Delft University of Technology
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Master of Science Thesis

Estimation of Relative Permeability Parameters in Reservoir Engineering Applications

by

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Delft, The Netherlands
September, 2009

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Estimation of Relative Permeability Parameters

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Abstract

In a reservoir application, there are a large number of parameters that can be considered for estimation during history matching of the simulated model to the production data. If traditionally, parameters like porosity and absolute permeability have been most often included in such applications, this study focuses on researching the possibility of estimating the relative permeability curves in an assisted procedure by means of the Ensemble Kalman filter (EnKF).

Stand alone estimation of the relative permeability parameters, as given by the Corey parametrization, as well as combined absolute permeability - relative permeability estimation experiments were performed on a synthetic study case and the ability of the EnKF to recover the true values of the parameters, given bottom-hole injector pressure and oil/water producer rates measurements, was tested.

The influence of the initial distribution of the relative permeability parameters on the value of the estimates and the reduction of uncertainty, as well as that of the number of the measurements and the length of the assimilation period, in terms of covering or not the water breakthrough moment, were also investigated.

Results show that some of the relative permeability parameters (the Corey oil coefficient) can be recovered from the measurements, while others (the relative permeability end points) are not very sensitive to data assimilation and that estimating relative permeability has a positive effect on the estimation of absolute permeability, at the loss of accuracy of the relative permeability parameters estimations.

Introduction

Reservoir management requires accurate predictions of reservoir behavior and the accuracy is influenced by the 'errors' that are introduced every stage of the prediction procedure: whether at the modeling stage, through the development of the governing system of equations and prescription of the physical, geological and geometrical parameters of the reservoir, at the simulation stage, when numerical methods need to be used to solve the analytical unsolvable governing system of equations, or during the history matching stage when measurements errors are introduced through assimilated production data and other reservoir observations. All these sources of error need to be taken into account and handled, in order to provide good predictions. These translate into properly quantifying the uncertainty and finding the means to reduce it.

One way to achieve these goals is through a better estimation of the reservoir parameters, considered so far in petroleum applications the main source of model error. Although the quality and quantity of the data field information keeps improving and increasing, it is not possible to identify the correct physical-geological description of the reservoir before the simulation stage, nor can this description be transferred error-free on the working reservoir grids. Therefore, history matching has to compensate for the initial uncertainty in the reservoir parameters and also for the numerical uncertainty caused by up-scaling issues.

Due to the large number of parameters, the limitation of computing power and also of the mathematical theories, as well as the nature of the production data that can condition the simulated model during history matching, the parameters that are considered for estimation have to be chosen 'wisely', taking into consideration at least two aspects: the impact of the parameters on the behavior of the reservoir and if they can be 'recovered' from the measurements that are available.

In this context the parameters that have been typically considered for estimation during history matching have been porosity and permeability. Recently, the interest steered towards other parameters, with the relative permeability curves being a popular choice. There have been already a series of studies on the impact of estimating the relative permeability parameters both on synthetic and real field applications, on two and three phase reservoirs, using a variety of data assimilation procedures, from gradient based ones to EnKF[2].

Relative permeability, describing the additional resistance to the flow of one phase (oil, water or gas) caused by the presence of another phase, can be modeled through different parameterizations, the most often used being the one proposed by Corey[1]. The relative permeability curves are functions of water saturation and their importance in the dynamics of a reservoir can be explained by the fact that given the water saturation, one can calculate the fraction of total flow that is water/oil/gas on the base of these curves. They are initially estimated through core plugs experiments in the laboratory and although these estimates have limited ability to characterize the entire reservoir, they are typically considered to do so. Since their influence on the behavior of the reservoir is of great importance and research[3] shows that they could be determined from production data, recent applications in reservoir predictions tend to put a greater emphasis on estimating the relative permeability curves.

In the current context of an increasing amount and variety of information coming from the production sites, as well as the interest of considering more reservoir parameters for updating, the traditional history matching methods can no longer be successfully applied as they are extremely time consuming when measurements become available at high frequency. In these conditions, sequential data assimilation methods such as the EnKF became popular for fast and continuous real time updating of reservoir predictions.

Various sequential data assimilation methods have been already developed and implemented, and, without considering it the most suitable one for reservoir applications, the following work will investigate the behavior of the EnKF. An extensive literature proved already that it can be easily implemented and successfully used as both a predictive tool and a way to estimate reservoir parameters, with significantly better results than the traditional methods.

EnKF

The original Kalman filter was designed to integrate measurements with a linear model and later on, an extended version was derived for non-linear systems as well, but it was rather restrictive when working with large state vectors and extremely unreliable when it came to highly non-linear systems. These issues were addressed by developing the Ensemble Kalman filter which accommodates nonlinearity and large state spaces at the same time. The EnKF is basically a Monte Carlo approach where errors are represented by an ensemble of realizations from which all necessary statistics can be derived.

The EnKF algorithm works as follows:

Initialization

As the EnKF method is based on a representation of the probability density of the state estimate $X(k)$ (where k refers to time) by a finite number of randomly generated system states, the first state consists of generating a fixed number N of ensembles:

$$\xi_i(0) \sim N(X_0, P_0) \qquad i = 1, \ldots, N$$

with X_0 the initial state of the system and P_0 the initial covariance matrix.

Forecast step

The second step consists of time-updating the ensembles, an operation given by the equation:

$$\xi_i^f(k) = F(\xi_i^a(k-1)) + G(k)w_k^i$$

with F being the model operator, subscripts f and a standing for forecast and analyzed, $G(k)$ the noise input matrix and w_k^i being the model error given by a white Gaussian system noise process with mean 0 and covariance matrix $Q(k)$.

The optimal state estimate and the square root of the covariance matrix of the estimation error are given by:

$$X^f(k) = \frac{1}{N}\sum_{i=1}^{N}\xi_i^f(k)$$

$$L^f(k) = [\xi_i^f(k) - X^f(k), \ldots, \xi_N^f(k) - X^f(k)]^T.$$

Analysis step

At this step, the forecast density is adjusted with the available observations by means of Bayes theorem, obtaining the conditioned density of the system state given the available set of measurements up to that moment:

$$\xi_i^a = \xi_i^f(k) + K(k)(Z(k) - M(k)\xi_i^f(k) + v_k^i)$$

with the Kalman gain given by the following equation:

$$K(k) = \frac{1}{N-1}L^f(k)^T M(k)^T [\frac{1}{N-1}M(k)L(k)L(k)^T M(k)^T + R(k)]^{-1}.$$

$Z(k)$ is the available set of measurements at the considered time step k, v_k is a white Gaussian noise with mean 0 and covariance matrix $R(k)$ to account for the measurement error and $M(k)$ is a measurement matrix describing the connection between the system vector and the measurement vector.

The model and the measurement noises, as well as the initial ensemble state are all considered to come from independent Gaussian distributions, a condition necessary for the entire theory of Kalman filtering to hold. Under these assumptions, the conditional distribution of the system state, given the available observations is Gaussian and completely determined by its mean and covariance matrix.

The EnKF is very attractive for history matching in reservoir engineering because it deals with measurements sequentially and its easy design makes it convenient for continuously updating reservoir models. In addition, it can work with the non-linear system of governing equations of a reservoir and the large state vectors associated with very fine grid numerical solutions which can contain both dynamic parameters (pressure and phase saturations that are the solution of flow equations) and static parameters(rock and fluid input parameters that are uncertain). Usually, the extended state vector in a reservoir application also contains the predicted production data because its inclusion simplifies the comparison with the measured data in the EnKF analysis step.

The role of the model operator used for forecasting the ensembles from the current time step to the next moment in time when measurements are assimilated, is played by a reservoir simulator, the current experiments being carried on using **simsim**, an educational software developed at TUDelft. For a detailed description of this simulator reference[4] can be consulted and a short presentation can also be found in Appendix 1.

One last aspect that should be mentioned about the specific implementation of the EnKF in reservoir applications is that, due to the fact that traditionally the model error is attributed only to poorly known input parameters, the model error $w(k)$ presented in the previous description of the algorithm is left out. This situation explains the importance and the interest placed on the research of finding the best estimates for the reservoir parameters.

Stand alone estimation of the relative permeability curves

Settings

The first experiment was performed in order to investigate only the capability of the EnKF to estimate efficiently the relative permeability parameters as given in the Corey representation, therefore a simple reservoir design was setup and the system state was chosen to contain only the dynamic variables and the relative permeability parameters, along with the predicted production data.

In the next table, the main properties of the reservoir are presented. It can be described as a quarter five spot design on a water-oil, two phase, two-dimensional reservoir with a injector in the NW corner (with prescribed rate and no pressure constraint) and a producer in the SE corner (with prescribed pressure and no rate constraints) with uniform absolute permeability and porosity fields. The numerical discretization is done on a 21 blocks by 21 blocks uniform cartesian square grid.

$c_o = 10^{-9} \quad 1/Pa$	$\mu_o = 0.5 \cdot 10^{-3} \quad Pa \cdot s$	$p_{prod}^{pres} = 280 \cdot 10^5 \quad Pa$
$c_r = 10^{-9} \quad 1/Pa$	$\mu_w = 10^{-3} \quad Pa \cdot s$	$rate_{inj}^{pres} = 0.001 \quad m^3/s$
$c_w = 10^{-9} \quad 1/Pa$	$\phi = 0.3$	
	$k = 5 \cdot 10^{-13} \quad m^2$	

Table 1: Input parameters for the reservoir used in stand alone estimation of the relative permeability parameters

The system state contains the dynamic variables which for a two-phase reservoir in the absence of capillary pressure are given by pressure and water saturation in each grid cell and the static variables, in this case the relative permeability parameters as given by:

$$k_{ro}(S_w) = k_{ro}^0 (1 - \frac{S_w - S_{wc}}{1 - S_{or} - S_{wc}})^{n_o}$$
$$k_{rw}(S_w) = k_{rw}^0 (\frac{S_w - S_{wc}}{1 - S_{or} - S_{wc}})^{n_w}$$

where:

- S_w is the water saturation
- S_{or} is the residual oil saturation ('oil saturation at the end of production')
- S_{wc} is the connate water saturation ('water saturation initially in the reservoir')
- k_{ro}^0 is the end point of oil permeability ('maximum relative permeability to oil')
- k_{rw}^0 is the end point of water permeability('maximum relative permeability to water')
- n_o, n_w are the Corey coefficients for oil and water.

The parameters considered for estimations are the two Corey coefficients n_o and n_w, the end points for the relative permeability curves k_{ro}^0 and k_{rw}^0, but also the residual oil and connate water saturations. Although S_{or} and S_{wc} characterize the reservoir at a larger scale and are not considered strictly relative permeability parameters, they model the relative permeability curves and therefore will also be included in this study.

Geological authenticity calls for assigning individual relative permeability curves to every grid cell, but for this study a common practiced simplification is applied, due to computational reasons: only one set of relative permeability parameters characterizes the entire reservoir.

The system state is completed by predicted bottom-hole injector pressure and oil and water rates for the producer.

The filter is initialized by generating 100 reservoir models. The dynamic variables are assumed known without uncertainty, there is no predicted data at this point and all the variability of the initial ensemble is given by the uncertainty in the relative permeability parameters. The pressure and water saturation are the same for all ensembles ($p = 300 \cdot 10^5\ Pa$, $s = S_{wc}$), while the relative permeability parameters are generated from the following distributions.

In order to investigate the sensitivity of the EnKF to the accuracy of the initial estimates for the relative permeability parameters, two types of distributions are considered:

1. a Gaussian distribution with the mean close, but not identical, to the true values of the reservoir parameters, and sufficiently large variance such that the initial ensembles cover the true design of the reservoir.

   In the following paragraphs, this distribution is identified for simplicity by the 'normal good distribution'.

2. a Gaussian distribution with a mean located further from the true values of the parameters, but with sufficient variance such that, in extremis, the initial ensembles can capture the measurements coming from the true reservoir.

   In the following paragraphs, this distribution is identified for simplicity by the 'normal wrong distribution'.

In addition, due to the fact that the estimates exhibited a drastic reduction in uncertainty during the history matching procedure, a uniform initial distribution on the interval $[1 \ldots 6]$ for n_o and n_w was also taken into consideration. Although it does not comply with the mathematical theory on which the EnKF was derived, the uniform distribution has the nice property of introducing a larger variability in the span of initial scenarios which could compensate the effect of the filter that, in the current setting, tends to over reduce the uncertainty.

The next table presents the complete description of the distributions used at the initialization step.

	normal good		**normal wrong**	
	mean	**var**	**mean**	**var**
n_o	3	0.5	4	0.5
n_w	3	0.5	4	0.5
k^0_{ro}	0.85	0.05	0.7	0.05
k^0_{rw}	0.65	0.05	0.77	0.05
S_{or}	0.18	0.05	0.1	0.05
S_{wc}	0.22	0.05	0.3	0.05

Table 2: Initial 'good' and 'wrong' Gaussian distributions of the relative permeability parameters

The experiments were done sequentially.

First, the two Corey exponents n_o and n_w were considered uncertain and estimated during history matching, while all the other reservoir parameters, including the rest of the relative permeability parameters, where considered known, one study case for each of the initial distributions proposed.

Secondly, n_o, n_w, plus the end points k^0_{ro} and k^0_{rw} were estimated during history matching, while keeping the rest of parameters fixed. Four study cases were investigate: two corresponding to the normal good and wrong initial distributions and two combinations of uniform distribution for n_o and n_w with normal good and wrong distributions for k^0_{ro} and k^0_{rw}.

The third experiment consisted of updating all six relative permeability parameters during the EnKF procedure, considering the same four initial scenarios as for the previous experiment.

At the analysis step, the measurements used to update the forecast come from a synthetic reservoir that has all the input parameters set to the values presented in the first table and the relative permeability parameters to:

$$n_o = 2 \quad n_w = 2 \quad k^0_{ro} = 0.9 \quad k^0_{rw} = 0.6 \quad S_{or} = 0.2 \quad S_{wc} = 0.2,$$

i.e. the 'true' values of the relative permeability parameters which the estimates need to match.

This reservoir model is denoted by 'the truth'. The measurements are generated from the 'true' production data by adding some measurement noise: 5% of the actual scale for the reservoir pressure ($10^7\ Pa$) and fluid rates ($10^{-3}\ m^3/s$). The diagonal covariance matrix $R(k)$ that accounts for measurement errors in the Kalman update was also set to these noise values.

Often, after the analysis step, the updated dynamic variables may not by physically meaningful or consistent with the static parameters, mainly due to the fact that the Kalman filter operates a linear update, while the flow equations are non-linear. This problem can be overcome by implementing an iterative filter, an extra confirmation step or simply a truncation procedure to assure that the dynamic variables stay within physical bounds. Much research has been done on the advantages and disadvantages of each of these methods, without concluding which approach is preferable, but since this issue is not the main concern of the current study, the simplest and least time consuming of the methods was chosen to fix the problem in this application, that being the truncation procedure.

At each Kalman update, a check is performed on the water saturations to assure that their values do not fall outside the interval $[S_{wc}, 1 - S_{or}]$ and if they do, the closest value within these interval is assigned to them.

An extra constraint on the updated relative permeability parameters also needed to be imposed: no negative values are allowed for the Corey parameters n_o and n_w to assure functionality of the simulator.

The sensitivity of the EnKF to production data can be investigated in the current setting where the measurements are restricted to bottom-hole pressures and fluid rates, only by varying the number of data assimilation steps and the length of the assimilation window. From this perspective, two scenarios were studied:

1. Measurements are assimilated only before water breakthrough occurs in the production well: history matching is done every 30 days for 17 months from the start of production.

2. Measurements are assimilated before and after water reaches the production well. Because for the considered reservoir the water breakthrough occurs late, the assimilation window is very large, history matching being performed every 30 days for 63 months from the start of production.

   Since in this scenario, most of the 63 assimilation steps take place before water breakthrough and the available measurements are of such nature that they are not likely to capture new information each of these steps until the water reaches the producer, a third scenario was derived at this point: on the same assimilation window, only 12 assimilation steps are performed, such that the number of measurements coming from before and after water breakthrough are better-balanced and the two time intervals are uniformly covered by the assimilation steps.

Findings

The main problems investigated were the capacity of the EnKF to recover the relative permeability parameters, the sensitivity of the filter to the initial ensemble distributions and the design of the history matching procedure.

The first issue addressed was the problem of whether or not the history matching procedure should pe performed on a assimilation window that captures the moment of water breakthrough. From a physical point of view, up until water breakthrough occurs, the measurements are in fact only injector bottom whole pressure and constant production oil rates, since the water has not yet reached the producer. Therefore, on this simple reservoir design, where every parameter is known without uncertainty, except for relative permeability parameters, measurements coming only from the water breakthrough moment rather provide reduced information about the movement of fluid in the reservoir and certainly not enough to characterize both relative permeability curves. In this situation, the filter is not expected to recover the true values of the parameters.

