Extreme-Value Analysis of Corrosion Data

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Outline

Motivation

Objective of the Thesis

Methods used

Data declustering

Examples of application

Framework for modelling extremes of corrosion

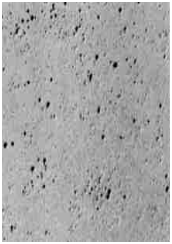
Conclusions and recommendations

Motivation

- in the oil industry, hundreds of kilometres of pipes and other equipment can be affected by corrosion
- it is extreme defect depth/wall loss that influences the system reliability ⇒ extreme-value methods are sensible for application
- only part of the system can be subjected to inspection ⇒ results extrapolation is needed

Motivation

- defects/wall losses caused by corrosion are likely to be locally dependent $\Rightarrow$ independence assumption questionable

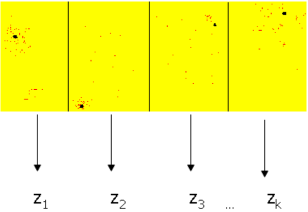


Objective of the Thesis

- present statistical methods to model extremes of corrosion data, taking into account local defect dependence
- spatial extrapolation of the results
- examples of application
- framework/guideline for modelling extreme-values of corrosion

Methods used

The Generalised Extreme-Value (GEV) distribution (block maxima data)

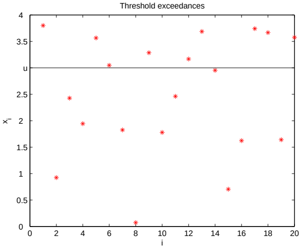


$$G(z) = \begin{cases} \exp\left\{-\left[1+\xi\left(\frac{z-\mu}{\sigma}\right)\right]_+^{-\frac{1}{\xi}}\right\}, & \xi \neq 0 \\ \exp\left\{-\exp\left[-\left(\frac{z-\mu}{\sigma}\right)\right]\right\}, & \xi = 0, \end{cases}$$

Methods used

The Generalised-Pareto (GP) distribution (excess over threshold data)

$$Y_i = X_i - u, \text{ for } X > u, \ i = 1, \ldots, n_u$$

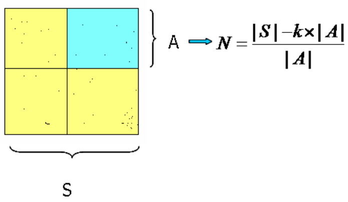


$$H(y) = \begin{cases} 1 - \left[1 + \frac{\xi y}{\bar{\sigma}}\right]_+^{-1/\xi}, & \xi \neq 0 \\ 1 - \exp\left(-\frac{y}{\bar{\sigma}}\right), & \xi = 0 \end{cases}$$

Methods used (extrapolation-GEV)

- return-level method

$$G(z_p) = 1 - p \Leftrightarrow Pr\{M > z_p\} = p = \frac{1}{N}$$

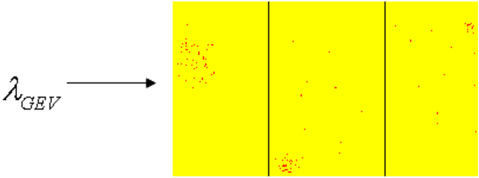


- implied distribution of the maximum corresponding to the not inspected area

$$Pr\{X_N \leq z\} = G_N(z) = G(z)^N$$

Methods used (extrapolation-GP)

based on Poisson frequency of threshold exceedances (Poisson-GP model)



- return-level method

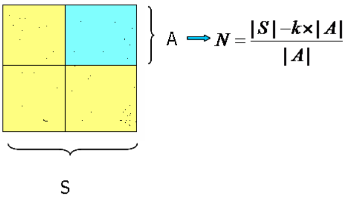
$$H(y_p) = 1 - p \Leftrightarrow Pr\{Y > y_p\} = p = \frac{1}{N_E}$$

$N_E = \lambda_{GEV} \times (|S| - k \times |A|)$ - expected number of exceedances on the not inspected area

Methods used (extrapolation-GP)

- implied distribution of the maximum corresponding to the not inspected area

$$Pr\{X_N \leq x\} = \exp\left\{-\lambda_{GEV}\left(1+\xi\frac{x-u}{\bar{\sigma}}\right)_+^{-1/\xi}\right\}^N$$



Methods used-summary

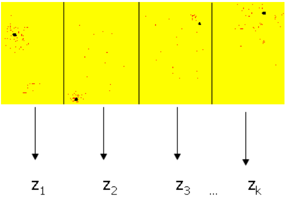
- two methods for statistical inference about extreme-values of corrosion
  - the GEV distribution (block maxima)
  - the GP distribution (excess over threshold data)
- two methods for spatial results extrapolation
  - return-level
  - distribution of the maximum corresponding to the not inspected area
- the GEV and GP distributions are closely related and theoretically, should give the same results

