9805	0	0.00038	2	1.9668	0	0.0011		0.0552	0	0.00305	1	0.9773	0	0.00052	1	1.009	0	8.1E-05
RY	23	1	0.9968	0	1E-05	1	0.9869	0	0.00017		0.0348	0	0.00121	1	0.9969	0	9.6E-06	1	0.9956	0	1.9E-05	6	5.9949	0	2.6E-05	1	0.9941	0	3.5E-05
UB	24		0.01	0	0.0001	2	2.0193	0	0.00037	5	4.9674	0	0.00106	2	1.999	0	1E-06	1	0.9824	0	0.00031	1	0.9788	0	0.00045		0.0431	0	0.00186
PT	25	4	3.9753	0	0.00061	2	1.9901	0	9.8E-05	1	1.0131	0	0.00017	1	1.0099	0	9.8E-05	2	2.0017	0	2.9E-06		0.0337	0	0.00114	1	0.9762	0	0.00057
XN	26		0.0206	0	0.00042	3	2.9877	0	0.00015	2	2.0061	0	3.7E-05	2	1.9812	0	0.00035	1	0.9771	0	0.00052	1	1.0126	0	0.00016	2	2.0148	0	0.00022
TB	27	1	0.9842	0	0.00025		0.0131	0	0.00017	2	1.9757	0	0.00059	3	3.0109	0	0.00012	4	4.0184	0	0.00034	1	0.9977	0	5.3E-06		0	0	0

sum error sam	0.0000	constraints
RMSE samples	0.0000	34
sum error fitting	0.0219	
RMSE fitting	0.0254	

VARIANT II

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
LO	21	2	1.9751	0.000225	0.00062		0.0021	0.0001638	4.4E-06		0.0178	2.12E-05	0.00032		0.0736	0.001475	0.00542	2	1.9824	0.00017	0.00031	1	0.9954	9.4E-05	2.1E-05	6	5.9536	8.5E-05	0.00215
DI	22	3	3.0099	0.000172	9.8E-05	3	2.9729	0.000228	0.00073	1	0.9903	9.6E-05	9.4E-05	2	1.9908	0.000576	8.5E-05		0.0385	0.00028	0.00148	1	1.0022	0.00062	4.8E-06	1	0.9954	0.00018	2.1E-05
RY	23	1	1.0139	0.000292	0.00019	1	0.9906	1.369E-05	8.8E-05		0.0177	0.000292	0.00031	1	1.0037	4.62E-05	1.4E-05	1	0.9925	9.6E-06	5.6E-05	6	5.9787	0.00026	0.00045	1	1.0029	7.7E-05	8.4E-06
UB	24		0.0022	6.08E-05	4.8E-06	2	1.9963	0.000529	1.4E-05	5	4.9614	3.6E-05	0.00149	2	2.0054	4.1E-05	2.9E-05	1	0.9889	4.2E-05	0.00012	1	1.0104	0.001	0.00011		0.0354	5.9E-05	0.00125
PT	25	4	3.9685	4.62E-05	0.00099	2	2.0141	0.000576	0.0002	1	1.0017	0.00013	2.9E-06	1	1	9.8E-05	0	2	1.9787	0.00053	0.00045		0.0293	1.9E-05	0.00086	1	1.0078	0.001	6.1E-05
XN	26		0.0345	0.000193	0.00119	3	2.9878	1E-08	0.00015	2	2.0173	0.000125	0.0003	2	1.9646	0.000276	0.00125	1	1.0014	0.00059	2E-06	1	0.992	0.00042	6.4E-05	2	2.0023	0.00016	5.3E-06
TB	27	1	0.9959	0.000137	1.7E-05		0.0363	0.0005382	0.00132	2	1.9937	0.000324	4E-05	3	2.9619	0.002401	0.00145	4	4.0175	8.1E-07	0.00031	1	0.9921	3.1E-05	6.2E-05		0.0025	6.3E-06	6.3E-06

sum error sam	0.0147	constraints
RMSE samples	0.0226	29
sum error fitting	0.0242	
RMSE fitting	0.0289	