Except for the simplest of the scenarios (estimating only the power coefficients) on which the EnKF was tested, all the experiments consolidated the initial believe that history matching is not efficient for estimating relative permeability parameters if performed on a time frame that does not contain the moment of water breakthrough.

In Figure 1, first pair of pictures represents the best result obtained, while the second pair is an example of the typical behavior of the estimates of relative permeability parameters when history matching does not capture water breakthrough.

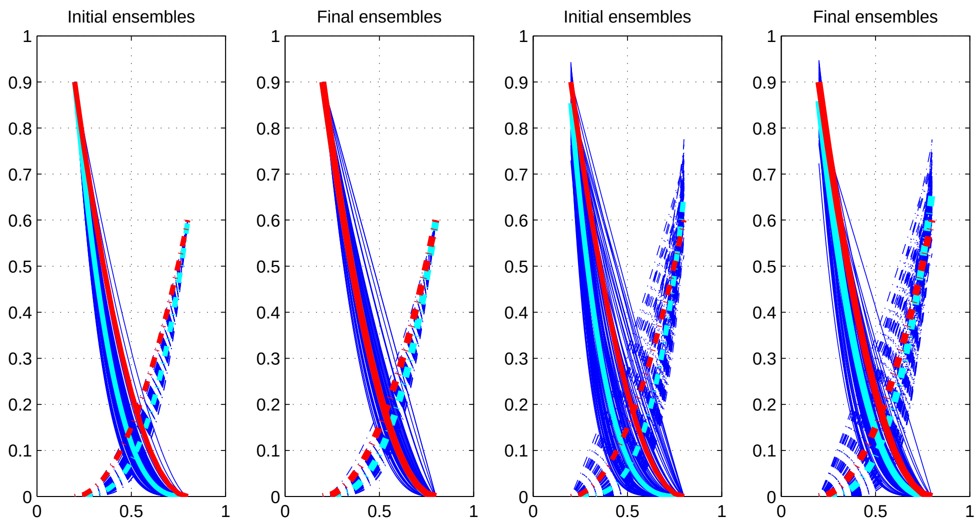

Figure 1: First pair: relative permeability curves using estimations of n_o and n_w starting from a normal good distribution. Second pair: relative permeability curves using estimations of n_o, n_w starting from a uniform distribution and k^0_{ro} and k^0_{rw} starting from a normal good distribution. All measurements done before water breakthrough. Dark blue: ensemble; light blue: mean of the ensemble; red: truth.

Typically, when measurements come only from before water breakthrough, the information is not enough to reduce the uncertainty: as it can be seen in Figure 1, the spread of the final ensembles is similar to the initial one. As for the values of the estimated parameters, they do not approach the true values significantly, apart from the case of estimating only the Corey coefficients with initial ensembles generated from a normal distribution with a mean closer to the true values of the parameters when the filter managed to correctly estimate the oil coefficient, as it can be seen in figure 1. Nevertheless, it did not manage to reduce the initial uncertainty.

Given these observations, the experiments further performed considered only assimilation windows that contain the moment of water breakthrough.

Performing the first experiments on a monthly history matching procedure that captures the moment of water breakthrough, the most striking feature of the results was that the Corey coefficients, and especially the one for oil, can be estimated correctly, but the uncertainty is reduced considerably, up to even 95% of the initial variance. Although the ability of the filter to recover the correct values of the parameters is a good result, reducing the uncertainty drastically is not particulary desirable for the forecasting task of the EnKF.

This issue was addressed by exploring the possibility of increasing the variance of the final distribution through: increasing the number of initial ensembles, generating the initial ensembles from a uniform distribution in order to introduce more variability and assimilating less information, since, as already mentioned, on known uniform absolute permeability and porosity fields, the measurements until water breakthrough will provide the 'same' information. Therefore, instead of assimilating very similar type of measurements every month until water breakthrough, the number of assimilation steps can be reduced while distributing them uniformly over the time period until water-breakthrough occurs.

None of these approaches changed the behavior of the filter: the Corey coefficient for oil can be recovered from measurements of oil/water rates and injector bottom-hole pressure and the trust in this estimation is extremely high. As most of the measurements come from before water-breakthrough, the coefficient for water is less correctly estimated than the one for oil and the uncertainty in this estimation is higher compared to the one for n_o, but still more reduced than for any of the other parameters.

Comparing the results of the monthly history matching procedure to the results of the 12-steps assimilation procedure, it was noticed a slight increase of the variance of the the final distributions for the second scenario. This observation combined with the fact that a 12-steps assimilation procedure is significantly less time expensive, makes the second scenario attractive. Unfortunately, comparing also the values of the estimated parameters, it can not be concluded that one scenario is better over the other one, as the values of the estimates are very similar. Therefore taking into account only the computational effort, it was decided that all the following observations should be illustrated with results from the experiments performed on the 12-steps history matching session.

For comparisons between the two assimilation procedure considered, Appendix 2 can be consulted: it contains complete information (tables and plots) about the estimated relative permeability parameters on the 63-steps assimilation procedure and it can be easily checked that not much difference between the two procedures was recorded, except for the over-all increased uncertainty in the estimates obtained form the 12-steps assimilation procedure.

As it can be seen by comparing the results presented in Tables 3, 4 and 5, the EnKF performs best when recovering the Corey coefficients alone. The power coefficients are most easily recovered from the available measurements and the trust in the estimations is very high compared to the estimations for the other parameters.

The oil coefficient is always correctly estimated, no matter which initial distribution was considered, which is to be expected since its value has a great impact on the movement of oil within the reservoir which is well captured and covered by the measurements. This is a valid observation for all experiments performed, but when considering more relative permeability parameters uncertain the quality of the estimations for n_o will decrease: for the first experiment the final mean estimates fall between 0.1% and 3% away from the true values, for the second experiment between 3% and 22%, while for the third experiment when all relative permeability parameters are estimated, between 1% to 19%. The reduction in uncertainty is as high as 95% of the initial variance for the first two experiments and between 50 and 80% for the last one.

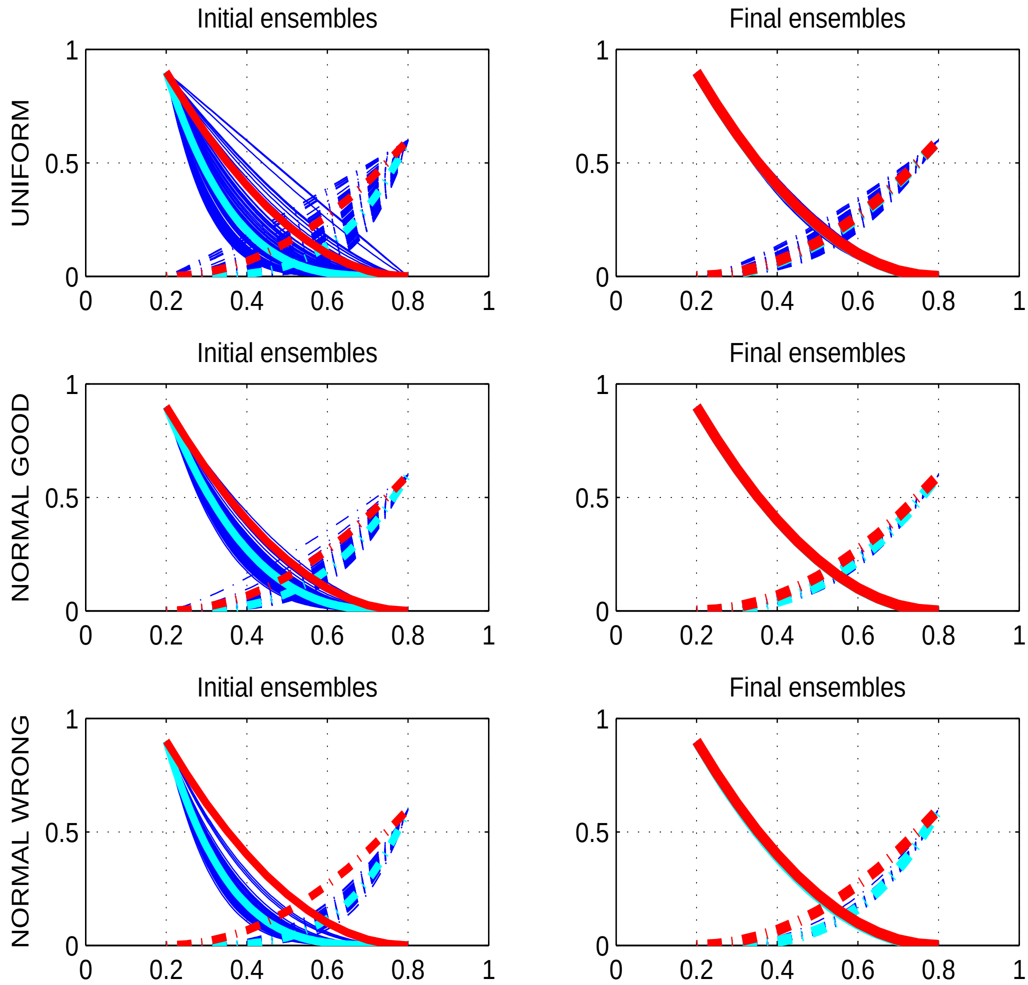

Figure 2: Relative permeability curves using estimations of n_o and n_w from the 12-steps history matching. Dark blue: ensemble; light blue: mean of the ensemble; red: truth.

		uniform		**normal good**		**normal wrong**	
	truth	**mean**	**std**	**mean**	**std**	**mean**	**std**
n_o	2	2.0058	0.0881	2.0018	0.0298	2.0573	0.0295
n_w	2	2.0725	0.3126	2.4353	0.1529	3.1745	0.1721

Table 3: Estimations of n_o and n_w from the 12-steps history matching procedure.

As for the water coefficient, providing a good initial distribution is of importance. This is visible from the experiments with only n_o and n_w estimated (as it can be seen in Table 3, for the normal wrong initial distribution the estimate is more that 50% away from the true value), but becomes more obvious for the next experiments: the estimations for n_w worsen as more of the relative permeability parameters are considered uncertain and included in the history matching procedure and the initial distribution becomes significant for the behavior of the filter. Nevertheless, the poorer estimates for n_w are compensated by a raise in uncertainty, which is generally 5-6 times larger than for n_o.

If estimating only n_o and n_w is a reasonable easy task for the EnKF, in terms of how many more of the rest of relative permeability parameters should be considered for estimation, the EnKF shows worse results and most of the experiments seem to indicate that estimating all the parameters that model the relative permeability curves is more efficient that estimating only n_o, n_w, k_{ro}^0 and k_{rw}^0: both Corey coefficients, and especially the one for water, are better estimated in the experiments where all parameters are considered unknown and the uncertainty is no longer over-reduced.

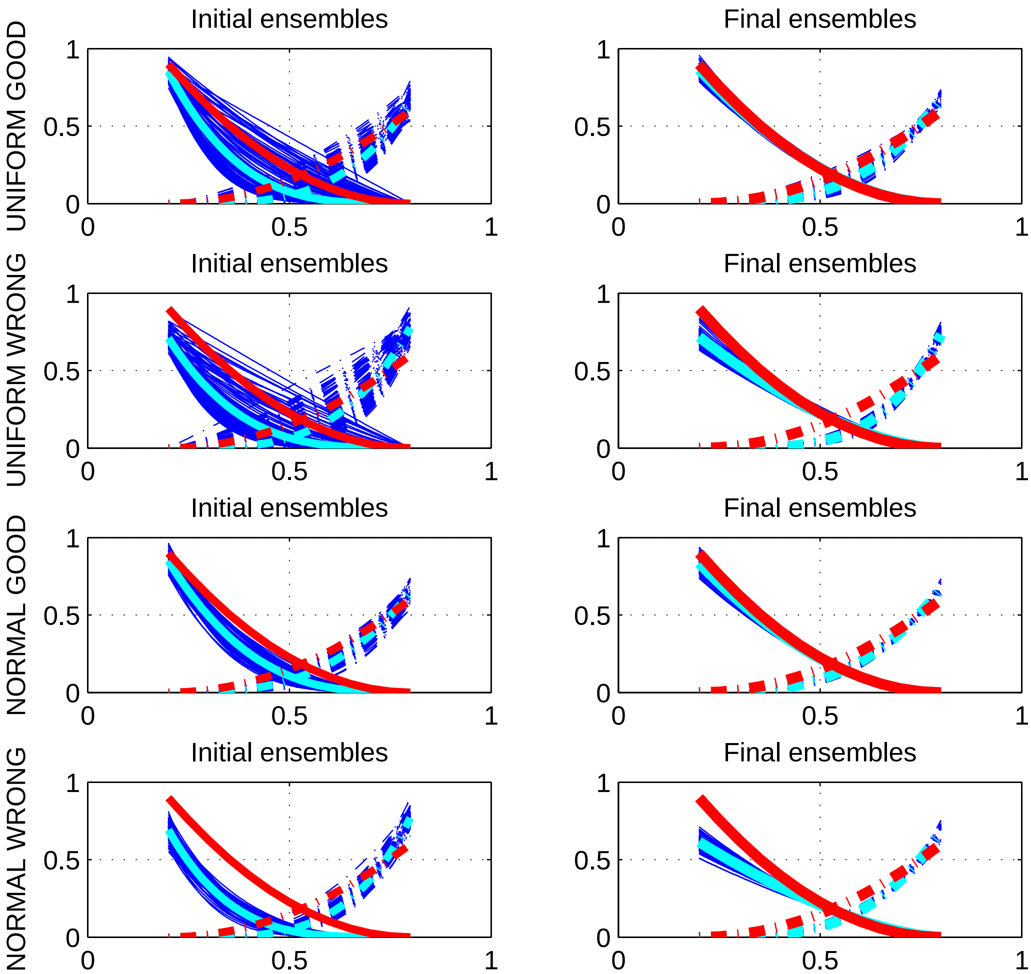

Figure 3: Relative permeability curves using the estimations of n_o, n_w, k_{ro}^0 and k_{rw}^0 from the 12-steps history matching.

		uniform good		uniform wrong		normal good		normal wrong	
	truth	**mean**	**std**	**mean**	**std**	**mean**	**std**	**mean**	**std**
n_o	2	1.9217	0.0702	1.6890	0.0839	1.9355	0.0555	1.5665	0.0639
n_w	2	3.1358	0.5095	4.1539	0.4179	2.9750	0.2538	3.3960	0.2352
k_{ro}^0	0.9	0.8671	0.0386	0.7171	0.0418	0.8358	0.0427	0.6145	0.0420
k_{rw}^0	0.6	0.6724	0.0312	0.7331	0.0353	0.6691	0.0254	0.6827	0.0291

Table 4: Estimations of n_o, n_w, k_{ro}^0 and k_{rw}^0 from the 12-steps history matching procedure

The end points for both relative permeability curves are less tuned during the assimilation procedure than any of the other parameters, with k_{rw}^0 being more responsive to the measurement updates: the final mean values reported in table 4 are less than 3% different from the initial mean values for k_{ro}^0 and 6% for k_{rw}^0, with the exception of the scenario where a initial normal wrong distribution is assumed (around 11% change), while those reported in table 5, are over-all less than 4% for both parameters.

The uncertainty reduces during history matching for both k_{ro}^0 and k_{rw}^0, slightly more for the water end point: 10-50% for the second experiment and somewhat less, only 10-20%, for the third experiment. In this conditions, since the values of the estimates do not change much from the initial ones, the quality of the initial estimates is obviously important: if the parameters can not be easily adjusted, then the reservoir predictions depend mostly on the initial estimations.

As for the two saturations, S_{wc} and and especially S_{or} are more responsive to history matching than the relative permeability end points: 10-20% variation of the initial mean of the ensemble for the estimated residual oil saturation and maxim 10% for S_{wc} were generally recorded at the end of the assimilation procedure. The reduction in uncertainty is similar for both parameters: 20-30% of the initial variance of the distribution.

