Quantifying Operational Risk within Banks according to Basel II

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Management Summary

According to Basel II banks should have a better understanding and quantification of their market, credit and operational risks. A bank should determine how much capital should be reserved as a sufficient buffer against unexpected losses. This thesis is concentrated specifically on the quantification of operational risks within banks using an Advanced Measurement Approach (AMA). It introduces a method to quantify operational risks within banks, which fulfills the requirements of Basel II under an Advanced Measurement Approach (AMA). One of the main used AMAs is the Loss Distribution Approach, but it requires sufficient operational loss data, which is not always at hand. Other mainly used AMAs are the Scenario-based AMA and the Scorecard Approach, but these methods need a great deal of unvalidated guesswork, which makes them less reliable.

This thesis describes and applies a method originating from the technology sector, which quantifies and validates experts opinions using only a small amount of loss data. In this way extra loss data is generated to get a reliable quantification of the operational risks within banks. The method is called LEVER and stands for Loss Estimated by Validating Experts in Risk.

A case study has been performed at BANK B to apply and test the LEVER method. BANK B's core business is Custody Services, Clearing and Settlement of securities. Within Custody Services the risks of two operations are studied: conversions and coupon/dividend payments.

It can be concluded that it is possible to use the LEVER method as an AMA according to Basel II. The LEVER method can combine internal loss data, external loss data and scenario analysis to quantify the operational risks. Furthermore, it gives the opportunity to 'zoom in' on the operational risk drivers as much as required. Finally the capital requirement for operational risks can be determined.

Preface

This thesis describes the results of a graduation project, carried out at Pricewaterhouse-Coopers (PwC) in the period November 2003 until July 2004, as part of the study Applied Mathematics at Delft University of Technology (TUDelft). The research was performed under the supervision of dr. de Swart and ir. Rabouw at PwC and prof. dr. Cooke at TUDelft. The graduations committee is composed of:

prof. dr. Roger Cooke	(Delft University of Technology)
dr. Jacques de Swart	(PricewaterhouseCoopers)
ir. Frank Rabouw	(PricewaterhouseCoopers)
dr. Svetlana Borovkova	(Delft University of Technology)

I would like to thank PwC for giving me the opportunity to work within their organization. In particular I would like to thank Jacques de Swart for his support and his useful comments from which I have learned a lot. Further, I would like to thank Frank Rabouw for his excellent support and his intensive supervision on my work.

From TUDelft I would like to thank Roger Cooke for his supervision and for "pushing at the right buttons". Further, I would like to thank Maarten-Jan Kallen for his useful comments on my first version and Bernd Kraan for the good discussions about measuring operational risk.

Finally I would like to thank BANK B for giving me the opportunity to do my case study within their organization and for the great cooperation. I am very grateful to the 10 experts of BANK B, who joined the workshops and put hard effort in filling in the questionnaire.

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Chapter 1

Introduction

Due to economical crises in the 1970 and 1980s, the Basel Committee was established by the central bank Governors of the G-10 countries to supervise internationally active banks. In 1988 the Committee introduced a capital measurement system commonly referred to as the Basel Capital Accord. Among other things this accord required a minimum capital to cover credit risk. Capital serves as a cushion against unexpected losses, which stimulates the public confidence in the banking system. The technical challenge for both banks and supervisors has been to determine how much capital to reserve as a sufficient buffer against unexpected losses. By the end of 1992 a fully implementable credit risk measurement framework with a minimum capital standard of 8% was the result. The Committee supplemented the 1988 Accord's original focus on credit risk with requirements for exposures to market risks. This supplemented version of the Accord will be named the Basel I Accord.

More recently, the issue of financial stability in the wake of economic integration and globalization, as highlighted by the 1997 Asian crisis, played an important role to further develop and refine the Basel Accord. As a result the Committee issued in 1999 a proposal for a New Capital Adequacy Framework, also known as the Basel II Accord, to replace the 1988 Accord. The proposed capital framework consists of three pillars:

1. Minimum capital requirements

2. Supervisory review of an internal assessment process and capital adequacy

3. Effective use of disclosure

An essential difference between the Basel II and Basel I Accords is that an extra risk measurement is included in the Accord: operational risk. A financial institution is required not only to allocate capital to cover both their market risk and credit risk, but also their operational risk in order to comply with Basel II. In the Accord operational risk is defined as: *"The risk of losses resulting from inadequate or failed internal processes, people and systems or from external events"*. The reason to include this specific risk is to make financial institutions more financially stable, which leads to more integrity and transparency. The Basel II Accord will entail significant and far-reaching implications for the financial institutions. The banks should have representative measurements and quantifications of the risks coming with their operations by the end of 2006. Basel II has defined three approaches, which banks can use to quantify their operational risks:

1. The Basic Indicator Approach

2. The Standardized Approach
3. Advanced Measurement Approach (AMA)

The Basic Indicator and the Standardized approaches are targeted to banks with moderate exposure to operational risk losses. Large international financial institutions facing a substantial exposure to operational risk are expected to implement the AMA over time. Basel II gives the institutions a certain degree of freedom to model their operational risk given some boundary conditions.

1.1 Scope

This thesis concentrates specifically on the quantification of operational risks within banks using an Advanced Measurement Approach according to Basel II (see Figure 1.1 for a visual representation of the scope of this thesis). It is important for both the bank and its clients to map out its risks as accurate as possible. Large internationally operating banks in particular aspire to implement an AMA to quantify their operational risks. The main issues are the uncertain nature of operational risks and especially the lack of historical operational loss data. Most internationally active banks have started registering losses due to operations within the bank or due to external events since 2001. Modelling operational risks statistically requires sufficient data, which are until now in most cases not at hand. This brings us to the main question of this thesis: “How can a bank quantify its operational risk based on scarce historical loss data (internal or external)?”

1.2 Objective

The objective of this thesis is to develop a method to quantify operational risks within a bank when historical operational loss data is scarce. This method is called the LEVER method, which stands for Loss Estimated by Validating Experts in Risk, and is based on performance based expert judgement presented in Cooke (1991). The method needs only a small amount of loss data to validate experts and subsequently uses the validated expert opinions to quantify the operational risks. Based on the quantified operational risks a capital requirement can be determined. This brings us to the second main question of this thesis: “Is it possible to use the LEVER method as an AMA according to Basel II and determine the capital requirement for operational risks?” This question will be answered based on a case study.

1.3 Outline of the report

In the next chapter operational risk according to Basel II is discussed in more detail. The approaches to determine the capital requirement proposed in Basel II are presented. Chapter 3 treats existing modelling methods to quantify operational risks and builds a mathematical framework. It discusses a method to model low frequency/high impact events. Chapter 4 describes the LEVER method, which stands for Loss Estimated by Validating Experts in Risk, based on the ‘Classical model’ introduced in Cooke (1991). Results of a case study in which the LEVER method is applied at a bank are discussed in Chapter 5. Finally the capital requirement according to Basel II is determined in Chapter 6. This thesis finishes with conclusions and recommendations.

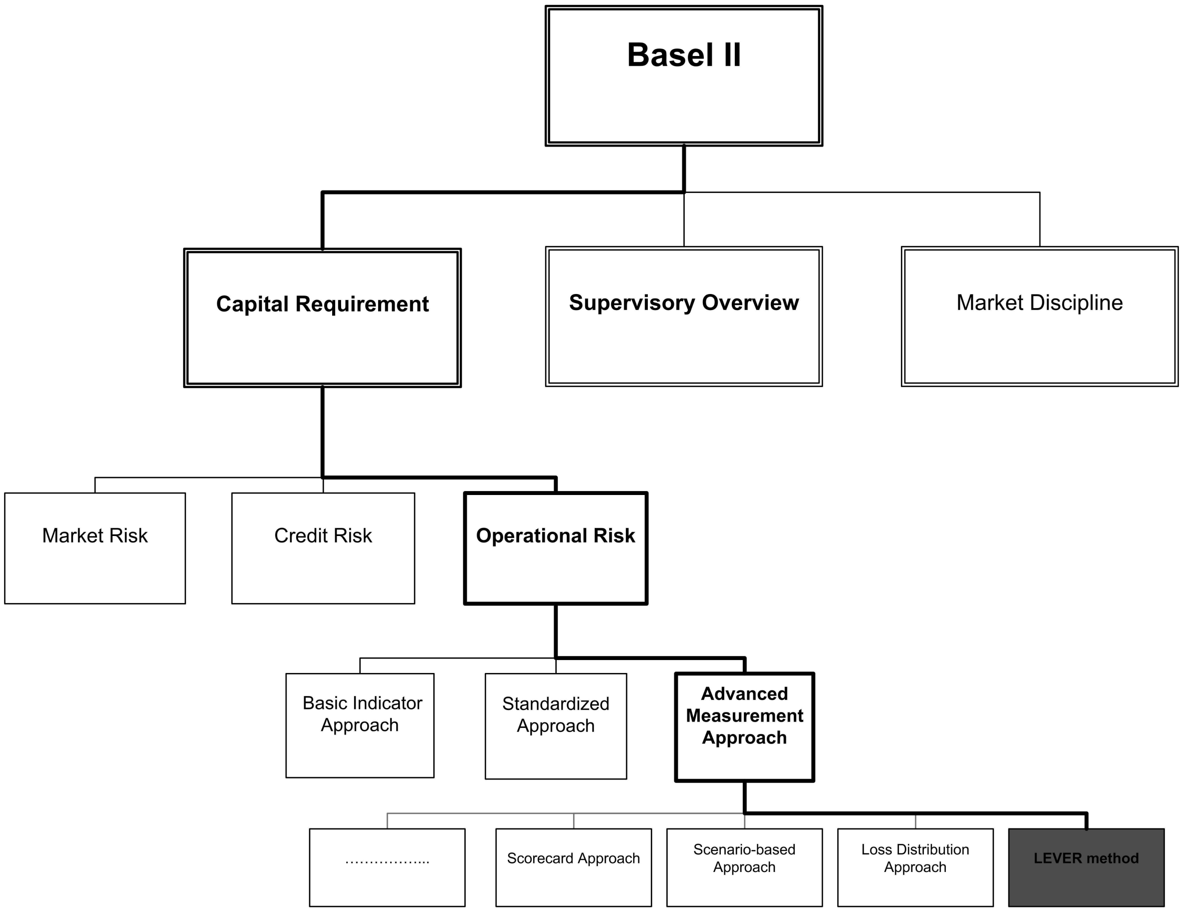

Figure 1.1: *Visual representation of the scope and the main objective "applying the LEVER method" of this thesis*

Chapter 2

Operational Risk according to Basel II

2.1 Introduction

The purposes of this chapter is not only to show the importance of operational risks within banks and why it is enclosed in the Basel II Accord, but also to present the main requirements and approaches on operational risk proposed by the Committee in Basel II. First a short description of the roots and the tasks of the Basel Committee on Banking Supervision is presented followed by the importance of operational risk. After the proposed approaches to quantify operational risk within a bank, the main quantitative standards of using an Advanced Measurement Approach are presented.

2.2 The Basel Committee on Banking Supervision

The Basel Committee was established by the central bank Governors of the Group of Ten countries at the end of 1974. Nowadays, the Committee's members come from twelve countries, which are represented by their central bank and also by the authority with formal responsibility for the prudential supervision of banking business where this is not the central bank. The Committee does not possess any formal supranational supervisory authority, and its conclusions do not have legal force. Rather, it formulates broad supervisory standards and guidelines and recommends statements of best practice in the expectation that individual authorities will take steps to implement them through detailed arrangements - statutory or otherwise - which are best suited to their own national systems. In this way, the Committee encourages convergence towards common approaches and common standards without attempting detailed harmonization of member countries' supervisory techniques.

As mentioned earlier the Basel Capital Accord from 1988, which has been progressively introduced in all countries with active international banks, is replaced by the New Capital Adequacy Framework (Basel II)) in 1999. A consultative document with a view to introducing the new framework at the end of 2006, was issued in April 2003 by the Committee in close collaboration with banks and industry groups (see Basel Committee on Banking Supervision (2003)).

Basel II consists of three pillars:

Pillar 1: Minimum capital requirements for market-, credit and operational risk.

Pillar 2: Supervisory review covering an institution's capital adequacy and internal assessment process.

Pillar 3: Market discipline through effective disclosure to encourage safe and sound banking practices.

The focus of this thesis is on the determination of the minimum capital requirements for operational risk as shown in Figure 1.1.

2.3 What is operational risk?

Imagine an employee of a bank on the trading department. Say, a client of the bank wants 10.000 stocks at €50 a share of a listed company. The trader accidently buys 100.000 stocks of that particular stock at €50 a share by mistyping. The moment the client or the bank discovers the error, which can be the same day or even a month after, the stock is worth €40. To rectify the mistake the bank must take a loss of €900.000. Such a loss can be defined as an operational loss, because the loss resulted from an inadequate operation of an employee of the bank. The bank's exposure to the risk of such a loss is called operational risk.

This leads us to a more general definition of operational risk as included in Basel II: *"The risk of losses resulting from inadequate or failed internal processes, people and systems or from external events"*. This definition includes legal risk, but excludes strategic and reputational risk. An example of an external event is a natural disaster, which can lead to enormous losses.

2.4 Why is operational risk included in Basel II?

Perhaps one of the most significant examples of an observation of a banks's operational risk, which is called a loss event, is the 827 million pound loss of the Barings Bank. This loss event was caused by one trader, Nick Leeson, and resulted in bankruptcy. Leeson took unauthorized speculative positions in futures and options on an unused error account and started loosing money in 1992, which accumulated and resulted in bankruptcy in 1996.

Among others, this incident was a provocation for the Basel Committee to include operational risk in Basel II. Moreover this is in line with the overarching goal of Basel II to promote the adequate capitalization of banks to encourage improvements in risk management, thereby strengthening the stability of the financial system.

2.5 Basel II requirements and approaches for operational risks

Once again, the Basel Committee does not possess supervisory authority and the Basel II Accord does not have legal force, but it is meant as guidelines for both the financial institutions and the authorities to encourage financial stability. Financial institutions that aspire to be Basel II-worthy should fulfil the Basel II requirements. For a total picture of the Basel II requirements one is referred to Basel Committee on Banking Supervision (2004). Within the scope of this thesis only the requirements for operational risks are of interest.

The Committee wants the banks to set aside a minimum capital to cover the operational losses depending on the exposure of the bank's operational risk. An approach to quantify these risks and determine the capital charge should be implemented at the end of 2006. The Basel Committee proposed the following three approaches for calculating operational risk capital requirement (CR) in a continuum of increasing sophistication and risk sensitivity:

1. The Basic Indicator Approach (BIA)
2. The Standardized Approach (SA)
3. Advanced Measurement Approaches (AMAs)

The Basic Indicator and the Standardized approaches are targeted to banks with moderate exposure to operational risk losses. Large international financial institutions facing a substantial exposure to operational risk are expected to implement over time an AMA. Banks are encouraged to move along the spectrum of available approaches as they develop more sophisticated operational risk measurement systems and practices. A bank will be permitted to use the BIA or the DA for some parts of its operations and an AMA for others provided certain minimum criteria are met. A bank will not be allowed to choose to revert to a simpler approach once it has been approved for a more advanced approach without supervisory approval (see Basel Committee on Banking Supervision (2004)). Below a more detailed description of the approaches is given.

2.5.1 Basic Indicator Approach

Banks using the Basic Indicator Approach (BIA) must hold capital for operational risk equal to a fixed percentage α of the average of positive annual gross income[1] (GI) over the previous three years. The capital requirement CR_{BIA} using the BIA can be expressed as follows:

$$CR_{BIA} = \alpha \times GI \tag{2.1}$$

where α is set at 15% by the Committee, corresponding the industry wide relative level of required capital.

2.5.2 Standardized Approach

The Standardized Approach (SA) represents a further refinement in the approach of the operational risk capital by dividing the banks' activities into standardized business lines. Within each business line, the capital requirement CR_{SA} is calculated by multiplying the average gross income $GI(i)$, generated by a business line i, over the previous three years by a factor β_i assigned to that business line (see Table 2.1). We have

$$CR_{SA}(i) = \beta_i \times GI(i) \tag{2.2}$$

The total capital requirement is then the summation of the regulatory capital requirements across each of the business lines, $i, \ldots, I$ (in this case $I = 8$):

$$CR_{SA} = \sum_{i=1}^{I} CR_{SA}(i) = \sum_{i=1}^{I} \beta_i \times GI(i) \tag{2.3}$$

[1]Gross income is defined by national supervisors and/or national accounting standards as:

$$\text{Gross Income} = \text{Net Interest Income} + \text{Net Non-Interest Income}$$

Business Lines	Beta factors
Corporate Finance	$\beta_1 = 18\%$
Trading & Sales	$\beta_2 = 18\%$
Retail Banking	$\beta_3 = 12\%$
Commercial Banking	$\beta_4 = 15\%$
Payment & Settlement	$\beta_5 = 18\%$
Agency Services	$\beta_6 = 15\%$
Asset Management	$\beta_7 = 12\%$
Retail Brokerage	$\beta_8 = 12\%$

Table 2.1: *Values of β per business line*

2.5.3 Advanced Measurement Approaches

Under the Advanced Measurement Approaches (AMAs), the regulatory capital requirement will equal the risk measure generated by the bank's internal operational risk measurement system using the quantitative and qualitative criteria for the AMAs discussed in The New Basel Capital Accord (p.125–p.131, 2003). Use of an AMA is subject to supervisory approval. In the next section the main quantitative criteria for the scope of this thesis are presented.

2.6 Basel II quantitative standards for AMAs

This section describes a series of standards for AMAs proposed by the Committee of Banking Supervision in Basel II (2003).

Soundness standard

- Given the continuing evolution of analytical approaches for operational risk, the Committee is not specifying the approach or distributional assumptions used to generate the operational risk measure for regulatory capital purposes. However, a bank must be able to demonstrate that its approach captures potentially severe 'tails' loss events. Whatever approach is used, a bank must demonstrate that its operational risk measure meets a soundness standard comparable to a one year holding period and a 99,9% confidence interval.

- Banks must have and maintain rigorous procedures for operational risk model development and independent model validation. The Committee will review progress by the end of 2006.

Detailed criteria

- Any internal operational risk measurement system must be consistent with the scope of operational risk defined in Basel II (see definition in section 2.3) and the loss event types shown in Table 2.3.

- A bank's risk measurement system must be sufficiently 'granular' to capture the major drivers of operational risk.

Business Lines
Corporate Finance
Trading & Sales
Retail Banking
Commercial Banking
Payment & Settlement
Agency Services
Asset Management
Retail Brokerage

Table 2.2: *Categorization of Business Lines*

- The bank may be permitted to use correlations between loss types or business lines if the correlations assumptions are validated.
- Any risk measurement system must include the use of internal data, external data, scenario analysis and factors reflecting the business environment and internal control systems.

Internal data

- Banks must track internal loss data, which is an essential prerequisite to the development and functioning of a credible operational risk measurement system.
- Internal loss data is most relevant when they are clearly linked to a bank's current business activities, technological processes and risk management procedures.
- A bank must be able to map its historical loss data into the regulatory categories, business lines and loss types (see Table 2.2 and 2.3). For a mapping of business lines and a detailed loss type classification the reader is referred to Annex 6 and 7 of Basel Committee on Banking Supervision (2004).
- The internally generated operational risk measures must be based on an observation period of a minimum of 5 years. However, when a bank first moves to the AMA, a three-year historical data window is acceptable.
- A bank must have an appropriate minimum gross loss threshold for internal loss data collection, for example €10.000.
- Aside from information on gross loss amounts, a bank should collect information about the date of the event, any recoveries of gross loss amounts, as well as some descriptive information about the drivers or causes of the loss event. The bigger the event the better the explanation should be.

External data

- A bank's operational risk measurement system must use relevant external data.
- A bank must have a systematic process for determining the situations for which external data must be used and the methodologies used to incorporate the data.

Loss Types
Internal Fraud
External Fraud
Employment Practices & Workplace Safety
Clients, Products & Business Practices
Damage to Physical Assets
Business Disruption & System Failures
Execution, Delivery & Process Management

Table 2.3: *Categorization of Loss Types*

Scenario analysis

- A bank must use scenario analysis of expert opinion in conjunction with external data to evaluate its exposure to high severity events.

- A bank should use scenario analysis to evaluate potential losses arising from multiple simultaneous operational risk loss events.

Business environment and internal control factors

- A bank's risk measurement framework must capture key business environment and internal control factors that can change its operational risk profile.

- The choice of each factor needs to be justified as a meaningful driver of risk, based on experience and involving the expert judgement of the affected business areas.

Remarks

A number of remarks on these standards are in order. To start with, the determination of a threshold of for instance €10.000 can give problems. If a bank has many losses just below the €10.000 and if you would sum these up the total could be €30 million. Say the sum of the losses greater than €10.000 could also give €30 million. In that case 50% of the losses would be left out of the operational risk measurement, which would result in a totally different model outcome. A bank should collect all the loss data they can get whatever the amount is and choose a threshold based on all the loss data.

A second remark is on the thorough explanation that must be given for operational losses. At banks where millions of transactions are processed on a daily basis, the number of operational losses might reach hundreds of thousands considering a large error rate. They are supposed to explain all the losses in great detail, but that will cost time and therefore extra money.

Chapter 3

Modelling Methods for Operational Risk

3.1 Introduction

The continuum of methods for modelling operational risks within banks are growing rapidly at this moment, because large internationally active banks in particular should have implemented an AMA at the end of 2006. In this chapter the most popular modelling methods among the Advanced Measurement Approaches are introduced. First the main definitions, the basic assumptions and notations are presented followed by a description of the Loss Distribution Approach (LDA). Then commonly used techniques to estimate the frequency and severity distributions, which are important components of the LDA, are discussed followed by a part on extreme value theory. This chapter finishes with a section on two qualitative approaches and a section on the shortcomings of the approaches presented in this chapter.

3.2 Definitions, assumptions and notations

Throughout this thesis a bank is assumed with different business lines $i = 1, \ldots, I$, as presented in Table 2.2, which wants to comply with the Basel II requirements. First a few definitions are given according to Basel II:

operational risk: The risk of losses resulting from inadequate or failed internal processes, people and systems or from external events.

operational loss event: An event that entails a financial impact (in euros) due to the banks' operational risk. An operational loss event is also called a loss event[1].

operational loss: A financial impact due to a loss event.