VARIANT III

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
LO	21	2	1.9914	1.69E-06	7.4E-05		0.0032	0.0001369	1E-05		0.8804	0.736164	0.7751		0.4668	0.186279	0.2179	2	0.6956	1.62282	1.70146	1	1.0189	0.00019	0.00036	6	5.9435	0.00037	0.00319
DI	22	3	2.9872	0.001282	0.00016	3	2.9866	1.96E-06	0.00018	1	1.1728	0.036979	0.02986	2	1.1023	0.74736	0.80587		0.7488	0.48108	0.5607	1	0.9941	0.00028	3.5E-05	1	1.0083	4.9E-07	6.9E-05
RY	23	1	0.989	6.08E-05	0.00012	1	0.9879	0.000001	0.00015		0.3362	0.090842	0.11303	1	0.7141	0.079976	0.08174	1	0.9835	0.00015	0.00027	6	5.9649	0.0009	0.00123	1	1.0244	0.00092	0.0006
UB	24		0.0051	2.4E-05	2.6E-05	2	1.9794	0.001592	0.00042	5	2.4087	6.546946	6.71484	2	2.609	0.3721	0.37088	1	2.9679	3.94221	3.87263	1	1.004	0.00064	1.6E-05		0.026	0.00029	0.00068
PT	25	4	3.9984	0.000534	2.6E-06	2	1.9941	0.000016	3.5E-05	1	0.8173	0.038338	0.03338	1	1.6363	0.392377	0.40488	2	1.5264	0.22591	0.2243		0.0352	2.3E-06	0.00124	1	0.9924	0.00026	5.8E-05
XN	26		0.0282	5.78E-05	0.0008	3	3.0127	0.000625	0.00016	2	0.9083	1.205165	1.19181	2	1.0947	0.785882	0.81957	1	2.9736	3.98601	3.8951	1	0.9829	0.00088	0.00029	2	1.9997	0.00023	9E-08
TB	27	1	1.0008	0.000276	6.4E-07		0.0361	0.000529	0.0013	2	4.4763	6.253	6.13206	3	3.3768	0.133883	0.14198	4	1.1042	8.49256	8.38566	1	1.0001	5.8E-06	1E-08		0.0057	3.2E-05	3.2E-05

sum error sam	36.3662	constraints
RMSE samples	1.3835	19
sum error fitting	36.4842	
RMSE fitting	1.3857	

VARIANT IV

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
LO	21	2	2.025	0.001218	0.00062		0.1816	0.0277889	0.03298		0.3534	0.109561	0.12489		0.2642	0.052441	0.0698	2	0.3325	2.67977	2.78056	1	1.8835	0.77159	0.78057	6	5.9597	9.6E-06	0.00162
DI	22	3	2.9857	0.001391	0.0002	3	1.8616	1.268777	1.29595	1	2.2177	1.530664	1.48279	2	1.2866	0.462672	0.50894		0.9496	0.79995	0.90174	1	0.7007	0.07651	0.08958	1	0.998	0.00012	4E-06
RY	23	1	1.0056	7.74E-05	3.1E-05	1	0.2486	0.5450869	0.5646		0.6158	0.337561	0.37921	1	1.031	0.001163	0.00096	1	2.2078	1.46943	1.45878	6	4.9036	1.19094	1.20209	1	0.9876	4.2E-05	0.00015
UB	24		0.0021	6.24E-05	4.4E-06	2	1.522	0.2473073	0.22848	5	2.4449	6.363006	6.52854	2	3.2372	1.533139	1.53066	1	2.4596	2.18212	2.13043	1	1.3015	0.10414	0.0909		0.0328	0.00011	0.00108
PT	25	4	3.9926	0.000299	5.5E-05	2	0.4666	2.3210523	2.35132	1	1.042	0.000835	0.00176	1	2.1595	1.32158	1.34444	2	1.7943	0.04301	0.04231		0.5262	0.24256	0.27689	1	1.0187	0.00181	0.00035
XN	26		0.0896	0.004761	0.00803	3	2.5141	0.224297	0.2361	2	1.1025	0.816493	0.80551	2	1.078	0.81577	0.85008	1	2.6161	2.68632	2.61178	1	1.599	0.34386	0.3588	2	2.0007	0.0002	4.9E-07
TB	27	1	0.8994	0.007191	0.01012		4.2055	17.576218	17.6862	2	3.2237	1.557504	1.49744	3	1.9435	1.139343	1.11619	4	0.64	11.4136	11.2896	1	0.0855	0.83211	0.83631		0.0025	6.3E-06	6.3E-06