Looking at the results for the two experiments where initial ensembles are sampled from the normal wrong distributions (Table 5, columns 2 and 4), it seems that, n_w, independently of its own better (uniform) or worse (normal wrong) initial distribution, is being simultaneously adjusted with S_{or} towards very poor final estimates: the means of the updated distributions for this two parameters are more than 50% away from the true values and two-three times further away than those obtained in the other two scenarios.

		uniform good		uniform wrong		normal good		normal wrong	
	truth	**mean**	**std**	**mean**	**std**	**mean**	**std**	**mean**	**std**
n_o	2	2.0326	0.3813	2.0570	0.2964	2.3607	0.2565	2.0225	0.1878
n_w	2	2.7611	0.8727	3.2440	0.7169	2.3768	0.3422	3.5696	0.3642
k_{ro}^0	0.9	0.8601	0.0454	0.7375	0.0517	0.8446	0.0381	0.6689	0.0389
k_{rw}^0	0.6	0.6586	0.0339	0.7653	0.0419	0.6897	0.0394	0.7800	0.0430
S_{or}	0.2	0.1581	0.0413	0.0821	0.0387	0.1575	0.0342	0.0918	0.0338
S_{wc}	0.2	0.2246	0.0378	0.2895	0.0387	0.1971	0.0319	0.2746	0.0305

Table 5: Estimations for n_o, n_w, k_{ro}^0, k_{rw}^0, S_{or}, S_{wc} for the 12-steps assimilation procedure

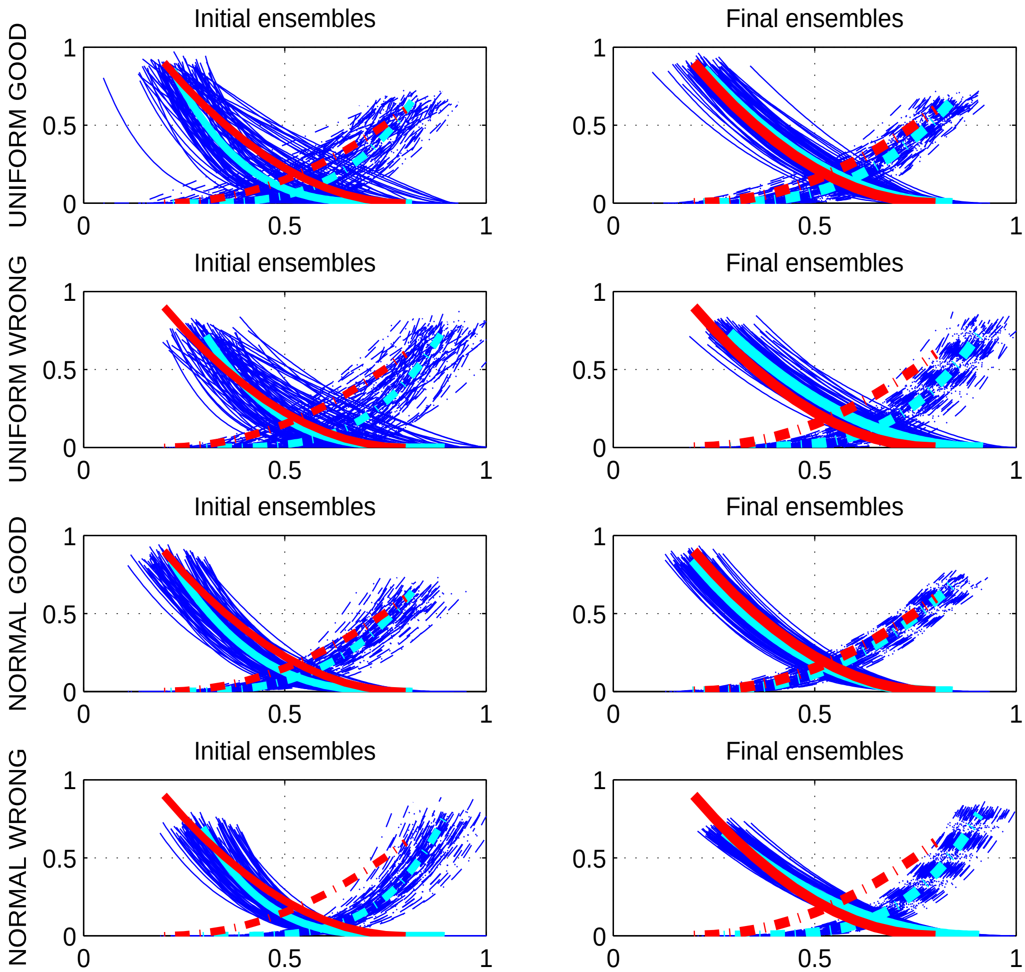

Figure 4: Relative permeability curves using estimations of n_o, n_w, k^0_{ro}, k^0_{rw}, S_{or}, S_{wc} from the 12-steps history matching.

Introduced as a possible solution to compensate the over-reduction in the uncertainty of the Corey coefficients, the uniform distribution as an initial guess for the EnKF does not change the results of the estimations in a significant way: although the initial variability is approximately three times larger than for the normal distributions considered, the filter still managed to reduce it accordingly to a standard deviation of the same order as for the scenarios having normal distributions as first guesses. The filters initialized using the uniform distributions always led to slightly, but not significantly, wider final distributions for n_o, with a more consistent increase for n_w, and in most experiments they did perform better than the filters initialized using the normal wrong distribution considered, yet these experiments do not seem enough to conclude that in the case no initial pertinent guess for the parameters is available, it is reasonable to decide, over the mathematical grounds of the EnKF, to use a uniform distribution.

Another possibility to correct the same issue was to consider a larger ensemble size, but performing the same experiments using 150 reservoir models did not improve the performance of the EnKF either. In fact, the benefits of using a uniform initial distribution for n_o and n_w are more consistent than those of increasing the ensemble size.

In Appendix 3, some additional plots can be consulted for a better understanding of the conclusions reached after performing all these sets of experiments. They represent the forecasts for the available measurements and compare the predictions done with the initial reservoir models to the ones done using the dynamic and static parameters obtained after the last assimilation step is performed.

Even for the less good estimations, the forecasts come very close to the truth, unfortunately, due to the over-reduction in uncertainty, for some of the less good estimations (such as the scenario using a normal wrong initial distribution estimating only the Corey coefficients - figures 14, 15 and 16 in Appendix 3) the spread of the final ensembles missed to capture the true measurements. Obviously, in this situation, the all relative permeability parameters estimations are preferable, since, even if they might lead to poorer means of the final ensemble, the uncertainty is larger, yet sufficiently reduced compared with the initial distributions, and have a better chance to cover the truth.

As it can be seen from figure 21 and 22, even the worst combinations of estimated parameters discussed before (for the uniform wrong and normal wrong initial scenarios) manage to provide forecasts for oil and water rates that capture the truth. In fact, the spread of the predictions for oil rates done using the estimates obtained starting from the normal initial distribution fails to cover the true rates when n_o, n_w, k_{ro}^0 and k_{rw}^0 are considered uncertain (figure 18), although coming very close, while for the experiments where all parameters are considered uncertain, the final distribution covers, in extremis, the truth. This can be seen as another argument to prefer estimating all relative permeability parameters over estimating only the Corey coefficients and end points.

Simultaneous estimation of relative and absolute permeability

Settings

For the second experiment the focus is set on studying the performance of the EnKF at estimating the relative permeability curves when other reservoir parameters are considered uncertain as well, and for this purpose the absolute permeability was chosen.

The reservoir preserves its main characteristics from the first study case, but it was equipped with four production wells placed at the corners and one injector in the center of the reservoir because the measurements coming from the previous quarter five spot designed proved not to be sufficient for adequate history matching. The producers are still constrained by prescribed bottom-hole pressure, and the injector by constant injection rate.

The next table presents the main properties of the reservoir and figure six the absolute permeability field.

$c_o = 10^{-9} \quad 1/Pa$	$\mu_o = 0.5 \cdot 10^{-3} \quad Pa \cdot s$	$n_o = 2$	$n_w = 2$	$p_{prod}^{pres} = 250 \cdot 10^5 \quad Pa$
$c_r = 10^{-9} \quad 1/Pa$	$\mu_w = 10^{-3} \quad Pa \cdot s$	$k_{ro}^0 = 0.9$	$k_{rw}^0 = 0.6$	$rate_{inj}^{pres} = 0.002 \quad m^3/s$
$c_w = 10^{-9} \quad 1/Pa$	$\phi = 0.3$	$S_{or} = 0.2$	$S_w c = 0.2$	

Table 6: Input parameters for the reservoir used in simultaneously estimation of the absolute and relative permeability parameters

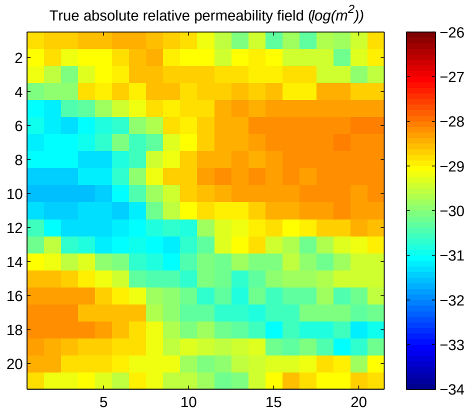

Figure 5: The true absolute permeability field

These reservoir parameters are the input configuration of 'the truth' and the measurements were generated following the same guidelines as in the first experiment.

The state vector should be completed with absolute permeability variables corresponding to each grid cell, but as research has shown, the natural logarithm of the absolute permeability is normally distributed, therefore the state vector is in fact augmented by the log-permeability.

The initialization of the filter follows as well the guidelines of the previous experiment. The initial absolute permeability fields are considered uncertain and are randomly chosen from the pre-generated set of absolute permeability fields that comes along with the **simsim** simulator. The next picture provides a visual representation of the mean and variance of the initial permeability fields.

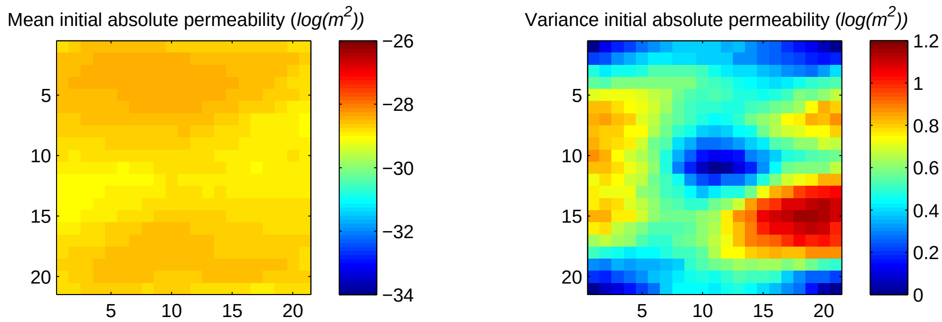

Figure 6: Mean(left) and variance(right) of the initial permeability ensemble

Having sorted out the initial ensembles issue, the same set of tests as in the previous experiment have been performed, investigating how well can the EnKF estimate first two, then four and finally all six relative permeability parameters simultaneously with the absolute permeability field. For a better comparison in terms of the quality of the prediction, two extra experiments were performed: the EnKF only updates the absolute permeability field, while the relative permeability curves are considered known; in the first experiment they are set to the true values and in the second experiment to some chosen wrong values:

$$n_o = 3,\ n_w = 3,\ k_{ro}^0 = 0.8,\ k_{rw}^0 = 0.5,\ S_{or} = 0.1,\ S_{wc} = 0.1.$$

The problem of un-physical updated dynamic variables was solved this time by adding an extra confirmation step after the measurement update is performed. Not being the purpose of this study to look into the influence that the choice of method to solve this issue has on the behavior of the EnKF, only a brief presentation of the confirmation approach is given here, with articles [5],[6] being good references for a better understanding.

The confirmation step takes place after the analysis step as follows: if the current time is t_0 and we integrated until time t_1, the flow simulator is re-run with the analysed static parameters (in this case relative and absolute permeabilty) from time t_0 to time t_1 and the newly flow simulated dynamic variables (pressures and saturations) replace the analyzed vectors from the Kalman filter and are used as the initial vectors for the next forecast step. This way the dynamic variables become physically meaningful.

Findings

The first issue addressed was again the problem of whether or not the assimilation procedure should capture the water breakthrough. Since, intuitively, for a combined absolute permeability-relative permeability estimation problem more information is needed for an efficient history matching procedure as there are significantly more uncertain parameters, but also having the experience of the previous study, the decision could not be but in favor of a longer assimilation procedure that captures the water breakthrough.

Having equipped the reservoir with 4 production wells, it was chosen to assimilate production data monthly until water breakthrough occurs in two of the producers (NW and NE corners), using the other two for testing the capacity of the filter to provide good forecasts that can capture the water breakthrough.

As in the previous case, the experiments performed on a shorter assimilation window lead to poor estimates for the parameters and little reduction in uncertainty.

Given the size of the state vector, it was also considered necessary to raise the number of ensembles and comparing results from the experiments with 100 ensembles and 150 ensembles it was indeed observed an improvement of the estimations for both absolute and relative permeability parameters. Therefore, the following observations are derived only from the results of the experiments performed using 150 ensembles and a 24 steps monthly history matching that covers the time of breakthrough in the production wells from the NW and NE corners of the reservoir.

The first observation to be made is that the more degrees of freedom of the state vector lead to poorer estimations of the relative permeability parameters and less reduction in uncertainty than those registered in the previous experiment.

The two Corey coefficients, n_o and n_w are not properly recovered from the measurements in any of the scenarios tested, but the final estimates do not necessary depart from the true values dramatically: most of them, although not all, stay within 50% away from the true values. The uncertainty in the estimations is not over-reduced as in the previous experiments, but it is notable and more significant than for any of the other parameters: it goes up to 80% reduction of the initial variability for the experiments where not all relative permeability parameters are estimated, but the medium reduction is around 50-60%. For the experiments where all parameters are estimated, the reduction in uncertainty is less: the final standard deviations for

n_w reported in table 9 for the scenarios with initial normal distributions are in fact only 10-20% reduced.

		uniform		**normal good**		**normal wrong**	
	truth	**mean**	**std**	**mean**	**std**	**mean**	**std**
n_o	2	2.7688	0.3182	2.8063	0.2018	2.8933	0.1849
n_w	2	1.7398	0.3087	2.2629	0.3303	2.9138	0.2966

Table 7: Estimations of n_o and n_w obtained while simultaneously estimating k.

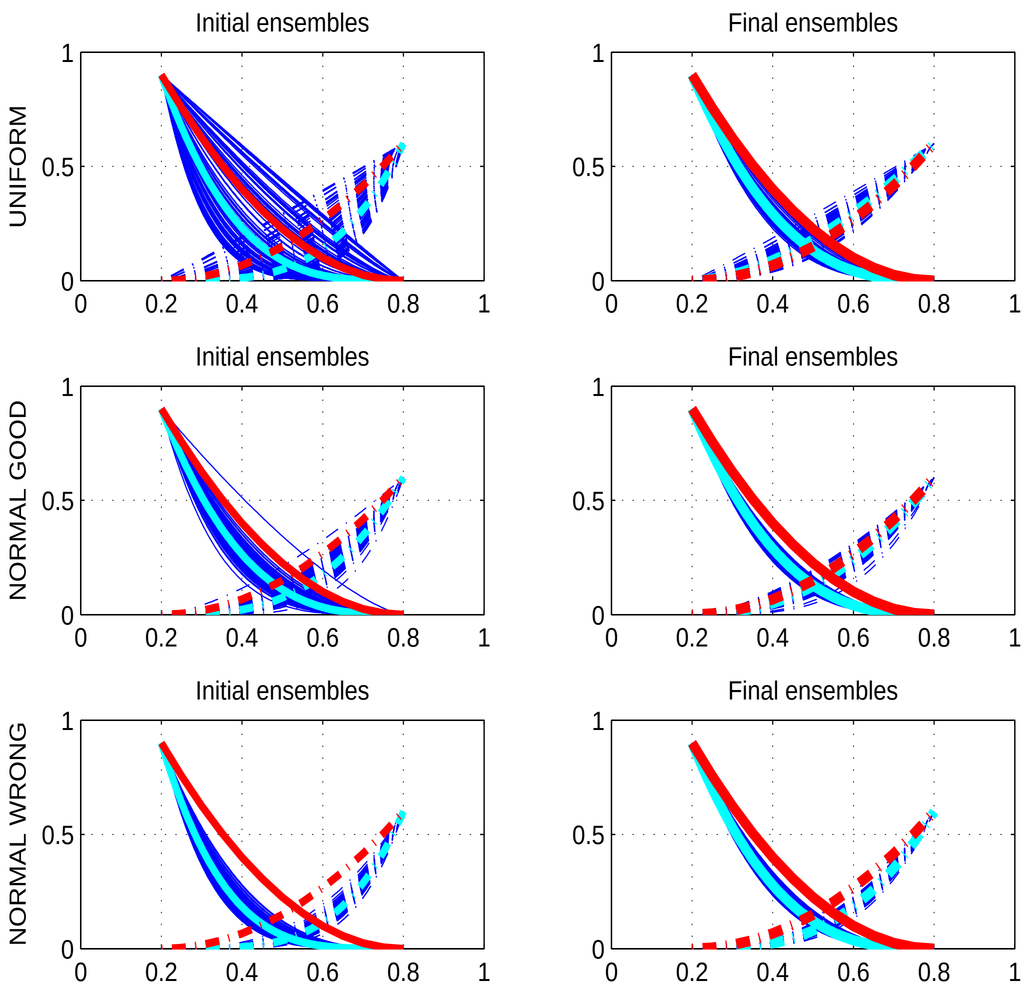

Figure 7: Relative permeability curves using estimations of n_o and n_w obtained while simultaneously estimating k.