Further, assume that operational losses are classified in loss types $j = 1, \ldots, J$, as presented in Table 2.3. Let $N_{t,\tau}(i,j)$ be a discrete non-negative random variable representing the number of loss events falling within business line i and loss type j in time interval $[t, t+\tau)$.

[1] According to Basel II, loss events should have a threshold u, which means that a loss event less than u (e.g. €10.000) will not be taken into account

Let $p_{ij}^{t,\tau}(k)$ be the probability of k loss events in interval $[t, t+\tau)$, which can be written as $p_{i,j}^{t,\tau}(k) = \mathrm{P}(N_\tau(i,j) = k)$ for $k = 0, 1, 2, \ldots$. The discrete cumulative distribution function of $N_{t,\tau}(i,j)$ is called the (loss) *frequency distribution* and corresponds to:

$$\mathcal{F}_{i,j}^{t,\tau}(n) = \mathrm{P}(N_{t,\tau}(i,j) \leq n) = \mathrm{P}(N_{t,\tau}(i,j) = 0) + \cdots + \mathrm{P}(N_{t,\tau}(i,j) = n) = \sum_{k=0}^{n} p_{i,j}^{t,\tau}(k) \quad (3.1)$$

Let $X_k(i,j)$ be a non-negative continuous random variable, which represents the financial impact (in euro) of loss event k, $k = 1, \ldots, N_{t,\tau}(i,j)$, for business line i and loss type j. $X_k(i,j)$ has a probability density function (also called probability distribution) $s_{X_k(i,j)}(x)$ for $k = 1, \ldots, N_{t,\tau}(i,j)$. The cumulative distribution function (also called distribution) of $X_k(i,j)$ is called the (loss) *severity distribution* and is denoted by $S_{k;i,j}(x)$, where x is an amount of loss in euros, so:

$$S_{k;i,j}(x) = \mathrm{P}(X_k(i,j) \leq x) = \int_0^x s_{X_k(i,j)}(z)dz \quad (3.2)$$

Further, assume $X_1(i,j), \ldots, X_{N_\tau(i,j)}(i,j)$ are independent identically distributed (iid) and independent of the number of events. This is reasonable to assume, because the impact a loss event can have is mostly not related to another event with a different impact.

Finally, it is assumed throughout the thesis that τ equals one year, i.e. $\tau = 1$, because the capital requirement is based on annual risk exposure (see the standards of section 2.5.3). τ will be suppressed from notation for a better legibility of the formulas.

3.3 Loss Distribution Approach

The Loss Distribution Approach (LDA) uses frequency and severity distributions based on operational losses to quantify operational risk and is at this moment one of the most used and discussed (see for instance Cruz (2002) and Frachot, Georges, and Roncalli (2001)) approach under the AMAs to measure operational risks. The LDA mainly relies on historical internal loss data, but it can also be used with external data or a mix of both. The Committee gave the following description of the 'internal methodology' in Basel Committee on Banking Supervision (2001a): '*Under the LDA, the bank estimates, for each business line and loss type cell, the distribution functions of the single event impact, which gives the severity distribution, and the event frequency for the next (one) year using its internal data, and computes the distribution function of the cumulative operational loss.*'

'The single event impact' means the financial impact on a bank caused by an event k that occurred within business line i and loss type j ($X_k(i,j)$). This event impact should be seen as a random variable with an underlying (cumulative) distribution. Based on observations a distribution function can be estimated as will be discussed in section 3.4. The obtained distribution is called the (loss) severity distribution ($S_{k;i,j}(x)$) or the impact distribution.

The 'event frequency' means the number of times a loss event took place independent of the impact that event had, corresponding a business line i and a loss type j in one year ($N(i,j)$). The obtained distribution is called the (loss) frequency distribution ($\mathcal{F}_{i,j}(n)$).

When the severity and frequency distributions for all business lines and loss types shown in Table 2.2 and 2.3 are obtained, these distributions will be aggregated to one distribution using a Monte Carlo simulation for instance. With this aggregated distribution a capital charge or capital requirement to cover the operational risks can be calculated.

To put it in mathematical terms we have the variable $X_k(i,j)$ representing the financial impact of loss event k for business line i and loss type j with $k = 1, \ldots, N(i,j)$, where $N(i,j)$ is the number of loss events in a year ($\tau = 1$). Then the total operational loss for the business line i and the loss type j in a year denoted by $L(i,j)$, is:

$$L(i,j) = \sum_{k=1}^{N(i,j)} X_k(i,j) \tag{3.3}$$

To keep the formulas more legible, the business line and loss type combination (i,j) will be suppressed from notation, so equation (3.3) turns in:

$$L = \sum_{k=1}^{N} X_k$$

The expected loss can be written as:

$$\mathbb{E}(L) = \mathbb{E}\left(\sum_{k=1}^{N} X_k\right) \tag{3.4}$$

Because of the assumptions that $X_1, \ldots, X_N$ are independent identically distributed and that these variables are independent of the number of events (see previous section), the expected loss can be written as:

$$\mathbb{E}(L) = \mathbb{E}[\mathbb{E}(L|N)] = \mathbb{E}\left[\mathbb{E}\left(\sum_{k=1}^{N} X_k\right)\right] = \mathbb{E}\left[\left(\sum_{k=1}^{N} \mathbb{E}(X_k)\right)\right] = \mathbb{E}[N \cdot \mathbb{E}(X_1)] = \mathbb{E}(N) \cdot \mathbb{E}(X_1) \tag{3.5}$$

This means the expected loss only depends on expected loss of one event and the expected number of events. Let $F_{i,j}(x)$ be the aggregated distribution of $L(i,j)$ then (leaving (i,j) out of notations):

$$\begin{aligned} F(x) &= P(L \leq x) \\ &= P(N=0) + P(N=1, X_1 \leq x) + P(N=2, X_1 + X_2 \leq x) + \cdots \\ &= P(N=0) + P(N=1)P(X_1 \leq x) + P(N=2)P(X_1 + X_2 \leq x) + \cdots \qquad (3.6) \\ &= p(0) + p(1)S^1(x) + p(2)P(X_1 + X_2 \leq x) + \cdots \qquad (3.7) \end{aligned}$$

where $p(k)$ and the severity distribution $S_k(x)$ for $k = 0, 1, 2, \ldots$ are as defined in section 3.2. In equation (3.6) the independence assumption between the number of events and the financial impact of each event is used. In equation (3.7) the definitions from section 3.2 are used as well.

For the term $P(X_1 + X_2 \leq x)$ and its companions, the definition of convolution should be introduced:

Definition 1. *For independent non-negative random variables X_1, X_2 with densities s_{X_1} and s_{X_2}, the convolution $s_{X_1} \star s_{X_2}$ is the density of $X_1 + X_2$ and is given by:*

$$s_{X_1+X_2}(x_1) = (s_{X_1} \star s_{X_2})(x_1) = \int_0^{x_1} s_{X_1}(x_1 - x_2)s_{X_2}(x_2)dx_2 \tag{3.8}$$

With this definition, the aggregated distribution $F(x)$ of L can be written as:

$$F(x) = \begin{cases} p(0) + \sum_{k=1}^{\infty} p(k)S^{k\star}(x) & x > 0 \\ p(0) & x = 0 \end{cases} \tag{3.9}$$

where $\star$ is the convolution operator on the distribution functions and $S^{n\star}$ is the n-fold convolution of S with itself, so

$$\begin{aligned} S^{1\star} &= S \\ S^{2\star} &= S \star S \\ S^{3\star} &= S^2 \star S \\ &\vdots \\ S^{n\star} &= S^{n-1} \star S \end{aligned} \tag{3.10}$$

In general, there does not exist a closed-formula to compute the aggregated distribution $F(x)$, i.e. $F(x)$ can not be solved analytically. Therefore a numerical method should be used. It is possible to use numerical integration for solving (3.9), but there is also a numerical method, which is very widely used, that combines distributions by simulations, called Monte Carlo simulation. For more detailed information about Monte Carlo simulation the reader is referred to Fishman (1996). From this aggregated loss distribution a capital requirement can be determined as will be shown in Chapter 6.
In the next section a technique is presented to estimate the distributions for frequency and severity based on underlying loss data. Thereafter a method called extreme value theory is discussed, which is often used to refine the LDA (see Cruz (2002) and King (2001)). This method models the low frequency/ high impact events.

3.4 Technique to estimate distributions for frequency and severity

In the previous section a general mathematical description is given for the random variables N and X with their distribution functions $\mathcal{F}$ and S respectively and the aggregated loss distribution $F(x)$. In this section a technique is discussed to estimate the frequency ($\mathcal{F}$) and severity (S) distributions for an underlying loss database using respectively the Poisson and the Lognormal distribution. It seems that these distributions model high frequency/low impact events quite well, but it has shortcomings when modelling low frequency/high impact events. To anticipate this shortcoming a method called extreme value theory is described to refine the LDA and deal with low frequency/high impact events.

3.4.1 Estimation of the frequency distribution using Poisson

There are many discrete distributions that can be used to model the frequency of operational loss events. However, the Poisson distribution is most commonly used in the LDA.

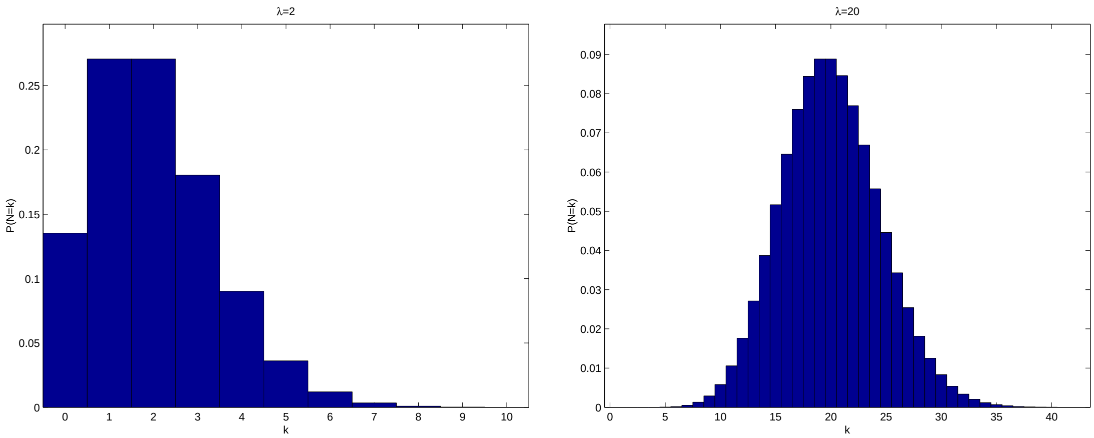

Figure 3.1: *Poisson probability distribution for $\lambda = 2$ (left) and $\lambda = 20$*

A random variable N has a Poisson probability distribution with intensity $\lambda > 0$ if N can only take values of $\{0, 1, 2, \ldots\}$ and if:

$$\mathrm{P}_\lambda(N = k) = p(k) = \frac{\lambda^k}{k!} e^{-\lambda}, \quad k = 0, 1, 2, \ldots \tag{3.11}$$

The parameter λ can be interpreted as the mean number of loss events per year. Figure 3.1 shows plots for different λ's. Using equation (3.1) the discrete cumulative distribution function (step function) can be written as:

$$\mathcal{F}_\lambda(n) = \sum_{k=0}^{n} p(k) = e^{-\lambda} \sum_{k=0}^{n} \frac{\lambda^k}{k!} \tag{3.12}$$

The parameter λ should be estimated based on an underlying data set, which can be internal or external frequency data. A frequently used estimator is the maximum likelihood estimator, but there are many other methods to estimate the parameters of a parametric statistical approach. When an estimation for λ, i.e. $\hat{\lambda}$, is obtained the so-called 'best fit' of the Poisson distribution to the underlying data set can be used as a frequency distribution $\mathcal{F}_{\hat{\lambda}}$.

3.4.2 Estimation of the severity distribution using Lognormal

The most commonly used distribution to approximate loss severity distributions is the Lognormal distribution. One of the reasons is that the Lognormal distribution, which is a transformed Normal distribution, can not take negative values. If the random variable $X = e^Z$ and Z would be normally distributed, then the distribution of X is said to be Lognormal. The Lognormal density function can be obtained from the Normal density function using this transformation, resulting in:

$$s_{\mu\sigma}(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\log x - \mu}{\sigma}\right)^2} \qquad \text{for } x > 0 \tag{3.13}$$

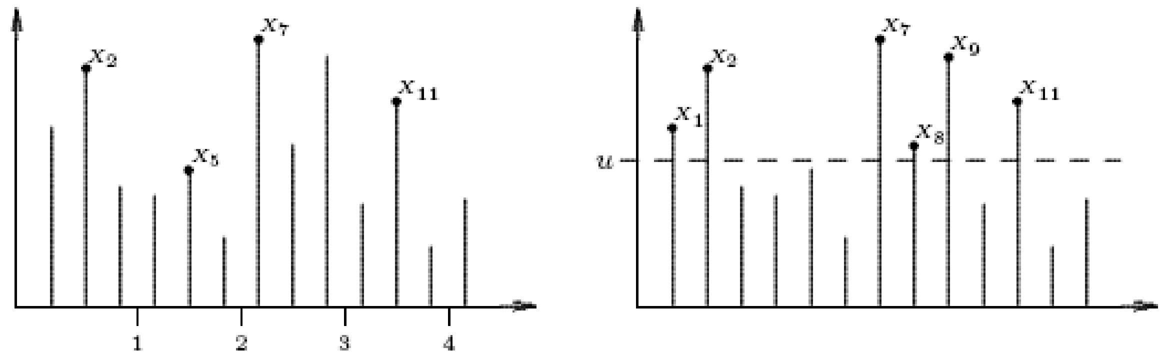

Figure 3.2: *Block-maxima (left) and excesses over a threshold u*

with parameters $\mu > 0$ and $\sigma > 0$. The Lognormal distribution function is the integral of the density function, given by:

$$S_{\mu\sigma}(x) = \int_0^x \frac{1}{t\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\log t - \mu}{\sigma}\right)^2} dt \tag{3.14}$$

3.4.3 Extreme Value Theory

To refine the LDA extreme value theory (EVT) is mostly used to handle low frequency/high impact events, typical in operational risk. EVT offers a parametric statistical approach to approximate high operational losses (high impact events), the so-called 'tails' of a distribution, such that a catastrophic loss can be taken into account. Unfortunately the probability such an event can occur is not always zero as the Barings Bank showed (see section 2.4). EVT deals only with the extreme values and ignores the majority of the underlying data and its measures in order to provide better estimates of the 'tails'. Its roots are in the physical sciences and since the 1990s it has been applied to insurance.

There are many reports written on extreme value theory applied on risk management issues (e.g. Embrechts and Samorodnitsky (1998)) and on value at risk calculations (e.g. Këllezi and Dilli (2000) and Fernandez (2003)). Among others, the books of Cruz (2002) and King (2001) treat extreme value theory in the scope of operational risk. In this thesis the main findings will be presented.

There are two principal distributions that are used in extreme value modelling:

1. Generalized extreme value distribution (GEV)
2. Generalized Pareto distribution (GPD)

The GEV considers the maximum (or minimum) the variable takes in successive periods, for example months or years. These selected observations constitute the extreme events, also called block (or per-period) maxima (e.g. the highest operational loss in a week or month). In the left panel of Figure 3.2, the observations x_2, x_5, x_7 and x_{11} represent these block maxima for four periods with three observations. The GPD focuses on the realizations (e.g. operational losses), which exceed a given (high) threshold (e.g. €10.000). The observations x_1, x_2, x_7, x_8, x_9 and x_{11} in the right panel of Figure 3.2, all exceed the threshold u and

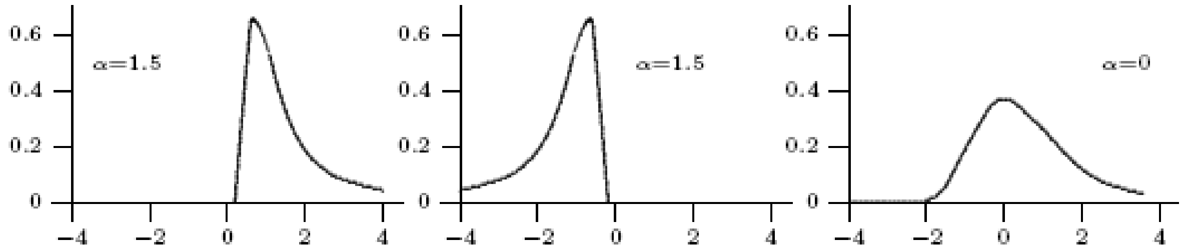

Figure 3.3: *From left to right, the Fréchet, Weibull and Gumbel densities*

constitute extreme events. Below, the fundamental theoretical results underlying the block maxima and the threshold method are presented.

Generalized extreme value distribution (GEV)

The limit law for block maxima, which is denoted by M_n, with n the size of a block (period), is given by the following theorem:

Theorem 1 (Fisher and Tippett (1928), Gnedenko (1943)). *Let (X_n) be a sequence of independent and identically distributed (iid) random variables and let $M_n = \max\{X_1, \ldots, X_n\}$. If there exists constants $c_n > 0$, $d_n \in \mathbb{R}$ and some non-degenerate distribution function $H(x)$ such that*

$$P\left(\frac{M_n - d_n}{c_n} \leq x\right) \to H(x) \qquad \textit{for } n \to \infty \tag{3.15}$$

then $H(x)$ belongs to one of the three standard extreme value distributions:

$$\Phi_\alpha(x) = \begin{cases} 0, & x \leq 0 \\ e^{-x^{-\alpha}}, & x > 0,\ \alpha > 0 \end{cases} \tag{3.16}$$

$$\Psi_\alpha(x) = \begin{cases} e^{-(-x^{-\alpha})}, & x \leq 0,\ \alpha > 0 \\ 0, & x > 0 \end{cases} \tag{3.17}$$

$$\Lambda_\alpha(x) = e^{-x^{-\alpha}}, \quad x \in \mathbb{R} \tag{3.18}$$

$\Phi_\alpha(x)$ is called the Fréchet distribution, $\Psi_\alpha(x)$ is the Weibull distribution and $\Lambda_\alpha(x)$ the Gumbel distribution. The shape of the probability density functions for the standard Fréchet, Weibull and Gumbel distributions is given in Figure 3.3. Jenkinson (1955) and von Mises (1936) suggested the following one-parameter representation:

$$H_\xi(x) = \begin{cases} e^{-(1+\xi x)^{-1/\xi}}, & \text{if } \xi \neq 0 \\ e^{-e^{-x}}, & \text{if } \xi = 0 \end{cases} \tag{3.19}$$

of these three standard distributions, with x such that $1 + \xi x > 0$. Equation (3.19) is called the generalized extreme value distribution (GEV) and is obtained by setting $\xi = \frac{1}{\alpha}$ for the Fréchet distribution, $\xi = -\frac{1}{\alpha}$ for the Weibull distribution and by interpreting the Gumbel distribution as the limit case for $\xi = 0$.

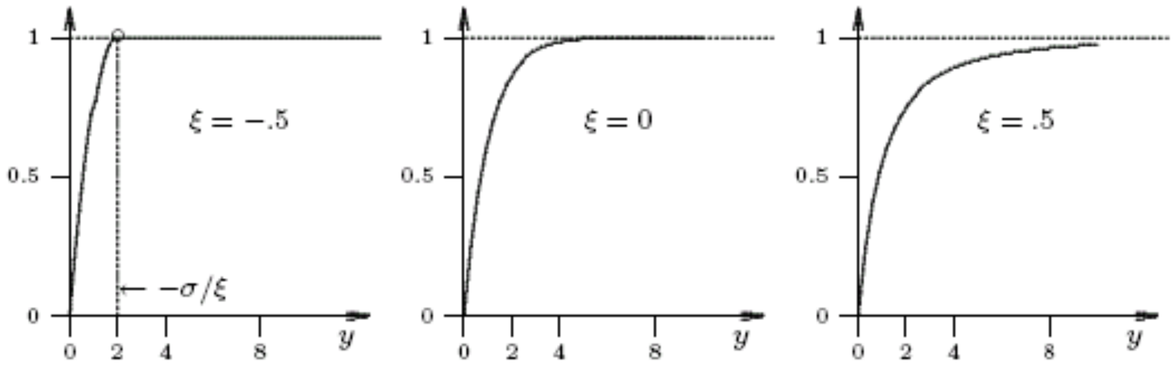

Figure 3.4: *Shape of the generalized Pareto distribution $G_{\xi\sigma}$ for $\sigma = 1$*

Generalized Pareto distribution (GPD)

In practice, modelling all block maxima is wasteful if other data on extreme values are available. Therefore, a more efficient approach, called the *peaks over threshold* method (POT), is to model the behavior of extreme values above a certain threshold.

The goal is to estimate a conditionalized distribution function S_u, called conditional excess distribution function, of values of x above a certain threshold u. So:

$$S_u(y) = \mathrm{P}(X - u \leq y | X > u), \qquad 0 \leq y \leq x_R - u \tag{3.20}$$

where $y = x - u$ are the excesses and $x_R \leq \infty$ is the right endpoint of S. S_u can be written in terms of S as follows:

$$S_u(y) = \frac{S(u+y) - S(u)}{1 - S(u)} = \frac{S(x) - S(u)}{1 - S(u)} \tag{3.21}$$

The estimation of S_u might be difficult, since it is mostly based on very little observations. The following theorem can be very helpful:

Theorem 2 (Pickands (1975), Balkema and de Haan (1974)). *For a large class of underlying distribution functions S the conditional excess distribution function $S_u(y)$, for u large, is well approximated by*

$$S_u(y) \approx G_{\xi\sigma}(y)$$

where

$$G_{\xi\sigma} = \begin{cases} 1 - \left(1 + \frac{\xi}{\sigma} y\right)^{-1/\xi}, & \text{if } \xi \neq 0 \\ 1 - e^{-y/\sigma}, & \text{if } \xi = 0 \end{cases} \tag{3.22}$$

for $0 \leq y \leq (x_R - u)$, is the so-called generalized Pareto distribution (GPD).

Figure 3.4 illustrates the shape of the generalized Pareto distribution $G_{\xi\sigma}$ when ξ, called the shape parameter or tail index, takes a negative, a positive and a zero value. The scaling parameter σ is kept equal to one. As one can not fix an upper bound for financial losses

Figure 3.4 shows that only distributions with shape parameter $\xi \geq 0$ are suited to model 'heavy tailed[2]' distributions.

In Chapter 6 the POT method will be applied to approximate the upper-quantiles of the loss distribution.

