Parameter Estimation in Reservoir Engineering Models via Data Assimilation Techniques

Mariya V. Krymskaya

TU Delft

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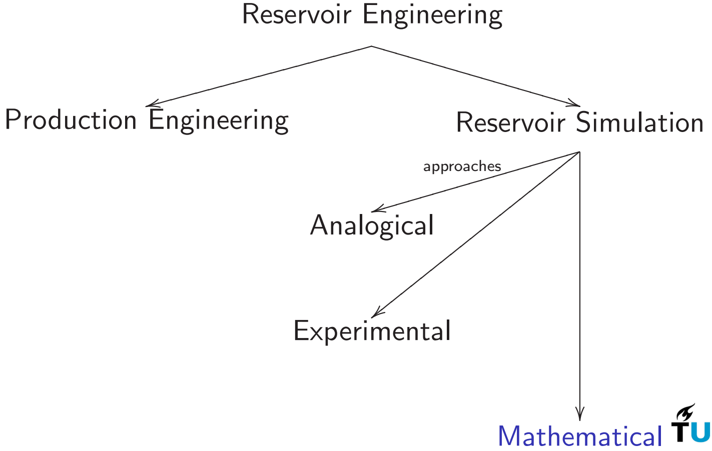
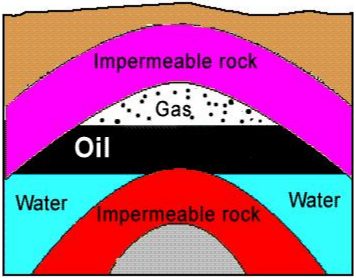
Structure of Reservoir Engineering



A Reservoir



General information

$>$ 40,000 oil fields in the world
300 *m* to 10 *km* below the surface
$2 - 500$ *million years* old

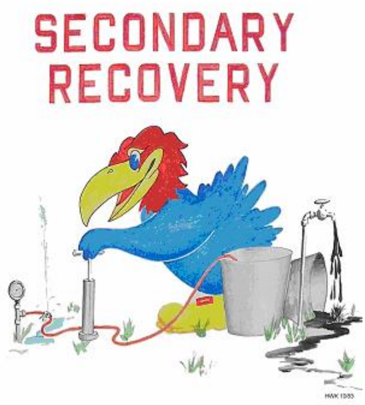
Ghawar oil field

the biggest among discovered
Location: Saudi Arabia
Recovery: since 1951
Size: 280×30 *km*
Age: 320 *million years* old
Production: 5 *million barrels* ($800,000\ m^3$) of oil per day

Oil Recovery

- Primary
  20% extracted

Oil Recovery



- Primary
  20% extracted
- Secondary (water flooding)
  25% to 35% extracted

Oil Recovery

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- Secondary (water flooding)
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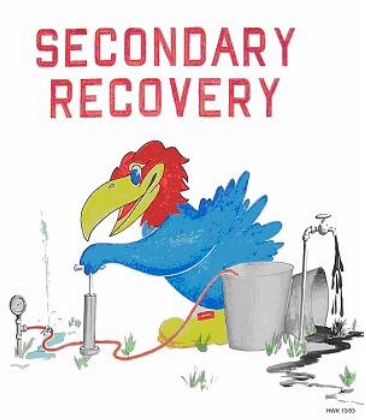
- Tertiary
  50% left

Oil Recovery



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Reservoir Properties

- Rock properties
  - Porosity
  - (Absolute) permeability
  - Rock compressibility

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Two-Phase Water-Oil Fluid Flow Model

- Mass balance equation for each phase
- Darcy's law for each phase
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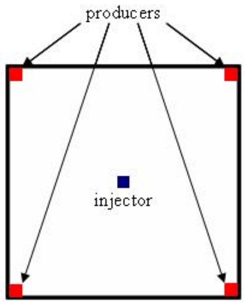
- Well model