sum error sam	63.1054	constraints
RMSE samples	2.6480	9
sum error fitting	63.5095	
RMSE fitting	2.6564	

VARIANT V

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
LO	21	2	0.2407	3.0604	3.09514		0.5037	0.2389254	0.25371		0.4106	0.150699	0.16859		0.2338	0.039442	0.05466	2	0.3711	2.55488	2.65332	1	3.2198	4.9049	4.92751	6	6.0203	0.00331	0.00041
DI	22	3	3.003	0.0004	9E-06	3	3.0081	0.000404	6.6E-05	1	1.7181	0.544054	0.51567	2	1.6093	0.127806	0.15265		0.8862	0.69056	0.78535	1	0.4697	0.25766	0.28122	1	0.3057	0.49463	0.48205
RY	23	1	0.0206	0.952966	0.95922	1	0.0869	0.81	0.83375		0.3636	0.108109	0.1322	1	0.3105	0.471145	0.47541	1	0.5658	0.18473	0.18853	6	6.0299	0.00122	0.00089	1	3.6227	6.90954	6.87856
UB	24		0.1496	0.019488	0.02238	2	2.1052	0.0073788	0.01107	5	4.9941	0.000713	3.5E-05	2	2.0071	6.56E-05	5E-05	1	1.3496	0.13484	0.12222	1	0.3002	0.4605	0.48972		0.0943	0.00262	0.00889
PT	25	4	3.991	0.000246	8.1E-05	2	0.3399	2.72316	2.75593	1	1.5088	0.245718	0.25888	1	2.7185	2.919314	2.95324	2	2.0357	0.00116	0.00127		0.3201	0.08202	0.10246	1	0.086	0.79246	0.8354
XN	26		2.4802	6.049632	6.15139	3	2.993	2.809E-05	4.9E-05	2	1.1066	0.8091	0.79816	2	1.1195	0.742527	0.77528	1	1.7926	0.66504	0.62821	1	0.6437	0.13609	0.12695	2	0.8645	1.32319	1.28936
TB	27	1	1.115	0.017109	0.01323		1.9633	3.80328	3.85455	2	0.8983	1.160791	1.21374	3	3.0014	9.03E-05	2E-06	4	3.9991	0.00037	8.1E-07	1	0.0166	0.96256	0.96708		0.0065	4.2E-05	4.2E-05

sum error validation	34.6127	constraints
RMSE validation	1.8605	10
sum error fitting	35.0653	
RMSE fitting	1.8726	

GROUP 1 - samples validation, errors of samples validation and errors of fitting, using weights for experts