3.5 Qualitative Approaches

There are many banks using (or implementing) more qualitative approaches to measure operational risk. The most popular among the qualitative approaches are the scenario-based approach and the scorecard approaches.

The scenario-based approach (sbAMA) is based on forward looking assessments of operational risks. The main thought of this approach is that loss data alone is an inadequate indication of current operational risk and should be adjusted by expert judgement. The sbAMA overlaps with the LDA and the scorecard approaches, i.e. it lies towards the middle of the spectrum. It overlaps the LDA in the sense that it models distribution by scenario type and business unit and the scorecard approach in the sense that it utilizes forward looking inputs.

The scorecard approaches come in many different appearances, but are all based on an assessment of specific operational risk drivers and controls. The Basel Committee gives a description of the scorecard approach in the working paper of September 2001 Basel Committee on Banking Supervision (2001b) and gives the banks a certain freedom in using this approach.

The main purpose of this approach is that banks after determining their initial operational risk capital (using for instance LDA) allocate the capital by means of scorecards to the loss types and the business lines on a top-down basis. These scorecards are mostly in the form of a questionnaire, where experts should give point estimates on measures of exposure or indicate risks. The scorecards attempt to capture the underlying risk profile and risk control environment of the various business lines. The difference between a statistical approach is that the scorecards intend to bring a forward-looking component, such that the severity of future losses can be reduced.

At this moment many banks are doing research on what approach they should use to quantify their operational risk. Many internationally active banks use a combination of a qualitative approach and the LDA. During last Basel Committee's Risk Measurement Group Conference (2003) on "Leading Edge Issues in Operational Risk Measurement" many large banks presented their approaches. The reader is referred to these presentations for more information, because this does not belong to the scope of this thesis.

The main issue which should be pointed out is that the qualitative approaches contain a great deal of guesswork compared to the quantitative approaches. To be more precise, if so-called 'experts' for instance fill in a questionnaire to quantify the impact of a scenario or to estimate the future loss on a given business unit, the 'experts' only give point estimates,

[2]The ordering of tails of distribution functions F_1 and F_2 will be defined by:

$$1 - F_1 > 1 - F_2 \Longleftrightarrow \lim_{x\to\infty} \frac{1 - F_1(x)}{1 - F_2(x)} > 1$$

then $1 - F_1$ has a *heavier tail* than $1 - F_2$ and hence describes a heavier risk than $1 - F_2$. The tails are equivalent if the above limit equals one.

i.e. ‘best guess’. To give a point estimate for an uncertain matter does not make sense (see for a discussion Cooke (1991)).

In the following chapter the LEVER method is presented, which deals with this shortcoming of the qualitative approaches. The LEVER method combines historical loss data and forward-looking assessments. It also makes use of expert opinions, but it assesses a range for uncertain quantities instead of a point estimate. Moreover, it first calibrates the ’experts’ based on their expertise of operational risk.

3.6 Shortcomings

The first main shortcoming of using statistical techniques for quantifying operational risk is the lack of operational loss data. The second is that LDA does not look forward and does not really reckon with the current situation.

Using a qualitative approach such as the scenario-based approach or a scorecard approach needs a great deal of guesswork, which makes it less reliable.

Thus, a method should be developed to fill the gaps of the existing loss distribution approach and the qualitative approaches. In the next chapter a technique mainly used in engineering fields is described to quantify uncertainties, which is the nature of operational risk. Chapter 5 introduces the LEVER method, a method to quantify operational risk within banks, which does not require much loss data (internal or external) to give a reliable quantification of the operational risks. In addition, using this method forward-looking scenarios can be quantified in a reliable way.

Chapter 4

Performance Based Expert Judgement

4.1 Introduction

As concluded in the previous Chapter the statistical methods and the scorecard approach have some shortcomings. The main issue of using a statistical approach to quantify operational risk is the lack of historical operational loss data. The issue of using a qualitative approach is the guesswork. So an (additional) method should be used that solves these shortcomings.

Expert judgement techniques are useful for quantifying models in situations in which, either because of cost, technical difficulties or the uniqueness of the situation under study, it has been impossible to make enough observations to quantify the model with "real data". Cooke developed an expert judgement technique based on the performance of these experts in Cooke (1991), which is mostly used for risk analysis in the nuclear and technology sector. It is a method to quantify uncertainties. Uncertainty is that which disappears when becoming certain. In practical scientific and engineering contexts, certainty is achieved through observation, and uncertainty is that which is removed by observation. Operational risks carry much uncertainty with them. For instance, the Barings Bank became certain in 1996 that they had to cope with large operational risks, because of the observation of the big loss due to the deeds of Nick Leason.

Due to the fact that operational risk observations (loss data) did not take place in many financial institutions, performance based expert judgement can be used to help quantifying operational risks. Expert judgement has always played a large role in science and engineering and is becoming an important part in finance. Increasingly, expert opinions are recognized as just another type of (scientific) data, and methods are developed for treating it as such. This Chapter will present a method to combine expert opinions based on the performance of the experts. First the method is discussed on a high level. The following subsections treat the method in more detail.

4.2 The Classical model

To combine expert judgements based on performance the 'Classical model' developed in Cooke (1991) can be used. The 'Classical model' constructs a weighted combination of subjective probability assessments of experts. The main aim of this method is to achieve rational consensus. The scientific foundation for subjective probability comes from the theory of rational decision making (for more detail of rational decision theory see Savage (1954) and Cooke (1991)). Since every rational individual has his own subjective probability it is necessary to find a way of building consensus. The fundamental principle of the 'Classical model' is to use performance based weighting to combine expert distributions by testing the expertise of the expert on the field of interest using so-called seed questions. These are questions for which the answers are known to the analysts but not to the experts. The main assumption of this method is that the future performance of an expert can be extrapolated from the past performance, i.e. if the expert performed well in the past the expert is likely to perform well in the future.

The subjective probability assessments are obtained by means of a questionnaire. This questionnaire exists of query variables, which include target variables and seed variables, for which the experts should give their quantile assessments. Target variables are the variables of interest and seed variables are the variables whose true values or realizations are known post hoc. The input quantiles can be chosen arbitrarily, but in most risk analysis applications and in the case study treated in the next chapter the 5%, 50% and 95% quantiles are used. Experts' assessments are scored with respect to calibration or statistical likelihood and informativeness, which will be discussed in the next subsections. These scores are used to compute weights which satisfy an "asymptotic strictly proper scoring rule". A scoring rule is called strictly proper if an expert receives his maximal expected score, in the long run, if and only if his stated assessment corresponds to his true belief (see Cooke (1991) (Chapter 9) for mathematical details of (strictly) proper scoring rules). The following subsections will describe the 'Classical model' in more detail to get a better grip on the method and on how it is used during the case study. This is also done in Bedford and Cooke (2001) and especially in Cooke (1991).

4.2.1 Calibration

Calibration denotes a correspondence between subjective probabilities and observed outcomes. In statistical terms the calibration of experts is done by testing the hypothesis: "Are the assessments given by the expert true", significantly using seed variables. The calibration score is then the p-value, representing the probability of rejecting the hypothesis wrongly.

Suppose experts are asked for their uncertainties over a number M, say $M = 100$, of seed variables, i.e. the outcome is known to the analyst but not to the expert. The expert gives his 5%-, 50%- and 95% quantile for each (for him) uncertain variable, so that for each seed variable four intervals, are given: 0%-5%, 5%-50%, 50%-95% and 95%-100%. A maximum calibration score is now received when 5 of the realizations of the 100 seed variables fall into the corresponding 0% to 5% interval of the expert, 45 realizations fall into the corresponding 5% to 50% interval, 45 realizations fall into the 50%-95% interval and 5 fall into the 95%-100% interval.

Let $p = (p_0, \ldots, p_n)$ be a probability mass function with n the number of assessed quantiles per query variable. Let $s = (s_0, \ldots, s_n)$ denote a sample mass function generated by M independent samples from the mass function p. Assume $p_i > 0$ for $i = 0, \ldots, n$, then the difference between the two mass functions can be measured by using the relative information of s with respect to p:

$$I(s,p) = \sum_{i=0}^{n} s_i \ln\left(\frac{s_i}{p_i}\right) \tag{4.1}$$

$I(s,p)$ is always nonnegative and takes its minimal value of 0 if and only if $s = p$. So a good expert should have a relative information score close to 0. $I(s,p)$ can be understood as a measure of the surprise someone experiences when he learns s while his belief was p.

It is known that for large M the distribution of $2M$ times the relative information is approximately χ^2 distributed with $n-1$ degrees of freedom (see Hoel (1971)), so

$$2MI(s,p) \sim \chi^2_{n-1}, \text{ as } M \to \infty \tag{4.2}$$

The calibration of expert e, $e = 1, \ldots, E$, with E the total number of experts, is then defined as the probability of getting an information score worse than that actually obtained under the assumption that the experts' true mass function is $(p_0, \ldots, p_n)$:

$$C(e) = 1 - \chi^2_{n-1}(2MI(s,p)) \tag{4.3}$$

where $\chi^2_{n-1}(2MI(s,p))$ is the value of the cumulative chi-square distribution with $n-1$ degrees of freedom at $2MI(s,p)$. Hence, the best possible calibration score can be one, which is the case when an empirical mass function s of an expert equals the hypothesized mass function p. In other words, a calibration score close to one means that experts' probabilities $(s_0, \ldots, s_n)$ are statistically supported by the set of seed variables. An example should make it more clear how the calibration score of one expert is calculated.

Example: Consider 10 seed questions (so $M = 10$) for which an expert should assess his 5%, 50% and 95% quantiles (so $n = 3$). It seems that the sample mass function, i.e. the empirical mass function of the expert is $s = [0.1, 0.3, 0.5, 0.1]$. The theoretical mass function is given by $p = [0.05, 0.45, 0.45, 0.05]$. The relative information can then be obtained by:

$$\begin{aligned} I(s,p) &= 0.1 \times \ln\left(\frac{0.1}{0.05}\right) + 0.3 \times \ln\left(\frac{0.3}{0.45}\right) + 0.5 \times \ln\left(\frac{0.5}{0.45}\right) + 0.1 \times \ln\left(\frac{0.1}{0.05}\right) \\ &= 0.0697 \end{aligned}$$

Then with $2MI(s,p) = 2 \times 10 \times 0.0697 = 1.39$ the calibration score becomes:

$$C = 1 - \chi^2_3(1.39) = 0.708 \quad \square$$

Calibration is a so-called "fast" function, i.e. adding or deleting a realization can give noticeable impact on the calibration score. The calibration score changing with a magnitude of 4 is not an exception.

Calibration should not be the only way to measure the quality of an expert opinion, because if an expert gives a very large range of possible outcome, i.e. his 5% and 95% quantile for instance are very wide stretched, the calibration score can be close to one, but the expert is not very informative.

4.2.2 Information

To measure the informativeness of an expert, a background measure is assigned to each query variable. The background measures are typically either uniform or loguniform. For these background measures it is necessary to choose an "intrinsic range" to truncate the range of possible outcomes.

Let $x_i(e)$ for $i = 1, \ldots, n$ be the values given by an expert e, $e = 1, \ldots, E$ for a query variable (suppressed from notation). Let

$$l = \min\{x_1(e), r|e\}, \qquad h = \max\{x_n(e), r|e\}, \tag{4.4}$$

where l and h are the lowest and the highest value assessed by all experts and r is the value of the realization (only for seed variables). The intrinsic range is obtained by adding a $k\%$ overshoot to the lower bound and the upper bound of the smallest interval containing all values of the quantile assessments of all experts. Thus,

$$x_0 = l - k\% \times [h - l] \tag{4.5}$$

$$x_{n+1} = h + k\% \times [h - l] \tag{4.6}$$

In practice, k is mostly chosen to be 10%. The relative information of expert e on a variable is given by:

$$I(e) = \ln(x_{n+1} - x_0) + \sum_{i=0}^{n} p_i \ln\left(\frac{p_i}{x_{i+1} - x_i}\right) \tag{4.7}$$

where $p = (p_0, \ldots, p_n)$ is the experts probability and the values $(x_{i+1} - x_i)$ are the background measures of the corresponding intervals. Informativeness per expert over all query variables is the average of the information scores per variable. The information score is a positive score and experts with a high score are preferred. In contrast with the calibration score information is a "slow" function, which means that large changes in the quantile assessments produce only modest changes in the information score. It would be an exception if the information scores vary more than a factor 3. An example can help understanding the intrinsic range and the calculation of the information score of a variable.

Example: Consider a trader and a mathematics student forecasting the opening price of the AEX the next day. Consider the following forecasts: The next day's opening price is

	5%	50%	95%
Trader1	325	340	380
Trader2	310	320	340

341.83. The intrinsic range can be obtained by, considering a 10% overshoot:

$$x_0 = 310 - 0.1 \times [380 - 310] = 303$$

$$x_{n+1} = 380 - 0.1 \times [380 - 310] = 387$$

So the intrinsic range over this variable is $[303, 387]$. The information score of Trader1 can be derived by, knowing that $p = [0.05, 0.45, 0.45, 0.05]$:

$$\begin{aligned} I(Trader1) &= \ln(387 - 303) + 0.05 \times \ln\left(\frac{0.05}{325 - 303}\right) + \cdots \\ &\cdots + 0.45 \times \ln\left(\frac{0.45}{340 - 325}\right) + 0.45 \times \ln\left(\frac{0.45}{380 - 340}\right) + 0.05 \times \ln\left(\frac{0.05}{387 - 380}\right) \\ &= 0.2821 \end{aligned}$$

The information score of Trader2 can be calculated in the same way resulting in:

$$I(Trader2) = 0.7385$$

which means that Trader2 is much more informative than Trader1, but he misses the realization, which will be taken into account more than the informativeness. $\square$

If the background measure is chosen to be loguniform then all the values $x_0, \ldots, x_{n+1}$ are logged before applying the same procedure for determining the information score.

4.2.3 Weighting and approximation of expert distributions

The calibration and information scores can be calculated for every expert e. In order to determine the performance based weight w_e that an individual expert gets the information and calibration score are combined as follows:

$$w'_e = C(e) \times I(e) \times 1_\alpha(C(e)) \tag{4.8}$$

$$w_e = \frac{w'_e}{\sum_{e=1}^{E} w'_e} \tag{4.9}$$

where $1_\alpha(x)$ is the indicator function which return 1 if $x \geq \alpha$ and 0 if $x < \alpha$. This function makes it possible that an expert receives zero weight when his calibration score falls below a predefined level α (the significance level). More about the so-called cut-off level α is discussed in the next subsection. From equation (4.8) it is clear that the calibration score is most important and the information score serves as a differentiator between equally well calibrated experts. The reason for this is that it should be prevented that very informative distributions, i.e. very narrow confidence bounds, compensate for poor calibration.

The weights in (4.8) are global weights, because they are the average of the information of all the query variables. The 'Classical model' also gives the possibility to weight the expert per item, called item weights. Using item weights the relative information $I(e)$ with respect to the background measure will be measured for each query variable separately and the weights will be calculated for each item. This allows an expert to downweight or upweight himself on individual items, according as his quantiles are further apart or, respectively, closer together. In the implementation of the 'Classical model' the experts are asked a limited number of quantiles for each query variable. In the case study described in the Chapter 5 the experts are asked for the 5%, 50% and 95% quantiles ($n = 3$). A choice can be made by the analyst to scale the query variables uniformally or loguniformally. If the scale is logarithmic then all the values $x_0, \ldots, x_{n+1}$ belonging to the quantiles and the upper and lower bound of the intrinsic range, are logged before applying the same procedure for determining the expert distributions as for uniform query variables.

Since the expert only gives a limited number of quantiles the inter-quantile range should be interpolated to get a (cumulative) distribution of the query variable. The distribution of expert e is according to the 'Classical model' approximated by linear interpolating the quantile information $(x_0, 0)$, $(x_1, 0.05)$, $(x_2, 0.5)$, $(x_3, 0.95)$ and $(x_4, 1)$. This is the distribution with the minimal information with respect to the uniform distribution on the intrinsic range that satisfies the experts quantiles. For instance, Figure 4.1 shows the interpolated distribution function of 'Trader1'.

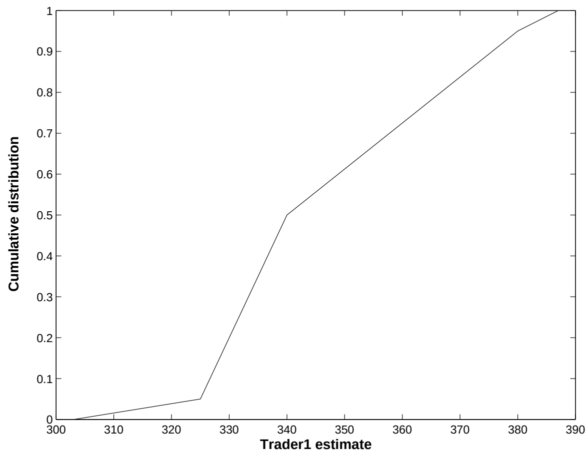

Figure 4.1: *Trader1's interpolated distribution function of his AEX forecast*

The above assumption on deriving the experts distributions is fairly arbitrary, but it generally makes little difference to the weights given to the experts. The reason is that the calibration scores which drive the weights the most, only depend on the quantiles and not on the interpolation. The information score only depends on quantiles and the choice of x_0 and x_{n+1}. The influence this assumption can have on especially the capital requirement is discussed in section 6.2.

Now, a distribution F_e for each expert on the current query variable (also called 'item') can be obtained. With these distributions a combined distribution function F_{dm} can be constructed by using the computed weights for each expert w_e:

$$F_{dm} = \sum_{e=1}^{E} w_e F_e \tag{4.10}$$

where E is the total number of experts, 'DM' stands for 'decision maker'. The DM will use the combined distribution in determining the uncertainty about a quantity.

4.2.4 Decision maker (DM)

In order to determine the weights per expert a choice should be made of the cut off level α. Then for each choice of α in equation (4.8), a distribution $F_{dm,\alpha}$ of the decision maker is defined. This distribution can also be scored with respect to calibration and information. Hence, a "virtual weight" $w_{dm}(\alpha)$ for the DM is defined as a function of α. This is the weight that the combined expert would receive when he gives the DM's distribution and is scored along with the actual experts. Since the DM's virtual weight depends on the significance level α, an α can be chosen such that this weight is maximized. The optimal virtual weight of the DM can be compared with the global weights of the expert or with the virtual weight of an item weight DM or even with an equal weight DM, i.e. each expert receiving the same weight.

For the processing of expert assessments, the calculation of calibration and information scores, the combination of expert assessments, the interpolation of the expert distributions and calculating different DM' weights the computer program Excalibur[1] can be used. This program is used to analyze the data obtained from the case study discussed in Chapter 5.

4.3 Practical issues using expert judgement

When asked to estimate probabilities or quantify uncertainties in numbers, people rely mostly on various rules of thumb, instead of performing mental calculations. Such rules of thumb are called heuristics. Heuristics can lead to predictable errors. The word "error" in this context most be used very carefully, since a degree of belief is subjective. By error in this sense is meant either a violation of the axioms of probability or an estimate that is not really in accord with the subject's beliefs and he/she would want to correct if the matter were brought to his/her attention.

Heuristics that lead to errors according to this definition are called biases. Another free translation of the word "bias" is "distortions of judgement through ideology". Psychologists also speak of a "motivational bias" if the subject distorts his/her judgement on purpose (as in lying). This bias can occur when a subject has a stake in the outcome of the study. In this section cognitive biases have to do with "misperceptions" of probabilities.

Much of the psychometric research into heuristics and biases has been performed by Kahneman, Tversky, Slovic and their colleagues and has been gathered in Kahneman et al. (1982). Two cognitive biases, overconfidence and anchoring, are discussed here, because these seemed to occur during the case study (Chapter 5). Other biases can also be found in Bedford and Cooke (2001).

4.3.1 Overconfidence

An expert stating his probability assessments biased by overconfidence means that the expert has presented his uncertainty with an interval that is (much) smaller than what he truly beliefs. A common consequence is that the realizations will too often fall outside the assessed probabilities.

4.3.2 Anchoring

Anchoring is the phenomenon that an expert fixes an initial value and then "adjusts" or "corrects" this value. The elicitation format can cause anchoring. Frequently the adjustment is insufficient. For instance when experts are asked to estimate the 5%, 50% and 95% quantiles of a distribution for an uncertain quantity X, they apparently fix on a central value for X and then adjust. The 5% and the 95% quantiles obtained in this way are frequently too close to the central value. This means that the true values will too often fall outside the given quantiles.

4.3.3 Guidelines

One can imagine that not only these phenomena, but also other biases should be avoided. There is no magic way to avoid these problems, but if the analyst and the expert are aware of these difficulties, it is possible to guard against them to some extend. Several guidelines

[1] Excalibur is a program developed at Delft University of Technology

are developed in Cooke (1991) to prevent most of the problems concerning eliciting expert opinions:

- The questions must be clear
- Prepare an attractive format for the questions and the graphic format for the answers
- Perform a dry run (i.e. a test session)
- An analyst must be present during the elicitation
- Prepare a brief explanation of the elicitation format and of the model for processing the responses
- Avoid coaching
- The elicitation session should not exceed 1 hour

The next chapter introduces the LEVER method, which uses performance based expert judgement to quantify operational risks within financial institutions.

Chapter 5

The LEVER Method

5.1 Introduction

In this chapter performance based expert judgement will be used to quantify operational risks within banks according to Basel II. Applying this technique in the field of operational risks within financial institutions is called the LEVER method. LEVER stands for Loss Estimation by Validating Experts in Risk. The idea of LEVER is that the method uses the scarce loss data, which can be both internal and external and leverage it to fulfill the advanced measurement approach (AMA) according to Basel II. See Figure 5.1 for a schematic representation of the LEVER concept.

To apply and test the LEVER method a case study is done at BANK B. BANK B is a specialized bank which provides securities services and related financial and information services. Their core business is Custody Services, Clearing and Settlement. The clients of BANK B have positions in shares and bonds and BANK B keeps these positions in custody. Among the Custody Services are Corporate Actions (conversions, stock-dividends) and Income Collections (coupon payments for bonds, dividend payments for shares).