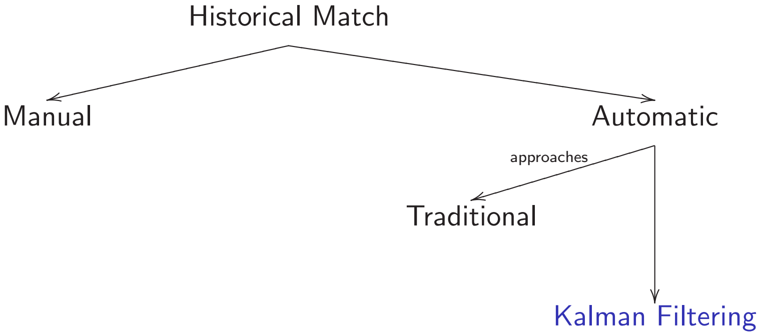
Model Discretization

$$\underbrace{\hat{\mathbf{E}}(\mathbf{X})}_{\substack{\uparrow \\ \text{accumulation} \\ \text{matrix}}} \dot{\mathbf{X}} - \underbrace{\hat{\mathbf{A}}(\mathbf{X})}_{\substack{\uparrow \\ \text{system} \\ \text{matrix}}} \underbrace{\mathbf{X}}_{\substack{\uparrow \\ \text{state} \\ \text{vector}}} - \underbrace{\hat{\mathbf{B}}(\mathbf{X})}_{\substack{\uparrow \\ \text{input} \\ \text{matrix}}} \underbrace{\mathbf{U}}_{\substack{\uparrow \\ \text{input} \\ \text{vector}}} = \mathbf{0}$$

$$\Downarrow$$

$$\mathbf{X} = \begin{bmatrix} \mathbf{p} \\ \mathbf{S} \end{bmatrix}$$

History Matching Process



Data Assimilation Problem Statement

- System

$$\mathbf{X}_{k+1} = \mathbf{F}(\mathbf{X}_k, \mathbf{U}_k, \mathbf{m}) + \mathbf{W}_k,$$
$$\mathbf{Z}_{k+1} = \mathbf{M}\mathbf{X}_k + \mathbf{V}_k$$

- Uncertainties

$$\mathbf{X}_0 \sim \mathcal{N}(\mathbf{X}_0, \mathbf{P}_0) \ - \ \text{uncertain initial state},$$
$$\mathbf{W}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}) \ - \ \text{model noise},$$
$$\mathbf{V}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}) \ - \ \text{measurement noise},$$

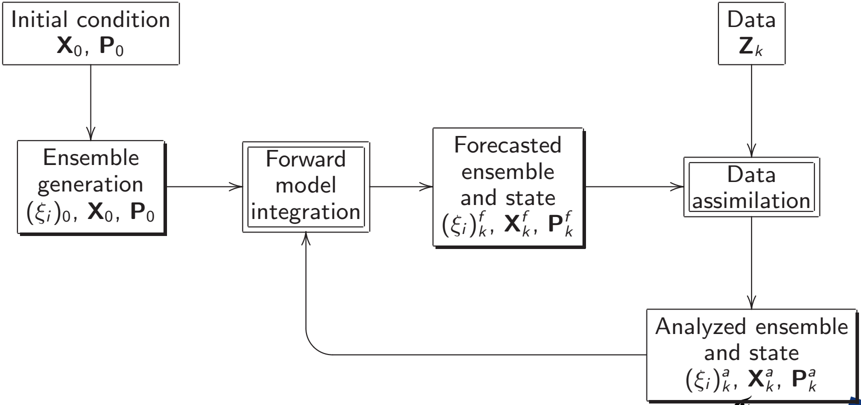
- Independency assumption

$$\mathbf{X}_0 \perp \mathbf{W}_k \perp \mathbf{V}_k$$

- State conditional pdf

$$(\mathbf{X}_k | \mathbf{Z}_1, \ldots, \mathbf{Z}_l) \sim \mathcal{N}(\text{mean}, \text{cov})$$

Ensemble Kalman Filter (EnKF)