Modelling extremes of dependent data

- for the stationary data characterised by the limited extend of long-range dependence at extreme levels, the extreme-value methods can be still applied
- in corrosion application it is reasonable to assume that pit depths are locally dependent $\Rightarrow$ extreme-value methods are applicable

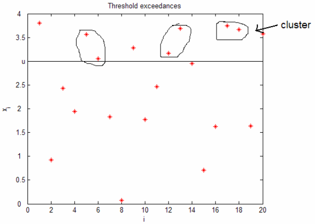
Modelling extremes of dependent data with the **GEV** distribution

- assuming local data dependence, block maxima of stationary data (for sufficiently large block sizes) can be considered as approximately independent
- the GEV distribution is used in its standard form



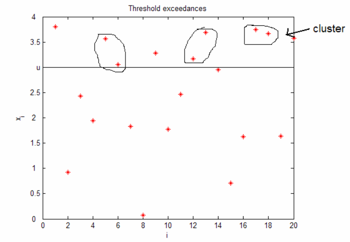
Modelling extremes of dependent data with the **GP** distribution

- neighbouring exceedances may be dependent, therefore the change of practise is needed
- one of the most widely adopted method is **data declustering** - filtering out dependent observations such that remaining exceedances can be considered as approximately independent



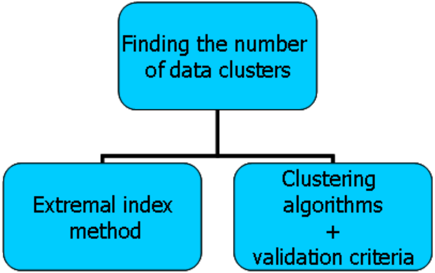
Modelling extremes of dependent data with the **GP** distribution-approach

- define clusters of exceedances
- identify maximum excess within each cluster
- assuming that cluster maxima are independent fit the GP distribution



Data declustering

Estimation of the number of data clusters



Data declustering

Extremal index method:

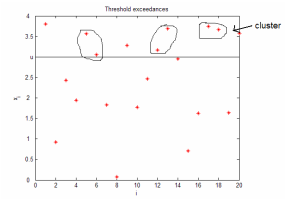
- the extent of short-range dependence of extreme events is captured by the parameter θ, called **extremal index**

$$\theta = \frac{1}{\text{limiting mean cluster size}}$$

- extremal index measures the degree of clustering of the process at extreme levels

Data declustering

Extremal index method



$$\theta = \frac{1}{\text{limiting mean cluster size}} \Rightarrow N_c = \theta \times N_e$$

where N_c - number of clusters, N_e - number of exceedances above threshold u

θ is estimated by the intervals estimator

Data declustering

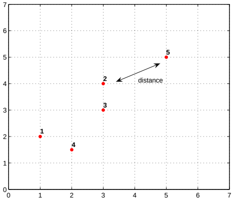
Finding number of clusters using clustering algorithm

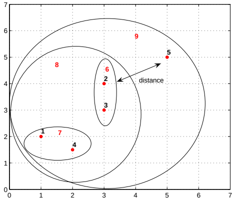
- define a validity criteria for the number N_c of found clusters
- run the clustering algorithm for a range of N_c
- as proper number of clusters choose the one for which the validity criteria are optimised

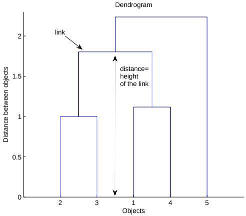
Agglomerative hierarchical clustering algorithm

- starts with single points as clusters
- at each step the two closest clusters are merged
- stops when only one cluster remains

Agglomerative hierarchical clustering algorithm







Data declustering

Validation criteria, that we used, aim at identifying clusters that are compact and well isolated:

- silhouette plot - maximum value indicates optimum
- Davies-Bouldin index - minimum indicates optimum

Data declustering-summary

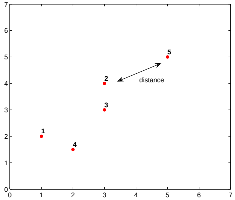
- two methods to estimate the number of data clusters
  - the extremal index method
  - clustering algorithm + validation criteria (Davies-Bouldin index, silhouette plot)
- prior to data declustering perform clustering tendency test