VARIANT I

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
SK	6	0.1739	0.1283	0	0.00208	0.4783	0.4595	0	0.00035	0.2173	0.1418	0	0.0057		0.0887	0	0.00787	0.0435	0.0175	0	0.00068	0.0435	0.0701	0	0.00071	0.0435	0.0941	0	0.00256
NA	7	0.2174	0.3054	0	0.00774	0.1304	0.0707	0	0.00356	0.0435	0.0266	0	0.00029		0.0175	0	0.00031	0.6087	0.5798	0	0.00084		0.0000	0	0		0.0000	0	0
FZ	8	0.2174	0.2459	0	0.00081		0.0262	0	0.00069	0.4783	0.3589	0	0.01426	0.3043	0.2286	0	0.00573		0.0727	0	0.00529		0.0564	0	0.00318		0.0113	0	0.00013
EV	9		0.0175	0	0.00031	0.0435	0.0566	0	0.00017		0.0793	0	0.00629	0.0435	0.0325	0	0.00012	0.0869	0.0946	0	5.9E-05	0.6087	0.4473	0	0.02605	0.2174	0.2721	0	0.00299
AS	10	0.0435	0.0630	0	0.00038		0.0599	0	0.00359		0.0509	0	0.00259	0.0435	0.0322	0	0.00013		0.0000	0	0	0.1739	0.3038	0	0.01687	0.7391	0.4903	0	0.0619
OS	11	0.0435	0.0355	0	6.4E-05	0.0435	0.0859	0	0.0018	0.0435	0.0394	0	1.7E-05	0.5652	0.5570	0	6.7E-05	0.2174	0.1854	0	0.00102	0.0869	0.0358	0	0.00261		0.0609	0	0.00371
LQ	12	0.3043	0.2045	0	0.00996	0.3043	0.2411	0	0.00399	0.2174	0.3031	0	0.00734	0.0435	0.0435	0	0	0.0435	0.0499	0	4.1E-05	0.0870	0.0866	0	1.6E-07		0.0713	0	0.00508

sum error validation	0.0000
RMSE validation	0.0000
sum error fitting	0.2199
RMSE fitting	0.0804

constraints
34

VARIANT II

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
SK	6	0.1739	0.1135	0.00022	0.00365	0.4783	0.4688	8.6E-05	9E-05	0.2173	0.1707	0.00084	0.00217		0.1204	0.001005	0.0145	0.0435	0.0267	8.5E-05	0.00028	0.0435	0.0524	0.00031	7.9E-05	0.0435	0.0476	0.00216	1.7E-05
NA	7	0.2174	0.2695	0.00129	0.00271	0.1304	0.0818	0.00012	0.00236	0.0435	0.027	1.6E-07	0.00027		0.0267	8.46E-05	0.00071	0.6087	0.5951	0.00023	0.00018		0	0	0		0	0	0
FZ	8	0.2174	0.2022	0.00191	0.00023		0	0.00069	0	0.4783	0.4866	0.01631	6.9E-05	0.3043	0.221	5.78E-05	0.00694		0.0302	0.00181	0.00091		0.058	2.6E-06	0.00336		0.002	8.6E-05	4E-06
EV	9		0.0000	0.00031	0	0.0435	0.0353	0.00045	6.7E-05		0	0.00629	0	0.0435	0.1693	0.018714	0.01583	0.0869	0.0882	4.1E-05	1.7E-06	0.6087	0.5088	0.00378	0.00998	0.2174	0.1984	0.00543	0.00036
AS	10	0.0435	0.0377	0.00064	3.4E-05		0	0.00359	0		0	0.00259	0	0.0435	0.259	0.051438	0.04644		0	0	0	0.1739	0.246	0.00334	0.0052	0.7391	0.4573	0.00109	0.07941
OS	11	0.0435	0.0470	0.00013	1.2E-05	0.0435	0.0845	2E-06	0.00168	0.0435	0.2311	0.03675	0.03519	0.5652	0.0762	0.231169	0.23912	0.2174	0.2163	0.00095	1.2E-06	0.0869	0.0724	0.00134	0.00021		0.2726	0.04482	0.07431
LQ	12	0.3043	0.3302	0.0158	0.00067	0.3043	0.3296	0.00783	0.00064	0.2174	0.0846	0.04774	0.01764	0.0435	0.1274	0.007039	0.00704	0.0435	0.0436	4E-05	1E-08	0.0870	0.0624	0.00059	0.00061		0.0222	0.00241	0.00049

sum error validation	0.5216
RMSE validation	0.1341
sum error fitting	0.5735
RMSE fitting	0.1406

