Technische Universiteit Delft

Project:

The Application of Continuous and Discrete Bayesian Belief Nets to Model the Use of Wake Vortex Prediction and Detection Systems

Master's Thesis
Risk and Environmental Modeling
Delft Institute of Applied Mathematics

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August 2006

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1. INTRODUCTION

1.1 Scope

In Europe, wake vortex prediction and detection systems are being introduced in order to increase airport capacity, while maintaining safety. The EC project ATC-Wake aims to develop a ground based system for ATC (Air Traffic Control) that would allow variable aircraft separation distances, as opposed to the fixed distances presently applied at airports. The EC project I-Wake aims to develop an on-board system for pilots in order to minimize the probability of a wake encounter. As part of these projects, quantitative safety assessments were performed. So far, the focus of these safety studies was the assessment of the risk related to the wake encounter itself.

However, for a quantitative safety assessment of the use of wake vortex prediction and detection systems, the following issues should also be considered:

- Humans working with these systems have to react on alerts, so as to ensure that a pilot will be able to initiate a wake vortex avoidance maneuver in time.

- If one or more of the system components provide a wrong or erroneous advice, there will be a higher risk on the presence of (severe) wake vortices. The consequences might be catastrophic, in case reduced separation is applied.

Therefore, there is a need to understand more clearly what the impact of hazards, human errors, and system failures is on the incident/accident risk related to the use of wake vortex prediction and detection systems such as ATC-Wake and also I-Wake.

1.2 Objectives

General Objective:

We aim to analyze the use of two new wake vortex prediction and detection systems. One ground based systems (ATC-Wake) and one on-board system (I-Wake), both used independently. We intend to apply a rich variety of mathematical models and methodologies based on continuous Bayesian Belief Nets (BBNs), discrete BBNs, and Fault Trees. It is our objective to provide insight into hazards and system failures related to the use of wake vortex prediction and detection systems. In this context, it is foreseen that the models will be used for the setting of requirements for these systems.

Particular Objectives:

This aim is to provide insights into the hazards and system failures related to the use of wake vortex prediction and detection systems. Three models, developed by NLR, will be analyzed in detail to support of the setting of requirements. These models are:

- A stochastic model, based on use of continuous Bayesian Belief Nets, for the initial aircraft separation time between aircraft taking off at a single runway;

- A Fault Tree model for the assessment of the impact of ground based system failures on a wake vortex detection, warning, and avoidance manoeuvre;

- A discrete Bayesian Belief Net for the assessment of the impact of an on-board system failure on a wake vortex avoidance manoeuvre.

The main ideas behind the use of Bayesian Belief Nets and the data requirements for the use of these three models will be investigated. Several questionnaires to elicit required data for the use of these models from operational experts will be proposed.

1.3 Modelling Approach

Bayesian belief nets (BBNs) become recently very popular models to represent high dimensional uncertainty distributions. A BBN is an acyclic directed graph in which nodes represent random variables and the arcs ‘influences’ between variables. We will use BBNs to analyse and evaluate the hazards and system failures related to the use of wake vortex prediction and detection systems. The proposed approach includes:

1. A graph and decision theory based model structure representing the stochastic initial aircraft separation time at the start of roll during ATC-Wake single runway departures is constructed. It enables the analysis of the safety aspects of the ATC-Wake system and operational concept in a causal way. The modelling approach will be based on the use of Continuous Bayesian Belief Nets.

2. A causal model for the assessment of the ATC Wake Detection, Warning and Avoidance Maneuver is introduced. This BBN will be applied to evaluate the risk of system failure for the departure operation. We will use two approaches to quantify such a model: Discrete Bayesian Belief Nets and Fault Trees.

3. A causal model for the assessment of the I-Wake Detection, Warning and Avoidance Maneuver probability is introduced. A Discrete Bayesian Belief Net BBN will be applied to evaluate the risk of system failure for the arrival operation.

In the model for the aircraft separation time the random variables are continuous. We will follow the copula – vine approach presented in [Kurowicka D., Cooke R.M. 2004]. It allows nodes having continuous invertible distribution functions. The influences are specified as (conditional) rank correlations, which are realized by copula that represents (conditional) independence as a zero (conditional) correlation. Updating such non-parametric BBN requires re-sampling the whole structure, which is very time consuming. To overcome this problem the structure is sampled once and used as data to quantify discretized (each node 10 states) version of continuous structure. This way the reduced assessment burden and modeling flexibility of the continuous BBNs is combined with the fast updating algorithms of discrete BBNs.

A different approach will be followed for the second model, representing a ground based wake vortex detection, warning and avoidance maneuver. From the problem statement, we will initially constrain the variables to take two values or two states. Moreover, we say that the failure of one the parent-nodes occurs if any one of its components fails. We shall compare two approaches that may be applied to model the hazard and system failures: the use of a Fault Tree and a discrete BBN respectively.

A third approach will be followed for the third model, representing an on-board wake vortex detection, warning and avoidance maneuver. Because of the statement of the problem, we will initially use a discrete BBN, since we will be able to constrain the variables to take two states. The use of discrete BBNs as most appropriate approach which may be applied to model the failure of aircraft/pilot performing a detection, warning, and avoidance maneuver will be motivated on basis of assumptions made.

1.4 Outline of the Thesis

This thesis describes the results of a graduation project, carried out as part of the Master of Science Programme in Applied Mathematics at Delft University of Technology. It describes and discusses a case study in which Discrete and Continuous BBNs are applied for analysis of wake vortex prediction and detection systems.

Chapter Two deals with ground based prediction and detection systems and its use during departures from single runways. The application of continuous BBNs, discrete BBNs, and fault trees to different parts of the ATC-Wake operation is presented and discussed. Chapter Three deals with the use of an on-board wake vortex detection, warning and avoidance system. Here, discrete BBN are used to support the setting of requirements for the different system components. Readers interested in the mathematical foundation of the applied methods may refer to Chapter Four and the references of this document for more information. Finally, conclusions and recommendations are given in Chapter Five. The Appendices contain the questionnaires for the elicitation of the required data from operational experts.

2. ATC-WAKE MODELS: CONTINUOUS AND DISCRETE BAYESIAN BELIEF NETS

For a description of the *current practice* Single Runway Departure (SRD) operation, as well as the SRD operation *as proposed in the ATC-Wake concept* refer to [Speijker et al. 2005b, 2005c, 2006b].

2.1 ATC-Wake: The Aircraft Separation Time Model

A mathematical model and techniques are introduced to incorporate the role of humans working with ATC-Wake. Thus, the so called aircraft separation time model is presented in this Section (see Figure 2.1 in Section 2.1.2). We should point out that a previous version of this model was developed by NLR, which was slightly adapted [Speijker et al 2005a]. We propose the use of a continuous Bayesian Belief Net to describe the relationships between the model components.

From the qualitative safety analysis of the ATC-Wake operation [Scholte et al. 2004, Speijker et al. 2005b], the following two factors were considered there the most relevant for the construction of a causal model:

- Aircraft separation time (at take off);
- Wake detection, warning, and avoidance manoeuvre.

It is assumed that these two stochastic phenomena are independent. Therefore, for each a Bayesian Belief Network (BBN) will be constructed. The continuous BBN for the Aircraft Separation Time is described in this Section. The discrete BBN for the Wake Detection, Warning, and Avoidance Maneuver is presented in Section 2.2.

For a quantitative assessment of the wake vortex induced risk related to the ATC-Wake operation with reduced separation, there are three main issues to consider:

- The controller working with the ATC-Wake system has to instruct the pilot to initiate a wake vortex avoidance manoeuvre, in case an ATC-Wake warning/alert is raised.
- If one or more ATC-WAKE system components provide(s) wrong or erroneous advice, there will be a higher risk on the presence of (severe) wake vortices. The consequences might be CATASTROPHIC, because reduced separation is applied.
- The separation distance/time will vary along the flight track, and will usually not be exactly the same as the separation minima advised by the Separation Mode Planner.

2.1.1 Brief Introduction to the Application of continuous BBNs for The Aircraft Separation Time Model

Bayesian belief nets (BBNs) become recently very popular models to represent high dimensional uncertainty distributions. BBN is an acyclic directed graph in which nodes represent random variables and the arcs 'influences' between variables[1]. We have used BBNs to build the Aircraft Take Off Separation Time model. Random variables in this model are continuous but not necessarily normally distributed. Hence the known normal BBNs [Cowell, R.G. *et al* 1999] cannot be applied here.

We could discretize our continuous variables and transform our problem to a discrete BBN. However, if we decide to choose not too crude discretization (more than 2 states) the assessment burden that we would have to deal with would be too cumbersome. If only two states for variables are used, the results will not be very precise.

The new approach to continuous BBNs using vines [Bedford T.J., Cooke R.M. 2002] and copula that represents (conditional) independence as a zero (conditional) correlation was introduced in [Kurowicka D., Cooke R.M. 2004]. It allows nodes having continuous invertible distribution functions. Hence this approach is not restricted to any parametric form (as normal BBNs). The influences are specified as rank correlations and conditional rank correlations. This approach allows traceable and defensible quantification methods but it comes at a price: these BBNs must be evaluated using Monte Carlo simulation.

We follow the copula – vine approach presented in [Kurowicka D., Cooke R.M. 2004] for the Aircraft Take Off Separation Time model. All marginal distributions and (conditional) rank correlations are specified. The BBN has to be sampled. We present a comprehensive description of the application of this methodology to the Aircraft Take Off Separation Time model.

Updating such non-parametric BBN requires re-sampling the whole structure. This is not as elegant as updating discrete BBNs and is very time consuming. To overcome this problem the structure is sampled once and used as data to quantify discretized (each node 10 states) version of continuous structure. This way the reduced assessment burden and modeling flexibility of the continuous BBNs is combined with the fast updating algorithms of discrete BBNs [Hanea A., Kurowicka D., Cooke R.M. 2005].

2.1.2 Description of the Model and Data

The causal model for the aircraft separation time is presented in Figure 2.1. The non-parametric continuous BBN for such a causal model and the explanation of every node are shown. Here, we will follow the 'copula – vine' approach to continuous BBNs [Kurowicka D., Cooke R.M. 2004] and associate nodes with continuous invertible

[1] A wider overview about BBNs and related mathematical definitions is presented in Chapter 4.

distributions, influences with (conditional) rank correlations. In order to quantify such a BBN using the 'copula – vine' approach, we need to specify all one-dimensional marginal distributions and the (conditional) rank correlations.

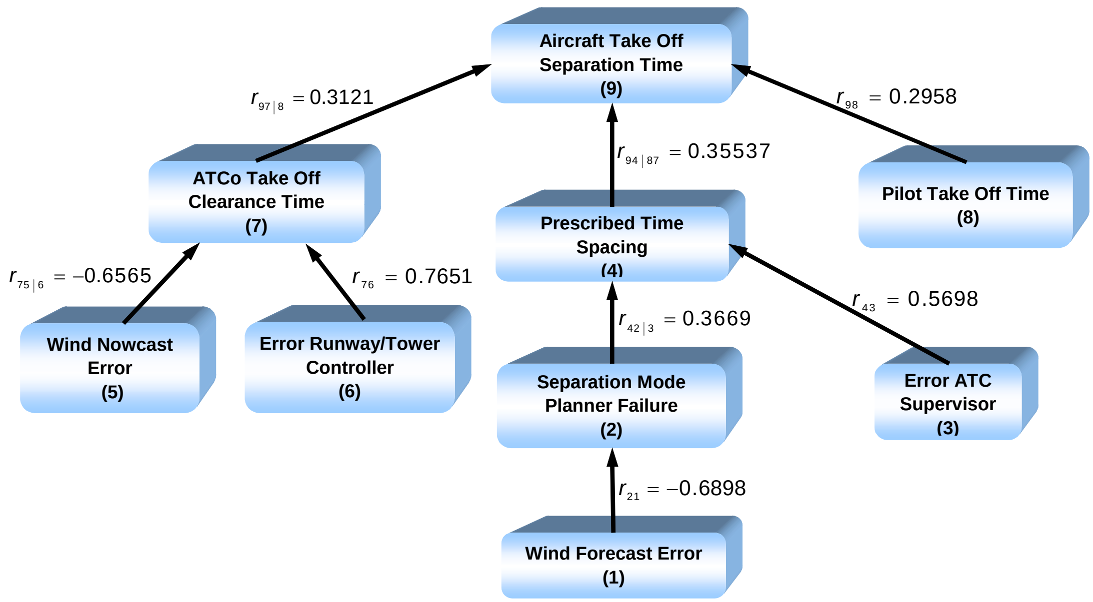


Figure 2.1: BBN for the aircraft separation time model

The explanation of the nodes in the BBN in Figure 2.1 is as follows:

- ***Aircraft Take Off Separation Time (9):*** Time difference between start of roll of the leader and the follower aircraft.
- ***ATCo Take Off Clearance Time (7):*** Time difference between start of roll of the leader and take off clearance of the ATCo for the follower aircraft.
- ***Pilot Take Off Time (8):*** Time difference between take off clearance of the ATCo and the start of roll of the aircraft.
- ***Prescribed Time Spacing (4):*** Separation Time Prescribed by the ATC supervisor for a departing leader and follower aircraft combination (in ATC-Wake Mode).
- ***Separation Mode Planner Failure (2):*** Time difference between output of the Separation Mode Planner (i.e. Separation Time Advise) and the separation time that should be advised.
- ***Wind Forecast Error (1):*** Meteo system wind profile forecast error at reference height (10 m altitude).
- ***Wind Nowcast Error (5):*** Meteo system wind profile nowcast error at reference height (10 m altitude).
- ***Error Runway/Tower controller (6):*** Time difference between Separation Time prescribed by the ATC Supervisor and Take Off Clearance Time.
- ***Error ATC Supervisor (3):*** Time difference between Separation Time prescribed by

the ATC Supervisor and the separation time that should be advised.

Throughout, when we talk about the univariate continuous random variables we denote them by the upper X_i's. Realizations of these random variables will be denoted by the lower cases x_i's where i refer to their corresponding number in the BBN. How to get the distributions of all nodes is described in Section 2.1.3. Since the influences are associated with (conditional) rank correlations (which does not depend on marginal distribution functions) then one may separate information about dependence and marginal distributions. We can easily transform variables to uniforms on $(0,1)$. This can be done as follows:

If F_i denotes the cumulative distribution function of the *i*-th node hence the variables X_i then $U_i = F_i(X_i)$ is a variable uniform on $(0,1)$.

After specification of the dependence structure on transformed to uniforms variables, they will be transformed back to their original distributions by applying inverse transformation $X_i = F_i^{-1}(U_i)$.

The protocol to assign (conditional) rank correlations to the arcs of the BBN; as well as, the procedure of sampling the structure for transformed variables are shown in the Section 2.1.4.

We point out that the probability distributions for the nodes in the BBN and (conditional) rank correlations are elicited using expert opinion [theory and methodology regarding structured expert judgment is found in Cooke R.M. 1991, Cooke R.M., Goossens L.H.J. 1999]. Initial data used to obtain insight were obtained through the application of questionnaires (See questionnaires and provided data in Appendix A.1).

2.1.3 Expert Distributions

Some marginal distributions for the Aircraft Take Off Separation Time Model have to be obtained from experts. In this context, we briefly present the Expert Judgment methodology which can be used to accomplish this task. An important step of the classical model [Cooke R.M. 1991] is the combination of all experts' assessments into one combined uncertainty assessment on each query variable. Here, we present the combination scheme named Equal weight decision maker, which gives each expert equal weights. Another scheme based on performance of experts on questions has successfully been applied in several studies [See Cooke, 1991; Cooke R.M., Slijkhuis K.A.; Cooke R.M., Goossens L.H.J., 1999; L.H.J. Goossens, R.M. Cooke 1996]. For practical reasons, we suggest the use of the equal weight combination scheme.

First of all, experts are asked to assess their uncertainty distribution via specification of a 5%, 25%, 50%, 75%, and 95%-iles for unknown values for each variable of interest. To build experts distribution we find minimum information distribution with respect to background measure satisfying expert's quantiles. The following procedure is used. Let $q_i(e)$ be the *i*% quantile of expert *e*. We assume that this minimum information distribution is restricted to a bounded interval. The intrinsic range or bounds for the variable distributions is obtained using the 10% overshoot rule: The smallest interval containing all assessments for a given item is overshot above and below. The expert's information scores are affected by the choice of the overshot; making this overshoot very large tends to suppress differences in the experts' information scores, however the effect is very low. First we find the lowest and the highest values named,

$$l = \min\{q_5(1), \ldots, q_5(6)\}, \; h = \max\{q_{95}(1), \ldots, q_{95}(6)\}$$

Then we set

$$q_l(e) = l - 0.1 \times [h - l],$$

and similarly,

$$q_h(e) = h - 0.1 \times [h - l],$$

The *intrinsic range* is thus $[q_l(e), q_h(e)]$. The distribution of expert *e* is then approximated by linearly interpolating the quantile information $(q_l(e), 0)$, $(q_5(e), 0.05)$, $(q_{25}(e), 0.25)$, $(q_{50}(e), 0.5)$, $(q_{75}(e), 0.75)$, $(q_{95}, 0.95)$, and $(q_h(e), 1)$. This is the distribution with minimum information (with respect to distribution on the intrinsic range) that satisfies the expert's quantiles [Cooke R.M., 1991]. The above procedure gives us a distribution function $F_{j,i}$ for expert *j* on variable *i*. We specify equal weight $\left(\frac{1}{e}\right)$ to each distribution and the combined distribution function is now $\sum_{j \cdots e}\left(\frac{1}{e}\right)F_{j,i}$, see [Cooke R.M., 2001; Bedford T.J., Cooke R.M. 2003].

The Expert Judgment methodology presented above can be used to fit distributions on quantiles given by experts (the questionnaire is formulated in Appendix A1.). Although, here initial data used to obtain insight are obtained by following another approach. In order to get marginal distributions, we use means and standard deviations of the marginal distributions. Hence, probability distributions for the nodes in the BBN have been estimated. Those considered adequate for the marginal distributions required are presented in Appendix A1[2].

In this respect, four variables were assumed to have the following gamma distributions: $X_4 \sim \Gamma\left(x_4 \mid 81, 1.1111\right)$, $X_7 \sim \Gamma\left(x_7 \mid 323.9741, 0.2778\right)$, $X_8 \sim \Gamma\left(x_8 \mid 9.0001, 3.3333\right)$, $X_9 \sim \Gamma\left(x_9 \mid 64, 1.8750\right)$. On the other hand, we assume that the variables having normal distributions are: $X_1 \sim N(0, 2.89)$, $X_2 \sim N(0, 100)$, $X_3 \sim N(0, 100)$, $X_5 \sim N(0, 0.0625)$, $X_6 \sim N(0, 25)$ (see Appendix A.1, ATC-Wake D3_5b and D3_6b for the details).

2.1.4 'Copula – Vine' Approach to Continuous BBN for The Aircraft Separation Time Model

We use the protocol presented in [Kurowicka D., Cooke R.M. 2004] to specify (conditional) correlations to be required from experts in the continuous BBN for the aircraft separation time. As we already said these correlations are assigned to the directed arcs of the BBN.

First we choose the sampling order 1, 2, 3, 4, 5, 6, 7, 8, 9 for the BBN structure, such that the ancestors of a node appear before that node in the ordering. This order is not unique; we could have chosen a different sampling order. Observe Figure 2.1, the node "Prescribed spacing", numbered 4 has as ancestors the nodes "Error ATC Supervisor", "Separation Mode Planner Failure", and "Wind Prediction"; thereby, they were placed in the ordering before node 4 as nodes 3, 2 and 1, respectively.