5.2 Case Study

In this case study only a part of one business line is used to apply the LEVER method. This business line is called Custody Services. Mapping this business line onto the definitions used by Basel II, it falls under Agency Services (see 2.2). Within Custody Services the risks of two operations are studied: conversions and coupon/dividend payments. Mapping the risk type on the Basel II definitions of Table 2.3 it will fall under Execution, Delivery & Process

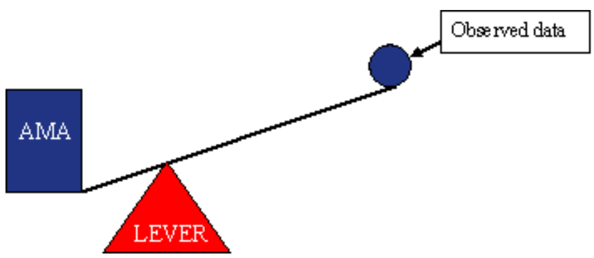

Figure 5.1: *Schematic representation of the LEVER concept*

Management. The purpose of this study is to apply the LEVER method starting with a pilot. If, evidently, the LEVER method is applicable on these two operations it can be used on other operations too.

For the execution of this case study a protocol is used for structured expert judgement described in Cooke and Goossens (2000). The application of the expert judgement elicitation at BANK B is described below.

Preparation for Elicitation

1. *Definition of case structure*
   The purpose of the elicitation is to quantify the operational risks that come along with conversion and coupon dividend payments.

2. *Identification of target variables*
   The target variables over which the uncertainty distributions are required are defined by the manager of Corporate Actions. There are six target variables, i.e. operational risks, identified for both conversions and coupon/dividend payments (see Table 5.8 and Table 5.9 in section 5.3).

3. *Identification of query variables*
   Query variables are variables which will be presented to the experts in the elicitation and for which they will quantify their uncertainty. Twelve target variables out of 19 query variables were defined (see Appendix B). Some target variables were split up into two query variables; one variable about the frequency of an event[1] and one about the impact an event can have. After assessing the experts distributions the two query variables will be combined using Monte Carlo simulation to calculate their aggregated distribution (see section 5.3).

4. *Identification of performance variables*
   The performance variables or seed variables are there to test the expertise of an expert and measure his ability to express uncertainty with numbers (see Chapter 4). A total of 16 seed variables were defined, eight about conversions and eight about coupon/dividend payments. A database with loss data of BANK B is used to define seed variables (see Appendix B). Thus, the experts are asked to give their retrodictions, i.e. the true value exists but is not known to the expert. The questionnaire is shown in Appendix A.

5. *Identification and selection of experts*
   The identification and selection of the experts were done by BANK B. Due to the fact that the elicitation is held for the first time in this field, as many experts as possible were selected to participate in the elicitation session.

6. *Definition of elicitation format document*
   The questions asked to the experts are shown in Appendix A. The questionnaire was split up in two parts, conversions and coupon/dividend payments. The conversions part contained 18 questions of which 8 seed and 10 target questions. The coupon/dividend part contained 8 seed and 9 target questions. To prevent ambiguity some definitions of terms used in the questions were explicitly defined.

[1] An event depends on the defined target variable. More details are discussed in the following subsection.

7. *Dry run exercise*
   A test session was held for seven employees of BANK B. This was done as preparation of the elicitation session during a workshop.

8. *Expert training session*
   An expert training session of an hour and a half was held during a workshop, in which the main ideas, the way to fill in the questionnaire and some examples were presented. At the end of the training session a questionnaire was given to practice, which was discussed afterwards. Two workshops with five participants for each workshop were given.

Elicitation

1. *Expert elicitation session*
   The elicitation session was given during the workshop. After the training session and a lunch the questionnaire was filled in. This took between 45 minutes and an hour.

Post-Elicitation

1. *Combination of expert assessments*
   In the following subsections the combination of assessments are discussed.

2. *Discrepancy and robustness analysis*
   A discrepancy and robustness analysis are treated in subsections 5.2.2 and 5.2.3.

3. *Feed back*
   The experts were given a report with their assessments, their calibration and information scores and their weighting factors. During a presentation the main and noticeable results were discussed.

4. *Post-processing analyzes*
   Post-processing analyzes were not applicable in this study.

5. *Documentation*
   A report to the management of BANK B has been written about the main results of the LEVER study. In addition, the main results were presented during a presentation.

5.2.1 Experts' assessments

In this subsection the expert assessments of the questionnaire shown in Appendix A are discussed. The experts were asked to fill in their 5%, 50% and 95% quantiles for each query variable. The 'Classical model' as discussed in Chapter 4 is applied to the expert assessments using the program Excalibur. Excalibur is also capable of assigning equal weights to the experts and calculating the "equal weights" decision maker (EqualDM), i.e. in mathematical terms:

$$F_{EqualDM} = \frac{1}{E}\sum_{e=1}^{E} F_e \tag{5.1}$$

as in (4.10) with $w_e = \frac{1}{E}$. Before using the optimization routine to compute the performance based weights, the output is shown using equal weights. Table 5.1 shows the calibration

1	2	3	4	5	6	7	8
ID	Calibration	Mean rel. info. total	Mean rel. info. seed	Num. seed	Unnorm. weight	Norm. weight w/o DM	Normalized weight with DM
expert 1	0,0001095	1,102	1,154	16	0,0001263	0,1	0,0005099
expert 2	4,418E-012	2,185	2,141	16	9,459E-012	0,1	3,818E-011
expert 3	0,1473	0,9112	0,903	16	0,133	0,1	0,5367
expert 4	4,918E-007	1,795	1,898	16	9,337E-007	0,1	3,768E-006
expert 5	0,003436	1,411	1,673	16	0,005747	0,1	0,0232
expert 6	3,435E-007	1,883	1,903	16	6,537E-007	0,1	2,639E-006
expert 7	4,016E-006	1,988	1,992	16	7,999E-006	0,1	3,229E-005
expert 8	2,407E-005	2,122	2,209	16	5,317E-005	0,1	0,0002146
expert 9	1,362E-005	0,7592	0,8834	16	1,203E-005	0,1	4,856E-005
expert10	4,918E-007	1,833	1,967	16	9,676E-007	0,1	3,906E-006
EqualDM	0,3381	0,3126	0,3219	16	0,1088		0,4393

Table 5.1: *Excalibur output using equal weights*

and information scores for the ten experts in this case study and the EqualDM. The second column of Table 5.1 shows the calibration score of each expert including the EqualDM. Note that the EqualDM is better calibrated (one is high and zero is low) than each expert individually. Expert 3 has the highest calibration score (0.1473) among the experts. Column 3 and 4 show the mean relative information scores respectively for all variables and for the seed variables only, of which there are 16 (shown in column 5). Note that the relative information score for both seed variables (0,3126) and all variables (0,3219) of the EqualDM is very low relative to the other experts. Expert 2 has a relatively high information score, but has a very low calibration score. This phenomenon indicates overconfidence or anchoring as described respectively in section 4.3.1 and 4.3.2. The last three columns show the weights for each expert. Column 6 shows the unnormalized weight of each expert, which is the product of the calibration score and the relative information score of the seed variables, so column 2 times column 4. Column 7 displays the normalized weight an expert gets if the decision maker is not joining the study. In this case all the experts without the decision maker get the same weight. The last column shows the weights of column 6 normalized, the decision maker included. Note that the EqualDM and expert 3 share almost all the weight (97,6%).

The results of using a performance based weighting scheme (see section 4.2.3) are shown in Table 5.2. Here global weights are used to compute the "global weight" decision maker (GlobalDM). Note that the calibration score of the GlobalDM of 0,4299 is almost three times higher than each expert individually. The calibration score of the GlobalDM as well as the information score is better than the EqualDM scores. This means that a performance based weight is preferred. All experts with a calibration score less than the significance level 0,003436 are unweighted as reflected by the zero's in column 6. Note that weights are assigned to expert 3 (96%) and 5 (4%) only. If the GlobalDM would be included in the study as a virtual expert then the GlobalDM would get a weight of 70%.

1	2	3	4	5	6	7	8
ID	Calibration	Mean rel. info. total	Mean rel. info. seed	Num. seed	Unnorm. weight	Norm. weight w/o DM	Norm. weight with DM
expert 1	0,0001095	1,102	1,154	16	0	0	0
expert 2	4,418E-012	2,185	2,141	16	0	0	0
expert 3	0,1473	0,9112	0,903	16	0,133	0,9586	0,286
expert 4	4,918E-007	1,795	1,898	16	0	0	0
expert 5	0,003436	1,411	1,673	16	0,005747	0,04143	0,01236
expert 6	3,435E-007	1,883	1,903	16	0	0	0
expert 7	4,016E-006	1,988	1,992	16	0	0	0
expert 8	2,407E-005	2,122	2,209	16	0	0	0
expert 9	1,362E-005	0,7592	0,8834	16	0	0	0
expert10	4,918E-007	1,833	1,967	16	0	0	0
GlobalDM	0,4299	0,7827	0,7587	16	0,3261		0,7016
ItemDM	0,4299	0,8113	0,7856	16	0,3377		0,7088

Table 5.2: *Excalibur output using performance based weights with significance level 0,003436*

Finally the other performance based weighting scheme, "item weights", is used for computing the decision maker. The results are also shown in Table 5.2, because the scores of the experts seemed to be same as using global weights. The decision maker is denoted as 'ItemDM' and has the same calibration score as the GlobalDM, but it has a better information score on both seed variables (0,7856) and all variables (0,8113). The normalized weight without the ItemDM (column 7) is not applicable in this case, because the weights are determined per item.

It is remarkable that the calibration scores are fairly low for most experts. Expert 3 with a calibration score of 0,1473 is the only expert having a p-value above 5%. Possible reasons for the low scores are:

- The experts have little experience with quantifying their uncertainties and in spite of the training session the experts had difficulties with it. This reveals itself in:
  - Overconfidence and anchoring, which are discussed in 4.3.1 and 4.3.2. For instance, expert 2 has a very high information score (2,141) on the seed variables, but a very low calibration score ($4,418 \cdot 10^{-12}$). A range graph gives a better feeling for these phenomenons. Figure 5.2 shows range graphs per calibration variable of expert 2. For each calibration variable (item), the upper and lower quantiles are given by "`[---]`"; the median is given by a "`*`" and the realization by a "`#`". It is clearly seen from Figure 5.2 that expert 2 gave too small intervals resulting in too many missing realizations. In other words expert 2 should be very surprised.
- The experts have a lack of expertise on both conversions and coupon/dividend payments. Some experts are only specialized in either conversions or coupon/dividend payments. This is shown in the next subsection (Filtering).
- A few seed questions appeared ambiguous, which can lead to different interpretations of the questions and causes a discrepancy in the assessments. For instance, the range

```
Expert no. :    2     Expert name:  expert 2
Items
  1(L)                   [-------*----]
Real   :::::::::#::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::

  2(L)            [----*--]
Real   :::::::::::::::::::::::::#::::::::::::::::::::::::::::::::::::::::::::::

  3(L)                     [-----*---]
Real   ::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::

  4(L) [----------------------------------------------------*---]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

  5(L)                                                     [-*]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::

  6(L)                                                               [----*--]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::

  7(L)                                                        [---*-]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::

  8(L)                                            [----*--]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::

 19(L)                                                         [*]
Real   ::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

 20(L)                                                [*]
Real   ::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

 21(L)                                                 [--*--]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::

 22(L)                                              [---*-]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::

 23(L)                                           [---*-]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

 24(L)                                       [*]
Real   :::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::::::::::::

 25(L)                                                    [----*]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::

 26(L)                                             [----*--]
Real   :::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::::
```

Figure 5.2: *Range graphs of expert 2 of the seed variables*

graphs in Figure 5.3 of target questions 11 and 21 (see Appendix B) shows a remarkable discrepancy between expert 2 and 4. Both experts are reasonably confident about this quantity, because of their small intervals, but the intervals are far from each other for

```
Item no.:   Item name: c: backvaluati Scale: LOG Experts
  1                                                    [--------*-----]
  2                                                                [-*]
  3 [-------------------------------------------*-------------]
  4 [-*]
  5                                                      [---*----------]
  6                                                      [------*--]
  7                                                     [--*--]
  8                                                 [--*--]
  9 [--------------------------------------------------------------*-----------]
 10                                                      [---*--]
Ite
[=============================================*==================]
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

    0.0001                                                                1E006
```

```
Item no.:   Item name: cd: backvaluat Scale: LOG Experts
  1                                                      [------*-]
  2                                                                   [*]
  3 [----------------------------------------------------*--------------]
  4 [-*]
  5                                                          [-*--------]
  6                                                           [*]
  7                                              [----*-]
  8                                                             [--*---------]
  9 [---------------------------------------------------------------------*----]
 10                                                      [--*-]
Ite
[=======================================================*===========]
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

    0.0001                                                                1E006
```

Figure 5.3: *Range graphs of target variables 11 and 21 of all experts*

both target questions. It seems that the experts had a different interpretation of the questions. This should be prevented at all times.

The other range graphs of the experts and the items are shown in Appendix C.2 and C.3.

Filtering

Based on the observations and possible reasons for the low score mentioned above the two expertise fields conversions and coupon/dividend payments are split up in two separate fields in this subsection. This is done to analyze whether some experts score significantly better at one field in compare with the other field. Again, the calibration score and information scores are calculated using the 'Classical model' based on the seed and target variables of both fields separately. An equal weight decision maker is used, because only the calibration and information scores of the experts are of interest.

First the scores based only on the seed and target variables of conversions are calculated, which are shown in Table 5.3.

1	2	3	4	5
ID	Calibration	Mean rel. info. seed	Num. seed	Unnorm. weight
expert 1	0,009073	1,085	8	0,009842
expert 2	2,1E-006	1,809	8	3,798E-006
expert 3	0,4291	0,6966	8	0,2989
expert 4	0,00348	1,66	8	0,005776
expert 5	0,01566	1,477	8	0,02313
expert 6	3,885E-008	2,026	8	7,871E-008
expert 7	0,0002533	1,852	8	0,0004693
expert 8	0,001547	1,877	8	0,002905
expert 9	0,00348	0,9441	8	0,003285
expert10	0,0001927	1,795	8	0,0003458
EqualDM	0,4291	0,2793	8	0,1199

Table 5.3: *Scores on conversions variables only*

Table 5.4 displays the calibration and information scores of the query variables of coupon/ dividend payments. Note that the number of realizations is 8. Cooke (1991) advises in to use 10 seed variables as a minimum, but in this case it is only used to investigate a difference between the performance of experts on both fields. Note also that the calibration scores shown in the Tables 5.3 and 5.4 can not be compared with the calibration scores shown in Table 5.1, because of the difference in number of seed variables[2].

1	2	3	4	5
ID	Calibration	Mean rel. info. seed	Num. seed	Unnorm. weight
expert 1	0,00567	1,223	8	0,006936
expert 2	3,723E-006	2,474	8	9,211E-006
expert 3	0,185	1,109	8	0,2053
expert 4	5,115E-005	2,137	8	0,0001093
expert 5	0,02651	1,868	8	0,04953
expert 6	0,185	1,78	8	0,3294
expert 7	0,0005755	2,131	8	0,001227
expert 8	0,01644	2,54	8	0,04176
expert 9	0,002029	0,8228	8	0,00167
expert10	0,001547	2,14	8	0,003312
EqualDM	0,5338	0,3646	8	0,1946

Table 5.4: *Scores on coupon/dividend payments variables only*

[2]To compare the scores a calibration power of 0,5 should be used to be able to compare the scores. For more information about this see Cooke (1991).

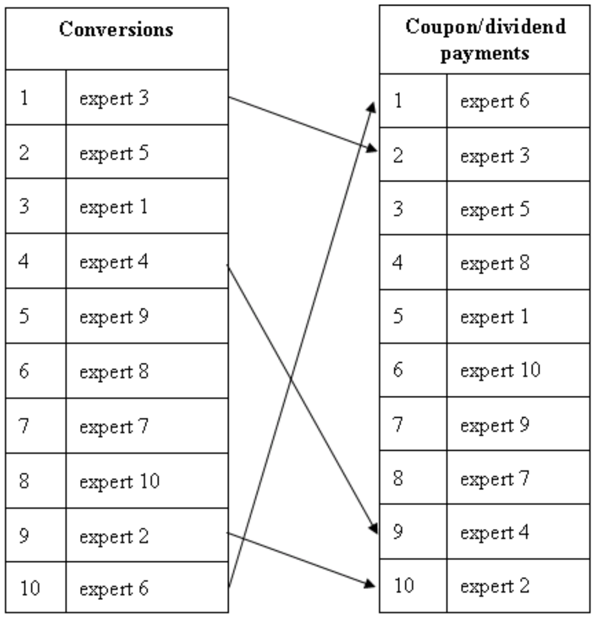

Figure 5.4: *Ranking of scores (calibration×information) on both conversions and coupon/dividend payments*

Comparing Table 5.3 and Table 5.4 it is remarkable that expert 6 has a very low calibration score on conversions ($3,885 \cdot 10^{-8}$) and a significant acceptable score on coupon/dividend payments. His calibration score is together with expert 3 the highest of all the experts and his information score is more than 1,5 times higher than of expert 3, which is preferable. Expert 4 has a much higher calibration score (factor 100) on conversions than on coupon/dividend payments. Note that expert 4 is much more informative on the seed variables of coupon/dividend payments (2,137) than of conversions (1,66). Presumably, overconfidence plays a role in this matter. Figure 5.4 shows the ranking of the unnormalized weights (column 5 of Table 5.3 and 5.4) of the experts on both conversions and coupon/dividend payments. From this filtering study it becomes clear that it is preferable to base the elicitation session on one field of expertise, but this will entail extra time and effort. Thus, a trade-off should be made when using the LEVER method within a bank between the scope of the field of expertise and the extra time and effort.

5.2.2 Discrepancy analysis

A discrepancy analysis is concerned with the agreement among the experts. In a discrepancy analysis the relative information of each experts' assessment per item is compared with the assessment of the decision maker for that item. In Table 5.5 the relative information of each expert with respect to the information of the EqualDM, which is an equal combination of the experts' assessments, is shown. The lower the score the more the DM agrees with the expert. It is seen that expert 1, 3 and 9 agree the most with the EqualDM.

1	2
ID	Rel. info. total
expert 1	0,7899
expert 2	1,836
expert 3	0,7208
expert 4	1,43
expert 5	1,016
expert 6	1,386
expert 7	1,593
expert 8	1,777
expert 9	0,9001
expert10	1,414
EqualDM	0

Table 5.5: *Discrepancy analysis relative to the EqualDM*

5.2.3 Robustness analysis

A robustness analysis is concerned about the question to what extend the decision maker's distributions depend on a particular expert or calibration variable. If for instance a seed variable is excluded from the analysis and the calibration and information score of the decision maker is drastically changed, the results are not very robust against the choice of the seed variables.

Experts

For robustness on experts, experts are excluded from the analysis one at the time and the scores of the resulting decision maker are recomputed. For instance if expert 3 was excluded from the study the calibration score in Table 5.6 would be 0,1481 and the relative information score of all variables 1,07. The relative information of these 'perturbed decision makers' may be compared to the overall agreement among the experts themselves. If the perturbed decision makers resemble the original decision maker more than the experts agree with each other, then we may conclude that the results are robust against the choice of the experts.

It is seen from column 3 of Table 5.6 that expert 3 and 5 only have influence on the calibration score if left out of the study as expected. The calibration score dropped with a factor 3, which is not unusual due to the fast function characteristic of calibration. Column 4 of Table 5.6 represents the relative information with respect to the decision maker (ItemDM). A score close to zero means that there is a great resemblance with the decision maker. The last column of Table 5.6 should be compared with the scores of Table 5.5. The effect of "perturbing" the model by removing an expert is small relative to the differences among the experts themselves. In case of removing expert 3 the relative information with respect to the decision maker is 0,7785 (based on all the variables), which is still lower than the average discrepancy, i.e. the average score of Table 5.5, between the experts. Thus, the results are robust against the choice of the experts.

1	2	3	4
ID	Rel. info/b total	Calibration	Rel. info/o total
expert 1	0,7801	0,4299	0,002338
expert 2	0,8004	0,4299	0,0008739
expert 3	1,07	0,1481	0,7785
expert 4	0,8113	0,4299	3,762E-008
expert 5	0,9055	0,1473	0,09695
expert 6	0,8043	0,4299	0,0006646
expert 7	0,8098	0,4299	0,0001499
expert 8	0,8113	0,4299	1,102E-007
expert 9	0,4548	0,4299	0,01673
expert10	0,8113	0,4299	5,625E-008
None	0,8113	0,4299	0

Table 5.6: *Robustness analysis on experts using item weights*

Items

For robustness on items the same procedure as with the experts is followed. Table 5.7 shows the results removing each calibration variable one at the time. The second column represents the item, i.e. seed variable, excluded from the model. The meaning of the abbreviations of the items can be found in Appendix B ("c:" stands for conversions and "cd:" stands for coupon/dividend payments; "None" means that no item is excluded). Note that if item 1, 2, 9 or 10 would be removed than the calibration score would be almost a factor 6 lower than the original calibration score of the decision maker. The last column shows a great resemblance with the original decision maker for each removed item. It can be concluded that the results are robust against the chosen items.

5.3 Decision maker's quantification of the risks

In this section the assessments of the item weight decision maker (ItemDM) on the target variables based on a log scale are used to quantify the risks defined by BANK B (see Table 5.8 and 5.9) . The quantile-assessments of the itemDM and the realization of the seed variables are shown in Appendix C.1. As mentioned before some defined operational risks within the field of conversions and coupon/dividend payments were split up in two target variables. The reason for this was to make it easier for the expert to assess his true beliefs. The variables were split up in frequency and severity. The way this is done will be shown below.