It can be easily seen that the trust in the estimation of n_o is higher than in the estimation for n_w and the values of the final estimates for n_o tend to be similar, independent of the initial distribution. Compared to the first experiment, the estimates for n_o are considerably worse and it could be even concluded that the filter can recover the water coefficients more easily than the one for oil: although this is a mixed behavior, examining, for example the values from table 8, it can be concluded that the best estimates for n_w are much better any of the good estimates for n_o. Intuitively, this could be explain by the fact that given the higher degree of freedom for the parameter space, more parameter combinations that match the measurements can be constructed, so the importance of the oil coefficient for the dynamics of the reservoir is not as persistent as in the first experiment when the absolute permeability was known.

		uniform good		**uniform wrong**		**normal good**		**normal wrong**	
	truth	**mean**	**std**	**mean**	**std**	**mean**	**std**	**mean**	**std**
n_o	2	2.5603	0.3138	2.8992	0.3750	2.9589	0.2267	3.0466	0.2118
n_w	2	2.1281	0.4960	3.1717	0.4759	2.2456	0.3253	2.9423	0.3034
k_{ro}^0	0.9	0.8471	0.0468	0.6967	0.0474	0.8295	0.0423	0.7344	0.0458
k_{rw}^0	0.6	0.6620	0.0403	0.8355	0.0453	0.6385	0.0371	0.8100	0.0354

Table 8: Estimations of n_o, n_w, k_{ro}^0 and k_{rw}^0 obtained while simultaneously estimating k.

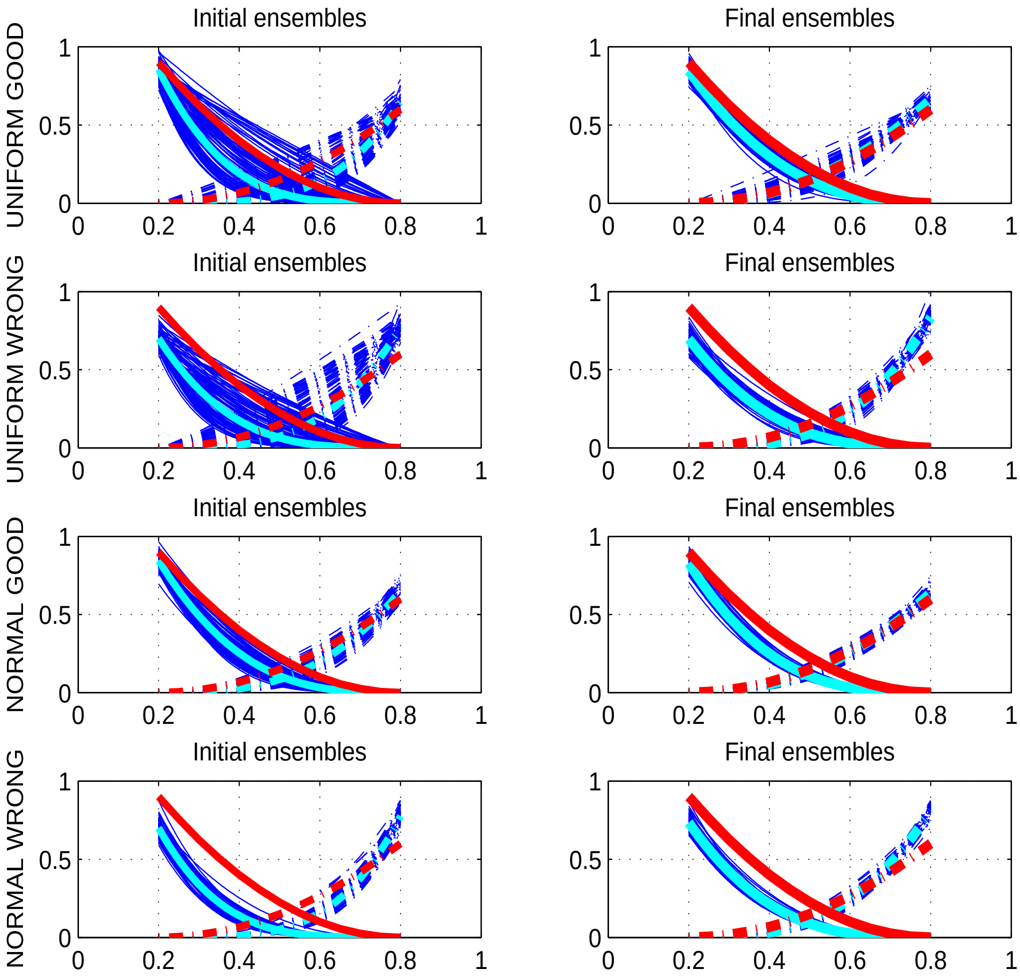

Figure 8: Estimations of n_o, n_w, k_{ro}^0 and k_{rw}^0 obtained while simultaneously estimating k.

As for the n_w, similar to the previous experiment, the initial distribution tends to play a more significant role in the estimation, if not so much its own initial distribution, certainly the initial distributions of the other parameters. For example, looking at the experiments when then Corey coefficients and end points for the relative permeability curves are estimated simultaneous (figure 8 and table 8), it can be noticed that when k_{ro}^0 and k_{rw}^0 are initially sampled from the normal wrong distribution, n_w is poorly estimated. Since k_{ro}^0 and k_{rw}^0 are less responsive to the data assimilation adjustment, the poor initial estimates are compensated by tuning n_w.

As already mentioned, the estimates for k_{ro}^0 and k_{rw}^0 preserve the same behavior noticed in the first experiment: they are not particulary receptive to measurement updates and this enforces the importance of the initial estimates. The end point for water remains more flexible than the one for oil and as it can be seen from both table 8 and 9, the reduction in uncertainty is similar and not significant for both end points.

The same conclusions as for the first experiment apply to S_{or} and S_{wc}: they are more responsive to history matching than the end points of the relative permeability curves and especially S_{or} gets tuned during the assimilation procedure to compensate for the poor estimation of the other parameters. Reductions in uncertainty around 20-30% of the initial variance are generally recorded.

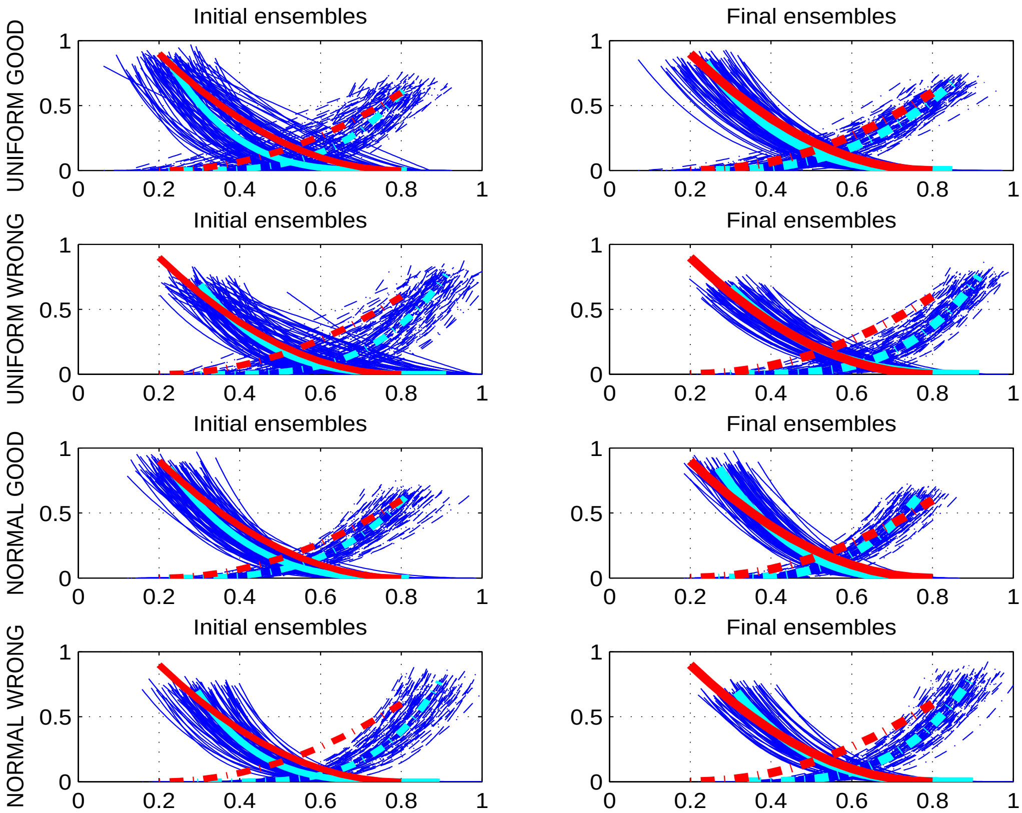

Figure 9: Relative permeability curves using the estimations of n_o, n_w, k_{ro}^0, k_{rw}^0, S_{or}, S_{wc} obtained while simultaneously estimating k.

		uniform good		uniform wrong		normal good		normal wrong	
	truth	**mean**	**std**	**mean**	**std**	**mean**	**std**	**mean**	**std**
n_o	2	3.0129	0.4658	2.7570	0.4833	2.6157	0.2982	3.2272	0.3151
n_w	2	2.5358	0.6781	3.4391	0.9671	2.9913	0.3860	3.2872	0.4012
k^0_{ro}	0.9	0.8300	0.0443	0.6673	0.0473	0.8454	0.0530	0.6897	0.0421
k^0_{rw}	0.6	0.6701	0.0424	0.7542	0.04325	0.6488	0.0411	0.8112	0.0423
S_{or}	0.2	0.1513	0.0395	0.0853	0.0356	0.2283	0.0356	0.1004	0.0423
S_{wc}	0.2	0.2371	0.0446	0.2984	0.0443	0.2699	0.0380	0.3145	0.0427

Table 9: Estimations for n_o, n_w, k^0_{ro}, k^0_{rw}, S_{or}, S_{wc} obtained while simultaneously estimating k.

The capability of the filter to recover the absolute permeability fields, in the context of uncertain relative permeability curves, can be judged visually by comparing the final estimates with the truth and also with the estimates provided by filters applied on reservoir models for which the relative permeability curves are considered known and equal to the true values or some other incorrect values. Also the comparison can be done through RMSE plots that can be consulted in Appendix 5.

In Figure 10 the estimated absolute permeability fields from all the experiments are plotted and comparing them with the truth, it can be concluded that, with more or less accuracy, the general patterns of the true k field are recovered for every experiment. And if they are compared with the estimation coming from the scenario with known wrong values for the relative permeability parameters, then it can be concluded that instead of incorrectly assuming some values for the relative permeability curves, it would be preferable to estimate them.

As the estimates are rather similar, it can not be concluded that one initial distribution of the relative permeability parameters leads to better estimates of the absolute permeability fields than another. But it can be observed that the absolute permeability estimates worsen when more relative permeability parameters are considered unknown: in figure eleven the resemblance to the truth reduces from the pictures on the third row, corresponding to the experiments where only n_o and n_w are estimated, to the fifth row, corresponding to the experiments where all relative permeability parameters are considered unknown and estimated.

Although the RMSE statistics may not necessary be the best means of comparing k field estimations, the plots in Appendix 5 confirm the previous conclusions and consolidate the observation that there can not be established a direct connection between the initial distribution of the relative permeability and the behavior of the absolute permeability estimates.

The forecast plots for all the experiments are presented in Appendix 4.

None of the final ensembles manage to properly capture the measurements coming from the producer placed in the SW corner. The forecasts made with the estimates obtained from the experiments where only n_o and n_w are estimated, which can be considered the best absolute-relative estimated permeability parameters, manage to approach the true values best. They are followed by those obtained when all relative permeability parameters are estimated, due to a larger variability in the final ensemble. Nevertheless, the predicted behavior of the reservoir is very similar for all experiments.

For the producer placed at the SE corner of the reservoir all forecasts capture the true measurements. The mean forecast does not necessary come close to the truth, but there is enough variability to cover the true reservoir model.

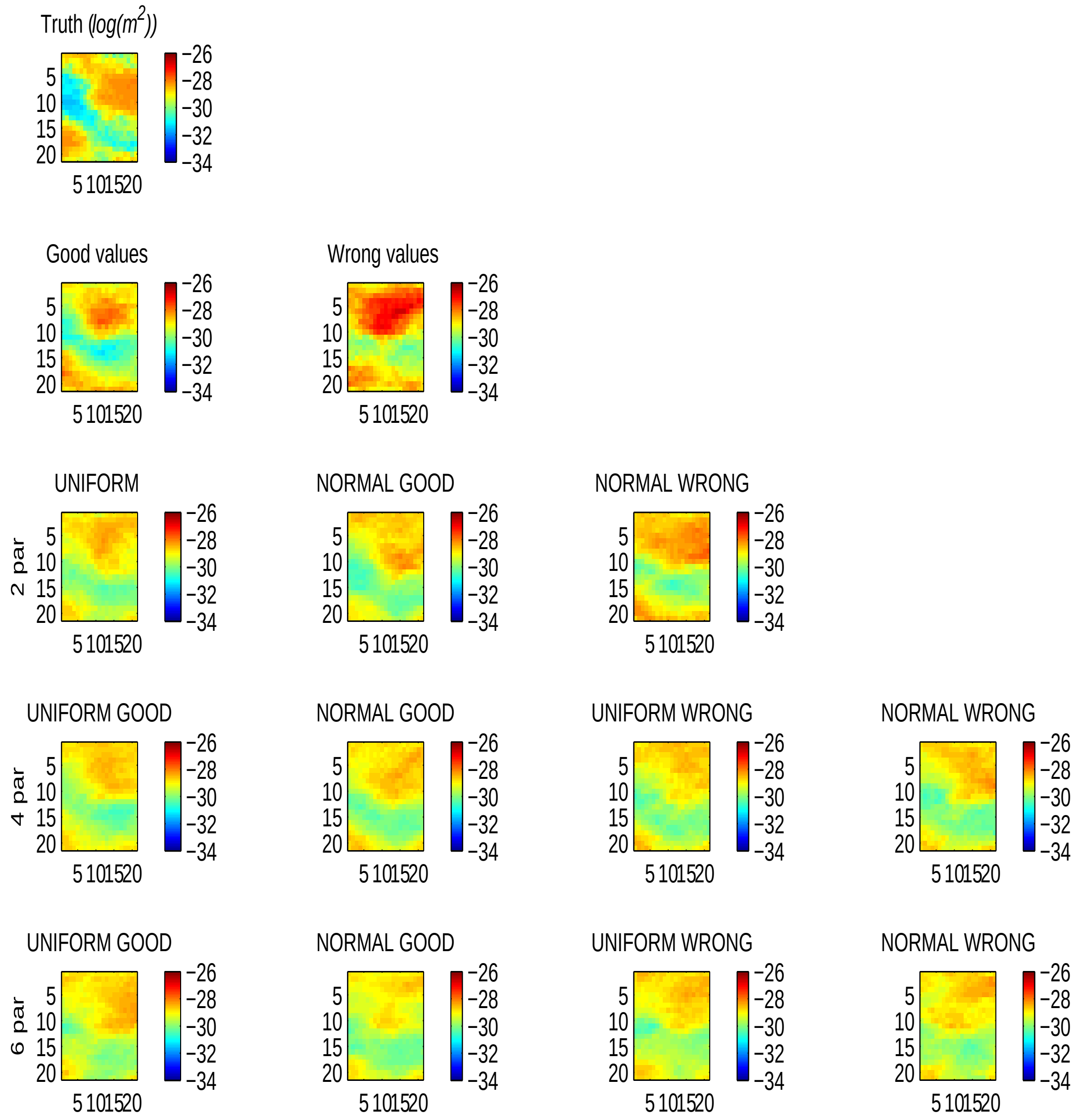

Figure 10: Estimations of absolute permeability. First row: the truth. Second row: estimations of absolute permeability alone. Third to six row: estimations of absolute permeability simultaneous with relative permeability

Conclusions

The possibility of estimating relative permeability parameters using the EnKF was investigated and it can be concluded that, given production data that comes from before and after the moment of water breakthrough, the relative permeability curves can be recovered, with more or less accuracy depending on the initial estimates.

The Corey coefficients seem to be the most easily recovered from the measurements: n_o is estimated correctly more often than not, rather independent of the initial distribution, and the trust in the estimation is high, which translates in having the smallest final uncertainty from all the relative permeability parameters. The estimates for n_w are not as consistent as for n_o and the uncertainty is less reduced. This could be explained by the fact that the measurements become more informative of the movement of water in the reservoir only after breakthrough, while they provide information about the oil movement over the entire assimilation window.

The residual oil and water connate saturations are less adjustable during history matching than the Corey coefficients, but more than the end points of the relative permeability curves which are rather not responsive to measurement updates. In this situation, as the values of k_{ro}^0 and k_{rw}^0 are not going to be adjusted much during history matching, it is important that a good initial estimate for them is provided.

It was also noticed, that the residual oil saturation and Corey coefficient for water are most sensitive to being tuned 'away' from the true values during history matching.