We write the complete factorization and underscore the nodes which do not have a direct "influence" with the conditioned variable, i.e., which are not its parents, and hence are not necessary in sampling it. This factorization is

$$\begin{aligned} P(1,2,3,4,5,6,7,8,9) = \; & P(1)\; P(2 \mid 1)\; P(3 \mid \underline{21})\; P(4 \mid 32\underline{1})\; P(5 \mid \underline{4321})\; P(6 \mid \underline{54321}) \\ & P(7 \mid 65\underline{4321})\; P(8 \mid \underline{7654321})\; P(9 \mid 874\underline{65321}) \qquad (1) \end{aligned}$$

If we drop the underscored variables, we obtain the standard factorization for the BBN given as follows [Pearl J. 1988, Jensen F.V. 1996]:

$$P(X_1, X_2, \ldots, X_9) = \prod_{i=1}^{9} P\left(X_i \mid pa(X_i)\right) \qquad (2)$$

where $pa(X_i)$ denotes the parents of variable X_i.

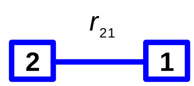
[2] These probability distributions were fitted by using the *disttool* of the Statistics Demo in Matlab.

To sample a distribution specified by a continuous BBN we use the sampling procedure for the D-vine [Kurowicka D., Cooke R.M. 2006]. For each part of the factorization we build a D-vine on K variables denoted by $\mathcal{D}^K = D(K, \mathcal{C}^K, \mathcal{I}^K)$. The ordering of the variables is very important. We start with the variable K; then the dependent variables, $\mathcal{C}^K$; and, at the end the independent variables, $\mathcal{I}^K$.

a) Let us start with the first term of the factorization, $P(1)$. Since variable X_1 neither has dependent variables, nor independent ones, $\mathcal{C}^1 = \mathcal{I}^1 = \phi$. Then, the D-vine for X_1 is trivial, we denote it by $\mathcal{D}^1 = D(1)$. To sample X_1, we can just sample a uniform random variable,

$$x_1 = u_1 . \tag{3}$$

b) Second part of the factorization gets a bit more complicated. We take $P(2|1)$.



$$\mathcal{C}^2 = \{1\}, \quad \mathcal{I}^2 = \phi \quad \Rightarrow \quad r_{21}$$

Figure 2.2: $\mathcal{D}^2$ for the BBN for the aircraft separation time with 9 variables

In Figure 2.2, we can see the D-vine $\mathcal{D}^2$ and sets of independent and dependent variables for X_2. There are no underscored variables, hence $\mathcal{I}^2 = \phi$. The set of dependent variables $\mathcal{C}^2$ consists of the variable X_1, so the ordering of $\mathcal{D}^2$ is as in Figure 2.2. To specify dependence between X_1 and X_2, it is required to assign a rank correlation r_{12} to the edge between X_1 and X_2 in $\mathcal{D}^2$ and equivalently to the corresponding arc in the BBN in Figure 2.1. The graphical representation of the sampling procedure is shown in Figure 2.3:

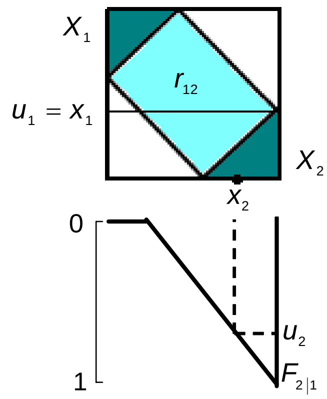


Figure 2.3: Graphical representation of sampling value of x_2 in D^2

We acquire a value of variable X_2, say x_2 in D^2. The horizontal axis represents the random variable X_2, and its parent X_1 is placed on the vertical axis. The diagonal band copula[3] [Cooke R.M., Waij R. 1986] realizes the correlation r_{12} between these random variables. Value $X_1 = x_1$ is known from the first term of the factorization, this allows us to calculate the conditional distribution of X_2 given variable $X_1 = x_1$, denoted by $F_{2|1}$. If we sample value of the independent uniform variable $U_2 = u_2$ and invert it with respect to $F_{2|1}$ then we get the desired value x_2. So, the sampled value of variable X_2 is obtained as

$$x_2 \quad = \quad F^{-1}_{2|1:x_1}(u_2). \tag{4}$$

Third part of the factorization can be now considered.

c) $P(3|\underline{21})$

[3] This copula will be used in the text only to visualize the sampling procedure, since it can be easily drawn. Although, for applications we will use Frank's copula [Frank M.J. 1979] as it does not add much information to the product of margins, enjoys the zero independence property and has a close form of conditional and inverse conditional distributions. For further details and mathematical background see Chapter 4.

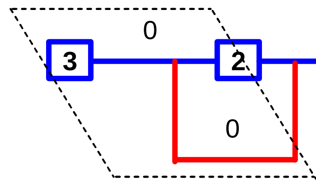


Figure 2.4: $\mathcal{D}^3$ for the BBN for the aircraft separation time with 9 variables

For the third part of the factorization K=3, and variables X_1 and X_2 are underscored, that is, X_1 and X_2 are independent of X_3. $\mathcal{C}^3 = \phi$ and $\mathcal{I}^3 = \{2,1\}$. Hence, the order of the variables is $\mathcal{D}^3 = D(3, 2, 1)$. Variables X_1 and X_2 were already sampled so we are now interested only in information about variable X_3, hence the information in the left-most part of the vine (stood out area in Figure 2.4). Both r_{32} , $r_{31|2}$ are equal to zero because X_3 is independent of X_1 and X_2.

Therefore, to sample random variable X_3 we just sample the value of the independent uniform variable U_3, say u_3

$$x_3 \quad = \quad u_3. \tag{5}$$

We turn to the fourth part of the factorization.

d) $P(4|32\underline{1})$

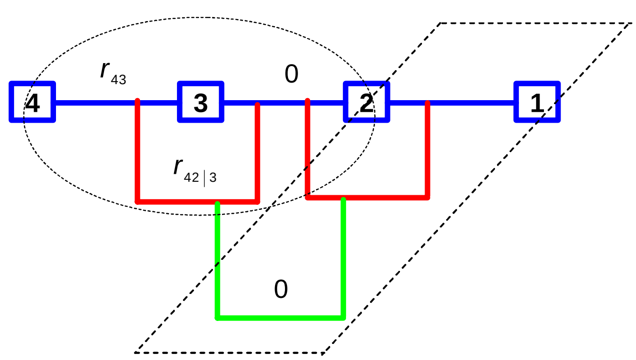


Figure 2.5: $\mathcal{D}^4$ for the BBN for the aircraft separation time with 9 variables

For the fourth term of the factorization $K = 4$; the set of dependent variables consists of variables X_2 and X_3, hence $\mathcal{C}^4 = \{3,2\}$; and, variable X_1 is underscored $\mathcal{I}^4 = \{1\}$, i.e., variable X_1 is independent of variable X_4 given X_2 and X_3. We have $\mathcal{D}^4 = D$ (4, 3, 2, 1). Notice that the order of the variables stays the same as in $\mathcal{D}^3$. We are only interested in information about variable X_4 as variables X_1, X_2 and X_3 were already sampled. We have that $r_{41|32} = 0$, due to independence between variables X_1 and X_4 given X_2 and X_3. The correlations r_{43} and $r_{42|3}$ need to be specified[4].

The equality of the top correlation in $\mathcal{D}^4$, $r_{41|32}$ to zero, makes quantile functions $F_{1|32}$ and $F_{4|32}$ independent , hence we can reduce $\mathcal{D}^4$ to a vine on three variables, in this case D (4, 3, 2) (circled area in Figure 2.5). Every time when some of the highest order (conditional) correlations of the left-most part of the vine are equal to zero, the D-vine can be reduced in a similar way. This simplifies the sampling of variable X_4 that does not depend on value of the variable X_1. From previous factorizations we know that the rank correlation r_{32} is equal to zero. The sampling procedure for the variable X_4, say x_4 is shown in Figure 2.6.

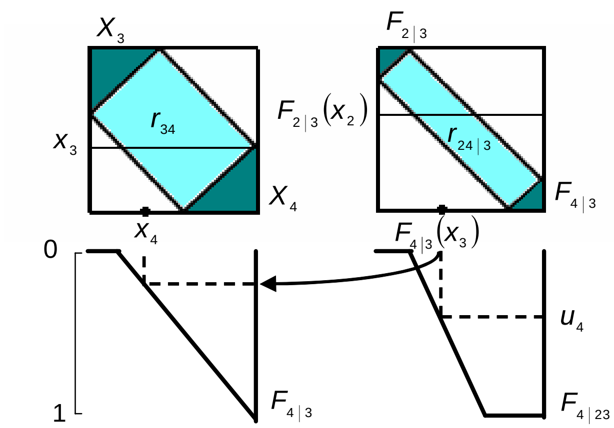


Figure 2.6: Graphical representation of sampling value of x_4 in $\mathcal{D}^4$

[4] Note that we can change the ordering in $\mathcal{D}^4$ to 4, 2, 3, 1, which allows us another possibility to specify conditional rank correlations, given as r_{42} and $r_{43|2}$. Hence, we have the following two possibilities to specify (conditional) rank correlations in $\mathcal{D}^4$.

$$K = 4,\ \mathcal{C}^4 = \{3, 2\},\ \mathcal{I}^4 = \{1\} \quad \Rightarrow \begin{Bmatrix} r_{43} \\ r_{42|3} \end{Bmatrix} \text{ or } \begin{Bmatrix} r_{42} \\ r_{43|2} \end{Bmatrix}$$

Since X_2 and X_3 were already sampled then values of $X_3 = x_3$ and $F_{2|3}(x_2)$ are known. We conditionalize copulas with correlations r_{43} and $r_{42|3}$ on value of $X_3 = x_3$ and $F_{2|3}(x_2)$, respectively. We calculate conditional cumulative distribution functions $F_{4|3}$ and $F_{4|23}$ (see Figure 2.6). We sample the value of the independent uniform variable U_4, say u_4 invert it with respect to $F_{4|23}$ and get value of the quantile $F_{4|3}$ which leads to x_4. Hence, x_4 is sampled as follows:

$$x_4 = F^{-1}_{4|3:x_3}\left(F^{-1}_{4|23:x_2}(u_4)\right). \tag{6}$$

Now, we consider the fifth term of Equation 1.

e) $P(5|\underline{4321})$

In this term, we have $K = 5$, the set of dependent variables is empty ($\mathcal{C}^{5} = \phi$) and the rest of the variables are underscored $\mathcal{I}^{5} = \{4, 3, 2, 1\}$, that is, variable X_5 is independent of X_1, X_2, X_3, X_4. We can then use the following ordering for $\mathcal{D}^5 = D$ (5, 4, 3, 2, 1), which after incorporating all zero correlations in the left most part of the vine simplifies to D (5). We are not required to specify any (conditional) rank correlation. Value x_5 of X_5 in $\mathcal{D}^5$ is found by simply sampling the value of the independent uniform random variable $U_5 = u_5$

$$x_5 = u_5. \tag{7}$$

Similarly, we can get value x_6 for the sixth term of the factorization.

f) $P(6|\underline{54321})$

We have K =6, $\mathcal{C}^6 = \phi$ and $\mathcal{I}^6 = \{5, 4, 3, 2, 1\}$, that is variable X_6 is independent of X_1, X_2, X_3, X_4, X_5. Then, the ordering of $\mathcal{D}^6$ is the following $\mathcal{D}^6 = D$ (6, 5, 4, 3, 2, 1), which simplifies to D (6). Hence

$$x_6 = u_6. \tag{8}$$

We present the seventh term of the factorization.

g) $P\left(7\,|\,65\underline{4321}\right)$

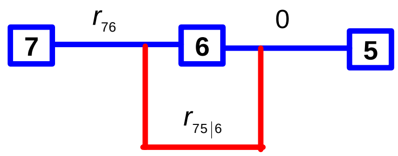


Figure 2.7: $\mathcal{D}^7$ for the BBN for the aircraft separation time with 9 variables

This part of the factorization has K=7, the set of dependent variables consist of two variables X_5 and X_6 then $\mathcal{C}^{\,7} = \{6, 5\}$ and there are four underscored variables $\mathcal{I}^{\,7} = \{4, 3, 2, 1\}$. Hence, $\mathcal{D}^7 = D$ (7, 6, 5, 4, 3, 2, 1), the order of the variables stays the same (7, 6, 5, 4, 3, 2, 1) as for the previous vines. So far, we have sampled variables X_1, X_2, X_3, X_4, X_5 and X_6, so we only need to incorporate the information about variable X_7 given in the left-most part of $\mathcal{D}^7$. Notice that, we have reduced $\mathcal{D}^7$ as we did for $\mathcal{D}^4$ to D (7, 6, 5). We must assign rank correlation r_{76} to the edge that connects variables X_7 and X_6 in $\mathcal{D}^7$ and equivalently to the corresponding arc in the BBN in Figure 2.1. We must also incorporate information about the conditional dependence of variables X_5 and X_7 given variable X_6 in form of conditional correlation $r_{75|6}$ [5], hence $r_{75|6}$ is assigned to the arc between X_7 and X_5 in the BBN in Figure 2.1. From previous factorizations we find that r_{65} is equal to zero.

Now the sampling procedure can be represented graphically as

[5] As we mentioned for $\mathcal{D}^4$, variables in $\mathcal{D}^7$ can be given in the different order (7, 5, 6), if it is the case r_{75} and $r_{76|5}$ are being needed. Hence, we have the following possibilities to specify (conditional) rank correlations in $\mathcal{D}^7$:

$$\mathcal{C}^{\,7} = \{6, 5\},\ \mathcal{I}^{\,7} = \{4, 3, 2, 1\} \Rightarrow \begin{Bmatrix} r_{76} \\ r_{75|6} \end{Bmatrix} \text{ or } \begin{Bmatrix} r_{75} \\ r_{76|5} \end{Bmatrix}$$

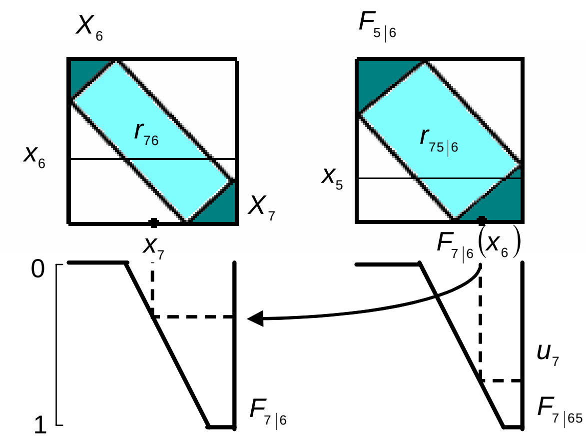


Figure 2.8: Graphical representation of sampling value of x_7 in D (7,6,5).

Figure 2.8 shows the sampling value of x_7 in D (7, 6, 5). It can be obtained in a way analogous to obtaining value x_4 . We get

$$x_7 \quad = \quad F^{-1}_{7|6:x_6}\left(F^{-1}_{7|65:x_5}(u_7)\right). \tag{9}$$

Now, we shall explain the case of the eighth part of the factorization.

h) $P(8|\underline{7654321})$

In this term, K = 8, the set of dependent variables is empty, $\mathcal{C}^{8} = \phi$ and $\mathcal{I}^{8} = \{7, 6, 5, 4, 3, 2, 1\}$, that is variable X_8 is independent of X_1, X_2, X_3, X_4, X_5, X_6 and X_7. Hence, we use the following ordering for $\mathcal{D}^8 = D$ (8, 7, 6, 5, 4, 3, 2, 1) which reduces to D (8). The sampling value of x_8 is obtained by just sampling the independent uniform variable U_8, say u_8

$$x_8 \quad = \quad u_8 . \tag{10}$$

Finally, the ninth part of the factorization is shown.

i) $P(9|874\underline{65321})$

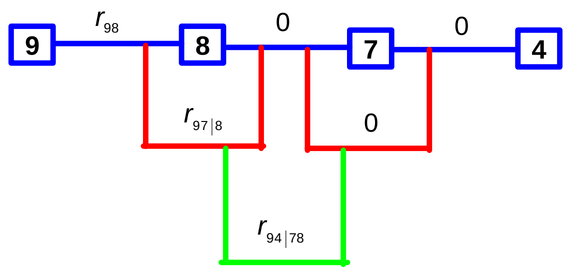


Figure 2.9: $\mathcal{D}^9$ for the BBN for the aircraft separation time with 9 variables

We can see in this term of the factorization that K =9, the set of dependent variables has three variables, $\mathcal{C}^9 = \{8, 7, 4\}$ and the underscored variables are $\mathcal{I}^9 = \{6, 5, 3, 2, 1\}$. Hence, the ordering of the variables is given as $\mathcal{D}^9 = D$ (9, 8, 7, 4, 6, 5, 3, 2, 1). Finally, following the same procedure as above, $\mathcal{D}^9$ is reduced to a sub-vine on four variables, namely, D (9, 8, 7, 4). We are only interested in the information about variable X_9. We can assign a rank correlation r_{98} to the edge of $\mathcal{D}^9$ and equivalently to the arc between variables X_8 and X_9 in BBN in Figure 2.1. We also need to incorporate the information about two conditional dependences $r_{97|8}$ and $r_{94|87}$ (we know values of variables X_7 and X_8 from $\mathcal{D}^7$ and $\mathcal{D}^8$, respectively, see Equations 9 and 10)[6].

Figure 2.10 shows the sampling procedure to realize (conditional) correlations in $\mathcal{D}^9$.

[6] As we said before, if we can change the order of the parents; we may have several possibilities to specify conditional rank correlations, namely,

$$\mathcal{C}^9 = \{8, 7, 4\}, \mathcal{I}^9 = \{6, 5, 3, 2, 1\} \Rightarrow \begin{Bmatrix} r_{98} \\ r_{97|8} \\ r_{94|87} \end{Bmatrix}, \begin{Bmatrix} r_{98} \\ r_{94|8} \\ r_{97|84} \end{Bmatrix}, \begin{Bmatrix} r_{97} \\ r_{98|7} \\ r_{94|78} \end{Bmatrix}, \begin{Bmatrix} r_{97} \\ r_{94|7} \\ r_{98|74} \end{Bmatrix}, \begin{Bmatrix} r_{94} \\ r_{98|4} \\ r_{97|48} \end{Bmatrix} \text{ or } \begin{Bmatrix} r_{94} \\ r_{97|4} \\ r_{98|47} \end{Bmatrix}$$

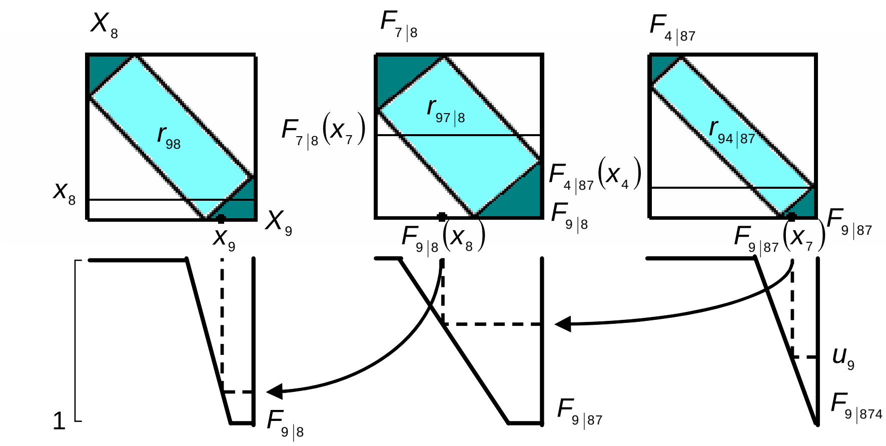


Figure 2.10: Graphical representation of sampling value of x_9 in D^9

Since X_4, X_7 and X_8 were already sampled then values of $X_8 = x_8$, $F_{7|8}(x_7)$ and $F_{4|87}(x_4)$ are known. We conditionalize copulas with correlations r_{98}, $r_{97|8}$ and $r_{94|87}$ on the values of $X_8 = x_8$, $F_{7|8}(x_7)$, and $F_{4|87}(x_4)$, respectively. We calculate conditional cumulative distribution functions $F_{9|8}$, $F_{9|87}$ and $F_{9|874}$ (see Figure 2.10). We sample the value of the independent uniform variable U_9, say u_9 invert it with respect to $F_{9|874}$ and get value of the quantile $F_{9|87}$ which is used to get quantile $F_{9|8}$, which leads to x_9 .