Simulated corroded surface

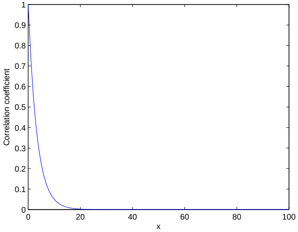
- application of the gamma-process model
- dependence in terms of the product moment correlation

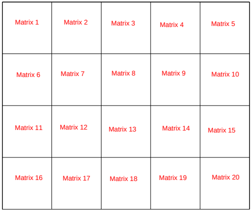
$$\rho(X_k, X_l) = \exp\left\{ -d \left(\sum_{i=1}^{2} |dist_i|^p \right)^{q/p} \right\}$$

$X_k = (x_k, y_k),\ X_l = (x_l, y_l),$
$dist_1 = |x_k - x_l|,\ dist_2 = |y_k - y_l|,$
$d = 0.3,\ p = 2,\ q = 1$



Simulated corroded surface





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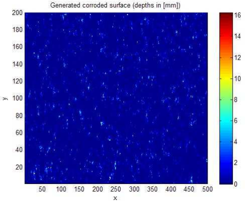
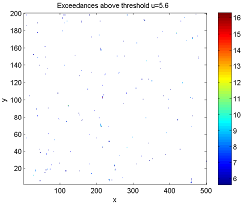
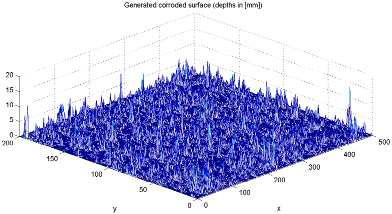
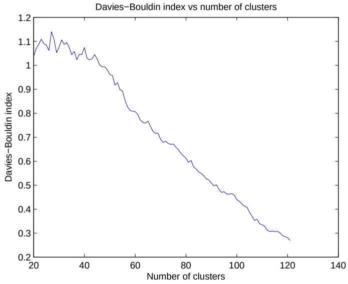
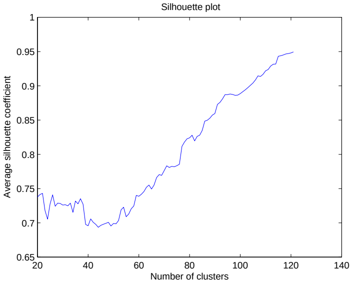






Simulated corroded surface - clustering algorithm





Simulated corroded surface - results

$\hat{\theta}$	n_c	$n_{c_{alg}}$
0.544	107	121

Table: The estimate of extremal index and determined number of clusters

$Number\ of\ clusters$	$AD^2_{up}\ p-v.$	$KS\ p-v.$
107	0.376	0.204
121	0.549	0.493

Table: Goodness-of-fit test results for different number of clusters

$\hat{\xi}$	$\hat{\hat{\sigma}}$	AD^2_{up} $p-v.$	KS $p-v.$
0.065	1.341	0.647	0.488

Table: GP fit to excess of dependent data

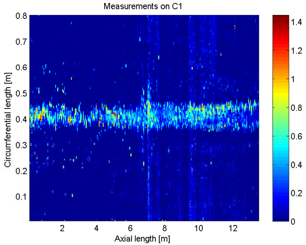
$\hat{\xi}$	$\hat{\sigma}$	AD^2_{up} $p-v.$	KS $p-v.$
-0.0137	1.6839	0.555	0.493

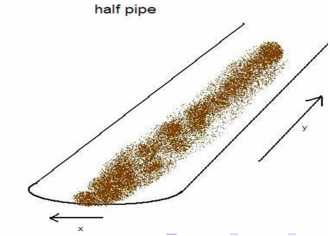
Table: GP fit to excess of declustered data

$\hat{\xi}$	$\hat{\sigma}$	$\hat{\mu}$	AD^2_{up} $p-v.$	KS $p-v.$
-0.007	1.627	5.637	0.621	0.899