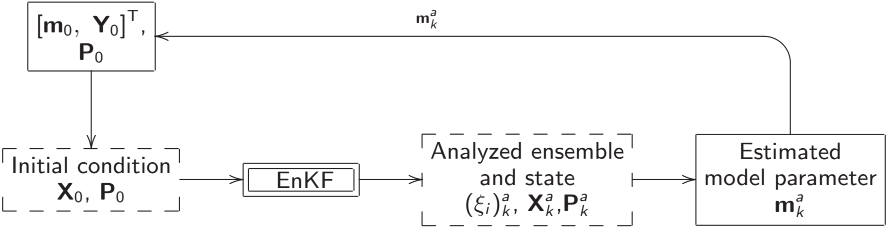


Parameter Estimation via EnKF

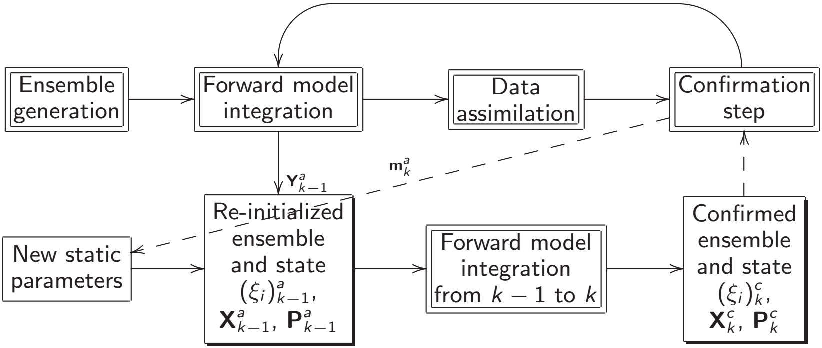
Augmented state vector

$$\mathbf{X} = \begin{bmatrix} \mathbf{p} \\ \mathbf{S} \end{bmatrix} \Rightarrow \mathbf{X} = \begin{bmatrix} \left.\log \mathbf{k}\right\} = \mathbf{m} \\ \left.\begin{matrix} \mathbf{p} \\ \mathbf{S} \end{matrix}\right\} = \mathbf{Y} \\ \mathbf{d} \end{bmatrix}$$

Iterative Ensemble Kalman Filter (IEnKF)



State Vector Feasibility



Re-scaling state vector

$$\mathbf{X} = \begin{bmatrix} \log \mathbf{k} \\ \mathbf{p} \\ \mathbf{S} \\ \mathbf{Y} \end{bmatrix},$$

$$\begin{matrix} \mathbf{A} \\ \mathbf{B} \end{matrix} = \left[\begin{array}{c|c|c|c} 10^{-7} & & & \\ \hline & 10 & & \\ \hline & & 10^{-7} & \\ \hline & & & 10^{3} \end{array}\right] \Rightarrow \begin{matrix} \mathbf{AML} \\ \mathbf{BL} \end{matrix}$$

Re-scaling state vector

Kalman gain

$$\mathbf{K} = \frac{1}{N-1} \qquad \mathbf{L}\mathbf{L}^{\mathsf{T}}\mathbf{M}^{\mathsf{T}} \qquad \left(\frac{1}{N-1}\mathbf{M}\mathbf{L}\mathbf{L}^{\mathsf{T}}\mathbf{M}^{\mathsf{T}} + \mathbf{R}\right)^{-1}$$

Re-scaling state vector

Kalman gain

$$\mathbf{K} = \frac{1}{N-1}\left(\mathbf{B}^{-1}\mathbf{B}\right)\mathbf{L}\mathbf{L}^{\mathsf{T}}\mathbf{M}^{\mathsf{T}}\left(\mathbf{A}\mathbf{A}^{-1}\right)\left(\frac{1}{N-1}\mathbf{M}\mathbf{L}\mathbf{L}^{\mathsf{T}}\mathbf{M}^{\mathsf{T}}+\mathbf{R}\right)^{-1}\left(\mathbf{A}^{-1}\mathbf{A}\right)$$

