MANAGEMENT SCIENCE
Vol. 51, No. 6, June 2005, pp. 995–1006
ISSN 0025-1909 | EISSN 1526-5501 | 05 | 5106 | 0995

DOI 10.1287/mnsc.1050.0370

Probabilistic Inversion of Expert Judgments in the Quantification of Model Uncertainty

Bernd Kraan
Department of Information, Technology and Systems, Delft University of Technology, CROSS, Mekelweg 4, 2628 CD Delft, The Netherlands, and Risk and Uncertainty Management (RandUM), Plein Delftzicht 51, 2627 CA Delft, The Netherlands, b.c.p.kraan@ewi.tudelft.nl

Tim Bedford
Department of Management Science, Strathclyde Business School, University of Strathclyde, 40 George Street, Glasgow G1 1QE, Scotland, tim@mansci.strath.ac.uk

Expert judgment is frequently used to assess parameter values of quantitative management science models, particularly in decision-making contexts. Experts can, however, only be expected to assess observable quantities, not abstract model parameters. This means that we need a method for translating expert assessed uncertainties on model outputs into uncertainties on model parameter values. This process is called *probabilistic inversion*. The probability distribution on model parameters obtained in this way can be used in a variety of ways, but in particular in an uncertainty analysis or as a Bayes prior. This paper discusses computational algorithms that have proven successful in various projects and gives examples from environmental modelling and banking. Those algorithms are given a theoretical basis by adopting a minimum information approach to modelling partial information. The role of minimum information is two-fold: It enables us to resolve the problem of nonuniqueness of distributions given the information we have, and it provides numerical stability to the algorithm by guaranteeing convergence properties.

Key words: multivariate distribution; uncertainty analysis; expert judgment; probabilistic inversion; relative information; minimum entropy; interior point method; credit scoring; environmental modeling
History: Accepted by Paul Glasserman, stochastic models and simulation; received September 15, 2003. This paper was with the authors 6 months for 2 revisions.

1. Introduction

Quantitative models are used to support decision making in many different areas of application. Over the last decade, uncertainty analysis has become an increasingly important part of the process of analyzing and understanding such models. A growing field of physicists, engineers, mathematicians, and modelers is focusing on this cluster of issues, as is evidenced by the interdisciplinary SAMO conferences (1998, 2001, 2004; the most recent conference proceedings are to be published freely online via the Los Alamos National Laboratory website) and the special issue of *Reliability Engineering and System Safety* (Vol. 85, 2004) on representations of epistemic uncertainty.

In an uncertainty analysis, one works with a joint probability distribution over the model parameters instead of "nominal" values for these parameters. The objective of using such a distribution is to take account of uncertainties in the model parameter values and, to some extent, even uncertainties about the model itself. With a distribution on parameters the model does not make deterministic predictions; rather, a distribution over the model outcomes is obtained. Concepts such as model validation, model calibration, and sensitivity of the model outcomes to model parameters take on a new meaning when viewed from the perspective of uncertainty analysis.

We specifically work with deterministic, and possibly nonlinear, models. One example of such a model that we consider here is the environmental model used by Harper et al. (1994) for the spread of the lateral plume in atmospheric dispersion. The plume spread in the lateral (often denoted "y") direction σ_y is modelled as a power-law function of downwind distance z from the source of a release:

$$\sigma_y(z) = A_y z^{B_y}, \tag{1}$$

where the coefficients A_y and B_y depend on the stability of the atmosphere at the time of the release. Expression (1) is not derived from underlying physical laws; rather, the coefficients are fit to data from tracer experiments where available. Clearly, there will be more variables, such as wind variation, plume meander, surface roughness, and vertical wind profile, which will influence lateral plume spread. However, Expression (1) is regarded as capturing the uncertainty associated with lateral plume spread well enough.

While the model given here is deterministic, there is uncertainty about the appropriate values of the