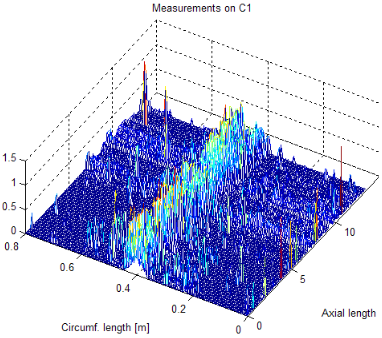
Table: GEV fit to block maxima data

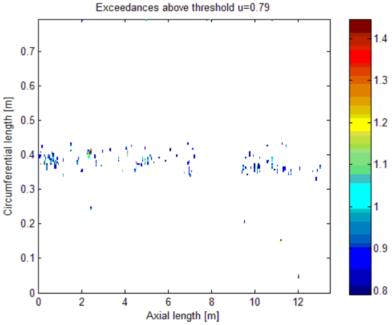
Real data example



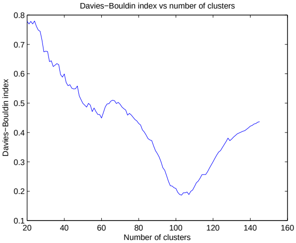


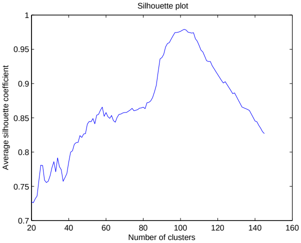
Real data example





Real data example





$\hat{\xi}$	$\hat{\sigma}$	$\hat{\mu}$	AD^2_{up} $p-v.$	KS $p-v.$
-0.082	0.182	0.757	0.713	0.986

Table: GEV fit to block maxima data

$\hat{\xi}$	$\hat{\tilde{\sigma}}$	AD^2_{up} $p-v.$	KS $p-v.$
-0.008	0.133	0.616	0.074

Table: GP fit to excess of dependent data

$\hat{\xi}$	$\hat{\tilde{\sigma}}$	AD^2_{up} $p-v.$	KS $p-v.$
-0.069	0.172	0.717	0.654

Table: GP fit to excess of declustered data

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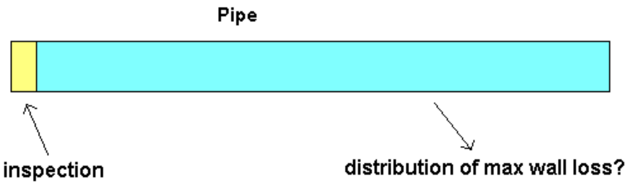
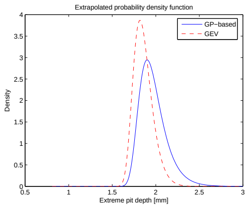
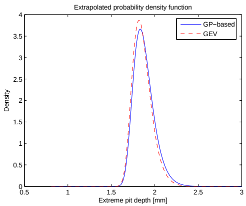

Pipe
inspection
distribution of max wall loss?


Extrapolated probability density function
GP−based
GEV
Density
Extreme pit depth [mm]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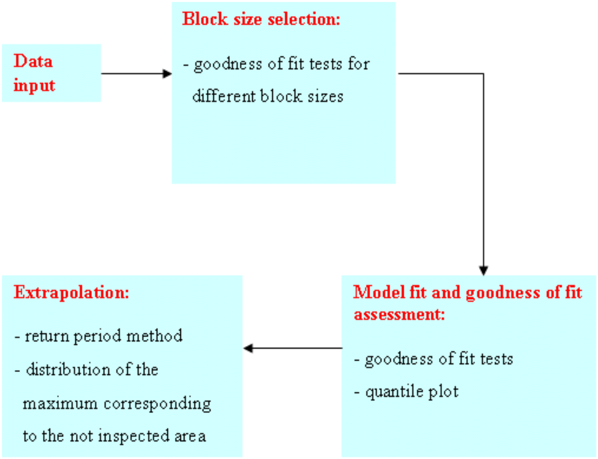
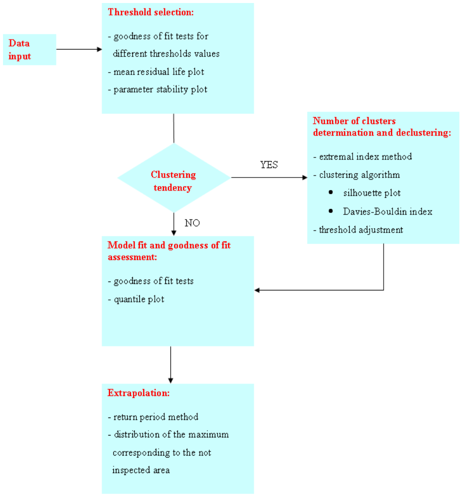


Framework for modelling extremes of corrosion with the GEV distribution



Framework for modelling extremes of corrosion with the GP distribution

Conclusions and recommendations

- the two applied distributions are closely related and lead to the consistent inference about extreme-values of corrosion
- data declustering improves the results given by the GP distribution
- the performance of other clustering algorithms could be checked
- in order to take into account corrosion nonstationarity due to space-varying environmental conditions, covariate-dependent extreme-value models with trends could be considered

THANK YOU FOR ATTENTION

Questions???