When relative permeability parameters are estimated simultaneously with absolute permeability, recovering the relative permeability becomes more difficult. Although the first experiment showed that the Corey coefficient for oil can be well estimated from the considered measurements, for an absolute-relative permeability estimation problem, the final distributions for n_o do not approach the truth significantly. The observations made previously for the other parameters are over-all still valid: k_{ro}^0 and k_{rw}^0 are the least adjustable during history matching with S_{or} and S_{wc} being more responsive to measurements updates, while the estimations of n_w are more dependent of the initial distribution. For all parameters, the uncertainty is reduced, but not of the same degree as in the first experiments.

The investigations also revealed that, from the point of view of recovering the absolute permeability field, estimating relative permeability parameters is preferable to considering them known and assigning them incorrect values. The results of the experiments do not suggest that the initial distribution of the relative permeability parameters has a strong influence on the quality of the estimation for the absolute permeability.

References

[1] R.H. Brooks, A.T. Corey, "Hydraulic properties of porous media"

[2] A. Seiler, G. Evensen, J.-A. Skjervheim, J. Hove, J.G. Vabo, StatoilHydro ASA, "Advanced Reservoir Management Workflow Using an EnKF Based Assisted History Matching Method", SPE 118906

[3] A.T. Watson, G.R. Gavalas, J.H. Seinfield, "Indentifiability of Estimates of Two-Phase Reservoir Properties in History Matching"

[4] J.D. Jansen, "System Theory for Reservoir Management (Lecture notes for course AES1490 Version 3u, October 2007)"

[5] Xian-Huan Wen, Wen.H. Chen, "Real-Time Reservoir Model Updating Using Ensemble Kalman Filter", SPE 92991

[6] M. Zafari, A.C. Reynolds, "Assessing the Uncertainty in Reservoir Description and Performance Predictions With the Ensemble Kalman Filter", SPE 95750

[7] A.W. Heemink, "Data Assimilation Methods - chapter five"

[8] M.V. Krymskaya,"Parameter Estimation in Reservoir Engineering Models via Data Assimilation Techniques" - Master of Science Thesis, TUDelft, 2007

Appendix 1 - The simsim simulator

Combining mass balance and Darcy's theory, the dynamics of a two phase (water and oil) reservoir under iso-thermal conditions and ignoring capillary pressures can be described by the following system of PDE's:

$$-\nabla \cdot [\frac{\alpha \cdot \rho_w \cdot k^0_{rw}}{\mu_w} \cdot \vec{\mathbf{K}}(\nabla p - \rho_w \cdot g \cdot \nabla d)] + \alpha \cdot \rho_w \cdot \phi[S_w \cdot (c_w + c_r)\frac{\partial p}{\partial t} + \frac{\partial S_w}{\partial t}] - \alpha \cdot \rho_w \cdot q_w = 0$$

$$-\nabla \cdot [\frac{\alpha \cdot \rho_o \cdot k^0_{ro}}{\mu_o} \cdot \vec{\mathbf{K}}(\nabla p - \rho_o \cdot g \cdot \nabla d)] + \alpha \cdot \rho_o \cdot \phi[(1 - S_w) \cdot (c_o + c_r)\frac{\partial p}{\partial t} - \frac{\partial S_w}{\partial t}] - \alpha \cdot \rho_o \cdot q_o = 0$$

where:

$-\nabla\cdot$ is the divergence operator
$-\nabla$ is the gradient operator
$-\alpha$ is the geometry factor
$-\rho_o, \rho_w$ are the fluids densities
$-\mu_o, \mu_w$ are the fluids viscosities
$-k^0_{ro}, k^0_{rw}$ are the relative permeabilities
$-\vec{K}$ is the permeability tensor
$-g$ is the acceleration of gravity
$-d$ is the depth
$-c_o, c_w, c_r$ are the compressibilities
$-\phi$ is the porosity
$-p$ is the oil/water pressure
$-S_w$ is the water saturation
$-q_o, q_w$ are the source terms
$-t$ is the time

The **simsim** software developed by professor J.D. Jansen implements the previous equations, assuming isotropic permeability, pressure independence of the parameters and absence of the gravity forces on a 2D grid using a block-centered finite difference spatial discretization and can solve the equation for time by integrating explicitly, implicitly or combining the two methods into IMPES.

It has a well model implemented following Paceman and implements the relative permeability curves in the Corey parametrization.

Appendix 2 - Results for the 63-steps assimilation procedure

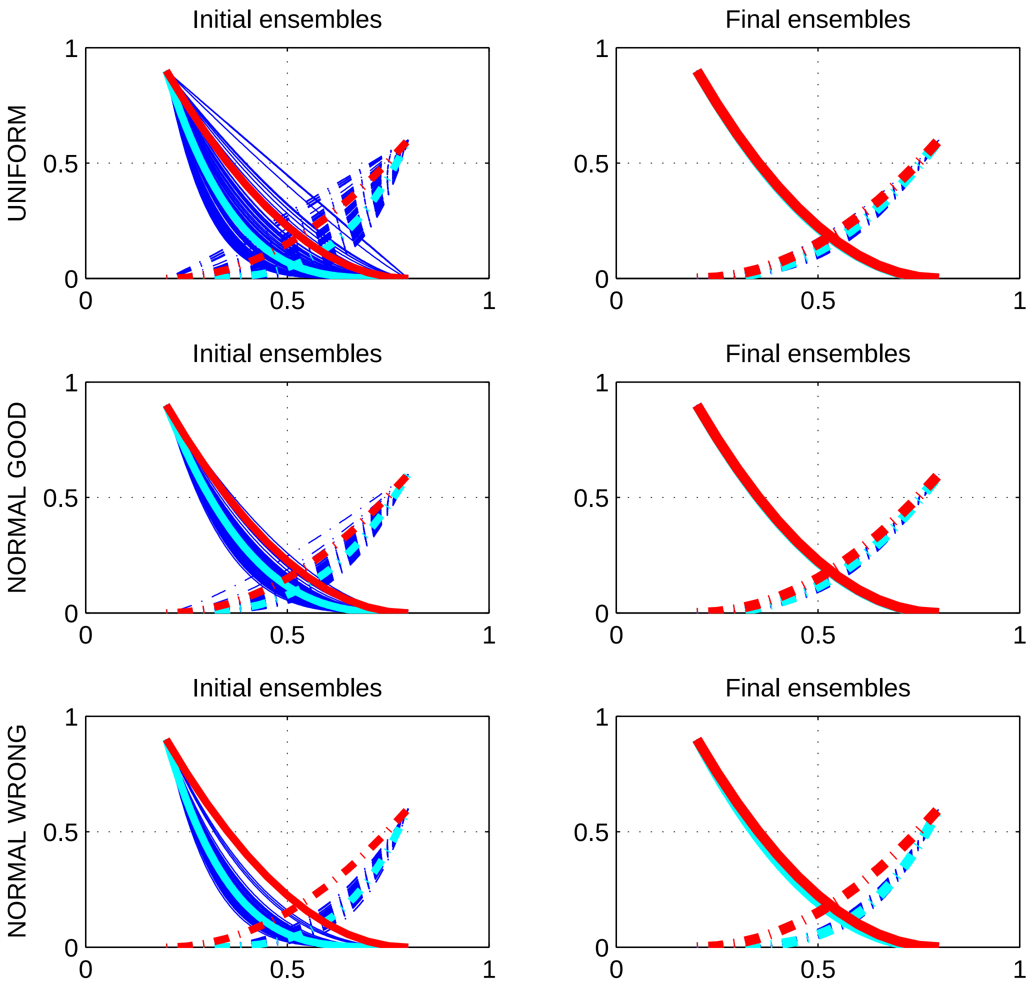

Figure 11: Relative permeability curves using estimations of n_o and n_w from the monthly 63-steps history matching procedure. Dark blue: ensemble; light blue: mean of the ensemble; red: truth.

		uniform		**normal good**		**normal wrong**	
	truth	**mean**	**std**	**mean**	**std**	**mean**	**std**
n_o	2	2.0328	0.0315	2.0263	0.0265	2.2033	0.0353
n_w	2	2.3368	0.2144	2.3800	0.1455	3.4861	0.2028

Table 10: Estimations of n_o and n_w from the 63-steps monthly history matching procedure

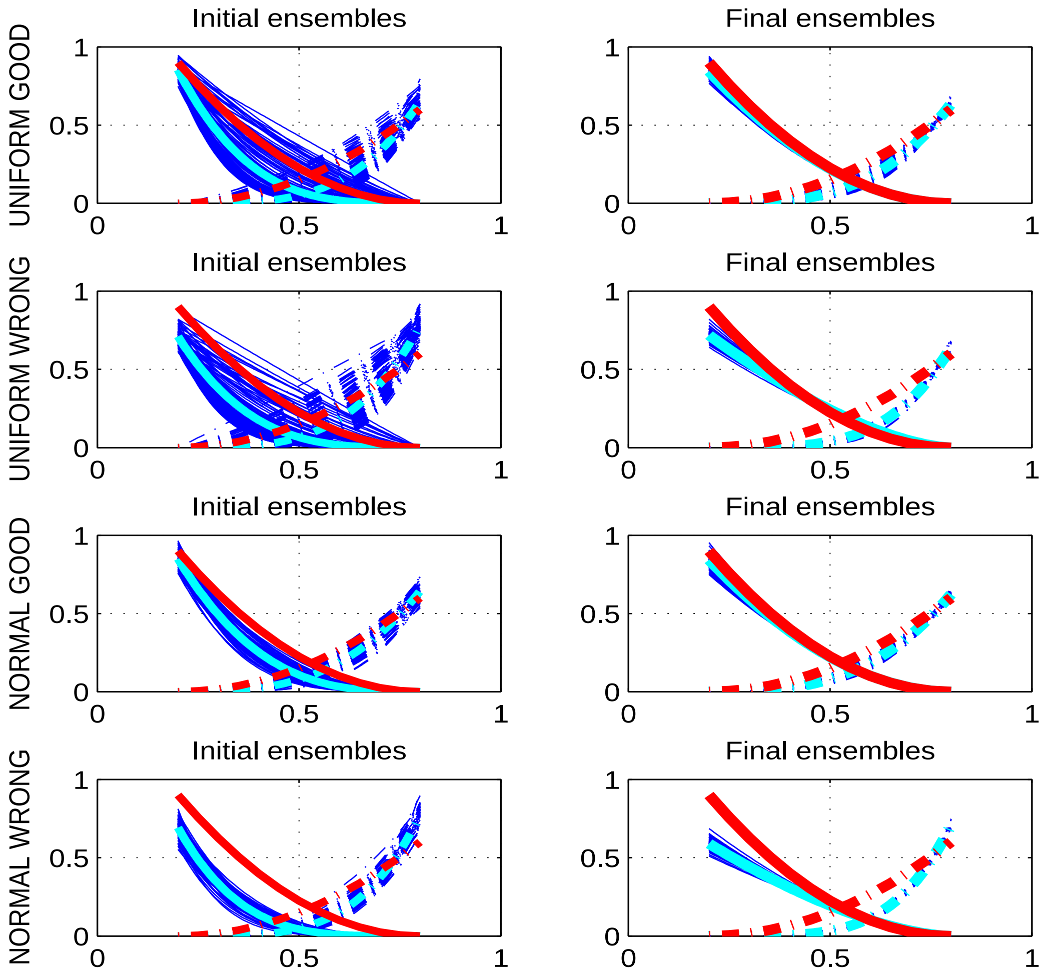

Figure 12: Relative permeability curves using the estimations of n_o, n_w, k^0_{ro} and k^0_{rw} from the 63-steps monthly history matching procedure.

		uniform good		**uniform wrong**		**normal good**		**normal wrong**	
	truth	**mean**	**std**	**mean**	**std**	**mean**	**std**	**mean**	**std**
n_o	2	1.9377	0.0606	1.566	0.0556	1.9301	0.0545	1.5982	0.0549
n_w	2	3.2339	0.4236	4.0654	0.3298	2.9677	0.2654	4.4458	0.2253
k^0_{ro}	0.9	0.8442	0.0377	0.7178	0.0318	0.8437	0.0383	0.5898	0.03002
k^0_{rw}	0.6	0.6362	0.0199	0.6441	0.0221	0.6232	0.0170	0.7106	0.0204

Table 11: Estimations of n_o, n_w, k^0_{ro} and k^0_{rw} on the 63-steps monthly history matching procedure

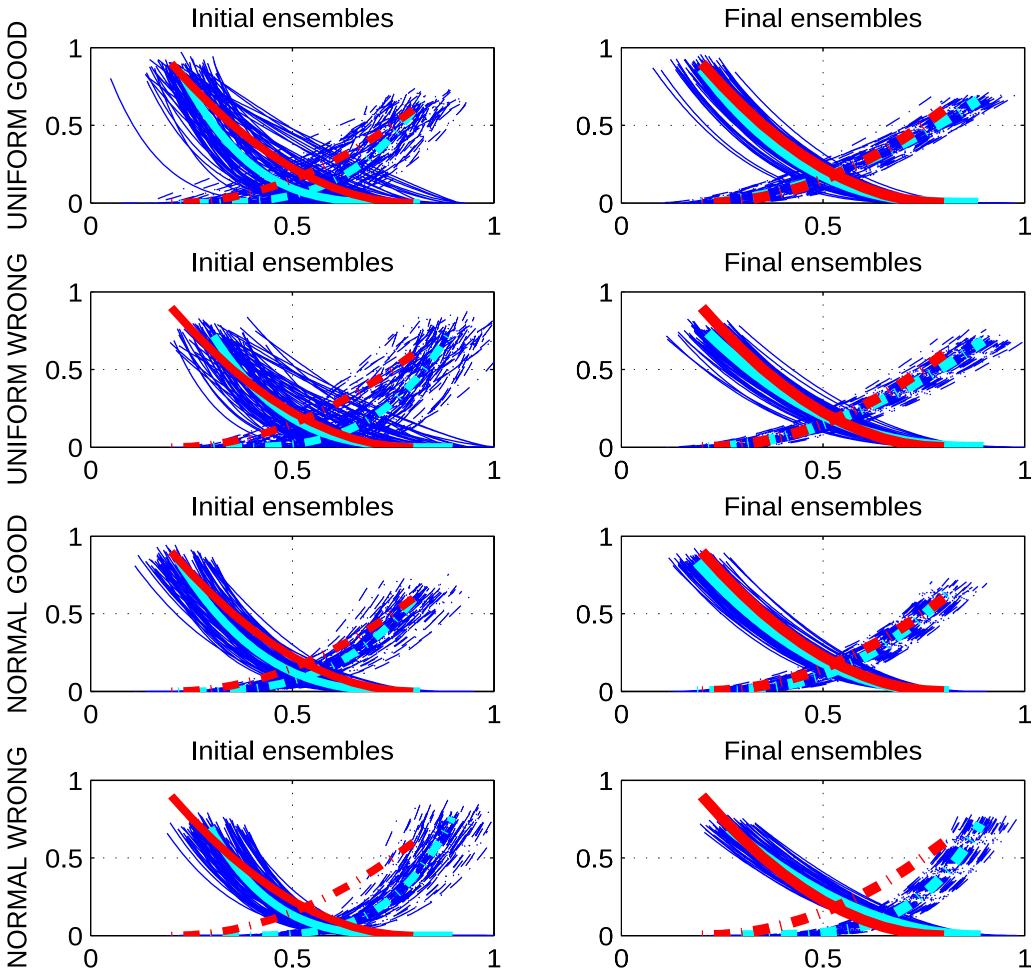

Figure 13: Relative permeability curves using the estimations of n_o, n_w, k^0_{ro}, k^0_{rw}, S_{or}, S_{wc} from the 63-steps monthly history matching procedure.

		uniform good		uniform wrong		normal good		normal wrong	
	truth	**mean**	**std**	**mean**	**std**	**mean**	**std**	**mean**	**std**
n_o	2	3.5062	0.2572	3.3726	0.2476	2.9874	0.2382	3.9874	0.1655
n_w	2	1.7148	0.2930	1.7726	0.1315	2.6115	0.3741	3.8380	0.3055
k^0_{ro}	0.9	0.8648	0.0431	0.7357	0.0374	0.8322	0.0375	0.6919	0.0331
k^0_{rw}	0.6	0.6607	0.0249	0.6843	0.0244	0.6459	0.0327	0.7201	0.0394
S_{or}	0.2	0.1155	0.0357	0.1025	0.0382	0.1875	0.0321	0.1088	0.0298
S_{wc}	0.2	0.1995	0.0347	0.2152	0.0366	0.1880	0.0310	0.2673	0.0305

Table 12: Estimations for n_o, n_w, k^0_{ro}, k^0_{rw}, S_{or}, S_{wc} from the 63-steps monthly assimilation procedure

Appendix 3 - Forecast plots for the 12-steps assimilation procedure

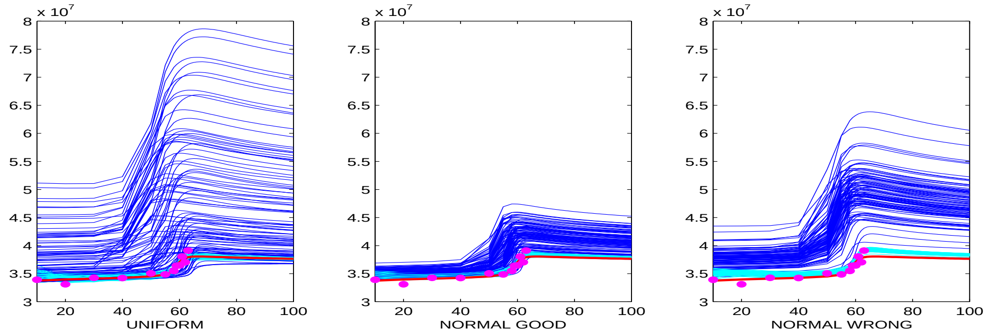

Figure 14: Bottom hole pressure forecasts with estimated n_o and n_w, starting from all the three initial distributions considered on 12-steps history matching. Dark blue: initial ensemble; light blue: final ensembles; red: truth; pink: measurements.