The sampling procedure of x_9 in D^9 yields,

$$x_9 \quad = \quad F^{-1}_{9|8:x_8}\left(F^{-1}_{9|7,\,8:x_7}\left(F^{-1}_{9|4,\,7,\,8:x_4}(u_9)\right)\right). \tag{11}$$

We conclude that the following rank correlations must be specified:

$$\{r_{21}, r_{43}, r_{42|3}, r_{76}, r_{75|6}, r_{98}, r_{97|8}, r_{94|87}\} \tag{12}$$

We have specified eight (conditional) correlations for the BBN structure shown in Figure 2.1 the same as the number of arcs in this BBN. Conditional independence properties of the BBN were used to simplify the sampling procedure in D-vines.

In principle, it is not necessary to draw D-vines to see which (conditional) correlations are necessary for calculations. One can follow the algorithm presented below:

- Find sampling ordering. An ordering such that all ancestors of node i appear before i in the ordering. A sampling ordering begins with a source node and ends with a sink node.

- Index the nodes according to the sampling order 1, …, *n*.
- Factorize the joint in the standard way (Equation 2) following the sampling order.
- Underscore those nodes in each condition, which are not parents of the conditioned variable and thus are not necessary in sampling it.
  The underscored nodes could be omitted thereby yielding the familiar factorization of the BBN as a product of conditional probabilities, with each node conditionalized on its parents (for source nodes the set of parents is empty).
- For each term i with parents (non-underscored variables) $i_1 \ldots i_{p(i)}$, associate the arc $i_{p(i)-k} \rightarrow i$ with the conditional rank correlation

$$r\left(i, i_{p(i)}\right); k = 0$$

$$r\left(i, i_{p(i)-k} \,\middle|\, i_{p(i)}, \ldots, i_{p(i)-k+1}\right);\ 1 \le k \le p(i) - 1 \tag{13}$$

  where the assignment is vacuous if $\{ i_1 \ldots i_{p(i)} \} = \phi$. Assigning conditional rank correlations for i = 1, …, n, every arc in the BBN is assigned a conditional rank correlation between parent and child.

In Section 2.1.5, the procedure of how the values of the required conditional and unconditional rank correlations are obtained is presented. There, an expert assessed excedence probabilities over original variables. We show in details how from these elicited excedence probabilities the (conditional) rank correlations become known.

2.1.5 Procedure to obtain the Values of the Required Conditional and Unconditional Rank Correlations

Assuming copula a unique joint distribution could be determined and sampled based on the previous protocol. The (conditional) rank correlations associated with each edge are determined. And these can be realized by the copula. For that, we do not only require one-dimensional marginal distributions, but also to quantify the uncertainty of the conditional dependencies of the BBN.

Thus far, we have obtained marginal distributions associated with the nodes of the continuous BBN for the aircraft separation time and we know which (conditional) rank correlations –influences in the BBN– are required. These (conditional) rank correlations could be non constant. This would complicate their realization and elicitation. In this way, it is convenient to work with constant conditional rank correlations. We consider the joint normal copulae where (conditional) rank correlations are constant and an appropriate close-functional form of the density function can be implemented in Matlab.

It is proven to be difficult for experts to assess (conditional) rank correlations directly [Kraan B. 2002]. Thereby, excedence probabilities are to be elicited. From the answers to the elicitation format shown in Appendix A.2, we will obtain the (conditional) rank correlations as follows.

We will start by describing the procedure to obtain the rank correlation r_{21} between variables X_1 and X_2. An expert assessed the $P\left(X_2 \geq x_{2_{50}} \middle| X_1 \geq x_{1_{50}}\right)$, i.e. probability that X_2 is bigger than or equal to its median given that X_1 is bigger than or equal to its median. An example of an appropriate question to elicit this probability is as follows [see questionnaire about Conditional and Unconditional Rank Correlations in Appendix A.2]:

Consider the relationship between the following two variables:

1. Suppose that the Wind Prediction was observed to be above its median value. What is your probability that the Separation Mode Planner Failure would also lie above its median value?
Probability [0, 1] : 0.25

This can be shortened as

X_1 : Wind Prediction **[m/sec]**	X_2 **: Separation Mode Planner Failure** [sec]
Suppose: $X_1 \geq x_{1_{50}}$ **; what is** $P\left(X_2 \geq x_{2_{50}} \middle| X_1 \geq x_{1_{50}}\right)$**?**	

Hence, expert has specified $P\left(F_{X_2}(x_2) \geq 1/2 \middle| F_{X_1}(x_1) \geq 1/2\right)$. We can also transform $F_{X_i}(x_i)$ to standard normal variables by applying the following $Y_i = \phi^{-1}\left(F_{X_i}(x_i)\right)$ i = 1, 2 and treat this problem as finding correlation ρ_{21} of joint normal distribution for which the conditional probability is $P(Y_2 \geq 0 | Y_1 \geq 0)$.

We require the rank correlation r_{21}. The standard bivariate normal distribution of transformed to standard normal random variables Wind Prediction and Separation Mode Planner Failure $-Y_1$, Y_2- has a density function which depends on their product moment correlation ρ_{21}. We first find ρ_{21} and then using Pearson transformation [Pearson K. 1907] we obtain corresponding value of r_{21} (see below). Density function for joint normal distribution is:

$$f(y_2, y_1) = f(\rho_{21}) = \frac{1}{2\pi\sqrt{1-\rho_{21}^2}} \exp\left(-\frac{\left(y_2^2 - 2\,\rho_{21}\,y_2\,y_1 + y_1^2\right)}{2\left(1-\rho_{21}^2\right)}\right) \qquad (14)$$

where ρ_{21} is a parameter between -1 and 1, $(Y_1, Y_2) \sim N\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}1 & \rho_{21}\\ \rho_{21} & 1\end{bmatrix}\right)$. The conditional probability

$$P(Y_2 \geq 0 \mid Y_1 \geq 0) = \frac{P(Y_2 \geq 0 \cap Y_1 \geq 0)}{P(Y_1 \geq 0)} = \frac{P(Y_2 \geq 0 \cap Y_1 \geq 0)}{1/2} = 2P(Y_2 \geq 0 \cap Y_1 \geq 0)$$

$$= 2\int_0^\infty\int_0^\infty f(y_2, y_1)\,dy_2\,dy_1 = \int_0^\infty\int_0^\infty \frac{1}{\pi\sqrt{1-\rho_{21}^2}} \exp\left[-\frac{\left(y_2^2 - 2\rho_{21}y_2y_1 + y_1^2\right)}{2\left(1-\rho_{21}^2\right)}\right] dy_2 dy_1. \qquad (15)$$

The following figure shows how the above conditional probability changes depending on the value of ρ_{21}. It was obtained by numerical integration in Matlab[7].

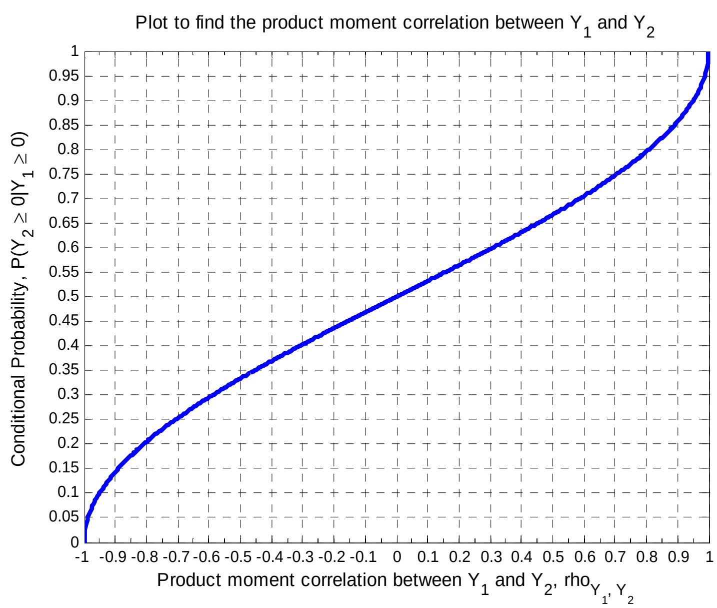


Figure 2.11: Conditional Probability $P(Y_2 \geq 0 \mid Y_1 \geq 0)$ *versus* ρ_{21}

[7] Two functions which depend on Y_1, Y_2 and ρ_{21} are created. The first function evaluates the integrand for the bivariate joint normal distribution, which is expressed by a double integral in Equation (15). This integrand accepts a vector Y_1 and a scalar Y_2 and returns a vector of values of the integrand. Then, an additional function uses a Matlab function named dblquad which numerically evaluates the double integral taking as integrand the previous created function. For that, we also need to specify the limits of integration over which the double integration and a tolerance are required. Being a time consuming task to evaluate the double integration from 0 to infinity in Matlab, the limits of integration were chosen from 0 to 5. It is pointed out that these limits of integration and a tolerance of 1×10^{-6} give a very good approximation of the results.

In Figure 2.11, the horizontal and vertical axis represent the product moment correlation between Y_1 and Y_2 and the conditional probability calculated with Equation 15, respectively. Hence, when we know value of $P(Y_2 \geq 0 | Y_1 \geq 0)$ the corresponding value of ρ_{21} can be read from this figure. Notice that if $P(Y_2 \geq 0 | Y_1 \geq 0) = 0$ then $\rho_{21} = -1$, if $P(Y_2 \geq 0 | Y_1 \geq 0) = 1$ then $\rho_{21} = 1$, and, if $P(Y_2 \geq 0 | Y_1 \geq 0) = 0.5$ then $\rho_{21} = 0$. The relationship shown in Figure 2.11 can be used to recover all unconditional rank correlations from BBN in Figure 2.1.

An expert stated that the value of $P\left(X_2 \geq x_{2_{50}} \middle| X_1 \geq x_{1_{50}}\right)$=0.25. Then from Figure 2.11 we read $\rho_{21} = -0.70672$. Hence, the required rank correlation with Pearson's transformation [Pearson K. 1907] is as follows:

$$r_{21} = \frac{6}{\pi} \arcsin\left(\frac{\rho_{21}}{2}\right) = -0.6898. \tag{16}$$

Now, we consider three variables: Separation Mode Planner Failure, Error ATC Supervisor, and Prescribed Spacing, X_2, X_3, X_4 respectively. We require the values of rank correlations r_{43} and $r_{42|3}$. We first find r_{43} by considering two variables Error ATC Supervisor and Prescribed Spacing. In this case, our expert assessed the $P\left(X_4 \geq x_{4_{50}} \middle| X_3 \geq x_{3_{50}}\right)$=0.7, then ρ_{43}=0.58786 (see Figure 2.11) and $r_{43} = 0.5698$.

To find $r_{42|3}$ we must consider Separation Mode Planner Failure, Error ATC Supervisor, and Prescribed Spacing that gives us after transformation to normals three-dimensional distribution of random vector (Y_2, Y_3, Y_4). Variables Y_2, Y_3 are independent. Using vector notation $\underline{y_{234}} = (y_2, y_3, y_4) \in R^3$, we can write the trivariate joint density function as

$$f\left(\underline{y_{234}}\right) = \frac{1}{\sqrt{8\pi^3 |V_{234}|}} \exp\left[-\frac{1}{2}\left(\underline{y_{234}}\right) V_{234}^{-1} \left(\underline{y_{234}}\right)'\right] \tag{17}$$

where $V_{234} = \begin{bmatrix} 1 & 0 & \rho_{42} \\ 0 & 1 & \rho_{43} \\ \rho_{42} & \rho_{43} & 1 \end{bmatrix}$ is a covariance matrix, with determinant $|V_{234}| = 1 - \rho_{42}^2 - \rho_{43}^2$. The value of ρ_{43}=0.58786 was already assessed. We must now find ρ_{42}. Expert is asked the following question $P\left(X_4 \geq x_{4_{50}} \middle| X_3 \geq x_{3_{50}}, X_2 \geq x_{2_{50}}\right)$. After transformation to normals we get that the $P(Y_4 \geq 0 | Y_3 \geq 0, Y_2 \geq 0)$ is provided.

Relationship between ρ_{42} and probability obtained from experts is

$$P(Y_4 \geq 0 \,|\, Y_3 \geq 0, Y_2 \geq 0) = 4\int_0^\infty\int_0^\infty\int_0^\infty \frac{1}{\sqrt{8\pi^3 |V_{234}|}} \exp\left[-\frac{1}{2}\left(\underline{y_{234}}\right) V_{234}^{-1} \left(\underline{y_{234}}\right)'\right] dy_2 dy_3 dy_4 . \qquad (18)$$

Knowing ρ_{42}, ρ_{43} and ρ_{32}, now we can calculate partial correlation $\rho_{42;3}$

$$\rho_{42;3} = \frac{\rho_{42} - \rho_{32} \cdot \rho_{43}}{\sqrt{1-\rho_{43}^2}\sqrt{1-\rho_{32}^2}} . \qquad (19)$$

For joint normal distribution partial and conditional correlations are equal, hence $\rho_{42;3} = \rho_{42|3}$ and with Pearson transformation we can obtain $r_{42|3}$. Figure 2.12 shows the relationship between $\rho_{42;3}$ and $P(Y_4 \geq 0 \,|\, Y_3 \geq 0, Y_2 \geq 0)$ for ρ_{43}=0.58786 corresponding to $P\left(X_4 \geq x_{4_{50}} \,\middle|\, X_3 \geq x_{3_{50}}\right)$=0.7. Notice that possible values of $P(Y_4 \geq 0 \,|\, Y_3 \geq 0, Y_2 \geq 0)$ are in the interval [0.40438, 0.99555]. This fact is explained in the following way. The information about $P\left(X_4 \geq x_{4_{50}} \,\middle|\, X_3 \geq x_{3_{50}}\right)$ tells us how much variability of X_4 is explained by X_3. The $P\left(X_4 \geq x_{4_{50}} \,\middle|\, X_3 \geq x_{3_{50}}, X_2 \geq x_{2_{50}}\right)$ gives how much more variability of X_4 can be explained by adding information about variable X_2 being bigger than its median. If one already explained significant portion of X_4 then it is not much left to explain by X_2. Notice that if $P\left(X_4 \geq x_{4_{50}} \,\middle|\, X_3 \geq x_{3_{50}}\right)$ is equal to 0.5 which gives that X_3 and X_4 are independent (information about X_3 does not constrain X_4), then the possible values for $P\left(X_4 \geq x_{4_{50}} \,\middle|\, X_3 \geq x_{3_{50}}, X_2 \geq x_{2_{50}}\right)$ are the whole interval $[0, 1]$.

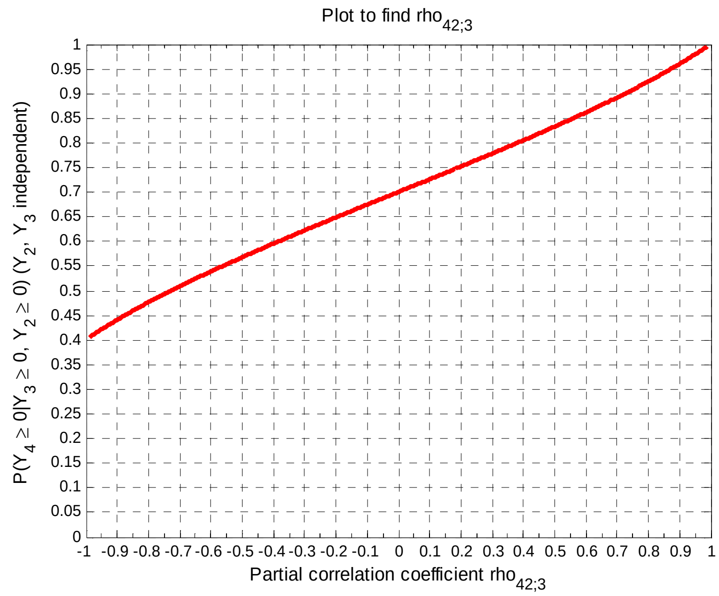


Figure 2.12: Conditional Probability $P(Y_4 \geq 0 \mid Y_3 \geq 0, Y_2 \geq 0)$ *versus* $\rho_{42|3} = \rho_{42;3}$, Y_2 and Y_3 independent

In this case, expert gave $P\left(X_4 \geq x_{4_{50}} \mid X_3 \geq x_{3_{50}}, X_2 \geq x_{2_{50}}\right) = 0.8$ then $\rho_{42|3} = \rho_{42;3}$ would be equal to 0.38184. This leads to $r_{42|3}$ as follows:

$$r_{42|3} = \frac{6}{\pi}\arcsin\left(\frac{\rho_{42|3}}{2}\right) = 0.3669 \tag{20}$$

Similarly, we can get r_{76} and $r_{75|6}$. Expert assessed the $P\left(X_7 \geq x_{7_{50}} \mid X_6 \geq x_{6_{50}}\right)$=0.8, then ρ_{76}=0.80874 (see Figure 2.11) and hence $r_{76} = 0.7951$.

When variables Wind Error, Error Runway/Tower Controller and Aircraft Traffic Controller Take Off Clearance Time are considered we get the following results. The relationship between $\rho_{75;6}$ and $P(Y_7 \geq 0 \mid Y_6 \geq 0, Y_5 \geq 0)$ for ρ_{76}=0.80874 corresponding to $P\left(X_7 \geq x_{7_{50}} \mid X_6 \geq x_{6_{50}}\right)$=0.8 is shown in Figure 2.13. Observe that possible values of $P(Y_7 \geq 0 \mid Y_6 \geq 0, Y_5 \geq 0)$ are in the interval $[0.60159, 0.99719]$. Here, the expert stated $P\left(X_7 \geq x_{7_{50}} \mid X_6 \geq x_{6_{50}}, X_5 \geq x_{5_{50}}\right)$=0.7 then $\rho_{75|6} = \rho_{75;6}$ would be equal to -0.67403. Thereby, the conditional correlation $r_{75|6}$ equals to -0.6565.

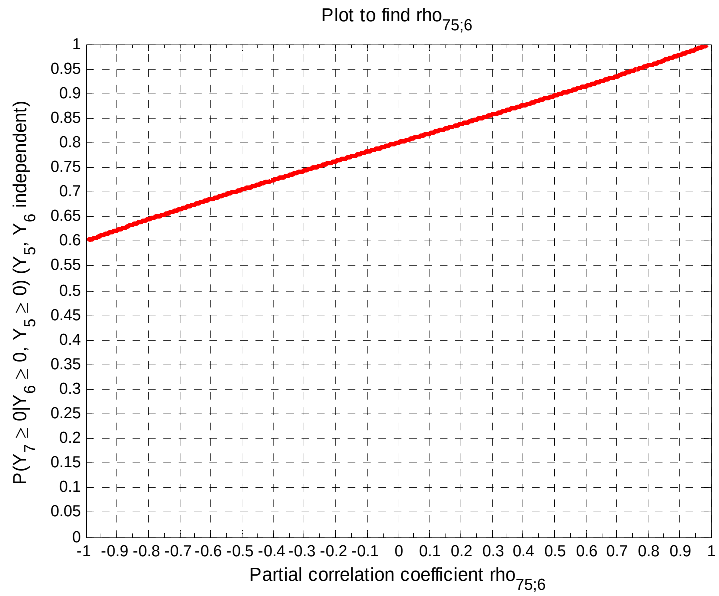


Figure 2.13: Conditional Probability $P(Y_7 \geq 0 \,|\, Y_6 \geq 0, Y_5 \geq 0)$ *versus* $\rho_{75|6} = \rho_{75;6}$, Y_5 and Y_6 independent

Now, the last correlations to be required are r_{98}, $r_{97|8}$, $r_{94|78}$. We start by computing r_{98} corresponding to variables Pilot Take Off Time and Aircraft Take Off Separation Time. Since $P\left(X_9 \geq x_{9_{50}} \,\middle|\, X_8 \geq x_{8_{50}}\right)$=0.6 then ρ_{98}=0.30854 and r_{98}=0.2958.