Conversions

The operational risks as defined by BANK B for conversions are:
Here RC_i for $i = 1, \ldots, 6$ are considered to be random variables with a distribution function. The way these variables were seperated and presented to the experts is explained "Errors due to manual processes (RC_1)". The corresponding elicitation questions looked as follows:

1	2	3	4
ID of excl. item	Rel. info/b total	Calibration	Rel. info/o total
c: ♯ claim	0,9359	0,08041	0,09803
c: € claim	0,9138	0,08041	0,09516
c: ♯ wo claim	0,8511	0,5435	0,02925
c: ave wo clai	0,8055	0,5435	0,01866
c: dif wo clai	0,8459	0,5435	0,01902
c: ♯ 2003	0,8796	0,5435	0,05035
c: diff mon ♯	0,7392	0,2356	0,1272
c: ♯ hours	0,8507	0,5435	0,0204
cd: ♯ claim	0,8991	0,07687	0,09194
cd: € claim	0,9075	0,08041	0,09891
cd: ♯ wo claim	0,9007	0,387	0,09513
cd: ave wo cla	0,8184	0,5435	0,02832
cd:dif wo cla	0,902	0,387	0,09443
cd: ♯ 2003	0,8384	0,5435	0,02929
cd: dif mon ♯	0,9011	0,08041	0,09912
cd: ♯ hours	0,7982	0,5435	0,05063
None	0,8113	0,4299	

Table 5.7: *Robustness analysis on items using item weights*

RC_1: Errors due to manual processes
RC_2: Recover actions due to incorrect instructions of VS-clients
RC_3: Manually recovered cum-ex positions before payment
RC_4: Manually recovered cum-ex positions after payment
RC_5: Adjusted errors
RC_6: Losses due to back valuation

Table 5.8: *Risks accompanying conversions*

1. "How many errors due to manual processes had to be corrected in an arbitrary month?" (N_{rc_1})

2. "How much time does it take to correct an arbitrary error due to manual processes (in minutes)?" (X_{rc_1})

The first question is about the frequency of the errors taken place and the second is about the severity an error can have. The expert was supposed to assess his 5%, 50% and 95% quantiles for the variables. With the use of the 'Classical model' a performance based decision maker was obtained as extensively described in the previous section. The distribution of the decision maker is then obtained by linear interpolating between the assessed quantiles (see section 4.2.3). So, a distribution function belongs to each target variable, which makes it a random variable. The example-items for instance provide a random variable N_{rc_1} for the number of errors (frequency) and a random variable X_{rc_1} for the time it takes to correct an error (severity) (using notations of Appendix B). These variables should be multiplied to get the risk variable $\widetilde{RC}_1$, which can be done by using a Monte Carlo simulation as discussed in section 3.3. The result is also a random variable with an underlying aggregated distribution. So:

$$\widetilde{RC}_1 = N_{rc_1} \cdot X_{rc_1} \tag{5.2}$$

The tilde on $\widetilde{RC}_1$ used in equation (5.2) is because this variable gives the risk of a month and RC_1 without tilde reflects the risk on a yearly basis. For the other operational risks the same procedure is followed. This gives for the separated variables:

$$\begin{aligned}
\widetilde{RC}_2 &= N_{rc_2} \cdot X_{rc_2} \\
\widetilde{RC}_3 &= N_{rc_3} \cdot X_{rc_3} \\
\widetilde{RC}_4 &= N_{rc_4} \cdot X_{rc_4} \\
\widetilde{RC}_5 &= N_{rc_5} \cdot X_{rc_5}
\end{aligned} \tag{5.3}$$

where the N- and X-family are respectively the assessed frequency and severity target distributions[3] of the ItemDM (see Appendix B for the definitions of all variables). Only $\widetilde{RC}_6$ is directly obtained from the decision maker's quantile assessment. To compare the risks with each other the units of the random variables should be the same. $\widetilde{RC}_1, \ldots, \widetilde{RC}_5$ have unit 'Time' and $\widetilde{RC}_6$ has unit 'Euro'. It is preferred to express the risks in terms of euro's, because the main goal of the case study is to compute the annual capital requirement, which is treated in Chapter 6. A few assumptions should be made to accomplish this:

- A working day contains 8 hours per person
- The costs of a working day for BANK B is €100,-
- A year has 260 working days

For instance, to get the random variable RC_1 in euro's on a yearly basis, a few scalar multiplications should be made:

$$RC_1 = 100 \cdot (12 \cdot N_{rc_1}) \cdot \left(\frac{X_{rc_1}}{8 \cdot 60}\right) = \frac{5}{2} \cdot N_{rc_1} \cdot X_{rc_1} \tag{5.4}$$

The same procedure is followed for risks $\widetilde{RC}_2, \ldots, \widetilde{RC}_6$, which gives:

$$\begin{aligned}
RC_2 &= \frac{325}{6} N_{rc_2} \cdot X_{rc_2} \\
RC_3 &= \frac{5}{24} N_{rc_3} \cdot X_{rc_3} \\
RC_4 &= \frac{5}{24} N_{rc_4} \cdot X_{rc_4} \\
RC_5 &= \frac{5}{2} N_{rc_5} \cdot X_{rc_5} \\
RC_6 &= 12 \widetilde{RC}_6
\end{aligned} \tag{5.5}$$

[3]It should be noted that X_{rc_3} and X_{rc_4} are assumed to be equally distributed. It is reasonable to assume this, because the recovering time of cum-ex positions before payment is likely to have the same behavior as the recovering time of cum-ex positions after payment. The same assumption holds for X_{rcd_3} and X_{rcd_4}.

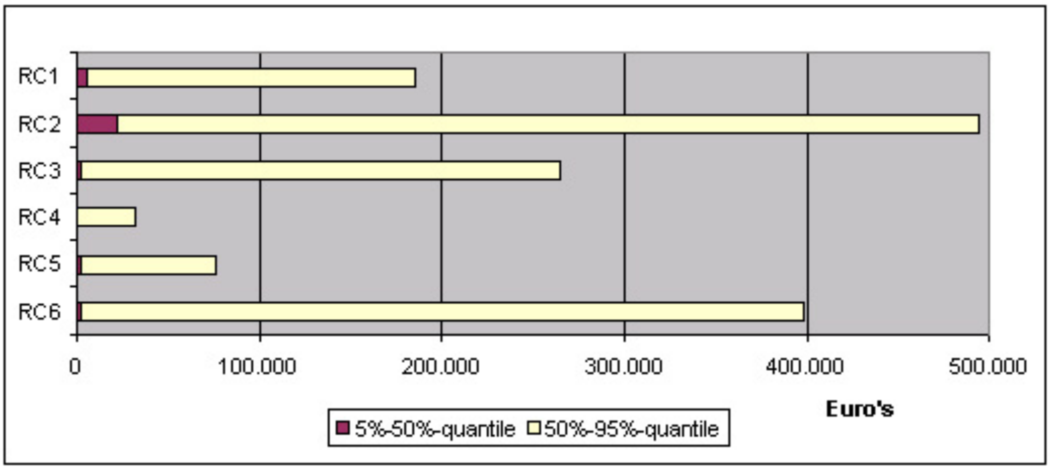

Figure 5.5: *90%-confidence intervals of the operational risks for conversions on a yearly basis*

Unicorn[4], a program which uses Monte Carlo simulations to combine distributions, is used to derive the aggregated distributions and the 90% quantile range of the operational risks $RC_1, \ldots, RC_6$ on a yearly basis in terms of euro's. The ranges are presented in Figure 5.5.

Note from Figure 5.5 that the 50%-95% quantile range is very large, i.e. the risks have a heavy tail as expected (see section 3.4.3). Note that RC_2 and RC_6 carry the most uncertainty, because they have the longest, i.e. heaviest tail (see footnote 2). Based on the assessments of the decision maker, the costs of recover actions due to incorrect instructions of VS-clients (RC_2) can with 95%-confidence not exceed €495.542,- on a yearly basis. According to the decision maker can the losses due to back valuation into accounts (RC_6) with 95%-confidence not exceed €397.560,- annually.

Coupon/dividend payments

The operational risks as defined by BANK B for coupon/dividend payments are:

RCD_1: Errors due to manual processes
RCD_2: Loss due to processing coupon/dividend payments within 2 systems
RCD_3: Manually recovered cum-ex positions before payment
RCD_4: Manually recovered cum-ex positions after payment
RCD_5: Adjusted errors
RCD_6: Losses due to back valuation

Table 5.9: *Risks accompanying coupon/dividend payments*

RCD_i for $i = 1, \ldots, 6$ are considered to be random variables with a distribution function. These variables are seperated and presented to the experts in the same way as the risk

[4]Unicorn is a program developed at the Delft University of Technology to combine distributions and investigate dependencies (see Kurowicka and Cooke (2004))

conversions variables. This results in:

$$
\begin{aligned}
RCD_1 &= \frac{5}{2} N_{rcd_1} \cdot X_{rcd_1} \\
RCD_2 &= RCD_2 \qquad (5.6) \\
RCD_3 &= \frac{5}{24} N_{rcd_3} \cdot X_{rcd_3} \\
RCD_4 &= \frac{5}{24} N_{rcd_4} \cdot X_{rcd_4} \\
RCD_5 &= \frac{5}{2} N_{rcd_5} \cdot X_{rcd_5} \\
RCD_6 &= 12\widetilde{RCD}_6 \qquad (5.7)
\end{aligned}
$$

Note that RCD_2 was not split up and was already in euro's on a yearly basis.
Again Unicorn is used to derive the 90% quantile range of $RCD_1, \ldots, RCD_6$ on a yearly basis in terms of euro's. The ranges are presented in Figure 5.6.

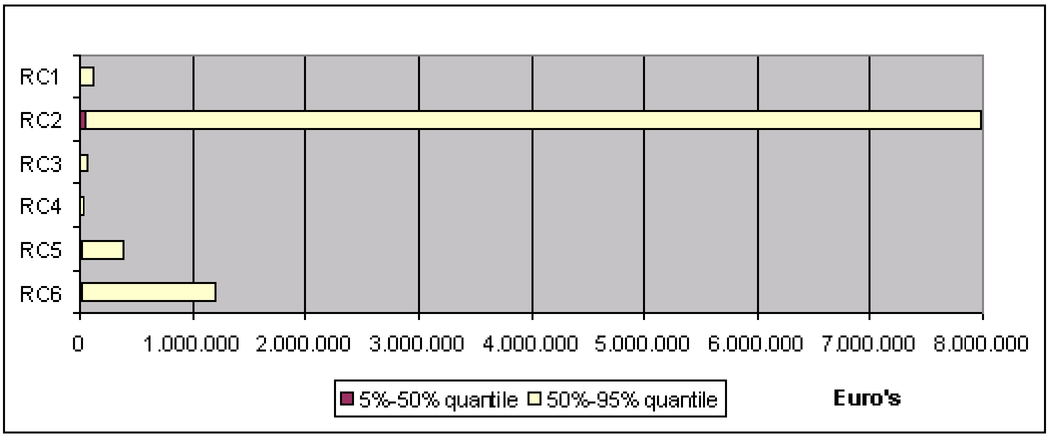

Figure 5.6: *90%-confidence intervals of the operational risks for coupon/dividend payments on a yearly basis*

Note from Figure 5.6 that RCD_2 has the longest tail and thus carries the largest uncertainty. According to the decision maker the losses due to processing coupon/dividend payments within 2 systems (RCD_2) can with 95%-confidence not exceed €7.970.000,- annually. Although losses due to back valuation (RCD_6) can also rise to €1.200.000,- annually, it is still almost a factor 6 less than the 95% quantile value of RCD_2.

5.4 LEVER as AMA

According to Basel II the method to quantify operational risk should use internal loss data, external loss data and scenario analysis. In the case study at BANK B only internal data is used for seed questions, but it is very well possible to use external data. Scenario analysis is neither used in the case study, because of the "pilot" nature of this research. It is not a hard task to put "what if" questions into the questionnaire as target questions and let the experts assess their uncertainty of the scenario's. It seems that the LEVER method is able to 'zoom in' on operational risks as much as needed, i.e. be sufficiently 'granular' to capture the major risk drivers. In this case study a few major risks of BANK B's core business were quantified. Further, it is assumed that the risks are independent. In practice it is not unlikely that the risks are correlated, but it is more important to have a good method to quantify operational risk in the first place. When banks get a good grip on their operational risks, it is possible to see if there are correlations between them. In the next chapter the aggregated operational loss distribution over one year will be computed and the capital requirement for BANK B based on the LEVER method is determined.

5.5 Conclusions

It can be concluded from the results of the case study that:

- The LEVER method is applicable within the field of operational risks of a bank, because the LEVER method gives a quantification of the investigated operational risks.
- The decision maker is more uncertain about the activities of coupon/dividend payments than of conversions.
- RC_2 and RCD_6 carry the most uncertainty for respectively conversions and coupon/dividend payments.
- The large quantile-range, shown in Figure 5.5 and 5.6, demonstrate large uncertainties about possible losses on certain operations within the bank. This means that a point estimation of an operational risk as used in a scorecard approach or scenario-based approach, misses information.
- From the filtering study it becomes clear that it is preferable to base the elicitation session on one field of expertise, but this will entail extra time and effort. Thus, a trade-off should be made when using the LEVER method within a bank between the scope of the field of expertise and the extra time and effort.
- Complications:
  - The questions of the questionnaire should be univocal.
  - The experts were not comfortable with expressing their uncertainties in numbers.

Chapter 6

Capital Requirement

6.1 Introduction

Once the risks are defined and the corresponding distributions are generated using the LEVER method, the capital requirement on operational risks should be computed according to Basel II (see section 2.5). Basel II requires a bank to set a capital aside to cover all the operational risks with a 99,9% certainty. First the aggregated operational loss distribution over one year should be determined using, for instance, the theory of the Loss Distribution Approach discussed in section 3.3. The 99,9% quantile of this loss distribution can be taken as the capital requirement. In this chapter the theory of section 3.3 and the results of the case study in Chapter 5 are used to determine the aggregated operational loss distribution belonging to the operations conversions and coupon/dividend payments of BANK B. Thus, this special case is studied, but it is also applicable on other operational risks within BANK B.

6.2 Operational loss distribution

The operational loss variable, denoted by L as in (3.3), can be written as a linear combination of the risks defined in section 5.3:

$$L = RC_1 + \cdots + RC_6 + RCD_1 + \cdots + RCD_6 \tag{6.1}$$

Let $F_L(x)$ be the distribution function of L and let $f_l(x)$ be the corresponding density function[1]. The density function $f_l(x)$ of the output variable L can be written as a convolution of all the risks (see section 3.3), if the risks are assumed to be independent. It is uncertain if the risks are independent, but for now it is reasonable to assume they are.

Let $F_{RC_i}(x)$ be the distribution of RC_i for $1 = 1, \ldots, 6$ and let $f_{rc_i}(x)$ be the corresponding

[1] A density function $f(x)$ only exists if the probability distribution $F(x)$ is a continuous function. In this case the distribution functions F_{RC_i}, $i = 1, \ldots, 6$, are not continuous, because they are a 100 point empirical distribution. If using a numerical method as Monte Carlo the distribution functions can be considered to approximate a continuous function.

density function then, using definition 1 in section 3.3:

$$
\begin{aligned}
f_{rc_1+rc_2}(x) &= \int_0^x f_{rc_1}(x-y) f_{rc_2}(y)dy \\
f_{rc_1+rc_2+rc_3}(x) &= \int_0^x f_{rc_1+rc_2}(x-y) f_{rc_3}(y)dy \\
&\vdots \\
f_{rc}(x) = f_{rc_1+\ldots+rc_6}(x) &= \int_0^x f_{rc_1+\ldots+rc_5}(x-y) f_{rc_6}(y)dy \qquad (6.2)
\end{aligned}
$$

where $f_{rc}(x)$ is the convoluted density function of the operational risks due to conversions. Then RC is a random variable with distribution $F_{RC}(x)$.

Let $F_{RCD_i}(x)$ be the distribution of RCD_i for $1 = 1, \ldots, 6$ and let $f_{rcd_i}(x)$ be the corresponding density function, then the same procedure as in (6.2) is followed resulting in:

$$
f_{rcd}(x) = f_{rcd_1+\ldots+rcd_6}(x) = \int_0^x f_{rcd_1+\ldots+rcd_5}(x-y) f_{rcd_6}(y)dy \qquad (6.3)
$$

as the convoluted density function of the operational risks due to coupon/dividend payments. Then RCD is a random variable with distribution $F_{RCD}(x)$.

Now, $f_l(x)$ can be written as a convolution of all the risks:

$$
f_l(x) = f_{rc+rcd}(x) = \int_0^x f_{rc}(x-y) f_{rcd}(y)dy \qquad (6.4)
$$

The expression in (6.4) can not be solved analytically, but can only be solved using a numerical method, e.g. Monte Carlo simulation (see section 3.3). Again Unicorn is used to compute the convoluted density function $f_l(x)$ and the aggregated loss distribution $F_L(x)$. The convoluted density functions $f_{rc}(x)$, $f_{rcd}(x)$ and $f_l(x)$ are shown in Figure 6.1. Note

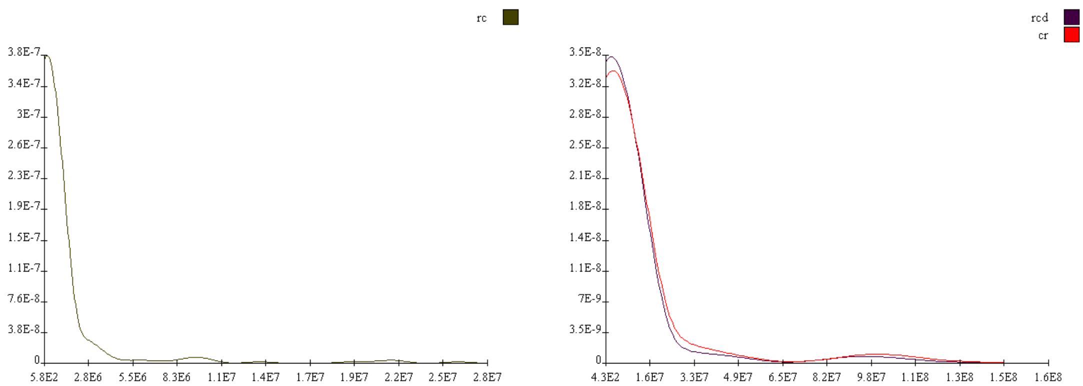

Figure 6.1: *The aggregated density functions $f_{rc}(x)$ (left) and $f_{rcd}(x)$ and $f_{or}(x)$ (right)*

that the behavior of the density functions f_{rcd} and f_l in Figure 6.1 is almost identical. This probably means that the variable RCD has the biggest influence on L. The main numerical

Variable	5% quantile	50% quantile	95% quantile	Mean
RC_1	1,73E+02	4,75E+03	2,01E+05	1,03E+05
RC_2	1,88E+02	1,93E+04	4,96E+05	2,45E+05
RC_3	2,73E+01	1,65E+03	2,54E+05	4,82E+05
RC_4	4,28E+00	1,73E+02	3,56E+04	2,93E+05
RC_5	4,24E+01	2,19E+03	8,71E+04	5,80E+04
RC_6	1,36E-03	1,75E+03	3,98E+05	2,03E+06
RCD_1	1,58E+02	3,58E+03	1,28E+05	2,57E+05
RCD_2	8,66E-04	6,19E+05	7,97E+06	3,53E+06
RCD_3	6,54E+01	1,38E+03	1,01E+05	2,12E+06
RCD_4	7,11E+00	3,41E+02	3,37E+04	7,84E+05
RCD_5	4,11E+02	1,21E+04	3,84E+05	1,85E+05
RCD_6	1,56E-03	2,62E+04	1,20E+06	2,02E+06
RC	1,16E+04	1,46E+05	7,52E+06	3,21E+06
RCD	1,70E+04	1,23E+06	4,67E+07	8,90E+06
L	1,04E+05	1,92E+06	8,44E+07	1,21E+07

Table 6.1: *Main quantiles and mean of the target and the combined variables in euro's using Monte Carlo simulation (100.000 runs)*

results are shown in Table 6.1 (all variables are in euro's).

As can be seen from Table 6.1 the 95% quantile of the operational loss L is €84,4 million and we did not even look at the 99,9% quantile as proposed by Basel II. Table 6.2

Quantile	L
0,95	8,44E+07
0,96	9,91E+07
0,97	1,02E+08
0,98	1,09E+08
0,99	1,28E+08
0,999	3,90E+08

Table 6.2: *The upper-quantiles of L*

shows the upper-quantiles of L and it can be seen that the 99,9% quantile is €390 million, which is an outrageous amount of money to set aside only for the risks of conversions and coupon/dividend payments. BANK B has an average gross income of the last three years of around €100 million. If BANK B chooses for a minimal capital requirement, the bank should use the Basic Indicator Approach (BIA). The BIA requires 15% of gross income to put aside for a banks operational risk, which would be around €15 million for BANK B.

Note from Table 6.1 that the mean of all variables are (much) higher then the median (50% quantile) and for some even higher then the 95% quantile ($RC_3, RC_4, RC_6, RCD_3, RCD_4$ and RCD_6). This is the case if the density functions of the variables have long tails as has been noticed in Figure 6.1. The means that are higher than the 95% quantiles show that

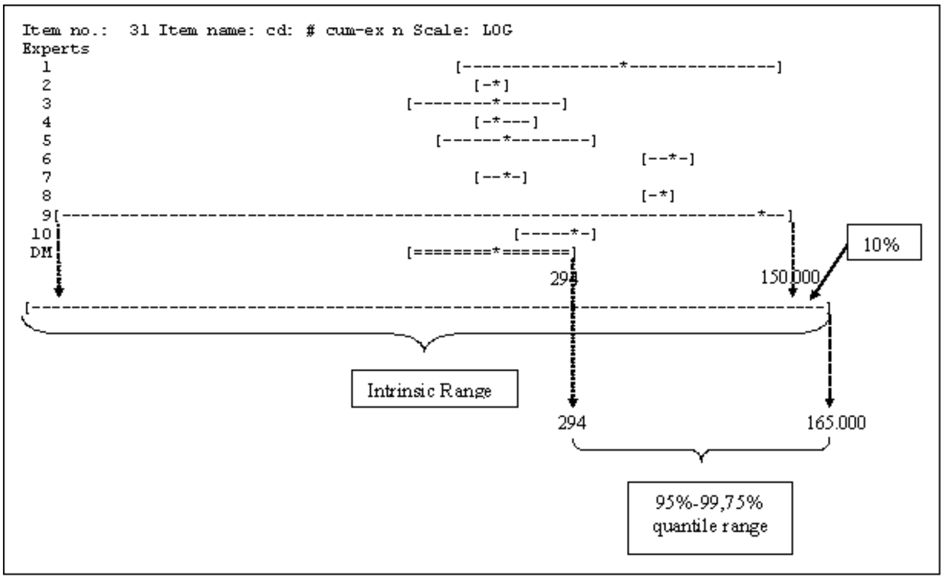

Figure 6.2: *Visual illustration of the determination of the intrinsic range and the upper quantile range of target variable 16*

the quantiles above the 95^{th} can have enormous values. For instance, the upper-quantiles of variable RCD_6 are shown in Table 6.3. The 95% quantile differs almost a factor 100 with the

Quantile	RCD_6
0,95	1,20E+06
0,96	3,64E+06
0,97	1,10E+07
0,98	3,34E+07
0,99	1,01E+08
0,999	1,20E+08

Table 6.3: *The upper-quantiles of RCD_6*

99% quantile, which is an implausible difference. Moreover, €101,2 million loss due to back valuation (RCD_6) is highly unlikely (much less than 1 percent). This would mean that this event only can cause bankruptcy.