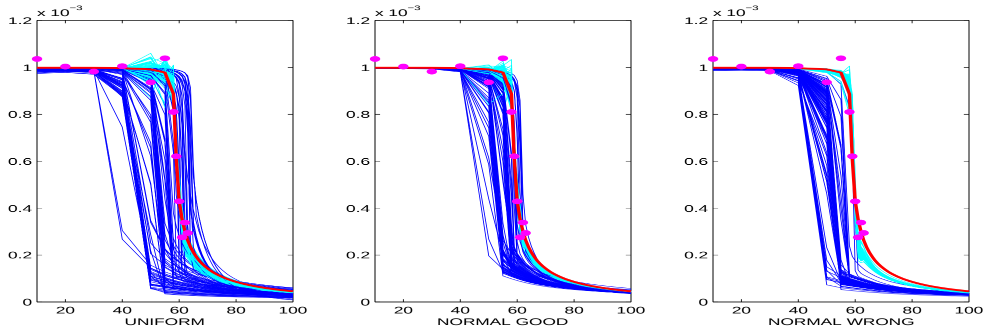

Figure 15: Oil rates forecasts with estimated n_o and n_w, starting from all the three initial distributions considered on 12-steps history matching. Dark blue: initial ensemble; light blue: final ensembles; red: truth; pink: measurements.

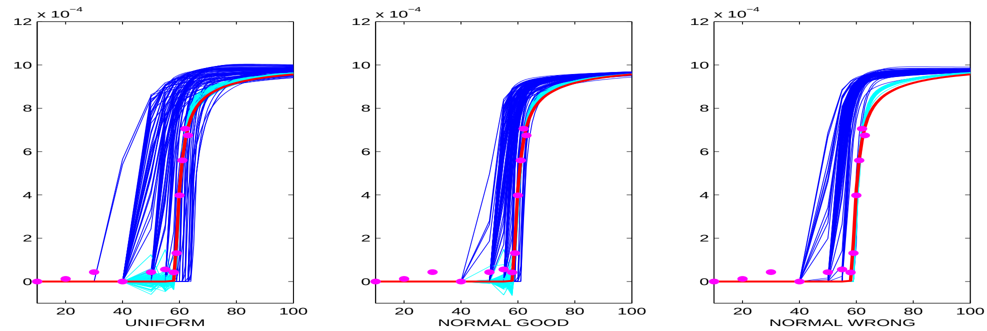

Figure 16: Water rates forecasts with estimated n_o and n_w, starting from all the three initial distributions considered on 12-steps history matching. Dark blue: initial ensemble; light blue: final ensembles; red: truth; pink: measurements.

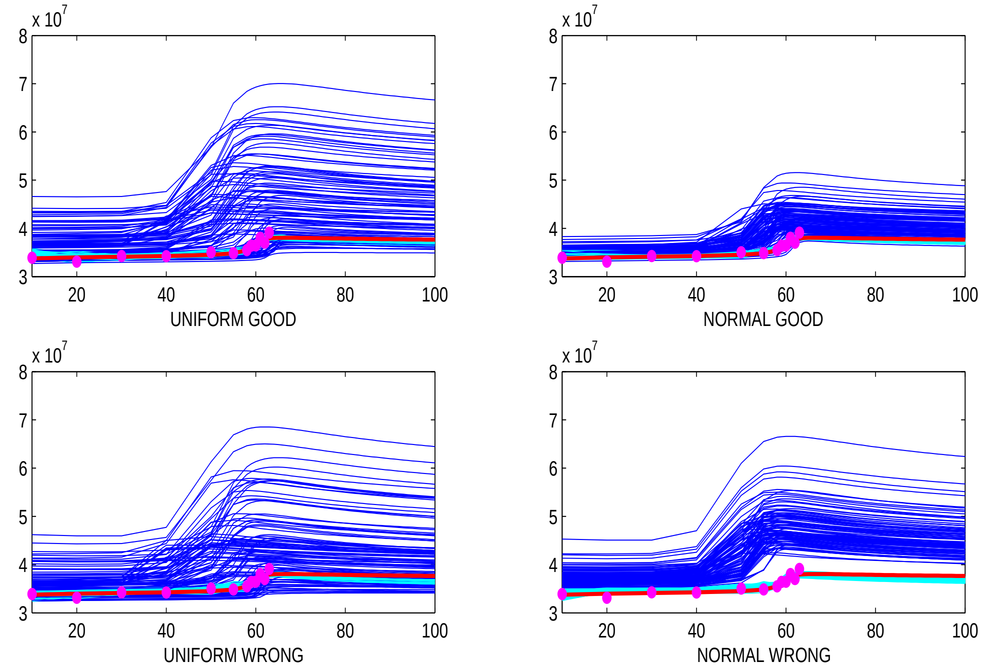

Figure 17: Bottom hole pressure forecasts with estimated n_o, n_w, k^0_{ro} and k^0_{rw}, starting from all the three initial distributions considered on 12-steps history matching. Dark blue: initial ensemble; light blue: final ensembles; red: truth; pink: measurements.

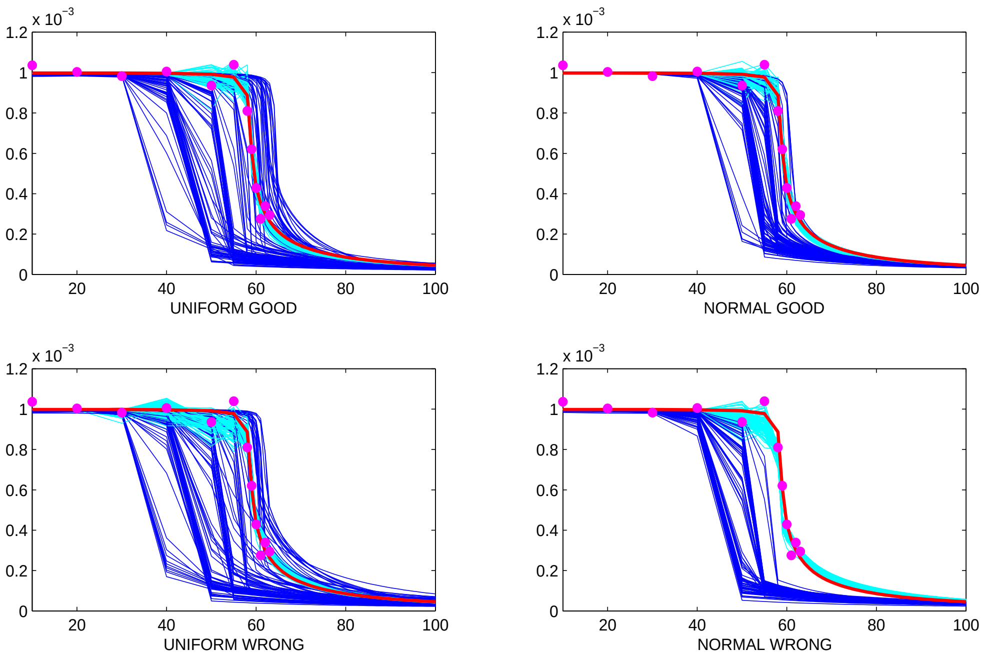

Figure 18: Oil rates forecasts with estimated n_o, n_w, k^0_{ro} and k^0_{rw}, starting from all the three initial distributions considered on 12-steps history matching. Dark blue: initial ensemble; light blue: final ensembles; red: truth; pink: measurements.

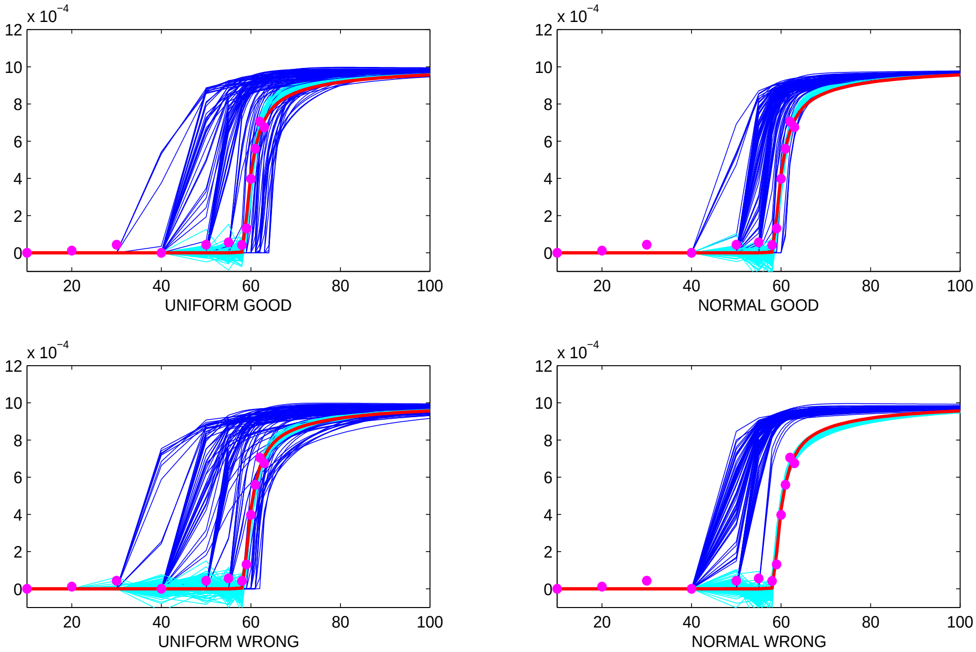

Figure 19: Water rates forecasts with estimated n_o, n_w, k^0_{ro} and k^0_{rw} starting from all the three initial distributions considered on 12-steps history matching. Dark blue: initial ensemble; light blue: final ensembles; red: truth; pink: measurements.

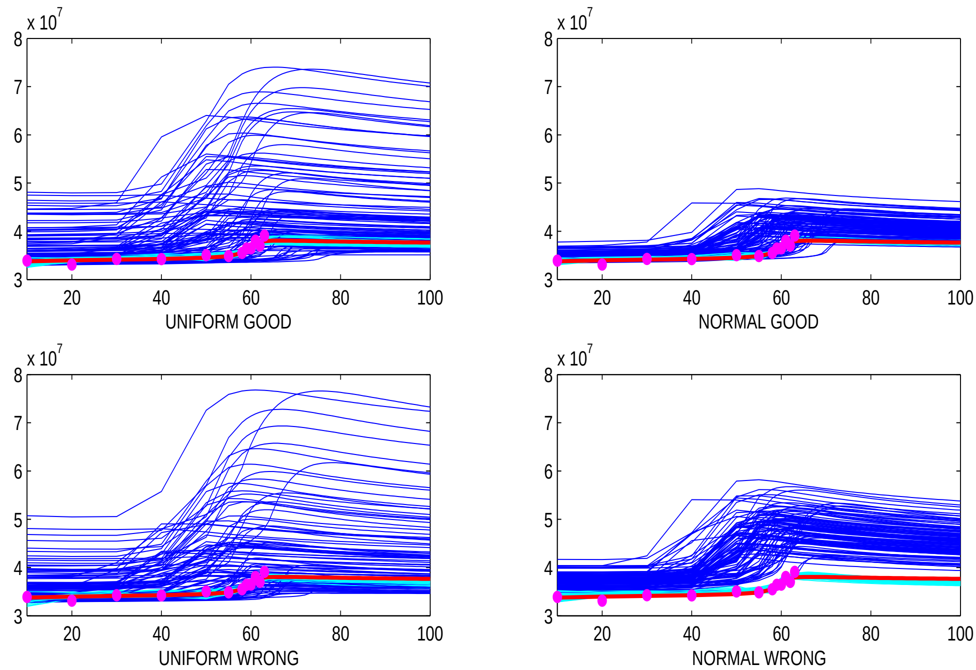

Figure 20: Bottom hole pressure forecasts with estimated n_o, n_w, k^0_{ro}, k^0_{rw}, S_{or} and S_{wc}, starting from all the three initial distributions considered on 12-steps history matching. Dark blue: initial ensemble; light blue: final ensembles; red: truth; pink: measurements.

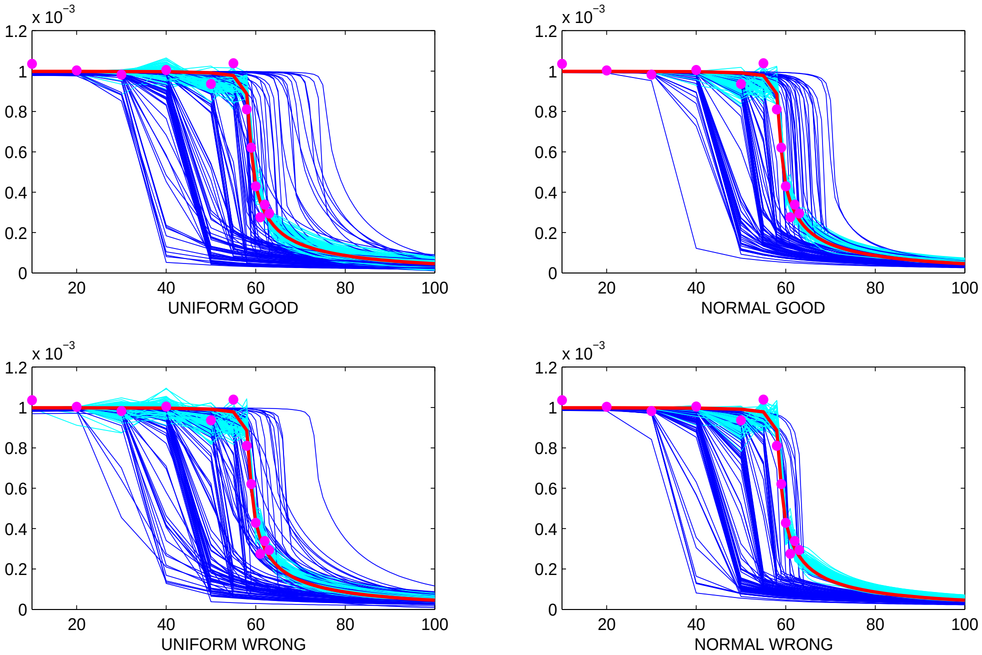

Figure 21: Oil rates forecasts with estimated n_o, n_w, k^0_{ro}, k^0_{rw}, S_{or} and S_{wc}, starting from all the three initial distributions considered on 12-steps history matching. Dark blue: initial ensemble; light blue: final ensembles; red: truth; pink: measurements.

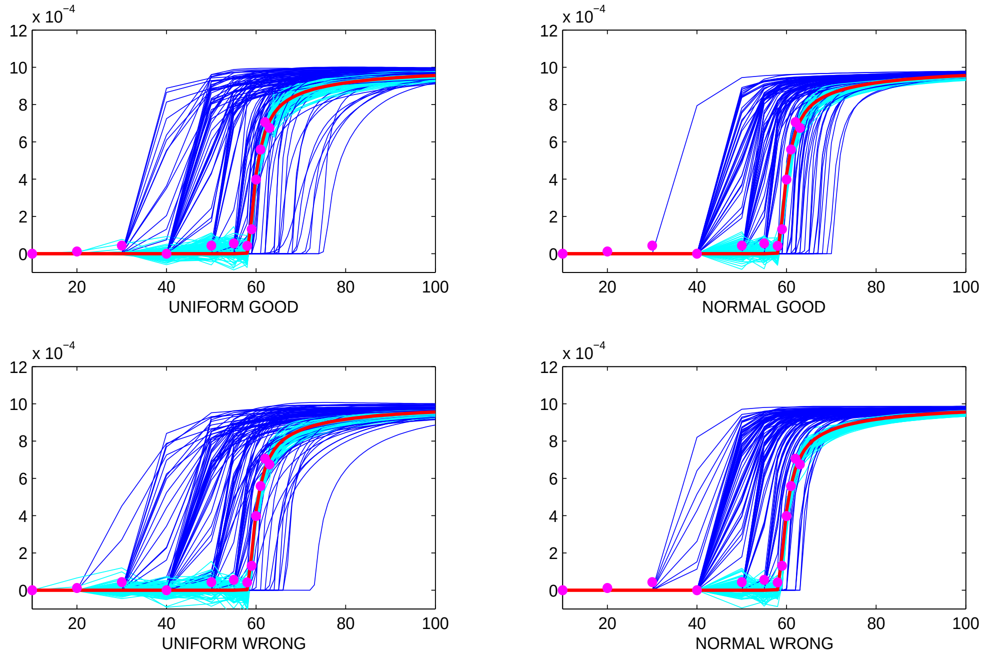

Figure 22: Water rates forecasts with estimated n_o, n_w, k^0_{ro}, k^0_{rw}, S_{or} and S_{wc}, starting from all the three initial distributions considered on 12-steps history matching. Dark blue: initial ensemble; light blue: final ensembles; red: truth; pink: measurements.