constraints
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VARIANT III

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
SK	6	0.1739	0.1741	0.0021	4E-08	0.4783	0.4767	0.0003	2.6E-06	0.2173	0.0459	0.0092	0.02938		0.0997	0.000121	0.00994	0.0435	0.1189	0.01028	0.00569	0.0435	0.0432	0.00072	9E-08	0.0435	0.0415	0.00277	4E-06
NA	7	0.2174	0.2182	0.0076	6.4E-07	0.1304	0.1298	0.00349	3.6E-07	0.0435	0.5415	0.26512	0.248		0.0816	0.004109	0.00666	0.6087	0.0286	0.30382	0.33652		0.0003	9E-08	9E-08		0	0	0
FZ	8	0.2174	0.2161	0.00089	1.7E-06		0.0042	0.00048	1.8E-05	0.4783	0.1236	0.05537	0.12581	0.3043	0.2536	0.000625	0.00257		0.399	0.10647	0.1592		0.0026	0.00289	6.8E-06		0.0008	0.00011	6.4E-07
EV	9		0.0022	0.00023	4.8E-06	0.0435	0.0428	0.00019	4.9E-07		0.0361	0.00187	0.0013	0.0435	0.0549	0.000502	0.00013	0.0869	0.0425	0.00271	0.00197	0.6087	0.6052	0.02493	1.2E-05	0.2174	0.2163	0.00311	1.2E-06
AS	10	0.0435	0.0433	0.00039	4E-08		0.0002	0.00356	4E-08		0.0129	0.00144	0.00017	0.0435	0.0166	0.000243	0.00072		0.0146	0.00021	0.00021	0.1739	0.1731	0.01708	6.4E-07	0.7391	0.7395	0.0621	1.6E-07
OS	11	0.0435	0.0431	5.8E-05	1.6E-07	0.0435	0.0423	0.0019	1.4E-06	0.0435	0.176	0.01866	0.01756	0.5652	0.391	0.027556	0.03035	0.2174	0.2587	0.00537	0.00171	0.0869	0.0884	0.00277	2.3E-06		0.0004	0.00366	1.6E-07
LQ	12	0.3043	0.303	0.0097	1.7E-06	0.3043	0.3039	0.00394	1.6E-07	0.2174	0.064	0.05717	0.02353	0.0435	0.1026	0.003493	0.00349	0.0435	0.1378	0.00773	0.00889	0.0870	0.0872	3.6E-07	4E-08		0.0015	0.00487	2.3E-06

sum error validation	1.0419
RMSE validation	0.2342
sum error fitting	1.0139
RMSE fitting	0.2310

constraints
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VARIANT IV

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
SK	6	0.1739	0.1737	0.00206	4E-08	0.4783	0.0687	0.15272	0.16777	0.2173	0.125	0.00028	0.00852		0.244	0.024118	0.05954	0.0435	0.2191	0.04064	0.03084	0.0435	0.1266	0.00319	0.00691	0.0435	0.0428	0.00263	4.9E-07
NA	7	0.2174	0.2183	0.00759	8.1E-07	0.1304	0.3615	0.08456	0.05341	0.0435	0.2917	0.07028	0.0616		0.0777	0.003624	0.00604	0.6087	0.0387	0.29279	0.3249		0.0121	0.00015	0.00015		0	0	0
FZ	8	0.2174	0.2148	0.00097	6.8E-06		0.2409	0.0461	0.05803	0.4783	0.1654	0.03744	0.09791	0.3043	0.1429	0.007344	0.02605		0.1287	0.00314	0.01656		0.1066	0.00252	0.01136		0.0007	0.00011	4.9E-07
EV	9		0.0042	0.00018	1.8E-05	0.0435	0.1083	0.00267	0.0042		0.1374	0.00338	0.01888	0.0435	0.2032	0.029138	0.0255	0.0869	0.1846	0.0081	0.00955	0.6087	0.1473	0.09	0.21289	0.2174	0.215	0.00326	5.8E-06
AS	10	0.0435	0.0424	0.00042	1.2E-06		0.0548	2.6E-05	0.003		0.0437	5.2E-05	0.00191	0.0435	0.0411	7.92E-05	5.8E-06		0.0438	0.00192	0.00192	0.1739	0.0351	0.0722	0.01927	0.7391	0.7392	0.06195	1E-08
OS	11	0.0435	0.0432	5.9E-05	9E-08	0.0435	0.0615	0.0006	0.00032	0.0435	0.1341	0.00897	0.00821	0.5652	0.1734	0.147149	0.15351	0.2174	0.2238	0.00147	4.1E-05	0.0869	0.3637	0.10752	0.07662		0.0003	0.00367	9E-08
LQ	12	0.3043	0.3034	0.00978	8.1E-07	0.3043	0.1044	0.01869	0.03996	0.2174	0.1027	0.04016	0.01316	0.0435	0.1178	0.00552	0.00552	0.0435	0.1611	0.01237	0.01383	0.0870	0.2085	0.01486	0.01476		0.0021	0.00479	4.4E-06