The reason for the absurd high 99% quantile values for some variables is the default tail-model of Excalibur on the interpolation of the 95%-100% quantile range. As explained in section 4.2.3, the intrinsic range of an expert per item is obtained by using the k% overshoot rule on top of the highest value given by an expert for that item. This obviously concerns the intrinsic range of the decision maker too. Thus, if an expert is uncertain about a target variable and gives a very large range, the minimal and maximal value of this expert will be used to derive the intrinsic range for that particular item, even if the expert is not weighted. See figure 6.2 for an illustration of this situation. Both the minimum and the maximum value of the intrinsic range of target variable 16 are based on the assessment of expert 9. From Table 5.2 it is known that expert 9 is not weighted.

6.3 Taylor approximation

In this case the default tail-model of Excalibur is not very appropriate. To solve this issue a more plausible assumption should be made on especially the right tail, i.e. the high-quantiles. The right tail starting from the 95% quantile should not be as long and heavy as it is using the Excalibur' tail-model. A simple method is to use a Taylor expansion to extrapolate the right end part starting from the 95% quantile of the decision maker's distribution F_{DM} for item i (for instance $F_{N_{rc_1}}$). A general Taylor expansion series can be written as:

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) + \cdots \tag{6.5}$$

where $f'(x)$ is the derivative of $f(x)$ and h is the step-size. Applying a third order Taylor expansion, Equation 6.5 can be written as:

$$Q(q) = Q(q_0) + (q-q_0)Q'(q_0) + \frac{(q-q_0)^2}{2!}Q''(q_0) + \frac{(q-q_0)^3}{3!}Q'''(q_0) + O(h^4) \tag{6.6}$$

with Q the quantile function of a target variable (the subscripts are suppressed from notation to keep the formula more legible) and $O(h^4)$ is the fourth-order symbol of Landau. The quantile function Q is the inverse function of its distribution function:

$$Q(q) = F^{-1}(q) = x \qquad 0 < q < 1 \tag{6.7}$$

where $F(x) = \mathrm{P}(X \leq x) = q$. The quantile function is introduced, because using this notation it is easy to numerically implement the extrapolation. In this case the scalar h of Equation 6.5 can be taken as 1% quantile, i.e. $q - q_0 = 1\%$, which makes Equation 6.6 easy to solve. The derivative functions can be numerically solved using a backwards differential:

$$Q'(q) = \frac{Q(q) - Q(q-h)}{h} \tag{6.8}$$

The reason the short description of the numerical method used is that it is not within the

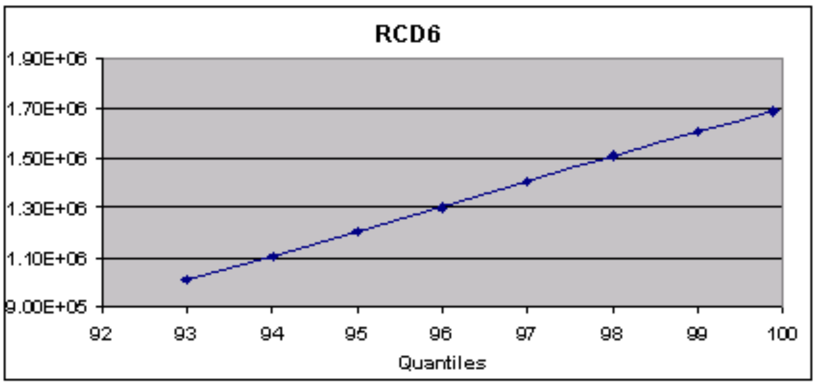

Figure 6.3: *Extrapolation using a Taylor expansion on the 96%-99,9% quantiles*

scope of this thesis. For more detail and proves on the used numerical methods the reader is referred to Brezinski and Redivo Zaglia (1991) or other numerical analysis books.

An example of the extrapolation using a Taylor expansion is given in Figure 6.3 of $RCD_{6,T}$. The notation of the variables is supplemented with an T to distinguish them with the variables using the default model. The corresponding quantiles of $RCD_{6,T}$ are given in Table 6.4. Comparing Table 6.4 with Table 6.3 show a large difference the corresponding quantiles as expected. For instance the 99% quantile differ almost a factor 100.

Quantile	$RCD_{6,T}$
0,95	1,20E+06
0,96	1,30E+06
0,97	1,41E+06
0,98	1,51E+06
0,99	1,61E+06
0,999	1,69E+06

Table 6.4: *The upper-quantiles of $RCD_{6,T}$ using Taylor extrapolation*

If using Equation (6.1) with the newly distributed variables and simulating them in Unicorn the main quantiles and mean are as in Table 6.5. Note that comparing the results with Table 6.1, the 95% quantile of the operational loss distribution L is now €8,49 million, which is almost a factor ten less than the corresponding result shown in Table 6.1. The 95^{th}-quantile of RC_T and RCD_T are also significantly smaller than the results based on the default tail-model of Excalibur. The median values do not differ as much, but the mean values show a

Variable	**5% quantile**	**50% quantile**	**95% quantile**	**Mean**
$RC_{1,T}$	1,75E+02	4,83E+03	8,76E+04	1,85E+04
$RC_{2,T}$	2,03E+02	1,91E+04	3,02E+05	6,72E+04
$RC_{3,T}$	2,63E+01	1,66E+03	1,30E+05	2,44E+04
$RC_{4,T}$	3,95E+00	1,83E+02	1,19E+04	2,16E+03
$RC_{5,T}$	4,41E+01	2,08E+03	4,14E+04	8,65E+03
$RC_{6,T}$	1,36E-03	1,75E+03	4,19E+05	6,18E+04
$RCD_{1,T}$	1,58E+02	3,43E+03	4,24E+04	9,46E+03
$RCD_{2,T}$	8,66E-04	5,84E+05	7,97E+06	1,74E+06
$RCD_{3,T}$	6,36E+01	1,35E+03	3,17E+04	6,08E+03
$RCD_{4,T}$	6,77E+00	3,15E+02	1,02E+04	2,00E+03
$RCD_{5,T}$	4,21E+02	1,15E+04	2,14E+05	4,52E+04
$RCD_{6,T}$	1,56E-03	2,62E+04	1,20E+06	2,09E+05
RC_T	1,11E+04	1,06E+05	6,26E+05	1,83E+05
RCD_T	1,51E+04	9,26E+05	8,28E+06	2,01E+06
L_T	8,27E+04	1,11E+06	8,49E+06	2,20E+06

Table 6.5: *Main quantiles and mean of the target and the combined variables in euro's based on the Taylor extrapolation*

remarkable difference. For instance, $RCD_{3,T}$ has a mean value of €6080 in Table 6.5, but the mean value of RCD_3 of €2,12 million in Table 6.1 is almost a factor 350 difference. Table 6.5

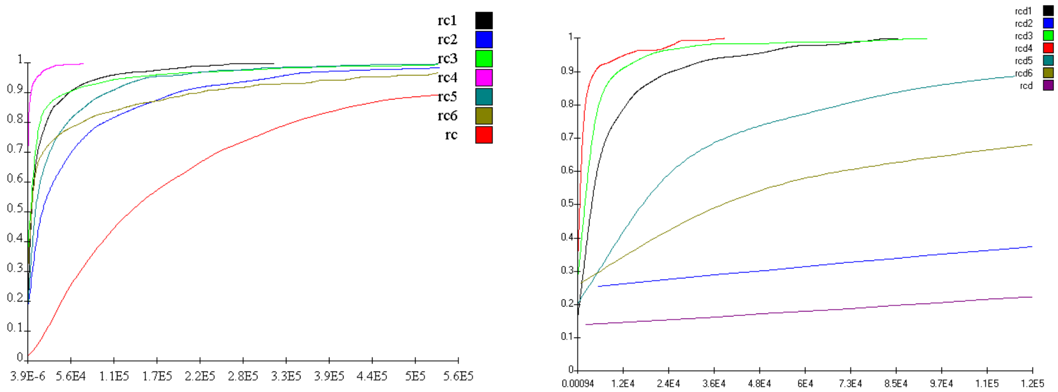

Figure 6.4: *Distribution functions $F_{RC_1,T}(x), \ldots, F_{RC_6,T}(x)$ and its aggregated distribution function $F_{RC,T}(x)$ (left) and the distribution functions $F_{RCD_1,T}(x), \ldots, F_{RCD_6,T}(x)$ and its aggregated distribution function $F_{RCD,T}(x)$ (right)*

shows more plausible numbers than the numbers based on the default tail-model of Excalibur, especially for the bank sector, where capital should be invested and not set aside. Figure 6.4 shows the distribution functions of the operational loss variables.

The upper-quantiles of the loss distribution using Taylor extrapolation are shown in Table 6.6. Note that the 99,9% quantile of L_T of €11,1 million is almost €370 million less than

Quantile	L_T
0,95	8,49E+06
0,96	8,80E+06
0,97	9,23E+06
0,98	9,74E+06
0,99	1,03E+07
0,999	1,11E+07

Table 6.6: *The upper-quantiles of L_T using Taylor extrapolation*

the corresponding quantile of L. This above shows that the tail model is of great essence, because it has a large influence on the final capital requirement.

6.4 Pareto fitting

Using a Taylor expansion to extrapolate the upper-quantiles is a fairly arbitrary assumption, but it was mainly used to show that upper-quantiles of the input variables of the model 6.1 can have a very large influence on the output variable L. Because of the 'heavy-tail' nature of operational risks it is more plausible to fit an extreme value distribution to the upper-quantiles (see section 3.4.3). Among the extreme value distributions the Pareto-type distributions have the heaviest tail behavior, which makes it an interesting candidates to apply to our model. Of the large class of Pareto-type distribution the Pareto distribution itself is chosen. Again the upper-quantiles from 96%- to 99,9% quantiles will be approximated.

A very useful graphical technique is a Quantile-Quantile plot (QQ-plot or quantile plot) to verify the goodness-of-fit of a model. It is a useful technique to compare distributions. A QQ-plot will be used to compare the Pareto distribution with the distributions of the target variables. Only one example will be given to show how it is done.

The Pareto distribution function with parameter α is given by:

$$F_\alpha(x) = 1 - x^{-\alpha} \tag{6.9}$$

The Pareto quantile distribution function $Q_\alpha(x)$ which is the inverse of its distribution function, is needed:

$$Q_\alpha(q) = F_\alpha^{-1}(q), \qquad 0 < q < 1 \tag{6.10}$$

It can easily be checked that $Q_\alpha(q)$ can be written as:

$$Q_\alpha(q) = (1-q)^{-\frac{1}{\alpha}}, \qquad 0 < q < 1 \tag{6.11}$$

In a quantile plot, one plots the sample quantiles against the quantiles of a given distribution, i.e. the Pareto distribution. A linear relation can be obtained by considering the logarithmic transform of both sides of equality (6.11):

$$\log Q_\alpha(q) = -\frac{1}{\alpha}\log(1-q) = \frac{1}{\alpha}\log Q_1(q) \tag{6.12}$$

because $Q_1(q) = (1-q)^{-1}$. Hence the plot

$$\Big(-\log(1-q), \log F \Big)$$

with F the distribution of a target variable, should be linear with slope $\frac{1}{\alpha}$ if the Pareto model would be a plausible fit to the underlying distribution. The best fit of a Pareto distribution to a heavy tail of an underlying distribution is probably above a certain high threshold u (see Theorem 2). So far, no automatic algorithm with satisfactory performance for the selection of the threshold is available. In Beirlant, Teugels, and Vynckier (1996) a few heuristic methods to approximate the threshold u are presented. In this case the threshold u is determined by using a QQ-plot and take that point from which a straight line is plausible to fit the remaining underlying distribution. For instance, Figure 6.5 shows the Pareto quantile plot against the distribution of RCD_6 until the 95^{th} quantile in steps of 1% quantiles. This plot shows a nice linear relation at the right end of the graph. From this figure the cut-off point at 86% quantile is chosen based on visual insight. Figure 6.6 shows the Pareto QQ-plot to RCD_6 from the 86% quantile to 99,9% quantile. A line is fitted through the points and the linear equation is shown together with the R^2 value[2]. Thus, the QQ-plot in Figure 6.6 indicates that it is plausible to fit the upper-quantiles of RCD_6 with a Pareto distribution with parameter $\alpha = \frac{1}{0{,}7449} = 1,34$.

With this fitted line the 96% to 99,9% quantiles can easily be computed. The results for $RCD_{6,Pa}$ are shown in Table 6.7. Again the notation of the variables is supplemented with an extra symbol, Pa, to distinguish them from the other models. Note that the tail is indeed

[2]R-squared value: A number from 0 to 1 that reveals how closely the estimated values for the trendline correspond to your actual data. A trendline is most reliable when its R-squared value is at or near 1. Also known as the coefficient of determination.)

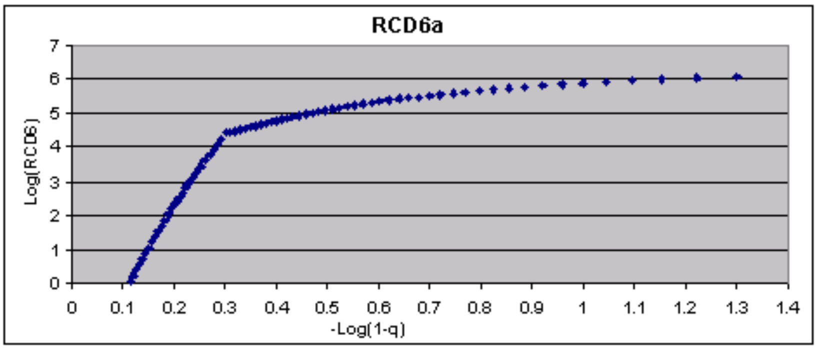

Figure 6.5: *Pareto QQ-plot to the distribution of RCD_6 until the 95% quantile*

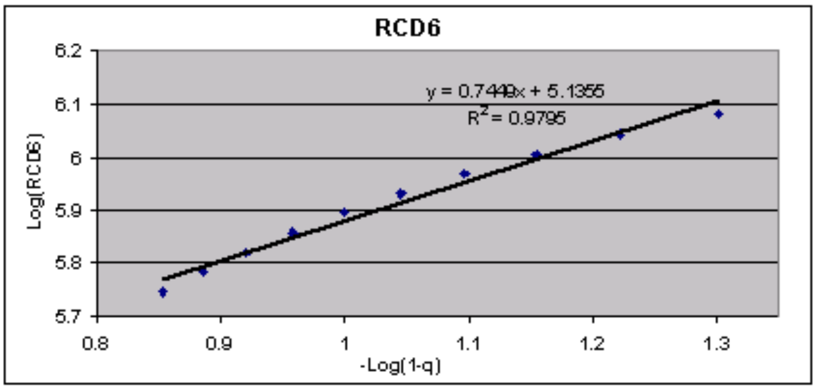

Figure 6.6: *Pareto QQ-plot against the 86%- to 95% quantiles of RCD_6 with a fitted line*

much heavier compared with a Taylor approximation, but much weaker compared with the default tail-model of Excalibur.

The main quantiles and the means of the variables using a Pareto tail-model are shown in Table 6.8. Note from Table 6.5 that the 95% quantile gives a loss of €9,63 million, which

Quantile	$RCD_{6,Pa}$
0,95	1,20E+06
0,96	1,50E+06
0,97	1,86E+06
0,98	2,52E+06
0,99	4,22E+06
0,999	2,35E+07

Table 6.7: *The upper-quantiles of RCD_6 using Pareto fitting*

is slightly more than the corresponding quantile of L_T. Again the difference with the loss of the 95th quantile of the default model is enormous. If we look at the 99,9% quantile shown in Table 6.9 the capital requirement using the Pareto model would be €38,8, which is more than a factor 3 higher than the corresponding quantile of L_T, but almost a factor 10 less than

Variable	5% quantile	50% quantile	95% quantile	Mean
$RC_{1,Pa}$	1,64E+02	4,85E+03	9,84E+04	2,15E+04
$RC_{2,Pa}$	2,06E+02	2,05E+04	3,48E+05	7,72E+04
$RC_{3,Pa}$	2,79E+01	1,72E+03	1,52E+05	4,33E+04
$RC_{4,Pa}$	4,25E+00	1,83E+02	1,30E+04	3,21E+03
$RC_{5,Pa}$	4,12E+01	2,01E+03	4,30E+04	9,41E+03
$RC_{6,Pa}$	1,36E-03	1,28E+03	4,19E+05	1,28E+05
$RCD_{1,Pa}$	1,61E+02	3,51E+03	4,49E+04	1,02E+04
$RCD_{2,Pa}$	8,66E-04	2,37E+05	7,97E+06	1,98E+06
$RCD_{3,Pa}$	6,11E+01	1,35E+03	3,35E+04	7,97E+03
$RCD_{4,Pa}$	7,42E+00	3,00E+02	1,23E+04	2,64E+03
$RCD_{5,Pa}$	4,00E+02	1,25E+04	2,44E+05	5,44E+04
$RCD_{6,Pa}$	1,56E-03	2,62E+04	1,20E+06	3,97E+05
RC_{Pa}	1,15E+04	1,14E+05	8,39E+05	2,83E+05
RCD_{Pa}	1,53E+04	9,04E+05	9,30E+06	2,45E+06
L_{Pa}	7,84E+04	1,20E+06	9,63E+06	2,73E+06

Table 6.8: *Main quantiles and mean of the target and the combined variables in euro's based on the Pareto fitting*

the capital requirement based on the default model. Thus, it can be concluded that using the Pareto distribution to fit the upper quantiles of the underlying target distributions results in a heavier tail compared with a Taylor approximation, but in a weaker tail compared with the default tail-model.

Quantile	L_{Pa}
0,95	9,63E+06
0,96	1,10E+07
0,97	1,33E+07
0,98	1,74E+07
0,99	2,00E+07
0,999	3,88E+07

Table 6.9: *The upper-quantiles of L_{Pa} using Pareto fitting*

6.5 Conclusions

The following conclusions can be made from this chapter:

- The mean and the 99,9% quantile of L result in €12,1 million and €380 million respectively using the default tail-model of Excalibur.
- Using a Taylor expansion to extrapolate the upper-quantiles is more plausible in this than the default tail-model.
- If using Taylor approximation the 99,9% quantile of the operational loss L_T is €11,1

million, which is almost a factor 35 less then the corresponding quantile when using the default tail-model.

- Using the Pareto distribution, which is an extreme value distribution, to fit the upper quantiles of the underlying target distributions results in a heavier tail compared with a Taylor approximation, but in a weaker tail compared with the default tail-model. The 99,9% quantile of the operational loss L_{Pa} is €38,8.

Thus, the tail model seems to be of great influence on the operational loss distribution and so on the capital requirement.

Remark

An additional remark on Table 6.1 and Table 6.5 should be made about the variables RC_6, RCD_2 and RCD_6. The quantile-values shown in the tables are obtained directly from the experts' assessments and show the largest quantile-range in their category. The other variables were separated in frequency and severity variables and aggregated using Monte Carlo simulation. The reason for this can be twofold: the risks are truly the most uncertain risks or the experts have difficulties with expressing their true beliefs if the target questions are not separated in frequency and severity variables.

Chapter 7

Conclusions and Recommendations

7.1 Conclusions

According to Basel II banks should have a better understanding and quantification of their market, credit and operational risks. This thesis is concentrated specifically on the quantification of operational risks within banks using an Advanced Measurement Approach (AMA). Especially large internationally active banks aspire to implement an AMA to quantify their operational risks.

Nowadays a continuum of AMAs are developed of which the main approaches are the loss distribution approach (LDA), the scenario-based approach and the scorecard approaches. Many banks use a combination of these approaches to get a complete comprehension of their operational risk. Unfortunately these approaches do not come without weaknesses. The main shortcomings of the LDA for quantifying operational risk is the requisite of a lot of loss data and the missing forward looking component. Using a qualitative approach such as the scenario-based approach or a scorecard approach needs a great deal of unvalidated guesswork, which makes it less reliable.

The LEVER method, which stands for Loss Estimated by Validating Experts in Risk, has been developed in order to be able to answer the first main question of this thesis:

"How can a bank quantify its operational risk based on scarce historical loss data (internal or external)?"

It seemed from the results of the case study that the LEVER method can give reliable quantifications of the operational risks within a bank based on only a small amount of loss data.

The large quantile-ranges for certain operational risks within the bank prove the large uncertainties that operational risks entail. This means that a point estimation of an operational risk as used in a scorecard approach or scenario-based approach, misses information. The LEVER method can help quantifying forward-looking scenarios in a reliable way.

The second main question of this thesis is:

"Is it possible to use the LEVER method as an AMA according to Basel II and determine the capital requirement for operational risks?"

It can be concluded that it is possible to use the LEVER method as an AMA according to Basel II. The LEVER method can combine internal loss data, external loss data and scenario analysis to quantify the operational risks. Furthermore it gives the opportunity to 'zoom in' on the operational risk drivers as much as required. As is shown in Chapter 6 the capital requirement for operational risks can be determined with the LEVER method.

A case study has been performed at BANK B to determine the capital requirement of two most important operational risks within their core business line (Agency services) and main risk type (Execution, Delivery & Process and Management). According to the default tail-model of Excalibur BANK B should keep a capital of €390 million (which is the 99,9% quantile of the capital requirement distribution) to be compliant with Basel II. This is of course an absurd amount of money, because the bank has only a Gross Income of around €100 (in 2002). In this case the bank should reserve 400% of his capital to be protected for only two out of many more of the operational risks. This result can be explained by two reasons. The first is that the model and especially the tail model does not fit the problem. The second reason is that the Basel II requirement to reserve the 99,9% of the capital requirement distribution is not appropriate. One would expect from a statistical point of view that with an uncertain matter as operational risk the possible outcome of an operational loss is very large, i.e. operational losses can lead to bankruptcy (see Barings Bank).

Using Taylor approximation as a tail model would give a capital requirement of €11,1 million according to the 99,9% quantile, which is more than 35× smaller than the default model. This result seems more plausible, but is still very high for just two operational risks within one business line and one risk type.

Finally a more sophisticated model, the Pareto distribution, is used to fit the upper-quantiles as a comparison. The Pareto fitting gives a 99,9% capital requirement of €38,8 million, which is still a high amount of capital for BANK B to reserve for their two core services.