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$$= \mathbf{B}^{-1}\underbrace{\frac{1}{N-1}\left(\mathbf{B}\mathbf{L}\right)\left(\mathbf{A}\mathbf{M}\mathbf{L}\right)^{\mathsf{T}}\left(\frac{1}{N-1}\mathbf{A}\mathbf{M}\mathbf{L}\left(\mathbf{A}\mathbf{M}\mathbf{L}\right)^{\mathsf{T}} + \mathbf{A}\mathbf{R}\mathbf{A}\right)^{-1}}_{\mathbf{K}_1}\mathbf{A}$$

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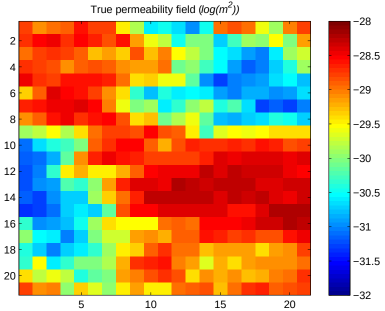
Ensemble update

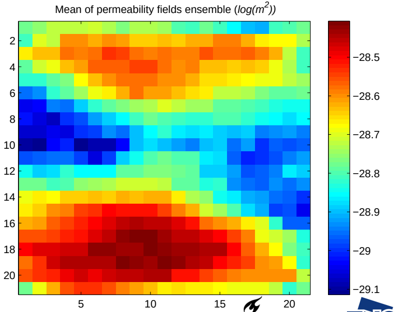
$$(\xi_i)_k^a = (\xi_i)_k^f + \mathbf{K}\left(\mathbf{Z} - \mathbf{M}(\xi_i)_k^f + \mathbf{V}^i\right)$$
$$= (\xi_i)_k^f + \mathbf{B}^{-1}\mathbf{K}_1\mathbf{A}\left(\mathbf{Z} - \mathbf{M}(\xi_i)_k^f + \mathbf{V}^i\right)$$
$$= (\xi_i)_k^f + \mathbf{B}^{-1}\left(\mathbf{K}_1\left(\mathbf{A}\mathbf{Z} - \mathbf{A}\mathbf{M}(\xi_i)_k^f + \mathbf{A}\mathbf{V}^i\right)\right)$$