On the other hand, Aircraft Traffic Controller Take Off Clearance Time, Pilot Take Off Time and Aircraft Take Off Separation Time: X_7, X_8 and X_9 are considered to calculate $r_{97|8}$. Our expert gave a value of $P\left(X_9 \geq x_{9_{50}} \,\middle|\, X_8 \geq x_{8_{50}}, X_7 \geq x_{7_{50}}\right)$=0.7, which is between the interval [0.20907, 0.99053]; then we will get the partial correlation $\rho_{97;8} = \rho_{97|8}$ =0.32538. From this partial correlation, we can get $r_{97|8}$=0.3121 (see Figure 2.14).

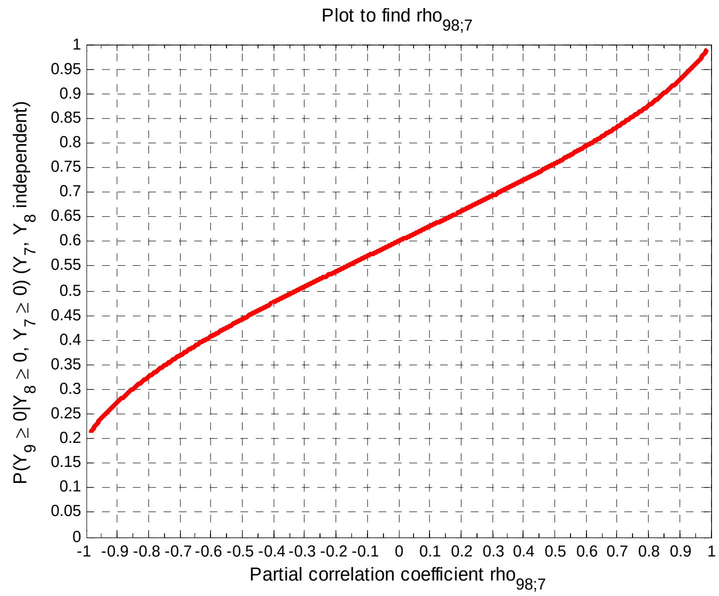


Figure 2.14: Conditional Probability $P(Y_9 \geq 0 \,|\, Y_8 \geq 0, Y_7 \geq 0)$ *versus* $\rho_{97|8} = \rho_{97;8}$, Y_7 and Y_8 independent

Now, to find $r_{94|78}$ we consider Prescribed Spacing, Aircraft Traffic Controller Take Off Clearance Time, Pilot Take Off Time and Aircraft Take Off Separation Time variables: X_4, X_7, X_8, X_9 that give us after transformation to normals four-dimensional distribution of random vector (Y_4, Y_7, Y_8, Y_9). Variables Y_4, Y_7 are independent, as well as Y_7, Y_8. Using vector notation $\underline{y_{4789}} = (y_4, y_7, y_8, y_9) \in R^4$, we can write the joint density function as

$$f(\underline{y_{4789}}) = \frac{1}{4\sqrt{\pi^4 |V_{4789}|}} \exp\left[-\frac{1}{2}(\underline{y_{4789}}) V_{4789}^{-1} (\underline{y_{4789}})'\right] \tag{21}$$

where the covariance matrix is given as $V_{4789} = \begin{bmatrix} 1 & 0 & 0 & \rho_{94} \\ 0 & 1 & 0 & \rho_{97} \\ 0 & 0 & 1 & \rho_{98} \\ \rho_{94} & \rho_{97} & \rho_{98} & 1 \end{bmatrix}$ with determinant $|V_{4789}| = 1 - \rho_{94}^2 - \rho_{97}^2 - \rho_{98}^2$. The values of ρ_{97}=0.30951 and ρ_{98}=0.30854 were already assessed. We must now find ρ_{94}. Expert is asked the following question $P\left(X_9 \geq x_{9_{50}} \,\middle|\, X_8 \geq x_{8_{50}}, X_7 \geq x_{7_{50}}, X_4 \geq x_{4_{50}}\right)$. After transformation to normals we get that $P(Y_9 \geq 0 \,|\, Y_8 \geq 0, Y_7 \geq 0, Y_4 \geq 0)$ is provided.

Relation between ρ_{94} and probability obtained from expert is

$$P\left(Y_9 \geq 0 \middle| Y_8 \geq 0, Y_7 \geq 0, Y_4 \geq 0\right)$$
$$= 2\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty \frac{1}{\sqrt{\pi^4 \left|V_{4789}\right|}} \exp\left[-\frac{1}{2}\left(\underline{y_{4789}}\right) V_{4789}^{-1} \left(\underline{y_{4789}}\right)'\right] dy_4 dy_7 dy_8 dy_9 \qquad (22)$$

Knowing ρ_{94} , ρ_{97} ρ_{98} and $\rho_{97\,;\,8} = 0.20454$, now we can calculate $\rho_{94\,;\,8}$ and hence $\rho_{94\,;\,87}$ and $r_{94|87}$.

Expert provided a value of $P\left(X_9 \geq x_{9_{50}} \middle| X_8 \geq x_{8_{50}}, X_7 \geq x_{7_{50}}, X_4 \geq x_{4_{50}}\right) = 0.8$, which is between the interval [0.40068, 0.99145]; then we will get the partial correlation,

$$\rho_{94\,;\,8} = \frac{\rho_{94} - \rho_{98} \cdot \rho_{84}}{\sqrt{1-\rho_{98}^2} \cdot \sqrt{1-\rho_{84}^2}} = 0.32447 \qquad (23)$$

which we need in order to compute

$$\rho_{94;87} = \frac{\rho_{94;8} - \rho_{97;8} \cdot \rho_{74;8}}{\sqrt{1-\rho_{97;8}^2} \cdot \sqrt{1-\rho_{74;8}^2}} = 0.37. \qquad (24)$$

For joint normal $\rho_{94;87} = \rho_{94\,|\,87}$. This value can be obtained from Figure 2.15. Hence, the value of $r_{94|87}$ is calculated as follows

$$r_{94|87} = \frac{6}{\pi} \arcsin\left(\frac{\rho_{94\,|\,87}}{2}\right) = 0.35537 \qquad (25)$$

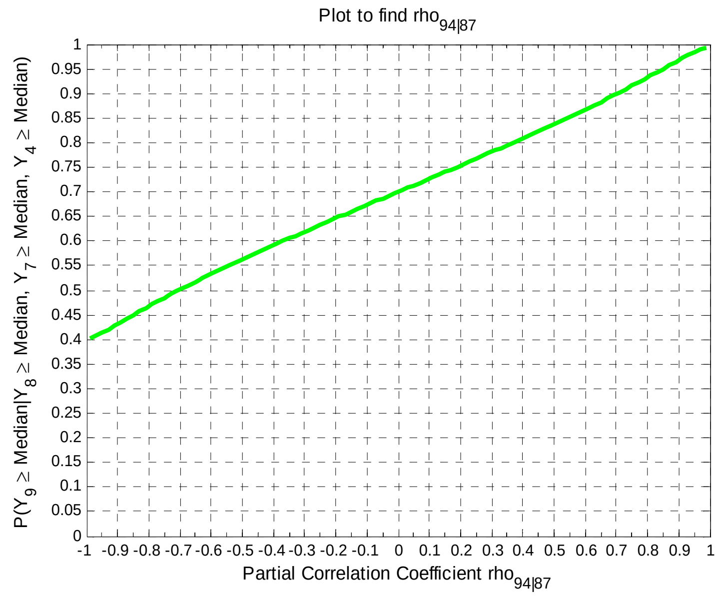


Figure 2.15: Conditional Probability $P(Y_9 \geq 0 \,|\, Y_8 \geq 0, Y_7 \geq 0, Y_4 \geq 0)$ *versus* $\rho_{94|87} = \rho_{94;87}$; Y_4, Y_7 and Y_7, Y_8 are independent

Therefore, all the values of the conditional and unconditional rank correlations required for calculations of the Aircraft Separation Time model were obtained. In Figure 2.1, (conditional) rank correlations obtained with the above procedure are assigned to each arc of the BBN. The rank correlation specification on a BBN plus copula determines the whole joint distribution [Kurowicka D., Cooke R.M 2004]. The following section analyzes updating the conditional probability of the Aircraft separation Time given some observations on certain variables.

2.1.6 Updating the BBN with Knowledge

In the previous section the (conditional) rank correlations required to sample the BBN structure of the Aircraft Separation Time model shown in Figure 2.1 were obtained. Continuous marginal distributions of each variable are derived as described in Section 2.1.3.

In this section, we aim to update our network given that some values of the variables become known.

If for instance, new policies are proposed to be implemented, updating the BBN structure allows us to evaluate the impact of such policies on our variable(s) of interest.

Updating can be performed in two different ways:

a. Updating with the Density Approach

If some of the variables become known, the results of sampling the aircraft separation time model conditional on these known values can be obtained by using the density approach [Kurowicka D, Cooke R.M. 2006; Hanea A., Kurowicka D, Cooke R.M. 2005].

The joint density for the aircraft separation time model is as follows [Kurowicka D, Cooke R.M. 2006; Hanea A., Kurowicka D, Cooke R.M. 2005]:

$$\begin{aligned} f(x_1,\ldots,x_9) = f(x_1)\cdot f(x_2)\cdot\ldots\cdot f(x_9) &= c_{12}\left(F_{X_1}(x_1), F_{X_2}(x_2)\right)c_{34}\left(F_{X_3}(x_3), F_{X_4}(x_4)\right) \\ &\cdot c_{42|3}\left(F_{X_2}(x_2), F_{4|3}\left(F_{X_4}(x_4)\right)\right)c_{67}\left(F_{X_6}(x_6), F_{X_7}(x_7)\right)c_{75|6}\left(F_{X_5}(x_5), F_{7|6}\left(F_{X_7}(x_7)\right)\right) \\ &\cdot c_{98}\left(F_{X_8}(x_8), F_{X_9}(x_9)\right)c_{97|8}\left(F_{9|8}\left(F_{X_9}(x_9)\right), F_{X_7}(x_7)\right)c_{94|87}\left(F_{9|87}\left(F_{X_9}(x_9)\right), F_{X_4}(x_4)\right) \end{aligned} \quad (26)$$

The bivariate copula used in Equation 26 is the Frank's copula[8]. The conditionalization can be accomplished by considering $x_8 = 60$. Having the density we can carry out updating.

In order to update the BBN structure we have to re-sample it. Each time, new evidence is obtained. We avoid re-sampling the whole structure several times in the Frank's copula – vine updating by doing so once with the copula – vine approach and then using Netica, which performs fast updating. This method called "Hybrid method for Continuous Bayesian Belief Nets" was introduced in [Hanea A., Kurowicka D, Cooke R.M. 2005].

b. Vines-Netica Updating

In the vines-Netica updating, the BBN structure can be sampled once with the 'copula – vine' approach described in Section 2.1.4. Then, conditional probability tables are created by incorporating this sample into a discrete BBN in Netica with ten states[9]. There updating can be performed. When the discrete BBN is already constructed, we might observe some events, then for certain variable(s) we know the value of, we enter that value as a *finding* (also known as "evidence"). Then Netica does *probabilistic inference* to find *beliefs* for all the other variables –indicating the subjective probabilities–. The final beliefs are sometimes called *posterior probabilities* (with *prior probabilities* being the probabilities before any findings were entered). This probabilistic inference done using Bayes' theorem and an improvement of the algorithm found in [Lauritzen S.L., Spiegelhalter D.J. 1998] is called *belief updating*.

[8] For a mathematical description of this copula see Chapter 4 or refer to [Frank M.J. 1979].
[9] The theoretical quantiles for each variable used to build the discrete BBN in Netica are found in Appendix A.1.

Now, some comparisons and results of these different ways of updating are presented. Figure 2.16 shows the BBN from model given in Figure 2.1 modelled in Netica. The variables are discretized in ten states having taken into account the 10, 20, … percentiles of their probability distributions (theoretical quantiles)[10].

Equal sized intervals from the samples over original variables −which bounds are the minimum and maximum of each sample− are used to create the BBN in the Figure 2.17. Although a discretization with equal intervals allows us to appreciate the continuous distributions of each variable, it is not convenient to capture changes in the model when updating is done. Therefore, the BBN in the Figure 2.16 is used in forthcoming calculations.

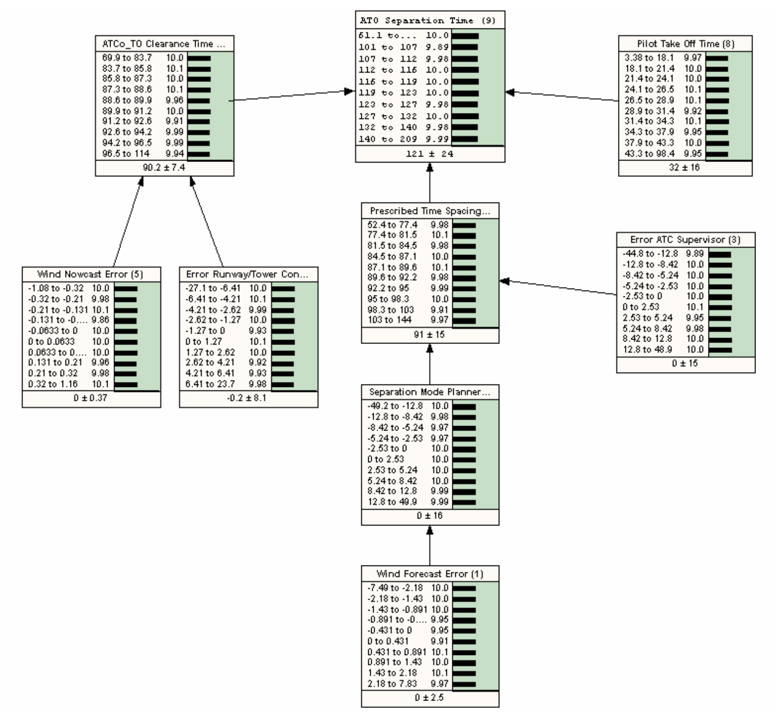


Figure 2.16: Aircraft Separation Time model in Netica using 4×10^5 samples[11]. Nodes were discretized with the values of the theoretical quantiles on the BBN. This BBN will be used in order to appreciate changes produced by conditionalizing the aircraft separation time with regard to a/some known values of the variables.

[10] See [Appendix A.1].

[11] For illustrative purposes, the case file incorporated in Netica to create those BBNs in Figures 2.15 and 2.16 has 4×10^5 samples which were obtained using the sampling procedure described in Section 2.1.4. Conditional probability tables are created instantaneously.

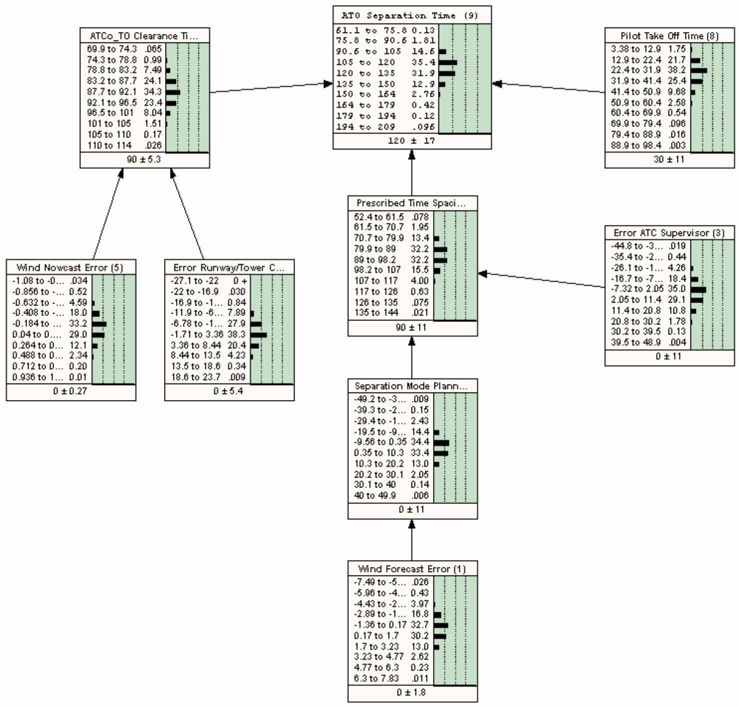


Figure 2.17: Aircraft Separation Time model in Netica using 4×10^5 samples. Equal intervals are taken into account to build the BBN in order to appreciate the continuous gamma and normal distributions of the variables.

Direct quantification of the discretized to 10 states for each variable BBN would require the specification of 12,150 probabilities in the conditional probability tables, whereas the quantification with continuous nodes requires nine algebraically independent (conditional) rank correlations and the specification of the nine marginal distributions. This demonstrates the reduction of assessment burden once we have quantified influences as (conditional) rank correlations. The probability tables of the discrete BBN are filled using the samples which are obtained for its continuous version.

In order to create the BBN for the Aircraft Separation Time model, we need to know which number of samples is appropriate to get a precise estimation of the conditional probability tables for the Aircraft Separation Time. If conditional probability tables are estimated with sufficient number of samples, then estimating them with slightly increased number of samples will not change this estimate much. Hence we take two sample files, say with N and M samples, respectively. We estimate both conditional probability tables and calculate the Maximum norm and Euclidean norm of both estimates[12]. By increasing the number of samples in both files we can observe that the Maximum norm and

[12] The maximum norm and Euclidean norm are calculated by using the following formulas: MN= $\max_{i,j} \left| a_{i,j} - b_{i,j} \right|$ and EN= $\sum_{i,j} \left(a_{i,j} - b_{i,j} \right)^2$, respectively.

Euclidean norm stabilizes at a level, the number of samples corresponding to it will be used to build probability tables. These results are shown in Table 2-1.

Interval (Number of samples in thousands)	Maximum norm	Euclidean Norm
10-25	0.44	6.46
25-50	0.23	5.57
50-75	0.22	4.65
75-100	0.19	4.01
100-200	0.19	3.38
200-300	0.14	2.61
300-400	0.10	2.21
400-500	0.10	1.95
500-600	0.08	1.77
600-700	0.07	1.62
700-800	0.07	1.52
800-900	0.08	1.43
900-1000	0.06	1.33

Table 2-1: The maximum norm and Euclidean norm of the difference between probability tables of the Aircraft Separation Time (X_9) created using different sample sizes are calculated.

The results of the maximum norm in Table 2-1 show that the biggest change in the maximum norm is from 0.44 to 0.23. After that, the maximum norm will vary slowly until the difference between probability tables for X_9 created using 1×10^5 and 2×10^5 samples is considered.

A good estimation of the conditional distribution of the aircraft separation time conditioned to certain values of X_8 is then obtained by considering a file of 1×10^5 samples. The next notorious change in the maximum norm is found when the difference between probability tables for X_9 created using 4×10^5 and 5×10^5 samples is considered. The maximum norm stabilizes in a value equal to 0.10.

On the other hand, the Euclidean norm stays constant until the difference between probability tables for X_9 created using 3×10^5 and 4×10^5 is considered. After that, the changes in the Euclidean norm are rough (see Table 2-1 and Figure 2.18). We conclude that 4×10^5 samples are enough for our calculations.

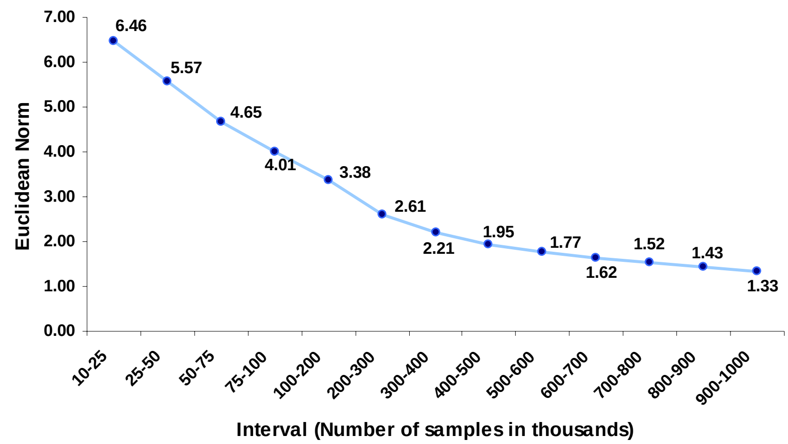


Figure 2.18: Euclidean norm of the difference between two probability tables of the Aircraft Separation Time (X_9) created using samples of different sizes.