Based on the quantitative results of the case study BANK B can best choose the Basic Indicator Approach (BIA) and keep 15% of their average positive Gross Income of the last three years (€15 million) as required capital for operational risks if a minimum of required capital is the goal. But the mutual competition plays an important role too in the sense that all the banks want to be the "best pupil in class". That is why most banks will choose for a more advanced approach to quantify their operational risks than the BIA.

7.2 Recommendations

7.2.1 Applying LEVER

If using LEVER it is recommended to keep the following in mind:

- The questions of the questionnaire should be univocal.
- The experts should be comfortable with expressing their uncertainties in numbers.

- It is preferable to base the elicitation session on one expertise field, but this will entail extra time and effort. Thus, a trade-off should be made when using the LEVER method within a bank between the scope of the expertise field and the extra time and effort.

- In the case study presented in this thesis the risks are assumed to be independent. In practice this assumption does not always hold. Before using the LEVER method, research should be done to the correlations of the operational risks.

- One should hide the seed questions of the questionnaire in order to prevent 'gaming', i.e. an expert should not be able to differentiate the seed questions and the target questions.

In this case BANK B is recommended to do research on how the large uncertainties can be reduced using 'controls' for instance.

7.2.2 Basel Committee

I would recommend the Basel Committee to review the requirement of reserving the 99,9% quantile of the overall operational risk distribution as capital. It makes more sense to require the 95% quantile or even the 90% quantile, because of the uncertain nature of operational risks.

7.2.3 Further research

A weakness of using the LEVER method is that it is sensitive for 'gaming', i.e. experts can influence the capital requirement. In other words, experts with incentives to manipulate the capital requirement are able to do that when they can differentiate the seed questions with the target questions. This is the reason not to use the LEVER method every year. An interesting sequel research will be to apply Bayesian 'updating'. The obtained capital requirement distribution of this thesis can then be used as the 'prior'. Then this prior can be updated with next years loss data to obtain the 'posterior' distribution. The year after having obtained the posterior this distribution can again be used as the prior to be updated, and so on.

In this thesis the studied operational risks within BANK B were assumed to be independent. In practice it is not unlikely that the risks are correlated. It would be interesting for BANK B and in general to do research on possible correlations between the risks.

Another interesting or even necessary research would be to do more research on the tail behavior of operational risks and in particular on the extreme value index ξ (see Beirlant, Teugels, and Vynckier (1996) for more detail). One should apply extreme value theory on a loss data set (if available) above a certain high threshold to examine if $\xi > 0$, $\xi = 0$ or $\xi < 0$, which all point to different distribution families. If $\xi > 0$ a plausible distribution to fit the underlying data would be of Pareto-type, which is the heavy-tailed family of distribution. One would expect that the operational risk behavior would have that character, but research should point this out.

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Appendix A

Questionnaire

Name: ____

Department: _______________________________________

Function: ____

Years employed at BANK B: __________________________

Years employed at Custody Services: _______________

Participated in RSA Custody Services: yes /   no

Please fill in your 5\%, 50\% en 95\% quantiles for the following uncertain values. Use the given definitions (see presentation or hand-out) for the italic words.

CONVERSIONS

1. What is the number of complaints of payments and stocks about conversions the second half year of 2003 (July-December)?

5%________   50%________   95%_______

2. What is the total amount claimed about conversions during the second half year of 2003 (July - December)?

5%________   50%________   95%_______

3. How many complaints-without-claim about conversions take place in February 2004?

5%________ 50%________ 95%_______

4. What was the average number of complaints-without-claim about conversions per day February 2004?

5%________ 50%________ 95%_______

5. What is the difference of the number of complaints-without-claim between the month with the least and the month with the most complaints-without-claim about conversions in the period April 2003-Februari 2004?

5%________ 50%________ 95%_______

6. How many conversions are closed in 2003?

5%________ 50%________ 95%_______

7. What is the difference in closed conversions between the month with the least and the month with the most closed conversions in 2003?

5%________ 50%________ 95%_______

8. How many hours are spend on conversions the second half year of 2003 (Juli-December)?

5%________ 50%________ 95%_______

9. How many errors due to manual processes on conversions have to be corrected in an arbitrary month?

5%________ 50%________ 95%_______

10. How much time does it take to correct an arbitrary error due to manual processes on conversions?

5%________ 50%________ 95%_______

11. How many recover actions due to incorrect instructions of

VS-clients take place on an arbitrary day?

5%________ 50%________ 95%_______

12. How much time does it take to recover an arbitrary incorrect instruction of VS-clients on an arbitrary day?

5%________ 50%________ 95%_______

13. How many cum-ex positions on conversions are manually recovered before payment in an arbitrary year?

5%________ 50%________ 95%_______

14. How many corrections of cum-ex positions on conversions take place after payment in an arbitrary year?

5%________ 50%________ 95%_______

15. How much time does a correction of an arbitrary cum-ex position on conversions take?

5%________ 50%________ 95%_______

16. How many errors on conversions are adjusted in an arbitrary month?

5%________ 50%________ 95%_______

17. How much time does it take to adjust an arbitrary error on conversions?

5%________ 50%________ 95%_______

18. How much loss due to back valuation does Bank B suffer in an arbitrary month on conversions?

5%________ 50%________ 95%_______

COUPON/DIVIDEND PAYMENTS

1. What is the number of complaints with claims about coupon/dividend payments the second half year of 2003 (July-December)?

5%________ 50%________ 95%_______

2. What is the total amount claimed about coupon/dividend payments during the second half year of 2003 (July - December)?

5%________ 50%________ 95%_______

3. How many complaints-without-claim about coupon/dividend payments take place in February 2004?

5%________ 50%________ 95%_______

4. What was the average number of complaints-without-claim about coupon/dividend payments per day February 2004?

5%________ 50%________ 95%_______

5. What is the difference of the number of complaints-without-claim between the month with the least and the month with the most complaints-without-claim about coupon/dividend payments in the period April 2003-Februari 2004?

5%________ 50%________ 95%_______

6. How many coupon/dividend payments are closed in 2003?

5%________ 50%________ 95%_______

7. What is the difference in closed coupon/dividend payments between the month with the least and the month with the most closed coupon/dividend payments in 2003?

5%________ 50%________ 95%_______

8. How many hours are spend on coupon/dividend payments the second half year of 2003 (Juli-December)?

5%________ 50%________ 95%_______

9. How many errors due to manual processes on coupon/dividend payments had to be corrected in an arbitrary month in 2003?

5%________ 50%________ 95%_______

10. How much time does it take to correct an arbitrary error due to manual processes on coupon/dividend payments?

5%________ 50%________ 95%_______

11. How much loss due to processing coupon/dividend payments within 2 systems is suffered in an arbitrary year?

5%________ 50%________ 95%_______

12. How many cum-ex positions on coupon/dividend payments are manually recovered before payment in 2003?

5%________ 50%________ 95%_______

13. How many corrections of cum-ex positions on coupon/dividend payments take place after payment in an arbitrary year?

5%________ 50%________ 95%_______

14. How much time does a correction of an arbitrary cum-ex position on coupon/dividend payments take?

5%________ 50%________ 95%_______

15. How many errors on coupon/dividend payments are adjusted in an arbitrary month?

5%________ 50%________ 95%_______

16. How much time does it take to adjust an arbitrary error on coupon/dividend payments?

5%________ 50%________ 95%_______

17. How much loss due to back valuation does Bank B suffer in an arbitrary month on coupon/dividend payments?

5%________ 50%________ 95%_______

Appendix B

Seed and Target Variables

Nr	Abbreviations	Seed questions
1	c: ♯ claim	What is the number of complaints of payments and stocks about conversions the second half year of 2003 (July-December)?
2	c: € claim	What is the total amount claimed about conversions during the second half year of 2003 (July - December)?
3	c: ♯ wo clai	How many complaints-without-claim about conversions take place in February 2004?
4	c: ave wo clai	What was the average number of complaints-without-claim about conversions per day February 2004?
5	c: dif wo clai	What is the difference of the number of complaints-without-claim between the month with the least and the month with the most complaints-without-claim about conversions in the period April 2003-Februari 2004?
6	c: ♯ 2003	How many conversions are closed in 2003?
7	c: diff mon ♯	What is the difference in closed conversions between the month with the least and the month with the most closed conversions in 2003?
8	c: ♯ hours	How many hours are spend on conversions the second half year of 2003 (Juli-December)?
9	cd: ♯ claim	What is the number of complaints with claims about coupon/dividend payments the second half year of 2003 (July-December)?
10	cd: € claim	What is the total amount claimed about coupon/dividend payments during the second half year of 2003 (July - December)?
11	cd: ♯ wo clai	How many complaints-without-claim about coupon/dividend payments take place in February 2004?
12	cd: ave wo cla	What was the average number of complaints-without-claim about coupon/dividend payments per day February 2004?
13	cd: dif wo cla	What is the difference of the number of complaints-without-claim between the month with the least and the month with the most complaints-without-claim about coupon/dividend payments in the period April 2003-Februari 2004?
14	cd: ♯ 2003	How many coupon/dividend payments are closed in 2003?
15	cd: dif mon ♯	What is the difference in closed coupon/dividend payments between the month with the least and the month with the most closed coupon/dividend payments in 2003?
16	cd: ♯ hours	How many hours are spend on coupon/dividend payments the second half year of 2003 (Juli-December)?

Nr	Variables	Abbreviations	Target questions
1	N_{rc1}	c: $\sharp$ errors	How many errors due to manual processes on conversions have to be corrected in an arbitrary month?
2	X_{rc1}	c: time errors	How much time does it take to correct an arbitrary error due to manual processes on conversions?
3	N_{rc2}	c: $\sharp$ recoverVS	How many recover actions due to incorrect instructions of VS-clients take place on an arbitrary day?
4	X_{rc2}	c: time recove	How much time does it take to recover an arbitrary incorrect instruction of VS-clients on an arbitrary day?
5	N_{rc3}	c: $\sharp$ cum-ex be	How many cum-ex positions on conversions are manually recovered before payment in an arbitrary year?
6	X_{rc3}	c: time cum-ex	How much time does a correction of an arbitrary cum-ex position on conversions take?
7	N_{rc4}	c: $\sharp$ cum-ex af	How many corrections of cum-ex positions on conversions take place after payment in an arbitrary year?
8	X_{rc4}	c: time cum-ex	How much time does a correction of an arbitrary cum-ex position on conversions take?
9	N_{rc5}	c: error adjus	How many errors on conversions are adjusted in an arbitrary month?
10	X_{rc5}	c: time adjust	How much time does it take to adjust an arbitrary error on conversions?
11	$\widetilde{RC}_6$	c: backvaluati	How much loss due to back valuation does Bank B suffer in an arbitrary month on conversions?
12	N_{rcd_1}	cd: $\sharp$ errors	How many errors due to manual processes on coupon/dividend payments had to be corrected in an arbitrary month?
13	X_{rcd_1}	cd: time error	How much time does it take to correct an arbitrary error due to manual processes on coupon/dividend payments?
14	RCD_2	cd: 2 systems	How much loss due to processing coupon/dividend payments within 2 systems is suffered in an arbitrary year?
15	N_{rcd_3}	cd: $\sharp$ cum-ex b	How many cum-ex positions on coupon/dividend payments are manually recovered before payment?
16	X_{rcd_3}	cd: time cum-e	How much time does a correction of an arbitrary cum-ex position on coupon/dividend payments take?
17	N_{rcd_4}	cd: $\sharp$ cum-ex a	How many corrections of cum-ex positions on coupon/dividend payments take place after payment in an arbitrary year?
18	X_{rcd_4}	cd: time cum-e	How much time does a correction of an arbitrary cum-ex position on coupon/dividend payments take?
19	N_{rcd_5}	cd: error adju	How many errors on coupon/dividend payments are adjusted in an arbitrary month?
20	X_{rcd_5}	cd: time adjus	How much time does it take to adjust an arbitrary error on coupon/dividend payments?
21	$\widetilde{RCD}_6$	cd: backvaluat	How much loss due to back valuation does Bank B suffer in an arbitrary month on coupon/dividend payments?

Appendix C

Excalibur output

C.1 ItemDM

```
Case name : case study                 15-9-2004             CLASS version W4.0
-------------------------------------------------------------------------------
Resulting solution (combined DM distribution of values assessed by experts)
 Bayesian Updates: no      Weights:  item       DM Optimisation:  yes
 Significance Level:   0.0034    Calibration Power:   1.0000
-----------------------------------------------------------------------
```

Nr.	Id	Scale	5%	50%	95%	Realizatii
1	c: # claim	LOG	5,318	133,3	590,3	10
2	c: $$ claim	LOG	1037	6,632E007	1,998E008	1,787E005
3	c: # wo claim	LOG	4,015	21,8	101,4	73
4	c: ave wo clai	LOG	1	2,029	6,48	3,6
5	c: dif wo clai	LOG	0,0001078	28,21	581,5	71
6	c: # 2003	LOG	51,11	562,8	4480	3605
7	c: dif mon #	LOG	5,019	105,1	643,6	281
8	c: # hours	LOG	1033	9006	1,996E004	1,21E004
9	c: # errors	LOG	5,001	22,32	170,9	
10	c: time errors	LOG	8	87,52	539,9	
11	c: # recoverVS	LOG	2,001	8,91	38,95	
12	c: time recove	LOG	1,004	61,38	317,3	
13	c: # cum-ex be	LOG	17,04	99,75	4996	
14	c: # cum-ex af	LOG	3	9,441	412,1	
15	c: time cum-ex	LOG	3,829	52,49	534	
16	c: errors adju	LOG	2,001	16,17	77,13	
17	c: time adjust	LOG	3,712	74,49	476,8	
18	c: backvaluati	LOG	0,0001135	145,5	3,495E004	
19	cd: # claim	LOG	5,015	78,01	546,2	2
20	cd: $$ claim	LOG	1006	1,5E007	1,169E008	1,257E004
21	cd: # wo claim	LOG	1,001	6,193	70,3	134
22	cd: ave wo cla	LOG	1	2,062	17,04	6,65
23	cd: dif wo cla	LOG	2,003	25,94	235,2	138
24	cd: # 2003	LOG	102,8	1,128E004	3,997E004	2,156E004
25	cd: dif mon #	LOG	100,3	3985	1,993E004	1149
26	cd: # hours	LOG	1015	6606	1,994E004	1,417E004
27	cd: # errors	LOG	5,001	16,49	99,95	
28	cd: time error	LOG	8,716	88,92	368,3	
29	cd: 2 systems	LOG	0,0008662	5,837E005	7,967E006	
30	cd: # cum-ex b	LOG	25,01	76,88	813,2	
31	cd: # cum-ex a	LOG	2,001	25,27	294,2	

```
  32|cd: time cum-e|LOG  |     7,529|     58,58|     539,8|          |
  33|cd: error adju|LOG  |        10|     97,39|     420,5|          |
  34|cd: time adjus|LOG  |     8,152|     59,33|       480|          |
  35|cd: backvaluat|LOG  | 0,0001302|      2185|     1E005|          |
----------------------------------------------------------------------
-----------------------------------------------------------------------
                                                      (c) 1999 TU Delft
```