sum error validation	1.4312
RMSE validation	0.3988
sum error fitting	1.5527
RMSE fitting	0.4154

constraints
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VARIANT V

	scenario	1st	obt pbty	error sam	error fit	2nd	obt pbty	error sam	error fit	3rd	obt pbty	error sam	error fit	4th	obt pbty	error sam	error fit	5th	obt pbty	error sam	error fit	6th	obt pbty	error sam	error fit	7th	obt pbty	error sam	error fit
SK	6	0.1739	0.0007	0.01628	0.03	0.4783	0.4928	0.00111	0.00021	0.2173	0.0621	0.00635	0.02409		0.0875	1.44E-06	0.00766	0.0435	0.0353	0.00032	6.7E-05	0.0435	0.3191	0.062	0.07596	0.0435	0.0026	0.00837	0.00167
NA	7	0.2174	0.2232	0.00676	3.4E-05	0.1304	0.1225	0.00268	6.2E-05	0.0435	0.0439	0.0003	1.6E-07		0.0002	0.000299	4E-08	0.6087	0.6102	0.00092	2.2E-06		0	0	0		0	0	0
FZ	8	0.2174	0.2196	0.00069	4.8E-06		0.0813	0.00304	0.00661	0.4783	0.4693	0.01219	8.1E-05	0.3043	0.0873	0.019966	0.04709		0.1031	0.00092	0.01063		0.0344	0.00048	0.00118		0.005	4E-05	2.5E-05
EV	9		0	0.00031	0	0.0435	0.0582	2.6E-06	0.00022		0.0976	0.00033	0.00953	0.0435	0.1588	0.015952	0.01329	0.0869	0	0.00895	0.00755	0.6087	0.5291	0.00669	0.00634	0.2174	0.1564	0.01339	0.00372
AS	10	0.0435	0.2378	0.03056	0.03775		0.0128	0.00222	0.00016		0.021	0.00089	0.00044	0.0435	0.0233	7.92E-05	0.00041		0	0	0	0.1739	0.0007	0.09187	0.03	0.7391	0.7043	0.0458	0.00121
OS	11	0.0435	0.0157	0.00039	0.00077	0.0435	0.1101	0.00059	0.00444	0.0435	0.1628	0.01523	0.01423	0.5652	0.5658	7.74E-05	3.6E-07	0.2174	0.0955	0.00808	0.01486	0.0869	0.0405	2.2E-05	0.00215		0.0097	0.00262	9.4E-05
LQ	12	0.3043	0.303	0.0097	1.7E-06	0.3043	0.1224	0.01409	0.03309	0.2174	0.1434	0.0255	0.00548	0.0435	0.0771	0.001129	0.00113	0.0435	0.1558	0.01121	0.01261	0.0870	0.0762	0.00011	0.00012		0.122	0.00257	0.01488

sum error validation	0.4511
RMSE validation	0.2124
sum error fitting	0.4198
RMSE fitting	0.2049

constraints
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Appendix C

Methodology

In elicitation procedure experts are required to order the scenarios based on their severity. After that, we compute the scores using 2.5. We need to have a starting distribution for B_i's, on which we apply probabilistic inversion. The natural choice for the starting distribution is the uniform distribution.