Experimental setup

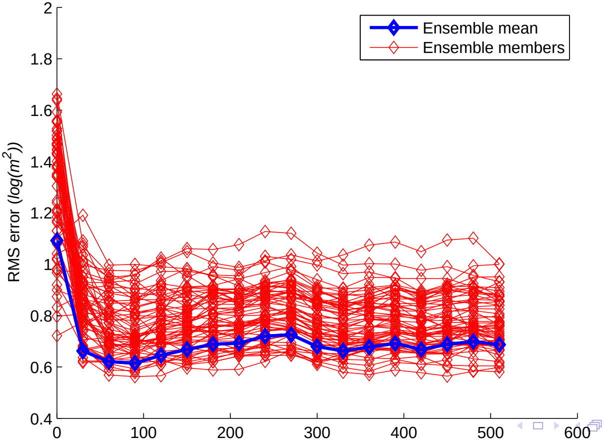
Twin experiment

Initialization

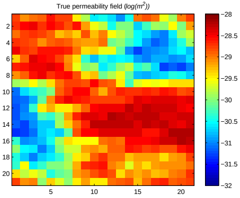


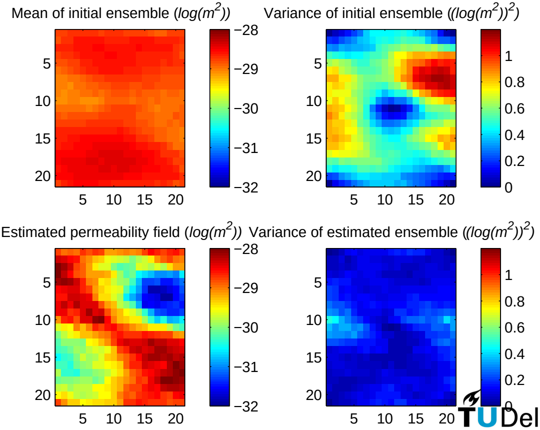


RMS Error in Model Parameter

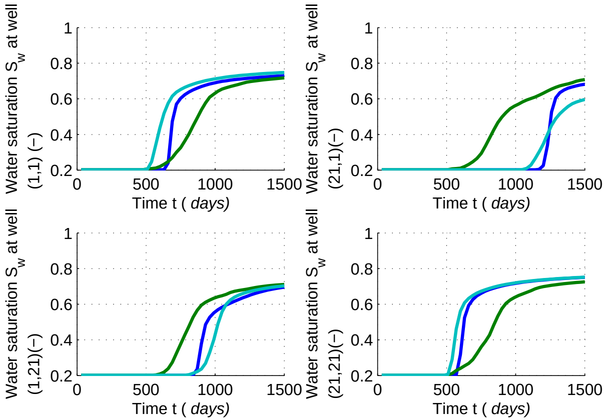


Estimated Permeability Field





Forecasted Reservoir Performance



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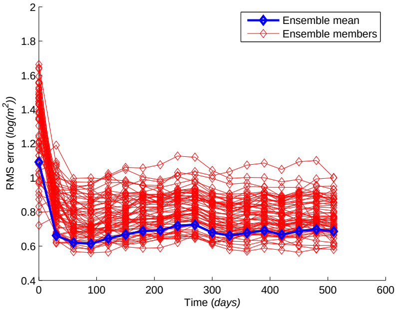

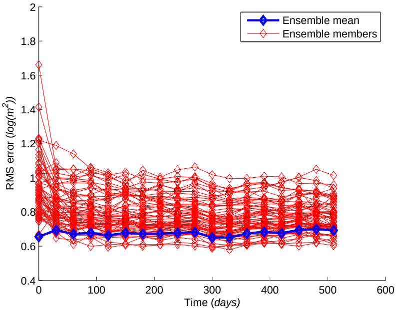

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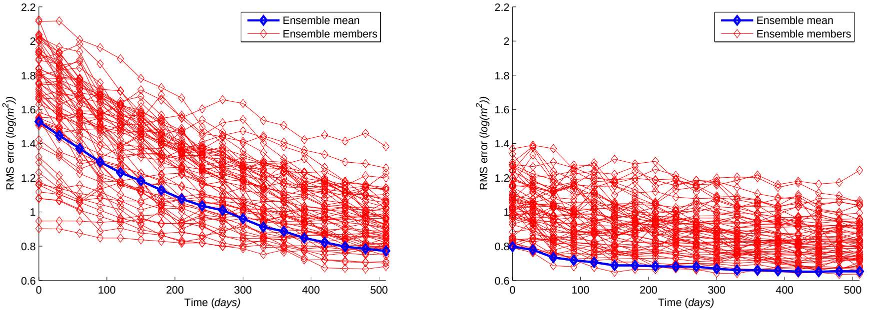

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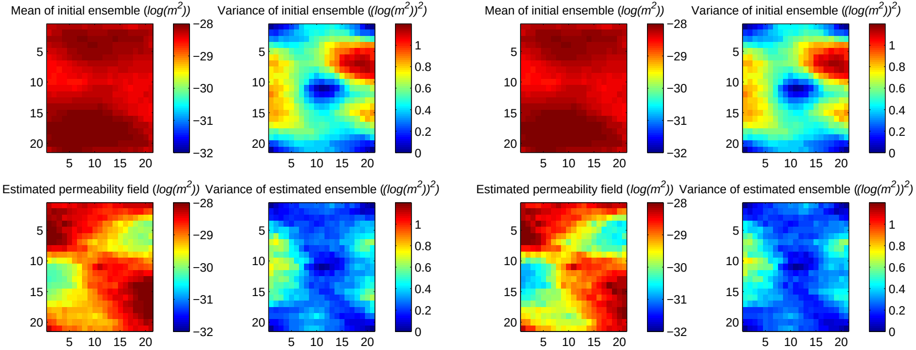

Conclusion

- Model calibration is essential
- EnKF provides reasonable parameter estimation
- There are cases at which IEnKF is superior to EnKF
- Further investigations on IEnKF sensitivities are required

Questions