C.2 Range graphs of the items

```
                          Range graph of input data
Item no.:   1 Item name: c: # claim Scale: LOG
Experts
  1 [--------------------------------------------*---------]
  2                    [-------*----]
  3 [--------------------------------------------*---------------------]
  4                                    [-----------------*-------------]
  5                                       [---------*------]
  6          [------------*---------]
  7                                                          [---------*------]
  8                                                 [----*---]
  9                       [---------------------*-------]
 10                                           [--------------------*----------]
Ite [=============================================*====================]
Glo [============================================*=====================]
EqualDM      [======================================*=======================]
Real:::::::::#:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
             10
     5                                                                    1000

Item no.:   2 Item name: c: $$ claim Scale: LOG
Experts
  1                                 [----------------------------------------*]
  2            [-----*--]
  3 [----------------------------------------------------------*---]
  4                            [----*-]
  5                                 [---------------*---]
  6            [----*---]
  7                    [--*--]
  8                 [------*----]
  9                                   [-*---]
 10                                 [----*--------------]
Ite [========================================================*=====]
Glo [=======================================================*======]
EqualDM         [===================*========================================]
Real::::::::::::::::::::::::::#::::::::::::::::::::::::::::::::::::::::::::::::
                           1.787E005
    1000                                                                 2E009

Item no.:   3 Item name: c: # wo claim Scale: LOG
Experts
  1 [------------------------------*-----------]
  2                      [-----*---]
  3          [---------------------*------------------]
  4                      [---------*-----------]
  5                            [---------------*------------]
```

```
  6                                                       [-----------*--------]
  7                                             [------*-------]
  8                                             [-----*---]
  9             [------------------------*------]
 10                                                   [------*-----]
ItemDM       [=======================*====================]
GlobalDM     [=======================*====================]
EqualDM        [===============================*=========================]
Real:::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::
                                                    73
     2                                                                     500

Item no.:   4 Item name: c: ave wo clai Scale: LOG
Experts
  1                                                        [-------------*---]
  2 [------------------------------------------------------*---]
  3                                                        [---*---]
  4                                                        [---------*---]
  5                                                               [--*--------]
  6                                                                [-----*---]
  7                                                        [-------*----]
  8                                                           [--*-]
  9 [------------------------------------------------------*------]
 10                                                                  [-*-]
ItemDM                                                     [===*=======]
GlobalDM                                                   [===*========]
EqualDM            [==============================================*==========]
Real:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::
                                                                   3.6
    0.0001                                                                  25

Item no.:   5 Item name: c: dif wo clai Scale: LOG
Experts
  1                                                    [---------*--]
  2                                                       [-*]
  3 [------------------------------------------------------*--------]
  4                                                            [--*--]
  5                                                        [-------*----------]
  6                                                              [-*]
  7                                                            [-*-]
  8                                                 [--*-]
  9 [-----------------------------------------------------------*]
 10                                                           [---*-]
Ite [======================================================*=============]
Glo [======================================================*============]
EqualDM             [===========================================*=========]
Real::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::
                                                                71
    0.0001                                                                2000

Item no.:   6 Item name: c: # 2003 Scale: LOG
Experts
  1                                            [----------------------*--------]
  2                                                                  [---*---]
  3 [-------------------------------*----------------]
  4                                                        [----*----]
  5                                                     [---------*--]
```

```
  6                                                        [-*--]
  7                                        [-*----]
  8                                                        [-*---]
  9                [----------------------*----------------]
 10                                      [-------*--]
Ite [=================================*============================]
Glo [================================*============================]
EqualDM             [=======================================*===============]
Real::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::
                                                               3605
    50                                                                   1E004

Item no.:   7 Item name: c: dif mon # Scale: LOG
Experts
  1                                                             [------------*]
  2                                                          [---*--]
  3                                             [------------*---]
  4                                                              [--*]
  5                                                              [--*]
  6                                                          [*]
  7                                                         [-*-]
  8                                                       [---*-]
  9 [-----------------------------------------------------------*-----]
 10                                                       [-*]
ItemDM                                          [============*=======]
GlobalDM                                        [============*=======]
EqualDM                        [===============================*==============]
Real:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::
                                                                 281
    0.0001                                                                7000

Item no.:   8 Item name: c: # hours Scale: LOG
Experts
  1         [----------------------------------------*---]
  2                                              [----*--]
  3     [---------------------------------------*------------------]
  4              [------*--]
  5                                                     [---*----]
  6                                                                      [--*---]
  7 [--*--------]
  8                          [-------------*-------]
  9                                             [-------------*----------]
 10                                                   [--*----]
ItemDM   [==========================================*==============]
GlobalD [========================================*=================]
Equal [===============================================*=====================]
Real:::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::
                                                      1.21E004
    800                                                                3.5E004

Item no.:   9 Item name: c: # errors Scale: LOG
Experts
  1                                                    [---------*---------]
  2                                                              [--*-]
  3                                                 [-----*------]
  4                                                    [-------*--]
  5                                                    [-------*--------------]
```

```
  6                                                              [------*--]
  7                                                         [*-]
  8                                                        [*]
  9 [--------------------------------------------------*------]
 10                                                           [--*----]
ItemDM                                              [=====*=========]
GlobalDM                                            [======*==========]
EqualDM                     [=================================*============]
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

    0.0001                                                                2000

Item no.:  10 Item name: c: time errors Scale: LOG
Experts
  1          [-------------------------*-------]
  2 [----------------*----------]
  3                  [----------------------*------------------]
  4                  [------*-------]
  5          [----------*------------------------------------]
  6                      [------*------]
  7                      [------*---]
  8                             [------*-----------]
  9                                            [-------------*----------------]
 10                      [-------------*-------]
ItemDM            [========================*===================]
GlobalDM         [=========================*===================]
EqualDM    [======================*==================================]
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

     2                                                                    2400

Item no.:  11 Item name: c: # recoverVS Scale: LOG
Experts
  1                                                          [-------*--------]
  2                                                      [---*---]
  3                                                  [-------*------]
  4 [------------------------------------------------*----]
  5                                                       [-*--]
  6                                                       [-*----]
  7                                                  [----*--]
  8                                           [---*--]
  9 [--------------------------------------------------*-----]
 10                                               [------*---]
ItemDM                                               [=====*========]
GlobalDM                                             [=====*========]
EqualDM           [========================================*==============]
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

    0.0001                                                                 300

Item no.:  12 Item name: c: time recove Scale: LOG
Experts
  1                                                      [--------*---]
  2                                                  [-------*----]
  3                                              [--------------------*--------]
  4 [--------------------------------------------------------*----]
  5                                                               [-----*-----]
  6                                                                    [-*----]
  7                                                                   [---*---]
  8                                                               [---*--]
```

```
  9 [--------------------------------------------------------------------*----]
 10                                                            [--*---]
ItemDM                                           [====================*=======]
GlobalDM                                         [====================*=======]
EqualDM              [==============================================*=========]
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
    0.0001                                                                 360

Item no.:  13 Item name: c: # cum-ex be Scale: LOG
Experts
  1                                            [------------------*-----]
  2                                                        [-*-]
  3                                                [---*--------------]
  4                                                    [---------*--]
  5                                            [-----*--------------]
  6                                                    [-----*--]
  7                                                             [*]
  8                                                  [-*---]
  9 [-------------------------------------------------------------------*-----]
 10                                                             [---*-]
ItemDM                                           [=====*==============]
GlobalDM                                        [======*==============]
EqualDM                        [================================*===========]
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
    0.0001                                                               5E004

Item no.:  14 Item name: c: # cum-ex af Scale: LOG
Experts
  1                                            [--------------*-]
  2                                                [*]
  3                                       [---*------------]
  4                                              [------*]
  5                                          [---*--------------]
  6                                                    [---*]
  7                                                         [-*-]
  8                                                          [--*]
  9 [----------------------------------------------------------------------*--]
 10                                            [-----*]
ItemDM                                    [===*==============]
GlobalDM                                  [====*=============]
EqualDM                       [=========================*===================]
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
    0.0001                                                               5E004

Item no.:  15 Item name: c: time cum-ex Scale: LOG
Experts
  1                [----------------*-------------]
  2 [-----*---]
  3                       [----------------*--------------------]
  4                [------*-----]
  5           [----*------]
  6                           [-----*------]
  7                       [---------*---]
  8                                 [------*------]
  9                                               [------------*--------------]
 10                           [-----*------]
ItemDM          [=========================*=====================]
```

```
GlobalDM          [======================*=====================]
EqualDM [==========================*==============================]
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
     1                                                                  2400

Item no.:  16 Item name: c: errors adju Scale: LOG
Experts
  1                                                 [----------*--]
  2                                                            [*-]
  3                                             [--------*----]
  4                                                       [----*-]
  5                                                    [----*---------------]
  6                                                               [------*--]
  7                                                              [-*--]
  8                                                    [---*-]
  9 [-------------------------------------------------------*-----]
 10                                                               [----*-]
ItemDM                                          [=================]
GlobalDM                                        [======*==========]
EqualDM                      [====================================*==========]
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
    0.0001                                                              1000

Item no.:  17 Item name: c: time adjust Scale: LOG
Experts
  1               [----------------*-----------------]
  2 [-----*---]
  3               [---------------------------*---------------]
  4                      [--------*---]
  5          [---------*-------]
  6                          [-----*------]
  7                          [-----*---]
  8                                       [------*---]
  9                                             [------------*--------------]
 10                          [------------*------]
ItemDM          [============================*=================]
GlobalDM         [============================*================]
EqualDM [============================*==============================]
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
     1                                                                  2400

Item no.:  18 Item name: c: backvaluati Scale: LOG
Experts
  1                                                 [--------*-----]
  2                                                             [-*]
  3 [------------------------------------------*------------]
  4 [-*]
  5                                                   [---*----------]
  6                                                   [------*--]
  7                                                  [--*--]
  8                                              [--*--]
  9 [-------------------------------------------------------------*-----------]
 10                                                   [---*--]
Ite [=============================================*=================]
Glo [============================================*================]
Equa [=====================================================*==============]
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
```

```
    0.0001                                                               1E006

Item no.:  19 Item name: cd: # claim Scale: LOG
Experts
  1                                                   [------------------*----]
  2                                                           [-*]
  3                                                   [-----------*----]
  4                                                                [---*]
  5                                                                      [-*--]
  6                                                         [----*--]
  7                                                                      [--*-]
  8                                                                      [-*]
  9 [------------------------------------------------------------*-----]
 10                                                              [------*--]
ItemDM                                                [============*========]
GlobalDM                                              [============*========]
EqualDM                         [=====================================*=======]
Real:::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::::
                                                   2
    0.0001                                                                1000

Item no.:  20 Item name: cd: $$ claim Scale: LOG
Experts
  1                                              [--------------------------*-]
  2                                                  [*]
  3                                        [-----------------------*----]
  4                                                         [--*--]
  5                                                    [-----------*-----]
  6                                       [--------*]
  7                                            [-*-]
  8                                        [---*-]
  9 [------------------------------------------------------*--]
 10                                        [---*-]
ItemDM                                     [======================*=====]
GlobalDM                                   [======================*=====]
EqualDM                   [==========================*=======================]
Real:::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::::
                                               1.257E004
    0.0001                                                             1.5E009

Item no.:  21 Item name: cd: # wo claim Scale: LOG
Experts
  1                                                      [---------------*----]
  2                                                   [--*--]
  3                                           [--------*------]
  4                                                         [----*--]
  5                                                      [--------*-----------]
  6                                                          [----*---]
  7                                                           [--*----]
  8                                                                 [-*]
  9 [----------------------------------------------------*----]
 10                                                              [------*--]
ItemDM                                        [========*==========]
GlobalDM                                      [=========*============]
EqualDM                      [===================================*===========]
Real::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::
                                                                     134
```

```
    0.0001                                                                1000

Item no.:  22 Item name: cd: ave wo cla Scale: LOG
Experts
  1                                                      [-------------*---]
  2                                                [---*-]
  3                                                [---*-------]
  4                                                [--------*--]
  5                                                      [--*-----------------]
  6                                                    [--*----]
  7                                                [-----*--]
  8                                                       [*-]
  9 [----------------------------------------------*-------]
 10                                                        [---*---]
ItemDM                                             [===*==========]
GlobalDM                                           [===*============]
EqualDM                   [================================*==============]
Real:::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::
                                                            6.65
    0.0001                                                                 200

Item no.:  23 Item name: cd: dif wo cla Scale: LOG
Experts
  1                                                              [---------*--]
  2                                             [---*-]
  3                                             [----------*-----]
  4                                                           [--*--]
  5                                                          [-----*-------]
  6                                                    [----*--]
  7                                                         [--*-]
  8                                                           [--*-]
  9 [-------------------------------------------------------------*-]
 10                                                 [-*]
ItemDM                                          [==========*=========]
GlobalDM                                        [==========*==========]
EqualDM                       [================================*=============]
Real:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::
                                                                   138
    0.0001                                                                2000

Item no.:  24 Item name: cd: # 2003 Scale: LOG
Experts
  1                                 [--------------------*--------------------]
  2                                         [*]
  3         [-----------------------*-------]
  4                              [*-]
  5                                    [*]
  6                                 [----------*---]
  7                                   [*-]
  8                                     [*]
  9 [--------------------*-------]
 10                         [--*-]
ItemDM      [========================*======]
GlobalDM    [========================*======]
EqualDM      [==========================*=========================]
Real::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::::::::::::::
                                     2.156E004
```

```
    20                                                                   3E007

Item no.:  25 Item name: cd: dif mon # Scale: LOG
Experts
  1                                                                   [-----*-]
  2                                                       [---*-]
  3                                                 [-------------*---]
  4                                                  [--*-]
  5                                                       [*]
  6                                                           [-*]
  7                                                               [-*-]
  8                                                         [*]
  9 [--------------------------------------------*----]
 10                                                       [-*-]
ItemDM                                              [=============*====]
GlobalDM                                            [=============*====]
EqualDM                    [=================================*================]
Real:::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::
                                                           1149
    0.0001                                                               2E005

Item no.:  26 Item name: cd: # hours Scale: LOG
Experts
  1                                            [-------------*----------]
  2                                               [----*--]
  3    [---------------------------*-------------------]
  4            [------*-]
  5                                [------*----]
  6                              [---------------------------------*----------]
  7 [-----*-------]
  8                   [----------*-----]
  9                                   [-----------*-------]
 10                                     [---*--]
ItemDM [=============================*=================]
Global [============================*==================]
EqualD [===================================*============================]
Real::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::::::
                                              1.417E004
    800                                                                  9E004

Item no.:  27 Item name: cd: # errors Scale: LOG
Experts
  1                                                     [---------------*-----]
  2                                                      [*-]
  3                                              [----*-------]
  4                                                 [----*-]
  5                                                 [----*--]
  6                                                     [----*-]
  7                                                      [----*-]
  8                                                      [-*-]
  9 [------------------------------------------------------*-----]
 10                                                        [--*--]
ItemDM                                           [====*=======]
GlobalDM                                         [====*=======]
EqualDM                     [==============================*===============]
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
    0.0001                                                                5000
```

```
Item no.:  28 Item name: cd: time error Scale: LOG
Experts
  1          [------------------*-------------------------]
  2 [--------*-------]
  3                  [----------------------*-------------]
  4          [-----------*----]
  5     [-------------------*--------------------------------]
  6                             [------*----]
  7                  [----------*------]
  8                             [------*-------]
  9                                            [-------------*----------------]
 10                      [-------------*-------]
ItemDM             [========================*==============]
GlobalDM        [===========================*==============]
EqualDM [==========================*=================================]
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
     2                                                                    2400

Item no.:  29 Item name: cd: 2 systems Scale: LOG
Experts
  1                                                 [------------*----]
  2                                                          [*]
  3       [-------------------------------------------------------------*-----]
  4                                                       [---*-]
  5 [-----------------------------------------------*------]
  6                                                         [---*-]
  7                                                   [---*-]
  8                                                  [----*]
  9 [-----------------------------------------------------------------*---]
 10                                                 [------*---]
ItemDM    [============================================================*======]
GlobalDM  [==========================================================*========]
EqualDM         [===========================================*===============]
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
    0.0001                                                               8E006

Item no.:  30 Item name: cd: # cum-ex b Scale: LOG
Experts
  1                                               [------------------------*--]
  2                                               [---*-]
  3                                          [---*---]
  4                                               [----*--]
  5                                               [----*----]
  6                                                       [-----*--]
  7                                                   [---*--------]
  8                                                    [--*-]
  9 [------------------------------------------------------------------*--]
 10                                                       [---*]
ItemDM                                       [===*=======]
GlobalDM                                     [===*========]
EqualDM                       [==========================*===================]
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
    0.0001                                                               5E005

Item no.:  31 Item name: cd: # cum-ex a Scale: LOG
Experts
```

```
  1                                         [----------------*--------------]
  2                                           [-*]
  3                                    [--------*------]
  4                                           [-*---]
  5                                       [------*--------]
  6                                                                [--*-]
  7                                           [--*-]
  8                                                                [-*]
  9 [---------------------------------------------------------------------*--]
 10                                               [-----*-]
ItemDM                                 [========*=======]
GlobalDM                               [========*========]
EqualDM                      [========================*=====================]
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
    0.0001                                                             1.5E005

Item no.:  32 Item name: cd: time cum-e Scale: LOG
Experts
  1   [--------------------*----------------------------]
  2         [----*--]
  3         [--------------------*-------------------------]
  4   [------------*-----]
  5 [---------*---------------------------------------------]
  6              [-------*-------]
  7              [-------*-----]
  8                      [-------*----]
  9                                        [----------------*-----------------]
 10              [---------------*-------]
ItemDM     [=======================*=========================]
GlobalDM  [========================*=========================]
EqualD [====================*=======================================]
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
     4                                                                    2400

Item no.:  33 Item name: cd: error adju Scale: LOG
Experts
  1                                                    [-----------------*----]
  2                                                                [-*--]
  3                                                    [---------*-----]
  4                                                            [--*-]
  5                                                      [--*-----------]
  6                                                              [----*-]
  7                                                      [-----*]
  8                                                           [--*]
  9 [------------------------------------------------*--------]
 10                                                         [---*---]
ItemDM                                                 [=======*=======]
GlobalDM                                               [=======*=======]
EqualDM                    [====================================*=========]
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
    0.0001                                                                2000

Item no.:  34 Item name: cd: time adjus Scale: LOG
Experts
  1         [------------------*------------------]
  2 [---*----]
  3                 [-----------------*---------------------]
```

```
  4          [----------------*----]
  5     [----------------*-----------------------------------]
  6                      [------*------]
  7                  [----------*---]
  8                             [------*-------]
  9                                            [-------------*----------------]
 10                      [-------------*-------]
ItemDM            [====================*=====================]
GlobalDM        [======================*=====================]
Equal [===========================*==================================]
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

     2                                                                     2400

Item no.:  35 Item name: cd: backvaluat Scale: LOG
Experts
  1                                                     [------*-]
  2                                                                  [*]
  3 [---------------------------------------------------*--------------]
  4 [-*]
  5                                                         [-*--------]
  6                                                          [*]
  7                                             [----*-]
  8                                                            [--*---------]
  9 [--------------------------------------------------------------------*----]
 10                                                     [--*-]
Ite [======================================================*===========]
Glo [====================================================*=============]
Equa [=======================================================*==============]
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

    0.0001                                                                1E006
```

C.3 Range graphs of the experts

```
                          Range graph of input data
Expert no. :    1     Expert name:  expert 1
Items
  1(L) [------------------------------------------*---------]
Real   :::::::::#::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::

  2(L)                               [---------------------------------------*]
Real   :::::::::::::::::::::::::#::::::::::::::::::::::::::::::::::::::::::::::

  3(L) [-----------------------------*----------]
Real   ::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::

  4(L)                                                      [------------*---]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

  5(L)                                                  [---------*--]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::

  6(L)                                         [---------------------*--------]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::

  7(L)                                                          [------------*]
```

```
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::

  8(L)        [--------------------------------------*---]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::

 19(L)                                                [-----------------*----]
Real   :::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

 20(L)                                           [-------------------------*-]
Real   :::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

 21(L)                                                   [--------------*----]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::

 22(L)                                                   [-------------*--]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::

 23(L)                                                           [---------*-]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

 24(L)                                [------------------*-------------------]
Real   :::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::::::::::

 25(L)                                                                [----*-]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::

 26(L)                                         [-------------*---------]
Real   ::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::::

Expert no. :    2     Expert name:  expert 2
Items
  1(L)                   [-------*----]
Real   :::::::::#:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::

  2(L)            [----*--]
Real   :::::::::::::::::::::::::#:::::::::::::::::::::::::::::::::::::::::::::

  3(L)                     [-----*---]
Real   :::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::

  4(L) [---------------------------------------------------*---]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

  5(L)                                                    [-*]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::

  6(L)                                                              [----*--]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::

  7(L)                                                       [---*-]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::

  8(L)                                           [----*--]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::

 19(L)                                                        [*]
Real   :::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::
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 20(L)                                                [*]
Real   :::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

 21(L)                                                 [--*--]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::

 22(L)                                              [---*-]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::

 23(L)                                           [---*-]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

 24(L)                                       [*]
Real   ::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::::::::::::

 25(L)                                                    [----*]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::

 26(L)                                             [----*--]
Real   ::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::::

Expert no. :    3     Expert name:  expert 3
Items
  1(L) [-----------------------------------------*---------------------]
Real   ::::::::#::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::

  2(L) [-------------------------------------------------------*--]
Real   ::::::::::::::::::::::::#::::::::::::::::::::::::::::::::::::::::::::::

  3(L)          [--------------------*-----------------]
Real   :::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::

  4(L)                                                      [---*---]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

  5(L) [---------------------------------------------------*--------]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::

  6(L) [------------------------------*--------------]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::

  7(L)                                            [-----------*---]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::

  8(L)     [-------------------------------------*------------------]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::

 19(L)                                                 [-----------*---]
Real   :::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

 20(L)                                       [---------------------*---]
Real   :::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

 21(L)                                          [-------*------]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::
```

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 22(L)                                              [---*-------]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::

 23(L)                                           [----------*-----]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

 24(L)         [----------------------*------]
Real   :::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::::::::::::

 25(L)                                               [------------*----]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::

 26(L)    [--------------------------*------------------]
Real   :::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::::

Expert no. :    4     Expert name:  expert 4
Items
  1(L)                                  [-----------------*-------------]
Real   :::::::::#::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::

  2(L)                           [---*--]
Real   :::::::::::::::::::::::::#::::::::::::::::::::::::::::::::::::::::::::::

  3(L)                     [---------*----------]
Real   ::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::

  4(L)                                                      [--------*---]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

  5(L)                                                         [--*--]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::

  6(L)                                                      [---*----]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::

  7(L)                                                            [--*]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::

  8(L)              [-----*--]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::

 19(L)                                                              [--*-]
Real   ::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

 20(L)                                                       [--*--]
Real   ::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

 21(L)                                                       [----*-]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::

 22(L)                                              [--------*--]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::

 23(L)                                                         [--*--]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

 24(L)                            [-*-]
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Real   ::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::::::::::

 25(L)                                              [--*]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::

 26(L)           [------*--]
Real   ::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

Expert no. :    5     Expert name:  expert 5
Items
  1(L)                                    [---------*------]
Real   ::::::::#:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::

  2(L)                              [---------------*---]
Real   ::::::::::::::::::::::::#:::::::::::::::::::::::::::::::::::::::::::::

  3(L)                          [--------------*-----------]
Real   :::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::

  4(L)                                                           [--*--------]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

  5(L)                                                     [------*----------]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::

  6(L)                                                  [--------*--]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::

  7(L)                                                          [-*-]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::

  8(L)                                                [---*-----]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::

 19(L)                                                                  [-*--]
Real   :::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::

 20(L)                                                [-----------*----]
Real   :::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::

 21(L)                                                  [-------*-----------]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::

 22(L)                                                  [--*----------------]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::

 23(L)                                                      [----*--------]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

 24(L)                                 [*]
Real   ::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::::::::::

 25(L)                                                    [*]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::

 26(L)                              [-----*----]
Real   ::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::
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Expert no. :    6     Expert name:  expert 6
Items
  1(L)          [-----------*---------]
Real   :::::::::#::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::

  2(L)            [---*---]
Real   :::::::::::::::::::::::::#::::::::::::::::::::::::::::::::::::::::::::::

  3(L)                                                    [----------*--------]
Real   ::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::

  4(L)                                                              [----*---]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

  5(L)                                                            [*]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::

  6(L)                                                      [-*-]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::

  7(L)                                                        *]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::

  8(L)                                                                 [--*---]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::

 19(L)                                                       [---*--]
Real   ::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

 20(L)                                     [--------*-]
Real   ::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

 21(L)                                                        [----*--]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::

 22(L)                                                  [--*----]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::

 23(L)                                                  [----*-]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

 24(L)                                [---------*---]
Real   :::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::::::::::::

 25(L)                                                         [*-]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::

 26(L)                            [---------------------------------*---------]
Real   :::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::::

Expert no. :    7     Expert name:  expert 7
Items
  1(L)                                                        [---------*-----]
Real   :::::::::#::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::

  2(L)                   [--*--]
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Real   :::::::::::::::::::::::::#::::::::::::::::::::::::::::::::::::::::::::::

  3(L)                                          [------*-------]
Real   ::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::

  4(L)                                                      [-------*---]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

  5(L)                                                         [-*-]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::

  6(L)                                      [--*---]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::

  7(L)                                                       [-*]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::

  8(L) [--*-------]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::

 19(L)                                                                   [--*-]
Real   ::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

 20(L)                                           [-*]
Real   ::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

 21(L)                                                         [--*---]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::

 22(L)                                              [-----*--]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::

 23(L)                                                       [-*--]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

 24(L)                                 [-*-]
Real   :::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::::::::::::

 25(L)                                                            [-*--]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::

 26(L) [-----*------]
Real   :::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::::

Expert no. :    8     Expert name:  expert 8
Items
  1(L)                                               [----*---]
Real   :::::::::#::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::

  2(L)                [------*----]
Real   :::::::::::::::::::::::::#::::::::::::::::::::::::::::::::::::::::::::::

  3(L)                                          [-----*---]
Real   ::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::

  4(L)                                                        [--*-]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::
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  5(L)                                              [--*-]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::

  6(L)                                                     [-*--]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::

  7(L)                                                    [--*-]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::

  8(L)                      [-----------*-------]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::

 19(L)                                                                  [-*]
Real   :::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

 20(L)                                      [---*]
Real   :::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

 21(L)                                                             [-*]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::

 22(L)                                                    [*]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::

 23(L)                                                        [--*]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

 24(L)                                  [*-]
Real   ::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::::::::::::

 25(L)                                                      [*]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::

 26(L)                  [---------*------]
Real   ::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::::

Expert no. :    9     Expert name:  expert 9
Items
  1(L)                      [--------------------*------]
Real   :::::::::#:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::

  2(L)                                [--*--]
Real   :::::::::::::::::::::::::#:::::::::::::::::::::::::::::::::::::::::::::

  3(L)            [----------------------*-----]
Real   :::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::

  4(L) [---------------------------------------------------*-----]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

  5(L) [-------------------------------------------------------*-]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::

  6(L)               [--------------------*----------------]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::
```

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  7(L) [--------------------------------------------------------*------]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::

  8(L)                                          [-------------*---------]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::

 19(L) [----------------------------------------------------------*----]
Real   ::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

 20(L) [---------------------------------------------------*---]
Real   ::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

 21(L) [--------------------------------------------------*----]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::

 22(L) [--------------------------------------------*-------]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::

 23(L) [----------------------------------------------------------*-]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

 24(L) [-------------------*------]
Real   :::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::::::::::::

 25(L) [-------------------------------------------*---]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::

 26(L)                                  [----------*-------]
Real   :::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::::

Expert no. :   10     Expert name:  expert10
Items
  1(L)                                         [--------------------*---------]
Real   :::::::::#::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::

  2(L)                               [-----*-------------]
Real   :::::::::::::::::::::::::#::::::::::::::::::::::::::::::::::::::::::::::

  3(L)                                                [------*-----]
Real   ::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::

  4(L)                                                               [-*-]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

  5(L)                                                         [---*-]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::

  6(L)                                    [-------*--]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::

  7(L)                                                     [*-]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::

  8(L)                                                 [--*---]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::

 19(L)                                                           [------*--]
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Real   :::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

 20(L)                                      [---*]
Real   :::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

 21(L)                                                          [------*--]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::

 22(L)                                                     [---*--]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::

 23(L)                                              [-*]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

 24(L)                       [--*]
Real   ::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::::::::::::

 25(L)                                                   [--*-]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::

 26(L)                                   [--*--]
Real   ::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::::::::::

Expert no. :  DM      Expert name:  ItemDM
Items
  1(L) [==========================================*===================]
Real   :::::::::#:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::

  2(L) [=====================================================*====]
Real   :::::::::::::::::::::::::#:::::::::::::::::::::::::::::::::::::::::::::

  3(L)          [====================*===================]
Real   :::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::

  4(L)                                                     [===*======]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::

  5(L) [===================================================*============]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::

  6(L) [==============================*============================]
Real   :::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::

  7(L)                                           [==========*======]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::

  8(L)     [========================================*==============]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::::

 19(L)                                                 [=========*========]
Real   :::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

 20(L)                                      [======================*===]
Real   :::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::

 21(L)                                         [=======*==========]
Real   ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::
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 22(L)                                                    [===*==========]
Real    :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#::::::::::::::::::

 23(L)                                                 [===========*========]
Real    ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::

 24(L)             [========================*======]
Real    ::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::::::::::::::

 25(L)                                                     [===========*======]
Real    ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::

 26(L)        [=============================*=================]
Real    ::::::::::::::::::::::::::::::::::::::::::::::::#:::::::::::::::::::::::::::::::
```