Appendix 4 - Forecast plots for the experiment concerning the simultaneous estimation ok absolute and relative permeability

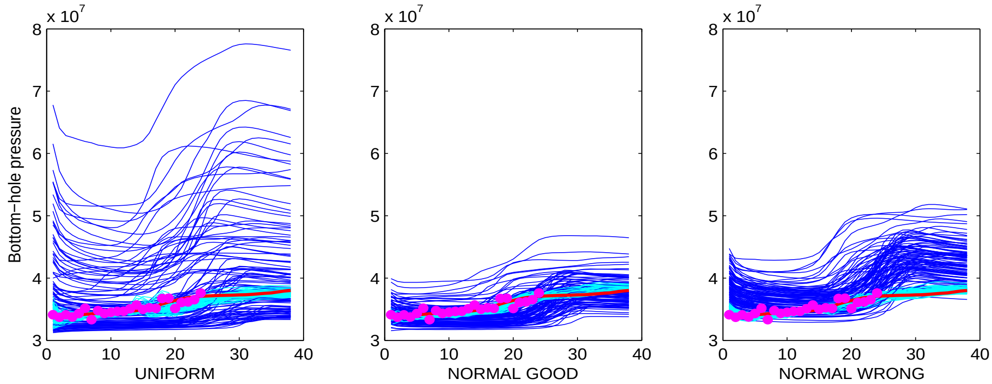

Figure 23: Bottom hole pressure forecasts with estimated n_o and n_w. Dark blue: initial ensemble; light blue: final ensembles; red: truth; pink: measurements.

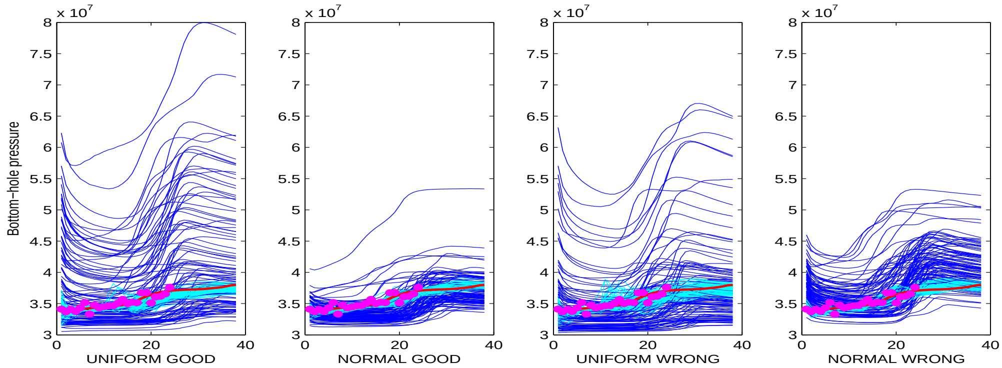

Figure 24: Bottom hole pressure forecasts with estimated n_o, n_w, k_{ro}^0 and k_{rw}^0. Dark blue: initial ensemble; light blue: final ensembles; red: truth; pink: measurements.

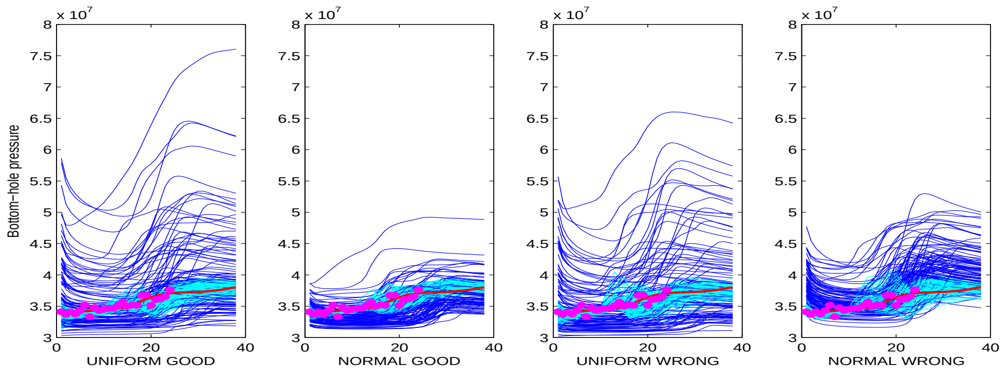

Figure 25: Bottom hole pressure forecasts with estimated n_o, n_w, k_{ro}^0, k_{rw}^0, S_{or} and S_{wc}. Dark blue: initial ensemble; light blue: final ensembles; red: truth; pink: measurements.

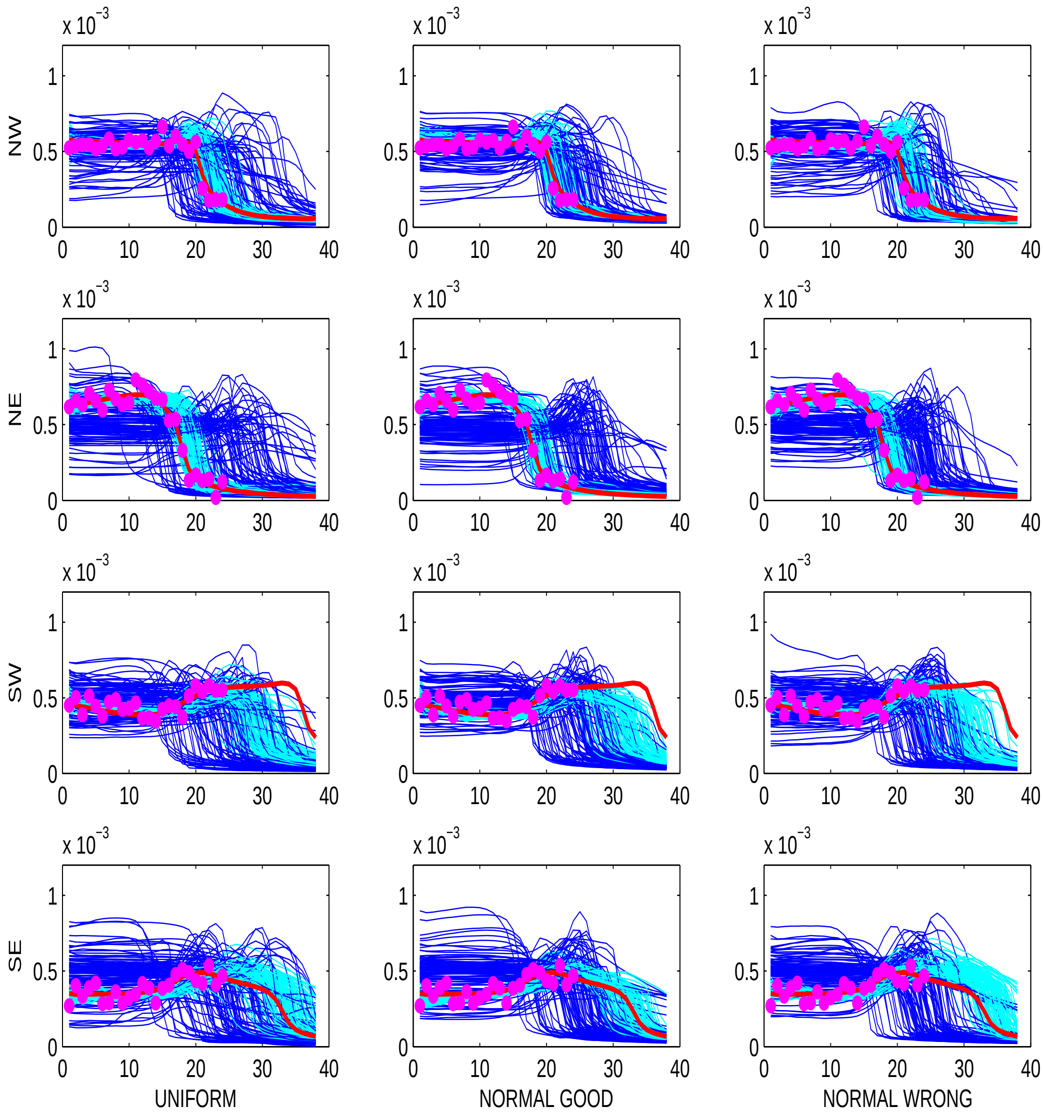

Figure 26: Oil rates forecasts with estimated n_o and n_w. Dark blue: initial ensemble; light blue: final ensembles; red: truth; pink: measurements.

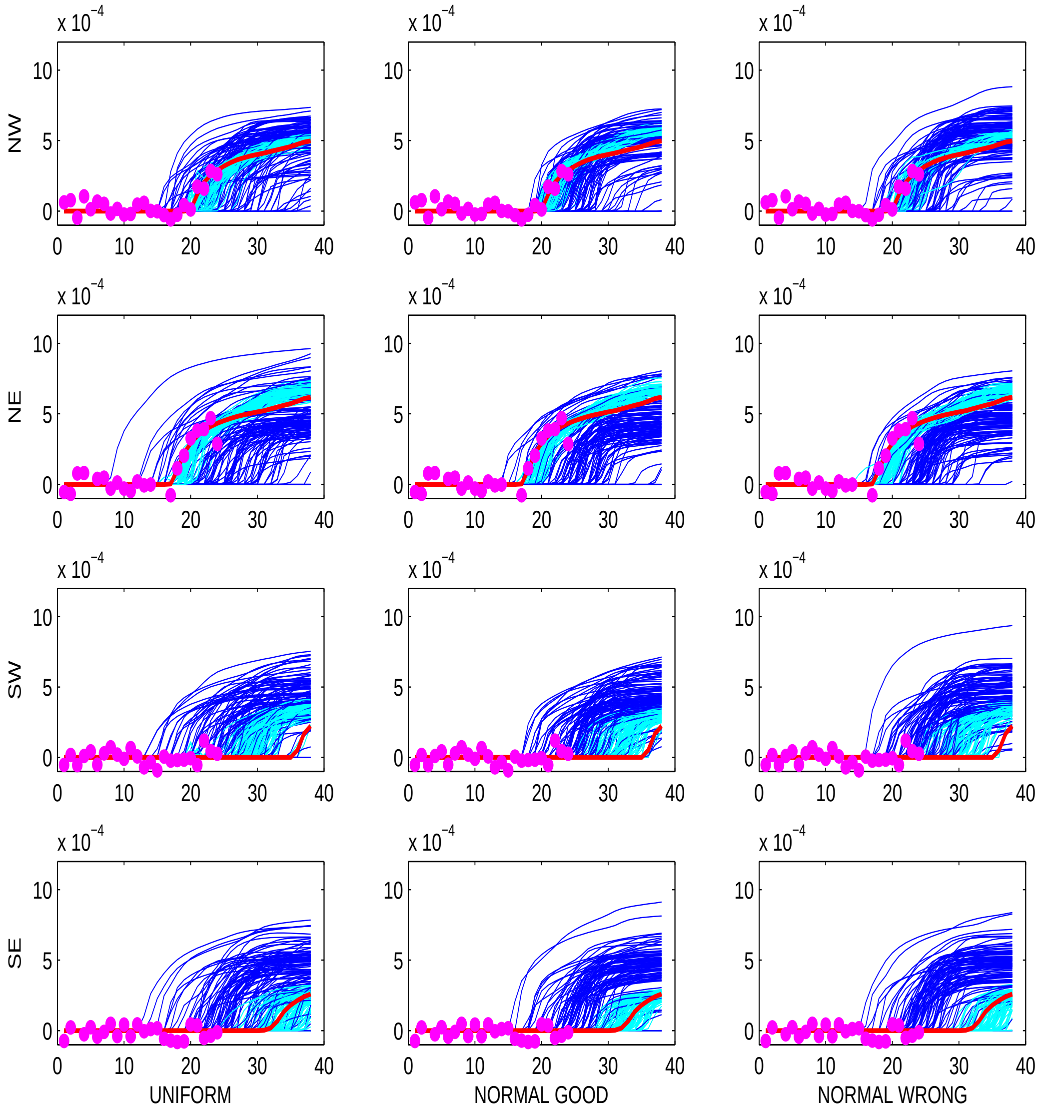

Figure 27: Water rates forecasts with estimated n_o and n_w. Dark blue: initial ensemble; light blue: final ensembles; red: truth; pink: measurements.

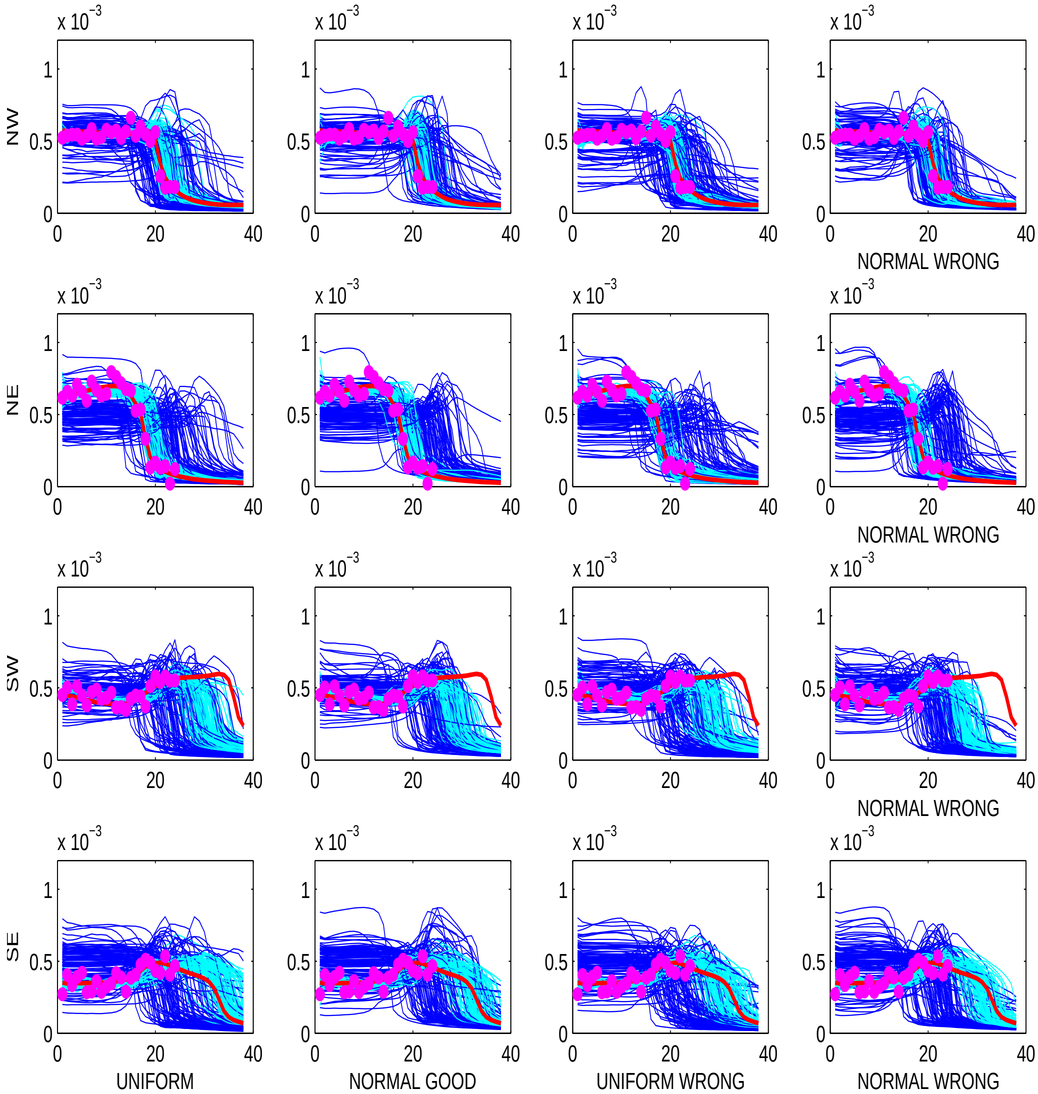

Figure 28: Oil rates forecasts with estimated n_o, n_w, k^0_{ro} and k^0_{rw}. Dark blue: initial ensemble; light blue: final ensembles; red: truth; pink: measurements.