Another way to find the precise number of samples to obtain a good estimation of the conditional probability tables is that proposed in [Hanea A., Kurowicka D, Cooke R.M. 2005], which is also implemented in our case. Thus, the conditional distribution of the Aircraft Separation Time (X_9) given some values of the Pilot Take Off Time (X_8) where only 1×10^4 samples were used is presented in Figure 2.19.

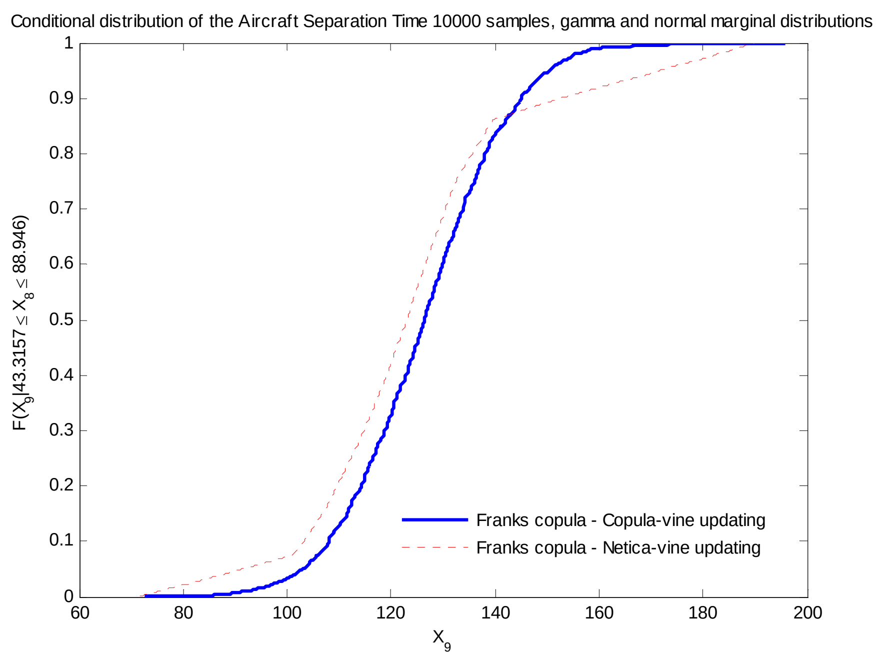


Figure 2.19: Comparison between the results of updating using Frank's copula and vines (solid blue lines) and Netica (dotted red lines). Sample files using 1×10^4 samples are created. The conditional distribution of $X_9 \,|\, 43.3157 \le X_8 \le 88.946$ is obtained.

There is a big difference between conditional distribution obtained using vine–copula method and Netica. This proves that 1×10^4 samples are not enough to build conditional probability tables.

If we now consider a sample file of 4×10^5 samples, as before a very good estimation of the conditional distribution of the aircraft separation time is obtained. These results are shown in Figure 2.20. Here, the agreement between the two methods is precise.

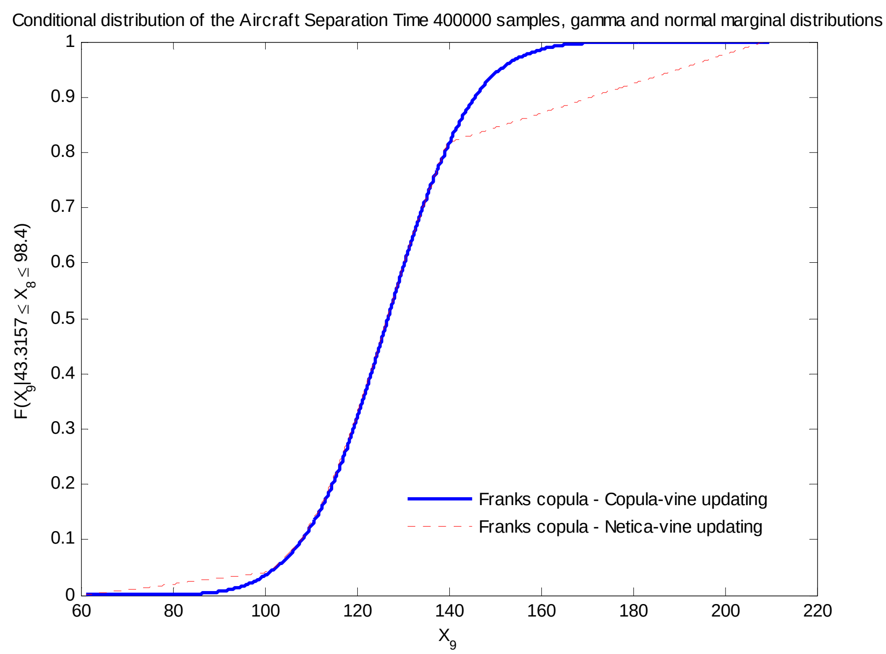


Figure 2.20: Comparison between the results of updating using Frank's copula and vines (solid blue lines) and Netica (dotted red lines). Sample files using 4×10^5 samples are created. The conditional distribution of $X_9 \,|\, 43.3157 \le X_8 \le 98.4$ is obtained[13].

From Figure 2.20, we can observe a discrepancy in the first and the last intervals of the discretization. There the results given by Netica-vine updating differ from those given by the Copula-vine updating. The discretization of the nodes was made according to their quantiles, thereby the first and the last intervals of the discretization for each variable are wider than the rest of the intervals (which are very narrow). For the variable Aircraft

[13] The maximum and minimum values of the samples differ when the number of samples differs. Because of this, the intervals for which X_8 belong to are not equal in Figures 2.19 and 2.20.

Separation time (X_9), the first and the last discretization intervals together amount 74% of the sample width. In order to plot the conditional distribution of the Aircraft Separation time (X_9) from Netica, uniform samples from each discretization interval are drawn. This is visible in Figure 2.20 as straight lines at the beginning and at the end of the conditional distribution of the Aircraft Separation time appeared.

After the sample file is imported in Netica, we conditionalize on high values of the Pilot Take Off Time (X_8). That is, those between its 0.9 and 1.0 quantiles equal to 43.3157 and 98.4, respectively (see Figure 2.21). Samples of the conditional distribution of the Aircraft Separation Time are created from Netica.

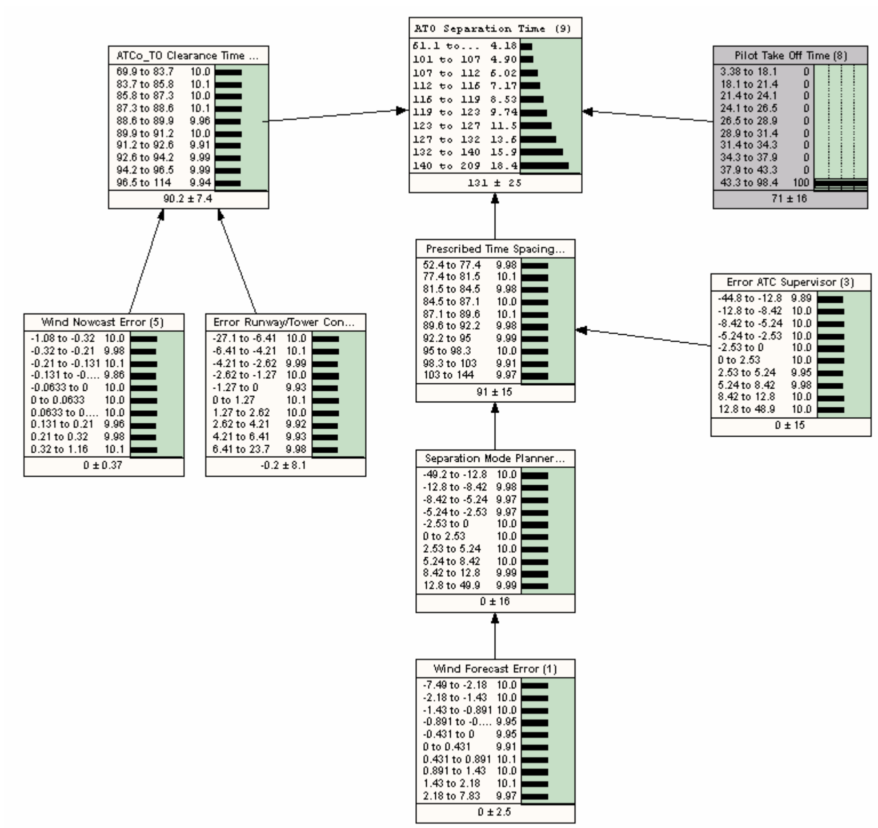


Figure 2.21: Conditional distribution of $X_9 \,|\, 43.3157 \le X_8 \le 98.4$ in Netica.

It is assumed that Take Off Clearance is provided by the tower (or runway) controller. The take off then may start, provided that the pilot has completed his/her checklists (i.e. is ready), at the Take Off Position at a certain distance from the runway threshold. The pilot selects the take off thrust at the starting time of his/her take off. The aircraft accelerates during the take off roll. It is up to the pilot to initiate the take off at a suitable moment after the take off clearance is given by the controller. If the pilot initiate the take of just after the Take Off Clearance is provided by the tower (or runway) controller the

Pilot Take Off Time (X_8) will have small values, if the decision of the pilot is to delay the aircraft to start to roll then the Pilot Take Off Time (X_8) will have big values.

As has been mentioned, the Pilot Take Off Time (X_8) influences directly the Aircraft Separation Time (X_9). The time difference between the moment when the leader and the follower aircraft start to roll is more probable to take high values than low values when the time difference between take off clearance given by the ATCo and the start of roll of the aircraft is big.

If the values of the Pilot Take Off Time (X_8) are in the interval [43.3157, 98.4], it is more probable that the values of the Aircraft Separation time (X_9) belong to the interval [140, 209]. Notice that values between its 70th to 100th percentiles are now very probable. Conversely, results could be obtained if we conditioned the Aircraft Separation time distribution on low values of X_8.

We present now the conditional distribution of the Aircraft Separation time (X_9) given the different values of the Prescribed Time Spacing (X_4). The probability of the time difference between the leader's starting to roll and the follower aircraft's starting to roll being high is high because of the big separation time prescribed by the ATC supervisor (in ATC-Wake mode). If the values of separation time prescribed by the ATC supervisor (X_4) are located between the 50th to 60th percentiles, the probability of high values of the Aircraft Separation time (X_9) meaningfully decreases.

There is much difference in the conditional probability of the Aircraft Separation time (X_9) while conditionalized on different values of X_4. In Figure 2.1, a rather low positive conditional rank correlation $r_{94|87} = 0.30864$ is observed (the unconditional rank correlation does not change much, $r_{94} = 0.3186$). The probability of the Aircraft Separation Time taking small values is high because of the low values of separation time prescribed by the ATC supervisor.

The conditional distribution of the Aircraft Separation time (X_9) is shown in Figure 2.22.

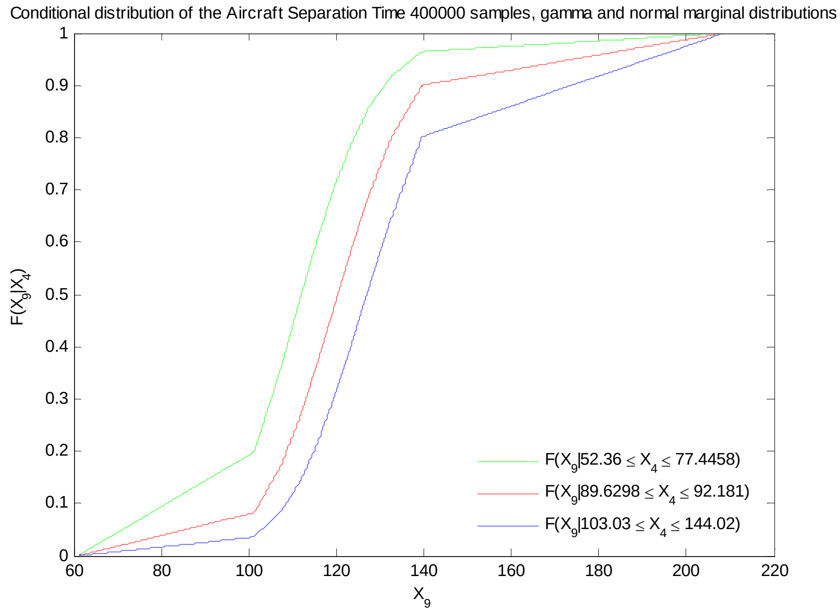


Figure 2.22: Comparison between the results of updating using Netica and 4×10^5 samples. Conditional distributions of $X_9 \left| 52.36 \le X_4 \le 77.4458 \right.$ (green solid line), $X_9 \left| 89.6298 \le X_4 \le 92.181 \right.$ (red solid line) and $X_9 \left| 103.03 \ge X_4 \ge 144.02 \right.$ (blue solid line)[14].

[14] As we said before the maximum and minimum values of the samples differ when the number of samples differs.

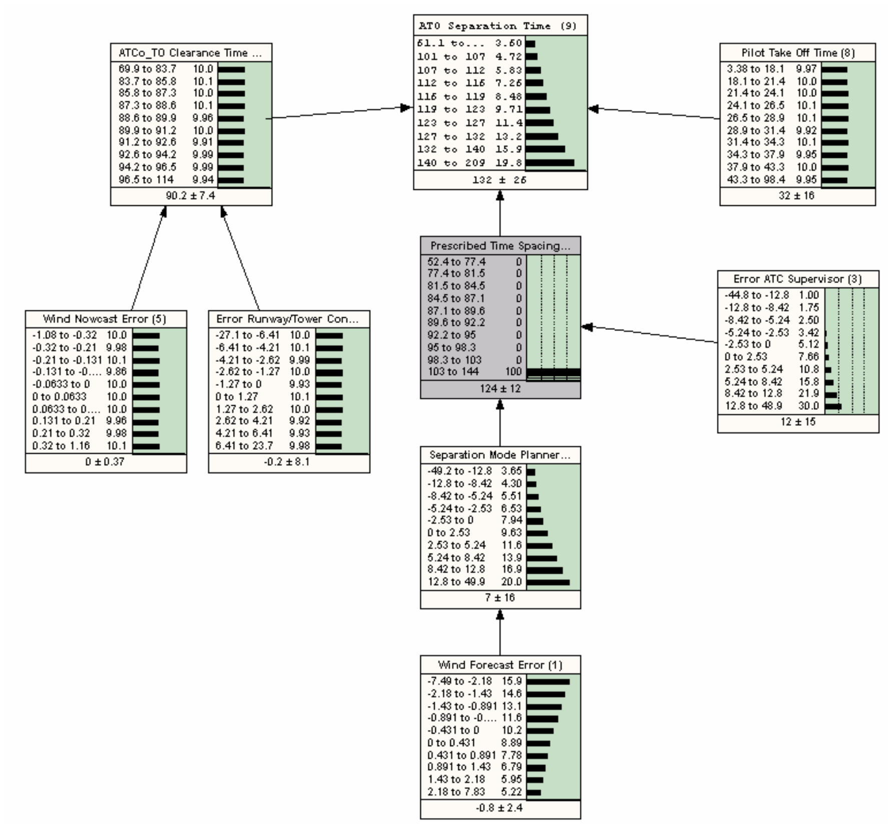


Figure 2.23: Conditional distributions of $X_9 \left| 103.03 \geq X_4 \geq 144.02 \right.$.

The results shown so far are based on the conditional distribution of the Aircraft Separation time (X_9) on certain known values of one variable. We now conditionalize on the values of the Wind nowcast error (X_5) which belongs to the interval [-1.08, -0.3204] −between its 0.0 and 0.1 quantiles− and the values of the Prescribed Time Spacing (X_4) which belongs to the interval [77.4458, 81.494] −between its 0.1 and 0.2 quantiles. The conditional distribution of the Aircraft Separation time (X_9) is obtained by using the two previous methods compared to that using the normal copula instead of Frank's copula (updating using Netica). Notice that the agreement between the three methods is very accurate. This is shown in Figure 2.24.

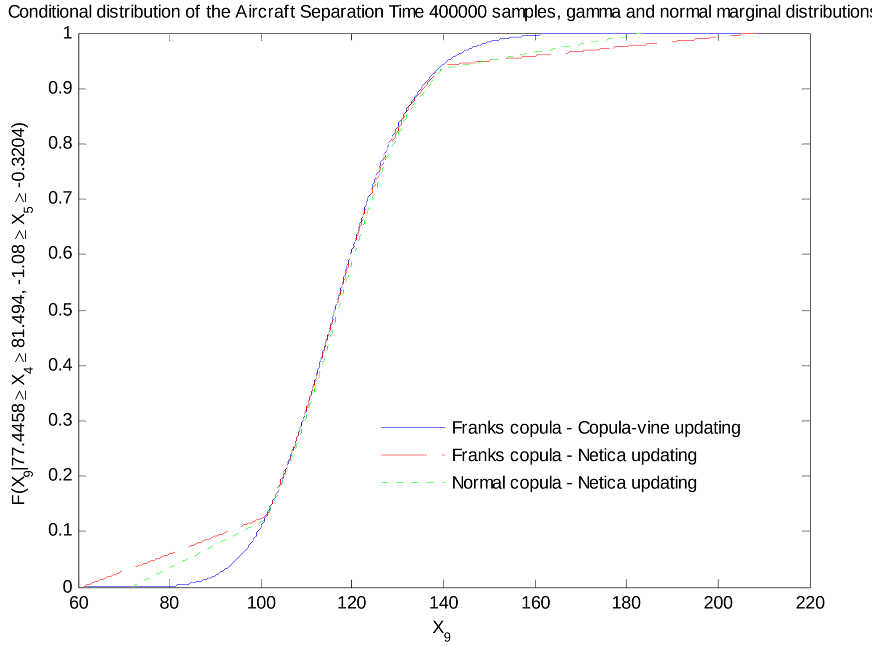


Figure 2.24: Conditional distribution of $X_9 \left| -1.08 \le X_5 \le -0.3204, 77.4458 \le X_4 \right.$ ≤ 81.494. Comparison between the results using Frank's copula and using vines updating with the copula − vine approach (green solid line), Netica updating (blue dotted line) and Joint normal copula and vines updating with Netica (red dotted line).

Wind nowcast error (X_5) represents the difference between actual wind (measured by the Detector) and predicted wind (as determined by the Meteo/weather systems). If the actual wind is stronger than the predicted wind, positive values of X_5 take place. In this case, the probability of the time difference between the leader's starting to roll and the follower aircraft's starting to roll being small is high. Conversely, the probability of the time difference between the moment when the leader and follower aircraft start to roll being in high quantiles is high because of the negative values of the difference between the actual wind and the wind predicted.

From the model, we can observe that the Predicted Time Spacing (X_4) greatly influence our variable of interest. If the separation time prescribed by the ATC supervisor is small, say, between its 0.1 and 0.2 quantiles, the effects of negative values of the wind nowcast error could be inverted as shown in Figure 2.25.