When we say that "scenario i is bigger than scenario j" we refer to the score of scenario i having a higher value than score of scenario j. For this we use the notation $S_1 > S_2$. The scores of scenarios i and j are computed using 3.1. We used indicator functions to obtain set of samples for which scenario i is on j^{th} position within a group of scenarios.
For instance, given k scenarios, we define the following functions for scenario S:

$$S_1R_1 = \mathbb{1}\{k, \#\mathbb{1}\{0, S_2, \ldots, S_k, S_1\}, k\} \tag{C.1}$$

$$S_1R_2 = \mathbb{1}\{k-1, \#\mathbb{1}\{0, S_2, \ldots, S_k, S_1\}, k-1\} \tag{C.2}$$

$$S_1R_3 = \mathbb{1}\{k-2, \#\mathbb{1}\{0, S_2, \ldots, S_k, S_1\}, k-2\} \tag{C.3}$$

In relation C.1 S_1R_1 is understood as: scenario S_1 is ranked on the first position (rank 1, R_1). We first explain the second indicator function.

$$\mathbb{1}\{0, S_2, \ldots, S_k, S_1\} = \begin{cases} 1 & \text{if } \ S_1 \geq S_i, \ i = i \ldots k \\ 0 & \text{otherwise} \end{cases}$$

This function returns 1 if all scenarios from S_2 to S_k are between 0 and S_1, hence S_1 is bigger than all of them, and 0 otherwise. Next we count the number of times for which S_1 is bigger than all the rest. In case this number is k (this means S_1 is bigger than all the rest), and the second indicator function returns 1.

$$\mathbb{1}\{k, \#, k\} = \begin{cases} 1 & \text{if } \ S_1 \geq S_i, \ i = i \ldots k \\ 0 & \text{otherwise} \end{cases}$$

In case this number is smaller than k, for instance $k-1$ the function returns 0. The number $k-1$ signifies that there is *one* scenario which is bigger than S_1. (if the number would be k-2 this means *two* scenarios are bigger than S_2).

S_1R_2 means that scenario S_1 is ranked on the second position (rank 2, R_2). The first indicator function returns 1 if S_1 is bigger than $k-1$ scenarios (hence there is *only one* scenario which is bigger than S_1, all the rest are smaller than S_1), and zero otherwise. Similarly we count the number of times for which S_1 is bigger than $k-1$ scenarios, and if this number is $k-1$ the first indicator functions returns 1, otherwise it returns 0.

In an analogous way we express the rankings of all scenarios. Next we proceed with generating samples from uniformly distributed random variables, B_i, necessary for probabilistic inversion. After the samples have been generated, we take the samples file and run probabilistic inversion program. We re-sample the file but imposing for each ranking of scenarios the experts assessments. In other words, if say 4 experts ranked S_1 on the first place, then the probability of S_1 to be on the first place is $\frac{4}{11}$ (remember that the total number of experts used in our project is 11). In probabilistic inversion program we impose 1-$\frac{4}{11}$. The same procedure is used for all scenarios rankings by experts. When we want to input probability x in our software tool used for probabilistic inversion, we must input 1-x. (i.e. if we want to input the probability $\frac{6}{11}$, we use in the program 1-$\left(\frac{6}{11}\right)$). We will impose these constraints by imposing them on indicator functions

In other words, probabilistic inversion algorithm re-weights the samples by imposing the experts preference on the scenarios, such that the probability of scenario 1 being bigger than all the other equals to the total number of experts who ranked scenario 1 as being bigger over the all the others divided by the total number of experts:

$$P\{S_1 > \{S_2, S_3, S_4\}\} = \frac{\#\{S_1 > \{S_2, S_3, S_4\}\}}{\text{the total number of experts}} \tag{C.4}$$

This way we obtain a new distribution for (B_1, B_2, B_3) which satisfies constraints in the form of probabilities of preferences.

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