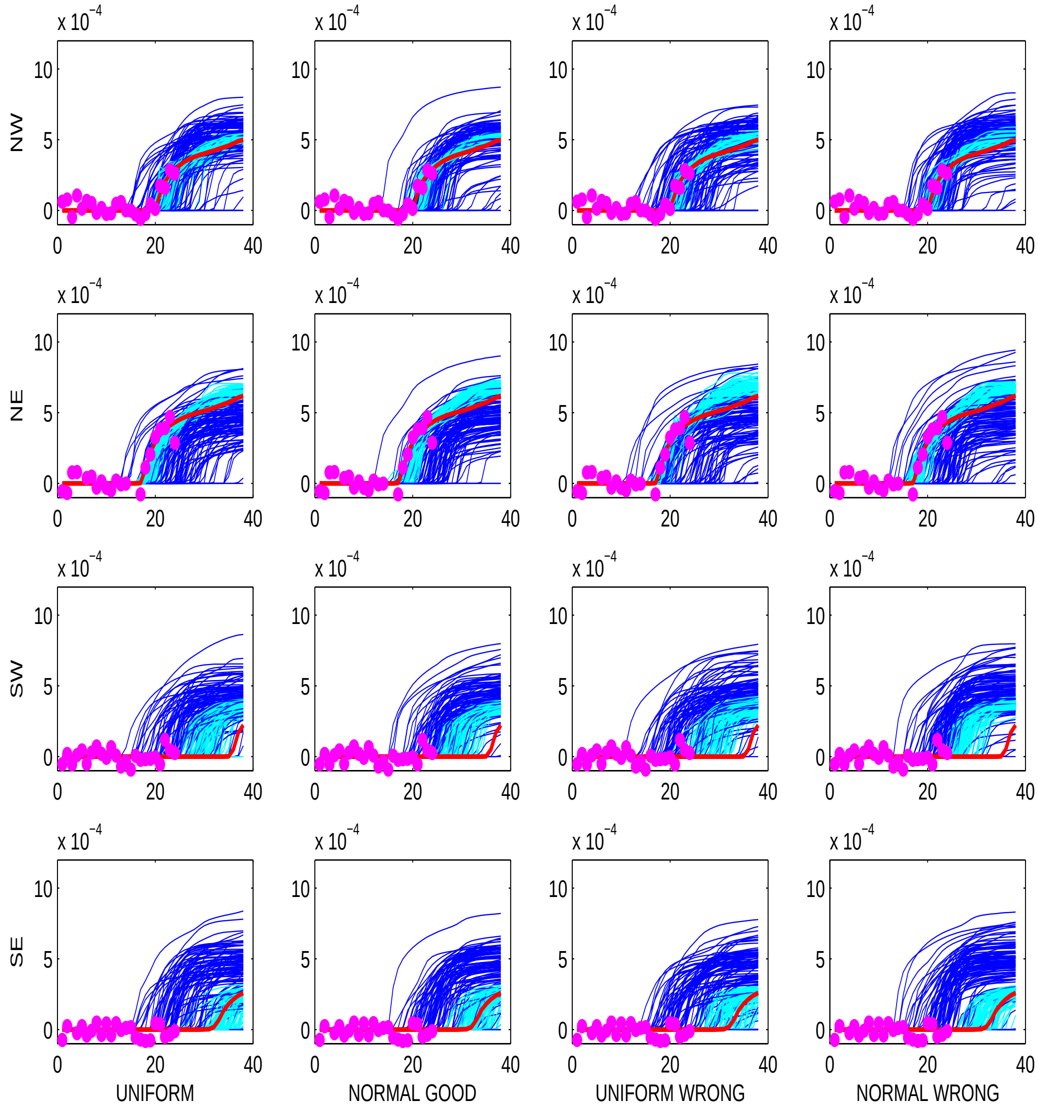

Figure 29: Water rates forecasts with estimated n_o, n_w, k^0_{ro} and k^0_{rw}. Dark blue: initial ensemble; light blue: final ensembles; red: truth; pink: measurements.

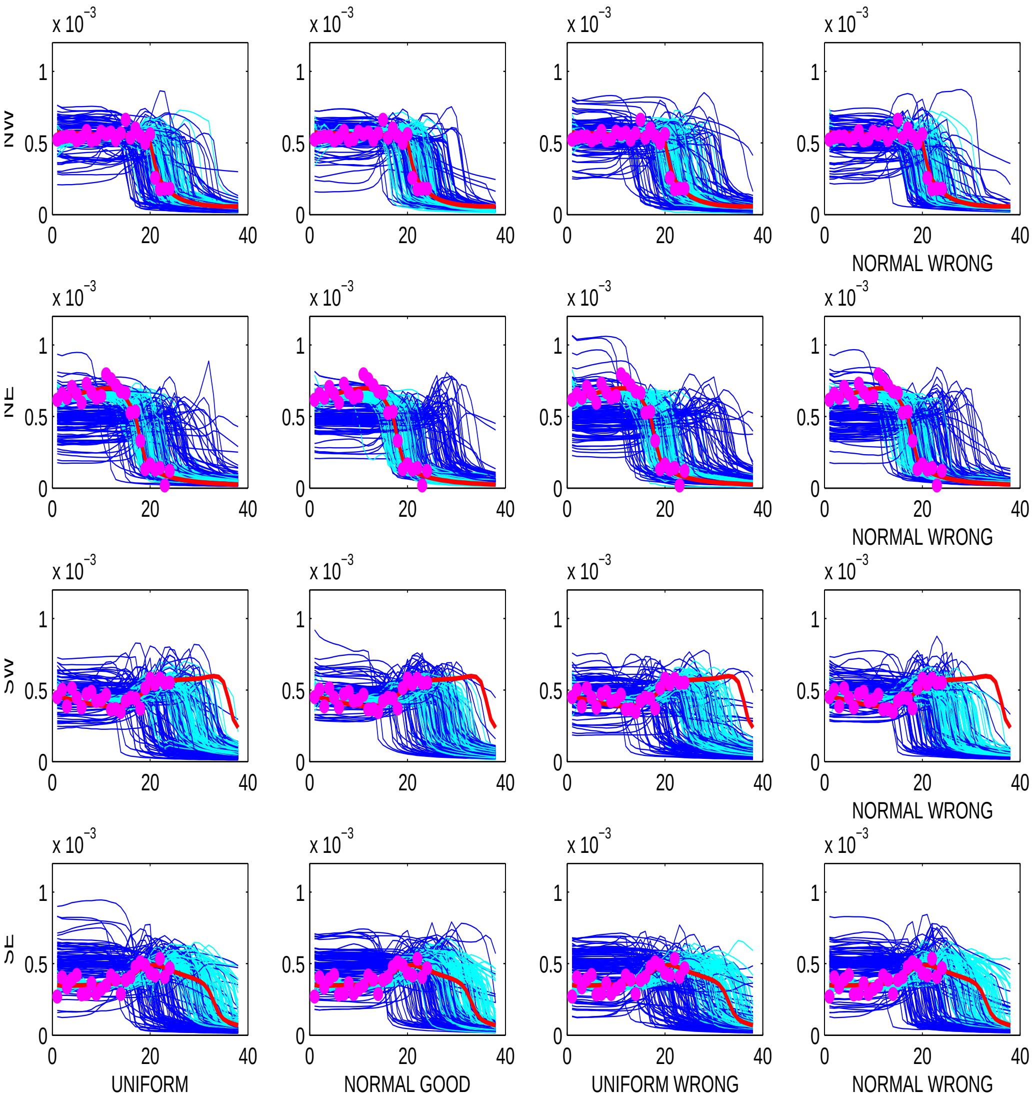

Figure 30: Oil rates forecasts with estimated n_o, n_w, k^0_{ro}, k^0_{rw}, S_{or} and S_{wc}. Dark blue: initial ensemble; light blue: final ensembles; red: truth; pink: measurements.

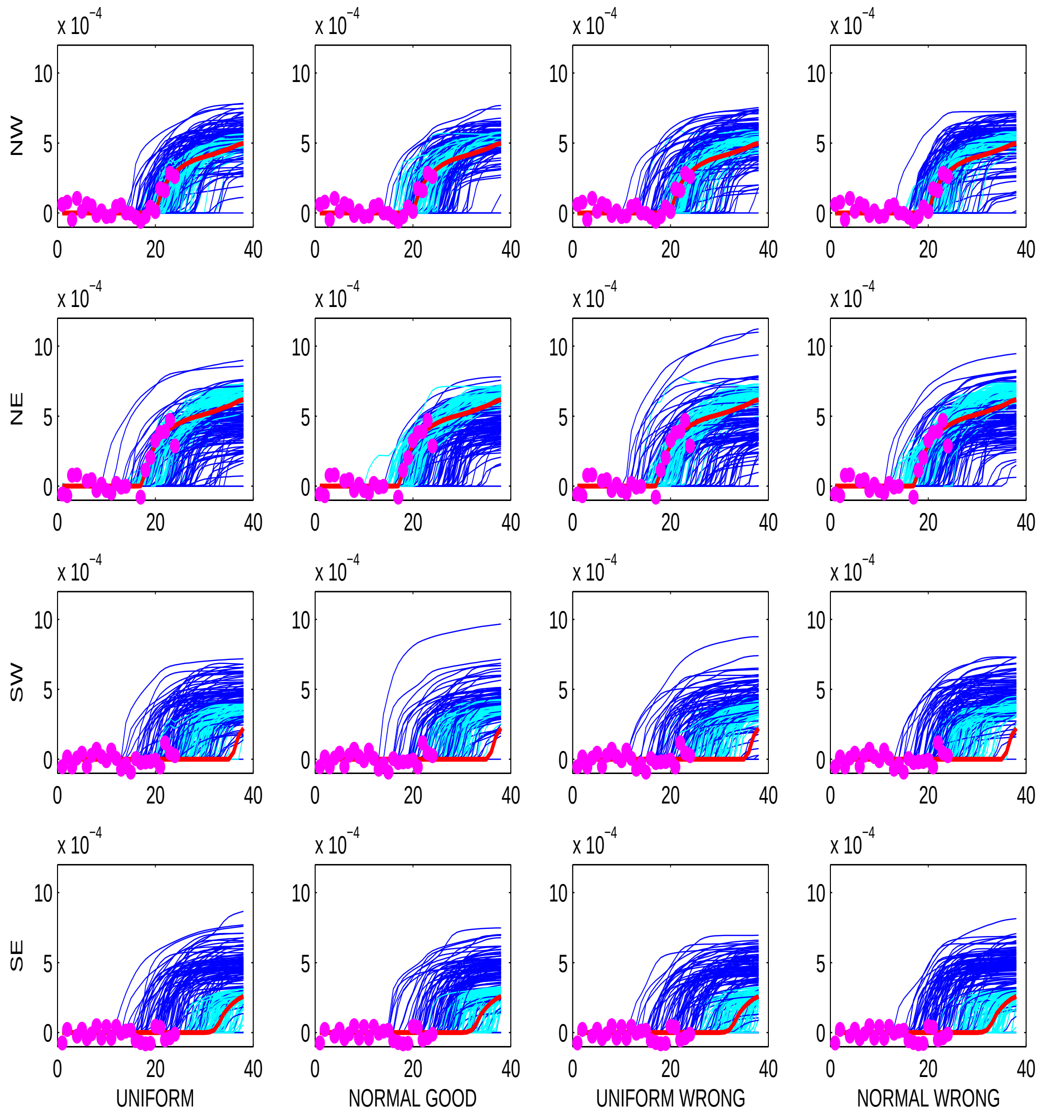

Figure 31: Water rates forecasts with estimated n_o, n_w, k^0_{ro}, k^0_{rw}, S_{or} and S_{wc}. Dark blue: initial ensemble; light blue: final ensembles; red: truth; pink: measurements.

Appendix 5 - RMSE plots for the estimated absolute permeability

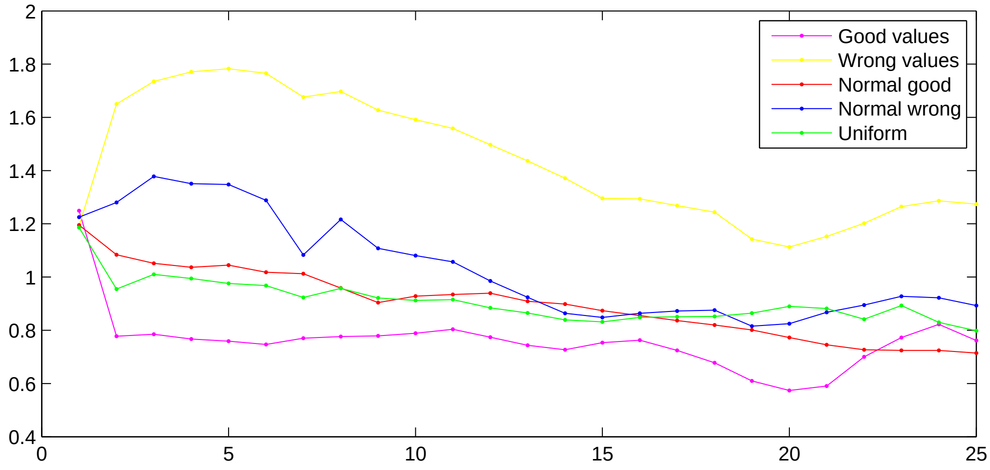

Figure 32: RMSE plots for the experiments where absolute permeability is estimated simultaneously with n_o and n_w.

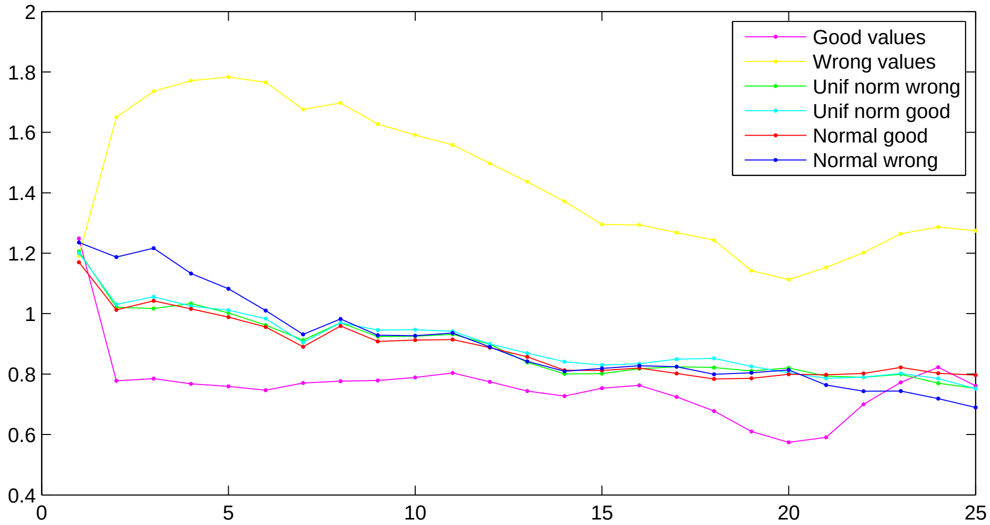

Figure 33: RMSE plots for the experiments where absolute permeability is estimated simultaneously with n_o, n_w, k^0_{ro} and k^0_{rw}.

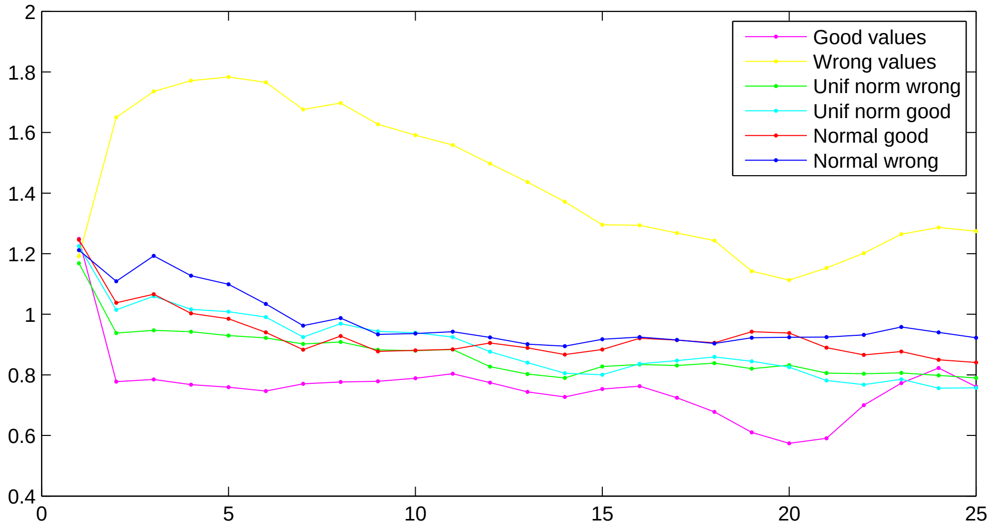

Figure 34: RMSE plots for the experiments where absolute permeability is estimated simultaneously with n_o, n_w, k_{ro}^0, k_{rw}^0, S_{or} and S_{wc}.