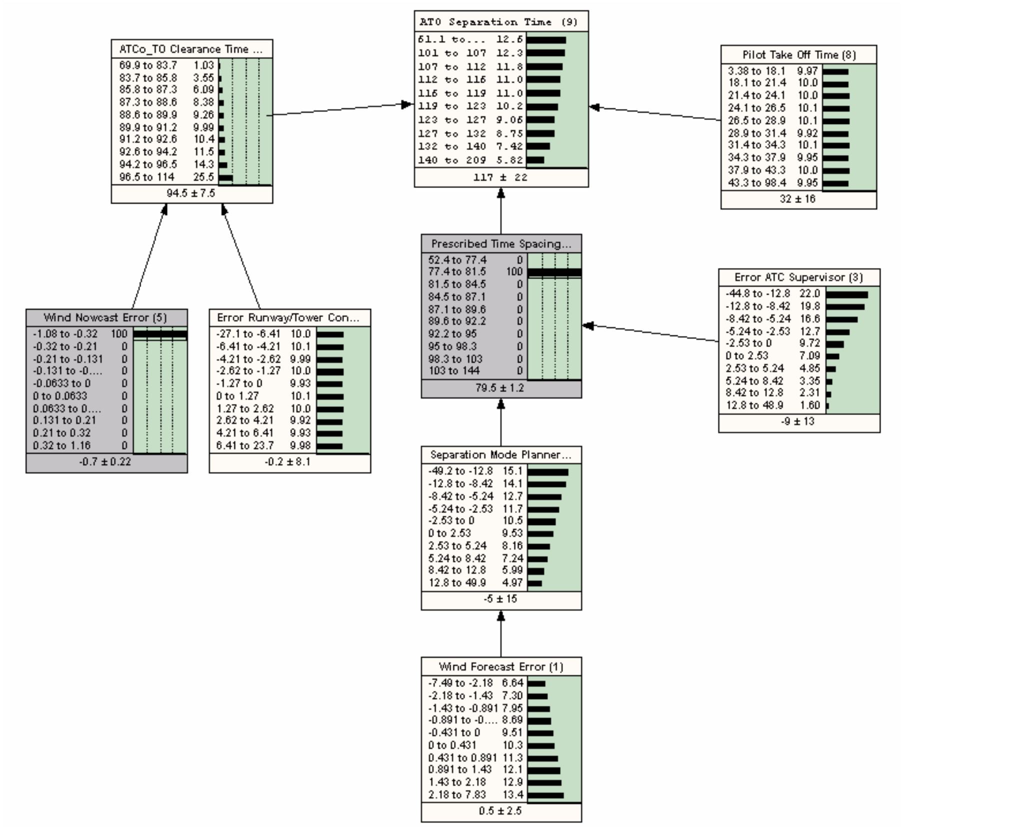


Figure 2.25: Conditional distribution of $X_9 \left| -1.08 \le X_5 \le -0.3204, 77.4458 \le X_4 \right. \le 81.494$ in Netica.

In Figure 2.26, the conditioning is performing on the values of the Prescribed Time Spacing (X_4) and the Pilot Take Off Time (X_8) which belong to their 0.4 and 0.5 quantiles and the values of the Aircraft Traffic Controller (X_7) which belong to its 0.5 and 0.6 quantiles.

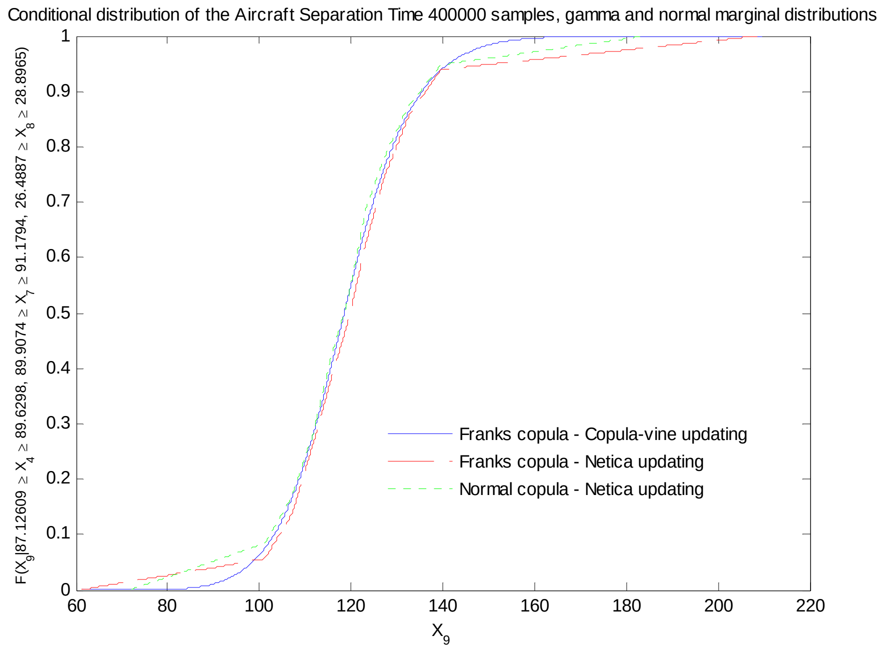


Figure 2.26: Conditional distribution of $X_9 \mid 87.12609 \leq X_4 \leq 89.6298, 26.4887 \leq X_7 \leq 28.8965, \; 89.9074 \leq X_8 \leq 91.1794$. Comparison between the results using Frank's copula and using vines updating using the copula − vine approach (green solid line), Netica updating (blue dotted line) and Joint normal copula and vines updating with Netica (red dotted line).

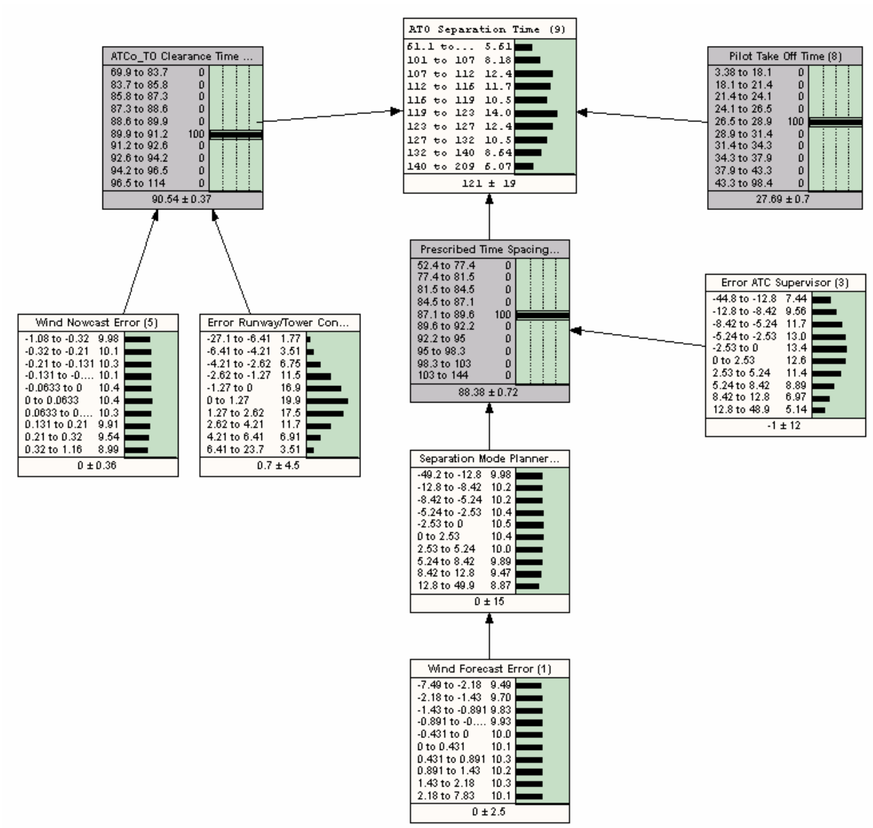


Figure 2.27: Conditional distribution of $X_9 \mid 87.12609 \le X_4 \le 89.6298,\ 26.4887 \le X_7 \le 28.8965,\ \ 89.9074 \le X_8 \le 91.1794$ in Netica.

Now the joint distribution of the input and output of the Aircraft Separation Time model is shown in Figure 2.30. We can conditionalize this whole joint distribution on low values of the Aircraft Traffic Controller Take Off Clearance Time (X_7) as presented in Figure 2.31. The visual representation allows us to observe the effect of this conditionalization on the whole joint distribution.

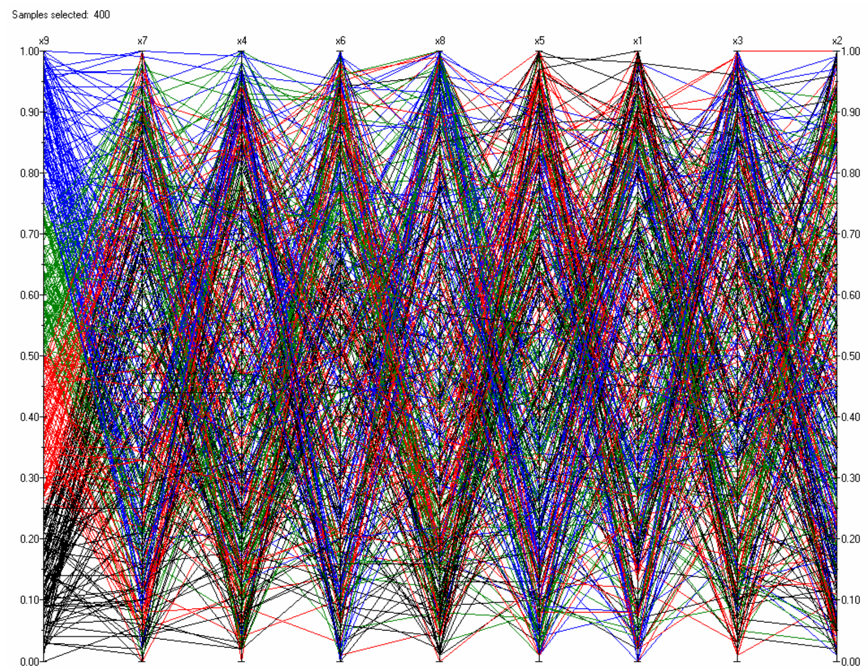


Figure 2.30: Cobweb plot of the joint distribution of the Aircraft separation time model

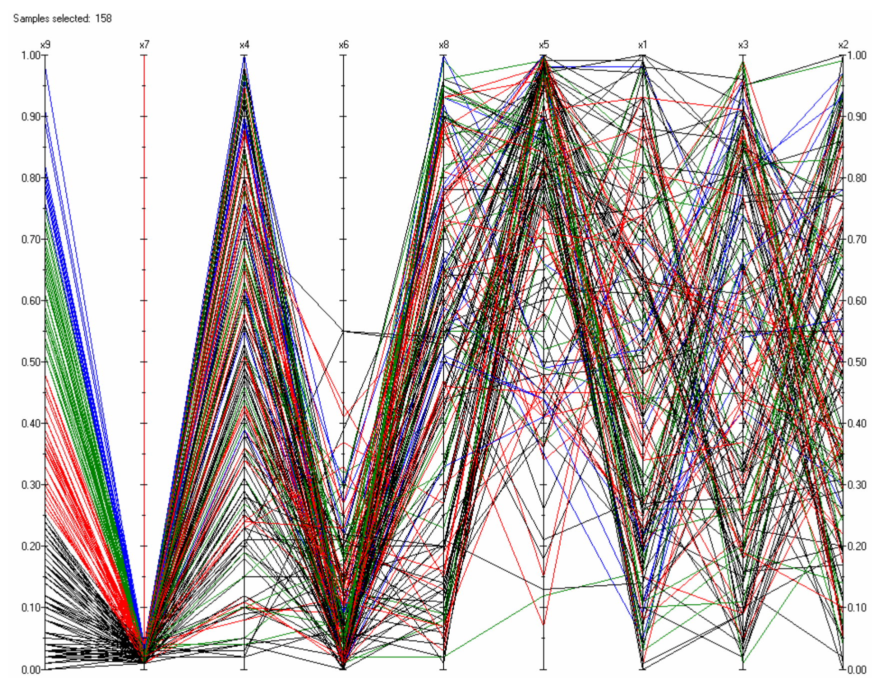


Figure 2.31: Cobweb plot of the Aircraft separation time (X_9) conditionalized in low values of the Aircraft Traffic Controller Take Off Clearance Time (X_7)

Finally, we should state that conditionalization is a "rough" way of carrying out a sensitivity analysis. Since, we could evaluate the importance of the variables by just "guessing" or conditionalizing on different values of the variables, which are relevant in our criterion or client's criterion. However, the appropriate way of investigating the importance of some variable(s) for the aircraft separation time is to calculate the correlation ratio. This is accomplished in the following section.

2.1.7 Sensitivity Analysis

To carry out the sensitivity analysis several statistics and sensitivity measures are obtained by using the Sensitivity Analysis program [Lewandowski D. 2005] as part of Unicorn[15], based on 4×10^5 samples derived from a continuous BBN created in UniNet[16].

The "predicted variables" are those whose behaviour we want to explain in terms of other variables, called the "base variables". Here we are interested in the variable Aircraft Separation Time (X_9), and we want to see how this variable depends on the variables $X_1, \ldots, X_8$.

Table 2-2 shows the sensitivity indices and statistics obtained by relating the Aircraft Separation Time (X_9) to each variable. These include: The product moment correlation, the Spearman rank correlation, the regression coefficient, the correlation ratio and the partial correlation coefficient[17].

[15] Unicorn (**Un**certainty Analysis with **Cor**relatio**n**s tool) developed at the Department of Mathematics of Delft University of Technology, The Netherlands.
[16] UniNet: BBNs software developed at the Department of Mathematics of Delft University of Technology, The Netherlands. From the continuous BBN (which is built by using the normal copula and canonical vines) created in UniNet, samples can be derived to be used immediately in the Sensitivity Analysis. It should be pointed out that the probabilities and samples derived from this BBN are not so different from the BBN created with Frank's copula and *D*-vines.
[17] Refer to [Kurowicka D., Cooke R.M. 2006; Bedford T.J., Cooke R.M. 2003; Lewandowski D. 2005] which contains mathematical definitions from sensitivity indices and statistics obtained in this Section.

(1) Id	(2) Predicted variable	(3) Base Variable	(4) Product moment correlation	(5) Rank Correlation	(6) Regression Coefficient	(7) Correlation ratio	(9) Partial correlation coefficient
1	X_9	X_7	3.078E-01	2.961E-01	9.342E-01	9.380E-02	1.688E-01
2	X_9	X_4	3.310E-01	3.186E-01	5.045E-01	6.189E-02	2.825E-01
3	X_9	X_6	2.395E-01	2.300E-01	7.324E-01	5.741E-02	-1.120E-03
4	X_9	X_8	3.012E-01	2.932E-01	4.517E-01	5.222E-02	3.388E-01
5	X_9	X_5	-1.298E-01	-1.244E-01	-7.854E+00	1.687E-02	4.247E-04
6	X_9	X_1	-7.313E-02	-7.075E-02	-6.542E-01	5.380E-03	-2.042E-03
7	X_9	X_3	1.964E-01	1.878E-01	3.005E-01	2.822E-04	3.275E-03
8	X_9	X_2	1.005E-01	9.667E-02	1.514E-01	3.861E-06	-2.726E-03

Table 2-2: Sensitivity indices for the predicted variable X_9 and a given base variable

Each row in Table 2-2 shows the sensitivity indices for a given base variable, for the predicted variable X_9. Note that the Aircraft Traffic Controller Take Off Clearance Time (X_7) has the highest correlation ratio to the Aircraft Separation Time (X_9); Separation Mode Planner Failure (X_2) has the smallest.

In the BBN for the Aircraft separation time model the variable Aircraft Traffic Controller Take Off Clearance Time (X_7), which represents the time difference between the leader's starting to roll and take off clearance of the ATCo for the follower aircraft is then considered the most representative variable to explain the time difference between the leader's starting to roll and the follower aircraft's starting to roll (X_9).

The product moment correlation matrix is shown below, the Aircraft Traffic Controller Take Off Clearance Time (X_7), the Prescribed Time Spacing (X_4), the Error Runway/Tower Controller (X_6) and the Pilot Take Off Time (X_8) variables have the highest positive correlations to the Aircraft Separation Time (X_9).

	X_9	X_3	X_6	X_5	X_7	X_1	X_2	X_4	X_8
X_9	1.000E+00	1.964E-01	2.395E-01	-1.298E+00	3.078E-01	-7.313E+00	1.005E-01	3.310E-01	3.012E-01
X_3	1.964E-01	1.000E+00	-1.396E+00	-1.819E+00	-1.798E+00	-1.015E+00	1.606E-03	5.875E-01	-1.559E+00
X_6	2.395E-01	-1.396E+00	1.000E+00	2.898E-03	7.793E-01	-7.325E+00	9.380E-04	-5.688E+00	-3.969E+00
X_5	-1.298E+00	-1.819E+00	2.898E-03	1.000E+00	-4.193E+00	2.241E-03	5.927E-05	-1.915E+00	-6.236E+00
X_7	3.078E-01	-1.798E+00	7.793E-01	-4.193E+00	1.000E+00	-2.006E+00	1.091E-03	-5.004E+00	-1.442E+00
X_1	-7.313E+00	-1.015E+00	-7.325E+00	2.241E-03	-2.006E+00	1.000E+00	-7.071E+00	-2.183E+00	-1.396E+00
X_2	1.005E-01	1.606E-03	9.380E-04	5.927E-05	1.091E-03	-7.071E+00	1.000E+00	3.078E-01	8.129E-04
X_4	3.310E-01	5.875E-01	-5.688E+00	-1.915E+00	-5.004E+00	-2.183E+00	3.078E-01	1.000E+00	-1.476E+00
X_8	3.012E-01	-1.559E+00	-3.969E+00	-6.236E+00	-1.442E+00	-1.396E+00	8.129E-04	-1.476E+00	1.000E+00

Table 2-3: Product moment correlation matrix

Table 2-4 shows the mean and standard deviation for the four detailed variables above.

Id	Predicted variable	Base Variable	E(X_9)	E(X_i)	Std(X_9)	Std(X_i)
1	X_9	X_7	1.198E-02	9.003E-01	1.517E-01	4.999E+00
2	X_9	X_4	1.198E-02	8.988E-01	1.517E-01	9.955E+00
3	X_9	X_6	1.198E-02	5.738E-03	1.517E-01	4.961E+00
4	X_9	X_8	1.198E-02	3.011E-01	1.517E-01	1.012E-01

Table 2-4: Some other statistics

2.2 ATC-Wake: Detection, Warning and Avoidance Maneuver Probability

With respect to the Wake Vortex Detection, Warning, and Avoidance Maneuver and following the recommendations from the interviewees with operational experts, a turn away from the Wake Vortex of a preceding aircraft has been considered as the only option to avoid an encounter.

The execution of the ATC-Wake detection, warning, and avoidance maneuver (e.g. turn away from a wake vortex (during departures)) depends –besides operational feasibility– on the probability of failure of the ATC-Wake system components. For the ATC-Wake system failures, a causal model has been constructed using discrete Bayesian Belief Nets (BBNs) (See figure 2.32).

It is shown that the resulting BBN might be represented by a fault tree [See ATC-Wake D3_5b].

2.2.1 Application of Fault Trees and Discrete BBNs for the Wake Detection, Warning and Avoidance Maneuver Probability

We aim to model a system which represents the wake detection, warning and avoidance maneuver failure. Moreover, we intend to quantify such a model. The problem is stated as follows:

The wake vortex detection warning and avoidance maneuver is performed by the aircraft/pilot, as requested by air traffic controllers. The aircraft/pilot can fail to perform the ATC-Wake DWA maneuver, either because the aircraft/pilot is not able to turn timely or the ATC-Wake warning fails. ATC-Wake DWA Failure happens if the failure of any of these components occurs. A failure of the ATC-Wake warning happens caused by either the controller does not provide a timely warning or because the Monitoring and Alerting ATC-Wake system component fails. Inaccurate information from the Monitoring and Alerting ATC-Wake system component is influenced by either the loss of DWA Tactical Function or an improper model prediction. Alternatively, it can be due to improper detector performance. An improper model prediction is yield either because of inaccurate or wrong WV Model Estimation or due to inaccurate Air Traffic Situation. It might even be because of faulty or inaccurate Meteo Nowcasting information. Finally, improper detector performance either derives from the wake vortex outside the detection range/scanning volume or from an inaccurate or faulty detection of wake vortices.

From the problem statement, we will be able to constrain the variables to take two values or two states. Different models can be used to represent this problem. We shall compare two approaches which may be applied to model the failure of the aircraft/pilot when performing the ATC-Wake DWA maneuver which amounts to discrete BBNs and Fault Trees. Throughout the Section 2.2, we will give a detailed description of the application of these methodologies to our particular case. Moreover we will stress the advantages and disadvantages of these methods.

2.2.2 Comparison of Discrete BBNs and Fault trees

We first present the BBN and the Fault trees that can be used to describe the problem stated in the introduction.

a) BBN

The problem above can be modelled using discrete BBNs as shown in Figure 2.32. This BBN has 13 nodes and 12 edges. The nodes correspond to binary variables with states 0 (not failure) and 1 (failure) specified in the problem and edges that represent relationships between variables. Each BBN node is labelled with a lowercase literal and the full name of the variables of interest. Names or labels are used in the text to refer to the variables. The BBN in Figure 2.32 shows the graphical representation of dependences and (conditional) independences in this problem. One reads that d, e, f are independent and they influence g (g is a child of d, e, f and d, e, f are called the parents of g; denoted as $pa(g)$). A graphical representation of BBN does not, however, give us all the information we need about joint distribution. To completely describe the joint distribution represented by BBN, we need to specify conditional probability tables of all variables in the BBN given their parents. Then the joint distribution can be given as

$$P(a, b, \ldots, T) = \prod_{w=a}^{T} P(w \mid pa(w)) \qquad (27)$$

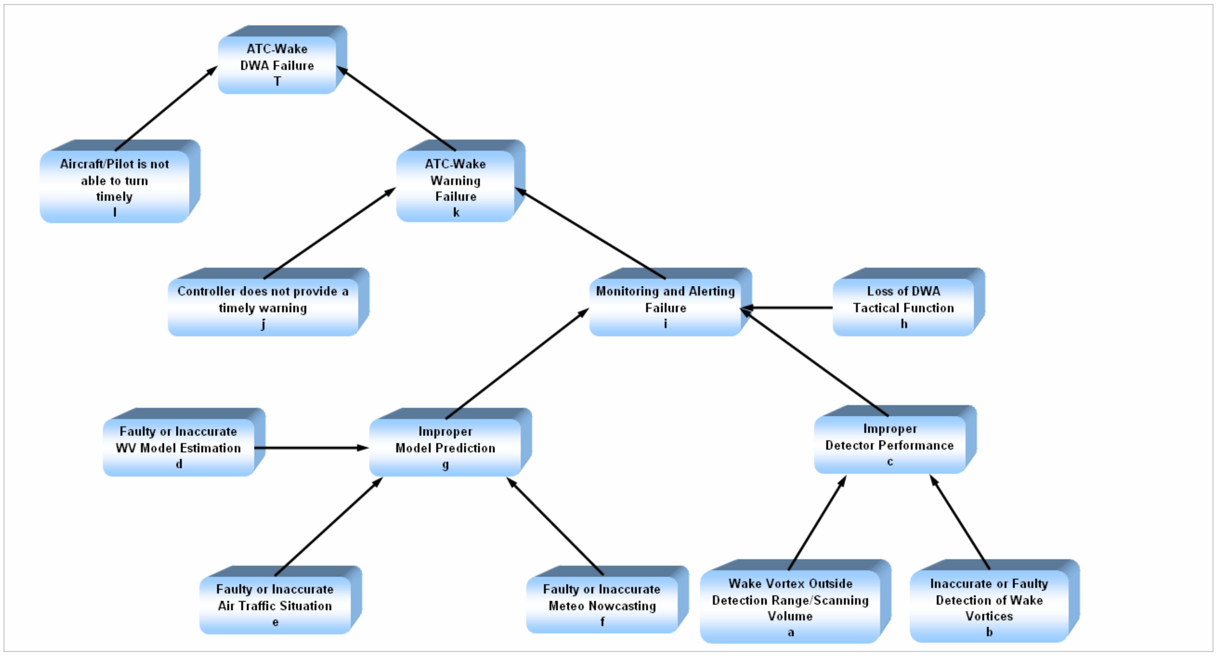


Figure 2.32: BBN for the WV DWA maneuver probability.

The nodes in this BBN have the following explanation:

- ***ATC-Wake DWA Failure (T):*** represents the probability of aircraft/pilot not able to initiate the ATC-Wake DWA maneuver (e.g. a turn away from WV of a preceding aircraft).
- ***Aircraft/Pilot not able to turn timely (l):*** represents the probability of an aircraft/pilot not able to perform the ATC-Wake DWA maneuver, when requested by the controllers.
- ***ATC-Wake Warning Failure (k):*** represents the probability of not providing a timely warning to the flight crew when one should be given. As a result, it is possible that a pilot reacts later to a wake encounter when one should occur.
- ***Controller does not provide a timely warning (j):*** represents the probability of the ATCo not providing an alert, when it is advised by the Monitoring and Alerting system.
- ***Monitoring and Alerting Failure (i):*** represents the probability of not providing a timely warning to the air traffic controllers when one should be given. As a result, the ATCo might NOT be able to initiate/instruct the pilot to perform an evasive action.
- ***Loss of DWA Tactical Function (h):*** represents the probability of an undetected loss of the Monitoring and Alerting Function. In case of a Detected Loss, the ATCos are aware that NO cautions/alerts will be given and a transition will be made to the ICAO Mode (the separation will increase, and the DWA maneuver will not be necessary).
- ***Improper Model Prediction (g):*** represents the probability that the predictions of Wake Vortex locations and strength are inaccurate/wrong.
- ***Inaccurate or Faulty WV Model Estimation (d):*** represents the probability that the

predictions of wake vortex locations and/or strengths made by the WV Model, on the basis of aircraft data and meteo data, are inaccurate/wrong. As a result, incorrect information is passed to ATC-Wake Predictor, causing improper functioning

- ***Inaccurate or Faulty Air Traffic Situation (e):*** represents the probability that the air traffic situation provided by the surveillance systems is inaccurate or wrong. As a result, incorrect information is passed to the Predictor, causing improper functioning.
- ***Inaccurate of Faulty Meteo Nowcasting (f):*** represents the probability that the meteorological conditions (i.e. nowcasting data) provided by the meteo systems are inaccurate or wrong. As a result, incorrect information is passed to the ATC-Wake Predictor, causing improper functioning.
- ***Improper Detector Performance (c):*** represents the probability that the ATC-Wake Detector (e.g. LiDAR) performs significantly less than the air traffic controllers expect (while they are not aware of the inaccuracies) (i.e. inaccurate/wrong alerts are given);
- ***Wake Vortex Outside Detection Range/Scanning Volume (a):*** represents the probability that the ATC-Wake Detector does not detect the wake vortices of the leading aircraft, because these are outside the scanning volume of the ATC-Wake Detector.
- ***Inaccurate or Faulty Detection of Wake Vortices (b):*** represents the probability that the ATC-Wake Detector does not detect wake vortices of the leading aircraft accurately, when these are inside the planned scanning volume of the ATC-Wake Detector(s).

b) Fault Tree

Figure 2.33 shows the fault tree (FT) that corresponds to the representation of the wake vortex detection warning and avoidance maneuver probability distribution. The nodes of the Fault tree are binary variables and are affine to the corresponding variables of the BBN. The structure of the FT is very similar to the BBN presented above. In contrast to BBN the fault tree relationships are represented by a symbol which appears several times in Figure 2.33 and is labelled with the uppercases *G1*, …, *G5*. These are the gate symbols OR. OR-gate means that the output event occurs if any one of the input events occurs. In Figure 2.33 only OR-gates can be seen. There are, however, other gates that could in principle be used e.g. the AND-gate (output event occurs if all input events occur) or the NOT-gate (output event occurs when input event does not occur). In a fault tree one can recognize two types of nodes: basic events (basic nodes) are shown in Figure 2.33 as circles and intermediate events (intermediate nodes) are represented as rectangles. The graphical representation of a Fault tree is not sufficient to construct the joint distribution. One must also specify distributions of basic nodes. The distribution specified by the Fault tree can be then calculated using a minimum cut set (MOCUS) algorithm that will be briefly described later.

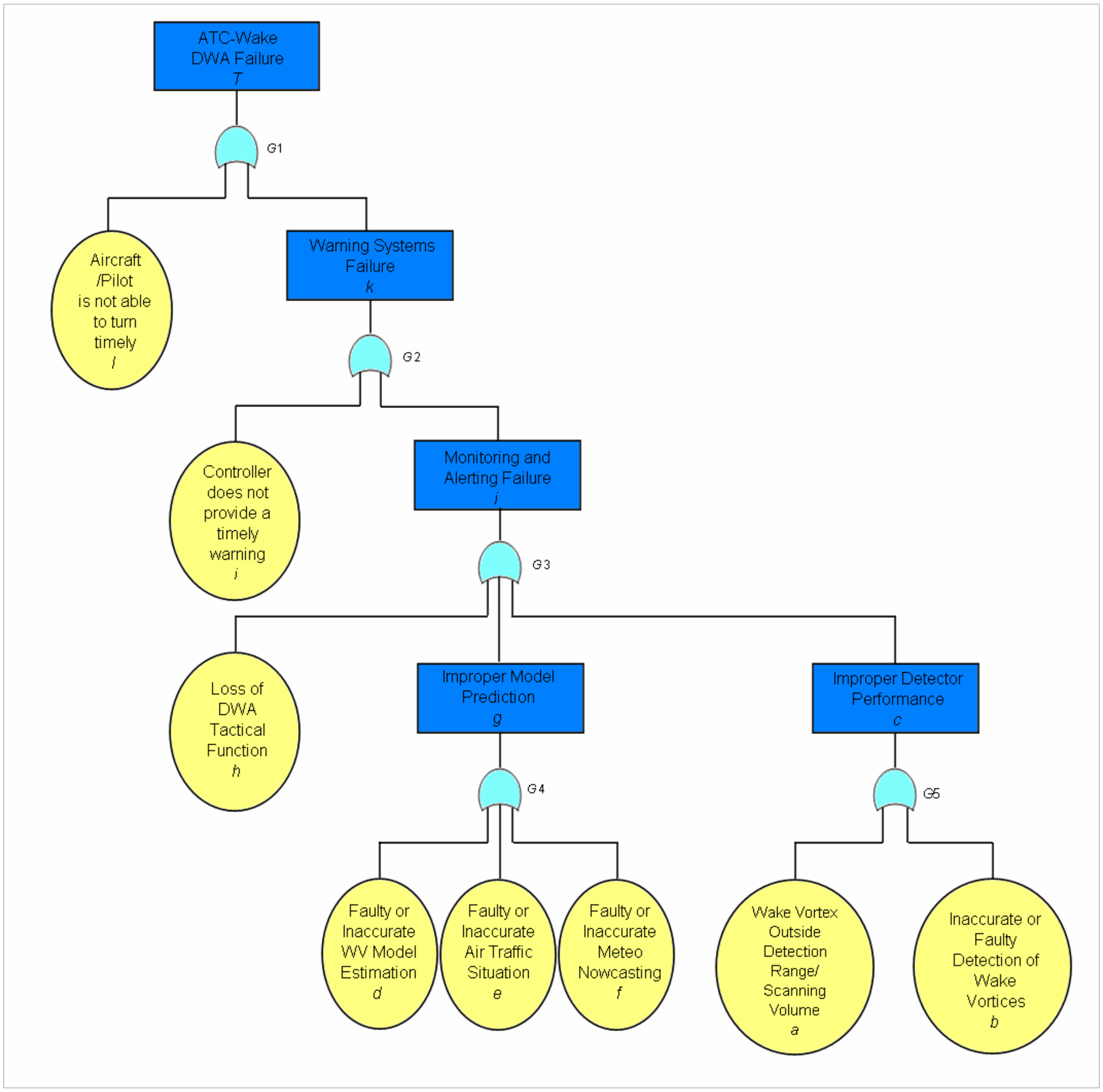


Figure 2.33: Fault tree for the WV DWA maneuver probability

c) Comparison of BBNs and FT

To compare BBNs and FTs we describe in detail the lower left-most part of the fault tree in Figure 2.33 and corresponding to this part a fragment of the BBN in Figure 2.32.

Figure 2.34 contains the lower right-most part of the FT. The basic events are Wake Vortex Outside Detection Range/ Scanning Volume (*a*) and Inaccurate or Faulty Detection of Wake Vertices (*b*). The OR-gate (*G5*) ensures that Improper Detector Performance (*c*) can be caused by failure of either *a* or *b*.

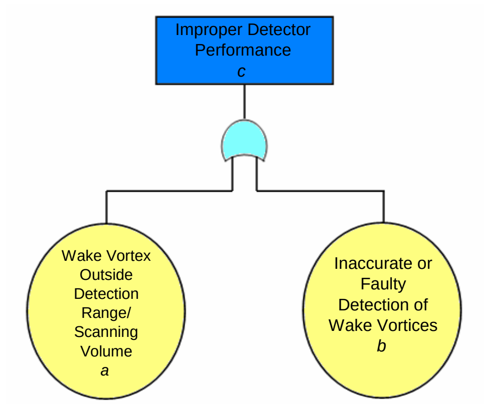


Figure 2.34: Improper Detector Performance caused by two basic events.

a and *b* are binary variables taking values 1 (failure) with probability 0.001 and 0 (not failure) with probability 0.999. Possible combinations of these variables are shown in Table 2.5.

a	*b*
1	1
1	0
0	1
0	0

Table 2-5: Information of *a* and *b* represented in a binary table.

Since the failure of *c* can be caused by failure of either *a* or *b* and because *a* and *b* are independent it is very easy to calculate the probability of failure of *c*.

$$P(c=1)=1-P(a=0,b=0)=1-P(a=0)P(b=0)=0.0020$$

For small FTs we can easily enumerate all combinations of basic events leading to failure of intermediate event. In general, however, this approach would not be appreciated. To perform fast the calculations in FTs the Boolean algebra with Boolean operators denoted as *a* and *b* as $a \cdot b$; *a* or *b* (*a*+*b*; not *a* as *a'*) is used. The general algorithm will not be described here. We will now show how to calculate the probability of Improper Detector Performance above using Boolean algebra (MOCUS algorithm). We consider event *c* as the top event caused by the failure of either event *a* or *b*. Since the top event is related to the basic events through an OR gate, we will give the Top Down approach to the MOCUS algorithm as follows:

$$\begin{aligned} c &= G5 \\ c &= a+b \end{aligned} \quad (28)$$

Clearly, if a and b fail we will get Detector performance failure. So, the set of events (cut set) leading to the failure of c is:

$$\{a, b\}$$

However from the description of the problem (OR gate) we know that the failure of either a or b is enough for the failure of c hence minimal sets leading to the failure of c are:

$$\{a\},\ \{b\}$$

These are called minimal cut sets. The notion of minimal cut sets is crucial when we have a large tree. After finding a minimum cut set the algorithm builds the minimum cut set representation of the FT (in general a smaller tree) which allows us to perform all the necessary calculations more efficiently.

We apply the inclusion-exclusion formula to equation 28 and get

$$P(c=1) = P((a=1)\bigcup(b=1)) = P(a=1) + P(b=1) - P(a=1)P(b=1) = 0.0020 \qquad (29)$$

Consider now the corresponding to FT in the Figure 2.34 fragment of BBN (see Figure 2.35). The probabilities of a, b are specified as in the FT above. From Figure 2.35 one can see that c is influenced by a and b but the nature of this dependence cannot be deduced from the graphical representation. How the probability of the state of c is affected by combinations of states of a, b must be read from conditional probability table of c given a, b. Such a conditional probability table may be very general and may describe many different types of dependencies between variables. Different probability tables will lead to different probability of failure of c. However if we assume that the failure of either event a or b necessarily leads to the failure of c the conditional probability table will be as shown in Table 2-6.

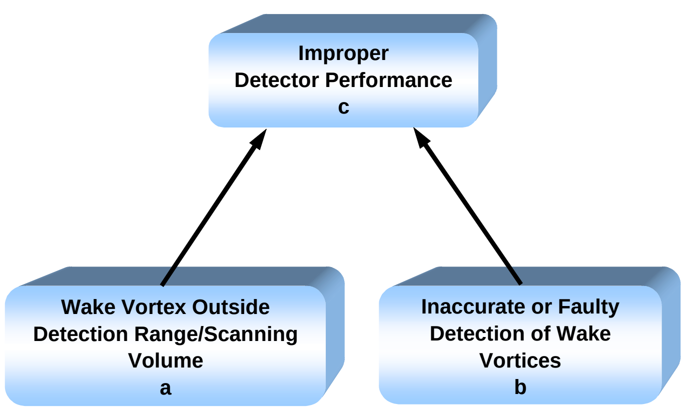


Figure 2.35: Discrete Bayesian belief net for the Improper Detector Performance Variable

Conditional Probability Table for $P(c \mid a, b)$				
a	1		0	
b	1	0	1	0
c=1	1	1	1	0

Table 2-6: Conditional Probability Table for the discrete BBN in Figure 2.35

The marginal distributions of *a* and *b* have been specified as:

a	0	1
	0.999	0.001

b	0	1
	0.999	0.001

Table 2-7: Marginal distributions for nodes *a* and *b* from the discrete BBN shown in Figure 2.35

Using formula 27 we can calculate the joint distribution of (*a*, *b*, *c*) as:

$$P(a, b, c) = P(a)\, P(b)\, P(c \mid a, b).$$

The probability P(*c*=1) can be calculated as a sum of the following probabilities:

$$P(a=1, b=1, c=1) = P(a=1)\, P(b=1)\;\; P(c=1 \mid a=1, b=1) = \left(10^{-3}\right)\cdot\left(10^{-3}\right)\cdot(1) = 10^{-6},$$
$$P(a=1, b=0, c=1) = P(a=1)\, P(b=0)\;\; P(c=1 \mid a=1, b=0) = \left(10^{-3}\right)\cdot(0.999)\cdot(1) = 0.999\times 10^{-3},$$
$$P(a=0, b=1, c=1) = P(a=0)\, P(b=1)\;\; P(c=1 \mid a=0, b=1) = (0.999)\cdot\left(10^{-3}\right)\cdot(1) = 0.999\times 10^{-3},$$
$$P(a=0, b=0, c=1) = P(a=0)\, P(b=0)\;\; P(c=1 \mid a=0, b=0) = (0.999)\cdot(0.999)\cdot(0) = 0.$$

Hence

$$\begin{aligned} P(c=1) &= P(a=1, b=1, c=1) + P(a=1, b=0, c=1) \\ &\quad + P(a=0, b=1, c=1) + P(a=0, b=0, c=1) = 0.0020 \end{aligned}$$

Notice that if one uses different probability tables then the probability of the failure of *c* will change (see the Example below).

EXAMPLE:

Consider three models with the same binary variables *a*, *b* shown in Figure 2.36. The variable *c* is also binary. We want to calculate P(*c*=1) for these three models.

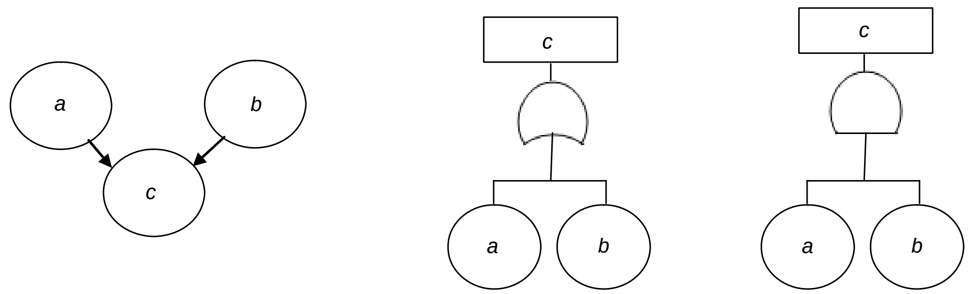


Figure 2.36: BBN, Fault Tree with an OR gate and Fault Tree with an AND gate.

To see how the probability of the failure of *c* changes in different models we first assume that *P*(*a*=1)=(*b*=1)=0.001. In the first case, *a* and *b* influence *c* and the following conditional probability table is specified:

Conditional Probability Table for *P* (*c* I *a* , *b*)				
a	1		0	
b	1	0	1	0
c=1	0.2	0.6	0.4	0.3

Table 2-8: Conditional Probability Table for $P(c \mid a, b)$

In the second case, the failure of event *c* occurs if *a* or *b* fail (OR –gate) and in the last case the failure of event *c* happens if both failures of events *a* and *b* occur (AND-gate). The probability of failure of *c* in OR-gate model was calculated above as 0.0020. Moreover we can easily see that for the AND-gate model the probability of failure of *c* will be equal to $P(a=1, b=1, c=1) = (10^{-3}) \cdot (10^{-3}) \cdot (1) = 10^{-6}$. For the BBN model we get

$$P(a=1, b=1, c=1) = P(a=1)\,P(b=1)\;P(c=1 \mid a=1, b=1) = (10^{-3}) \cdot (10^{-3}) \cdot (0.2) = 2 \times 10^{-7}$$
$$P(a=1, b=0, c=1) = P(a=1)\,P(b=0)\;P(c=1 \mid a=1, b=0) = (10^{-3}) \cdot (0.999) \cdot (0.6) = 5.994 \times 10^{-4}$$
$$P(a=0, b=1, c=1) = P(a=0)\,P(b=1)\;P(c=1 \mid a=0, b=1) = (0.999) \cdot (10^{-3}) \cdot (0.4) = 3.996 \times 10^{-4}$$
$$P(a=0, b=0, c=1) = P(a=0)\,P(b=0)\;P(c=1 \mid a=0, b=0) = (0.999) \cdot (0.999) \cdot (0.3) = 0.2994$$

Summing these probabilities we get that in this case the probability of failure of *c* is equal to 0.3004. Taking different conditional probability tables in the BBN model one can obtain a whole variety of distributions for *c*.

Next we calculate in an analogous way the joint distribution of the lower left-most part of the fault tree corresponding to event *g*, Improper Model Prediction. The probabilities of the failures of *d*, *e*, *f* are all equal to 0.001. Thereby, the Probability of Improper Model Prediction in FT can be calculated as:

$$P(g=1)=1-P(d=0)P(e=0)P(f=0)=0.002997\ .$$

In BBN we must specify the following conditional probability table:

Conditional Probability Table for *P(g I d, e, f)*								
d	1				0			
e	1		0		1		0	
f	1	0	1	0	1	0	1	0
g=1	1	1	1	1	1	1	1	0

Table 2-9: Conditional Probability Table for $P(g\,|\,d, e, f)$

As in the case above we calculate *P*(*g*=1) using formula 27.

The upper parts of the BBN and Fault Tree structures for the wake vortex detection, warning and avoidance maneuver are shown in Figure 2.37 and 2.38. Probabilities of failures for nodes and intermediate events *c* and *g* from the discrete BBN and the Fault Tree were already obtained. We start with FT and then show corresponding calculations in BBN.

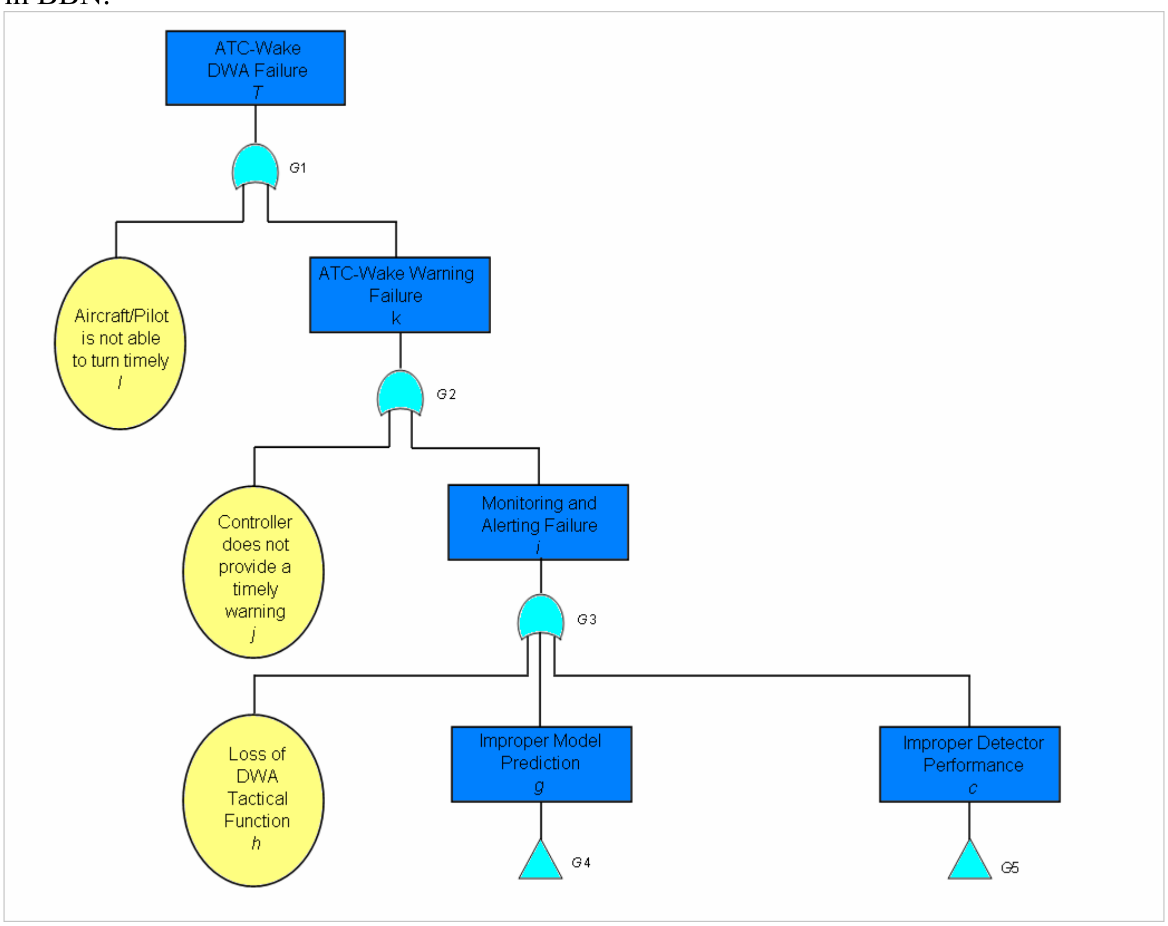


Figure 2.37: Upper part of the Fault Tree structure

Since the ATC-Wake Monitoring and Alerting Failure is caused by the failure of either *c* or *g* or else *h* we can get the probability of Monitoring and Alerting Failure in the fault tree as:

$$
\begin{aligned}
P(i=1) = P((c=1)\bigcup(g=1)\bigcup(h=1)) &= P(c=1)+P(g=1)+P(h=1) \\
&\quad - P(c=1)P(g=1) - P(c=1)P(h=1) - P(g=1)P(h=1) \\
&\quad + P(c=1)P(g=1)P(h=1) \\
&= 0.005985
\end{aligned}
$$

Having the probability of ATC-Wake Monitoring and Alerting Failure and knowing that the Controller does not provide a timely warning with probability 0.001, we can calculate

$$P(k=1) = P((i=1)\bigcup(j=1)) = P(i=1)+P(j=1)-P(i=1)P(j=1) = 0.006979$$

Finally, the desired probability of ATC-Wake DWA Failure is $P(T=1)=0.007972$.

$$P(T=1) = P((k=1)\bigcup(l=1)) = P(k=1)+P(l=1)-P(k=1)P(l=1) = 0.007972$$

Similar calculations can be made using the BBN in Figure 2.38.

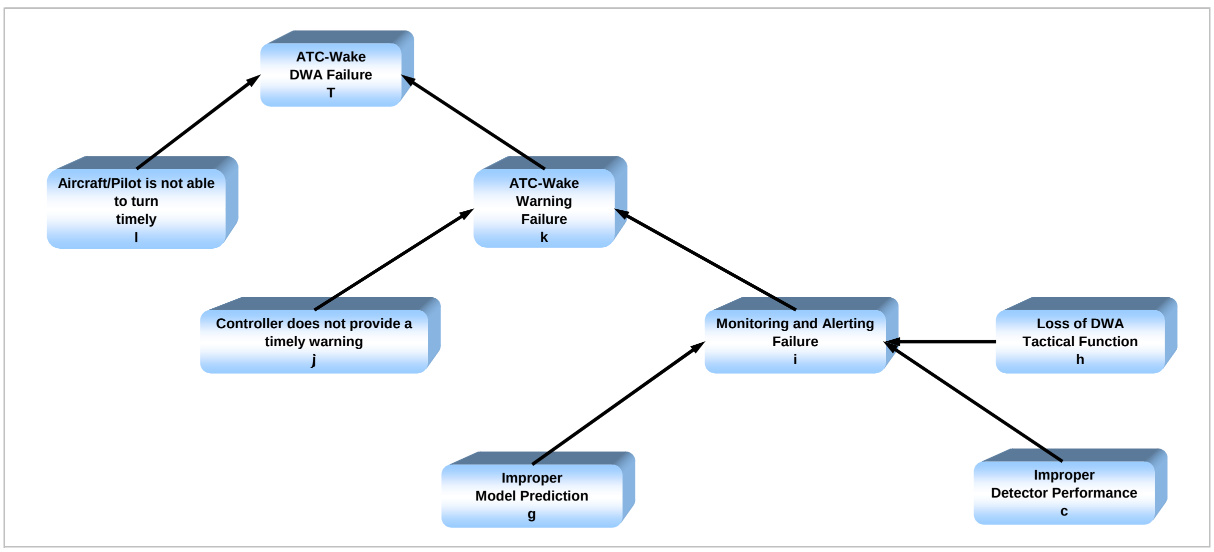


Figure 2.38: Upper part of the BBN structure

To calculate the probability of Monitoring and Alerting Failure given the states of the variables *c*, *g* and *h*, we specify the following conditional probability table and an additional marginal distribution for *h*:

Conditional Probability Table for *P (i l c, g, h)*								
c	1				0			
g	1		0		1		0	
h	1	0	1	0	1	0	1	0
i=1	1	1	1	1	1	1	1	0

h	
0	1
0.999	0.001

Table 2-10: Conditional Probability Table for $P(i \mid c, g, h)$ and marginal distribution for node *h*

Then, to calculate the probability of the failure of *i* given events *c*, *g*, and *h*, we first use formula 27 to get the joint distribution of (*c*, *g*, *h*, *i*)

$$P(c, g, h, i) = P(c)\, P(g)\, P(h)\, P(i \mid c, g, h)$$

After that we can calculate the probability of the Monitoring and Alerting Failure given variables *c*, *g*, and *h* by summing all probabilities in (*c*, *g*, *h*, *i*) for which *i*=1:

$$P(i = 1) = \sum_{c,\, g,\, h} P(c, g, h, i = 1) = 0.005985$$

To calculate the probability of the ATC-Wake Warning Failure we specify the following conditional probability table and an additional marginal distribution of *j*:

Conditional Probability Table for *P (k l i , j)*				
i	1		0	
j	1	0	1	0
k=1	1	1	1	0

j	
0	1
0.999	0.001

Table 2-11: Conditional Probability Table for $P(k \mid i,\ j)$ and marginal distribution for node *j*

The probability of the ATC-Wake Warning Failure is:

$$P(k = 1) = \sum_{i,\, j} P(i,\ j, k = 1) = 0.006979$$

And, finally we specify

Conditional Probability Table for *P (T l* k, l)				
k	1		0	
l	1	0	1	0
T=1	1	1	1	0

l	
0	1
0.999	0.001

Table 2-12: Conditional Probability Table for $P(T \mid k, l)$ and marginal distribution for node *l*

The probability of the ATC-Wake DWA Failure is:

$$P(T=1)=\sum_{k,l} P(k,l,T=1)=0.007972$$

2.2.3 Model Calculations

In this section we combine calculations done in Section 2.2.2 for smaller parts of FT and BBN.

We will start calculations for the fault tree model of wake vortex detection warning and avoidance maneuver in Figure 2.33. These calculations are to be done for several risky specifications. Initial data used to obtain insight is given in Appendix A.3.

We will apply the MOCUS Algorithm for finding Minimal Cut Sets. This algorithm substitutes each gate formula and applies Boolean Laws. The outcome of this algorithm is a list of cut sets with the property that every minimal cut set is included as a subset of the list of cut sets. After the final iteration all duplicated cut sets must be removed and all cut sets must be checked to insure that they are minimal cut sets.

We are going to apply the MOCUS Top Down algorithm. We will start at the top event T in Figure 2.33. We substitute the Boolean expression for each gate and calculate as follows

$$\begin{aligned}
&T = G1 = l + k = l + G2\\
&T = l + (j + G3) = l + j + G3\\
&T = l + j + (h + g + c) = l + j + h + g + c = l + j + h + G4 + G5\\
&T = l + j + h + (d + e + f) + (a + b)\\
&T = l + j + h + d + e + f + a + b
\end{aligned} \tag{30}$$

Hence, we get the minimal cut sets as shown in Section 2.2.2. The minimal cut sets are listed below.

Minimal cut sets: $\{l\}$, $\{j\}$, $\{h\}$, $\{d\}$, $\{e\}$, $\{f\}$, $\{a\}$, $\{b\}$.

With the minimal cut sets we can build a cut set representation for our problem (see Figure 2.39). We can observe that the Top event can be obtained by the union of eight basic events joined by an OR gate, which tells us that the ATC-Wake DWA Failure is caused by failures of any one of eight basic events.

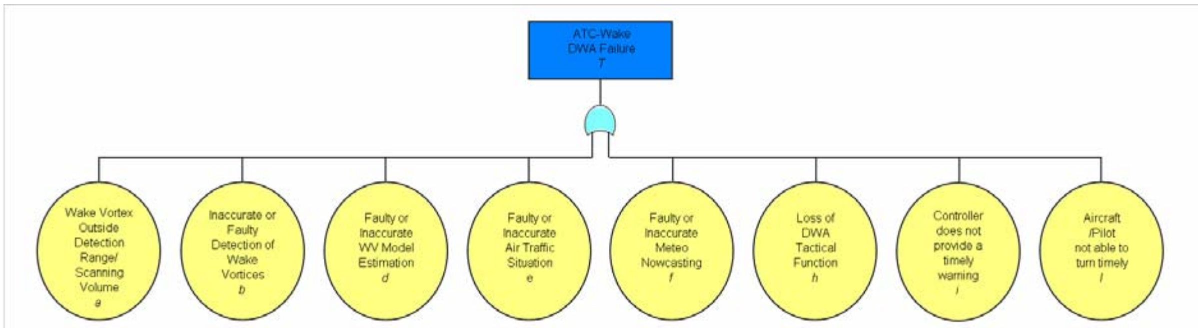


Figure 2.39: Cut set representation of the fault tree

These events are not mutually exclusive so the probability of the ATC-Wake DWA Failure with the formula:

$$P\left(a\bigcup b\bigcup d\bigcup e\bigcup f\bigcup h\bigcup j\bigcup l\right)=\sum_{x=a}^{l}P(x)-\sum_{x\neq y}P\left(x\bigcap y\right)+\sum_{x\neq y\neq z}P\left(x\bigcap y\bigcap z\right)-\cdots+(-1)^{7}P\left(a\bigcap\cdots\bigcap l\right) \qquad (31)$$

where all the summation literals $x, y, z, \ldots$ belong to $\{a, b, d, e, f, h, j, l\}$ in our case.

This is known as the inclusion-exclusion formula. If we develop such a formula we will have 2^8-1=255 terms to calculate the probability of the ATC-Wake DWA Failure. Probabilities of Failures of events in FT are summarized in the following table.

a	b	c	d	e	f	g	h	i	j	k	l	T
0.001	0.001	0.001999	0.001	0.001	0.001	0.002997	0.001	0.005985	0.001	0.006979	0.001	0.007972

Table 2-13: Probabilities of failure for all variables

Main ideas behind BBNs and probability tables necessary to quantify the BBN in Figure 2.32 were presented in Section 2.2.2. Joint distribution represented by this structure can be calculated with formula 27. There exist many BBN software packages supporting fast calculations and specification of BBNs. In Figure 2.40, the screen shot of the BBN of the WV DWA maneuver in Netica is shown. Marginal probabilities of all variables (in percentages) are given. We can see that the probability of ATC-Wake DWA Failure is about 0.008.

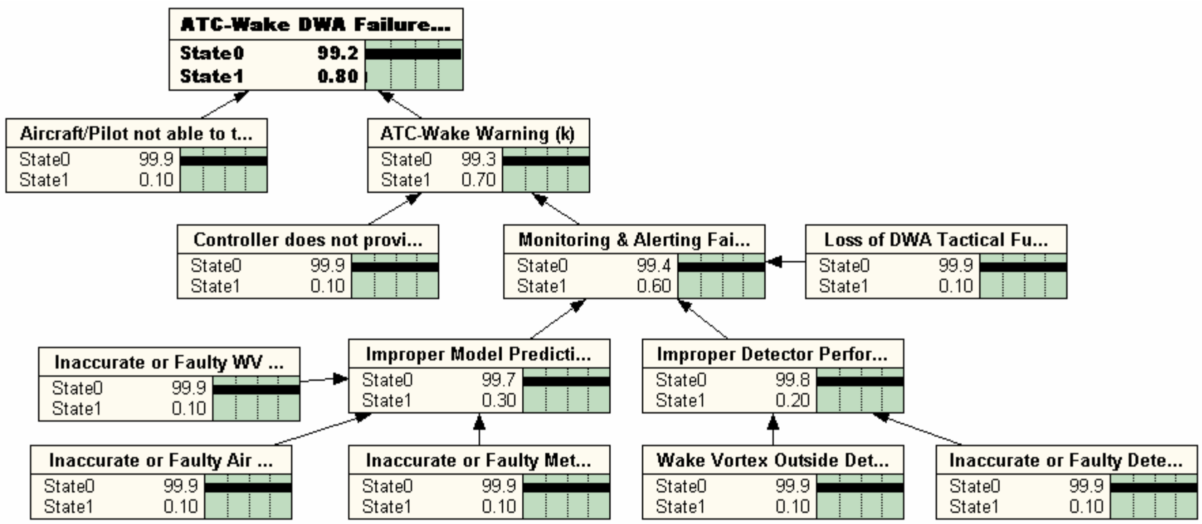


Figure 2.40: BBN for the WV DWA maneuver probability.

In Section 2.2 we have shown relationships between Fault trees and BBNs. Problem that was used to stress similarities of BBNs and FTs could have been handled by both as all variables were binary and dependencies between them were of the form easily handled by OR-gates. Moreover there were no dependencies between basic events. We have shown that this problem can be modelled by fault tree and the corresponding BBN can be easily built.

The graphical representation of FTs contains more information about underlying joint distribution than the directed acyclic graph for BBN. However, it is much more restrictive in dependence structures that can be described. BBN allows their nodes to have more than two states. It does not restrict 'basic' variables to be independent and allows much richer set of dependence structures to be used. It comes with the price of course. Quantification of discrete BBNs is very cumbersome.

Finally, we can point out that the choice of model always depends on the statement of the problem to solve. If the assumptions of FTs cannot be fulfilled we must turn to richer model e.g. BBNs. If however the problem can be handled with FTs (as the one presented in this section) we recommend use of the simplicity of FTs.

3. I-WAKE MODEL: DISCRETE BAYESIAN BELIEF NET

For a description of the *current practice* approach operation, as well as the approach operation *as proposed in the I-Wake concept* refer to [De Jong et al. 2005, Speijker et al. 2006a].

3.1 I-Wake: On-board Wake Vortex Detection Warning, and Avoidance Probability

The execution of the I-Wake detection, warning, and avoidance maneuver (e.g. a missed approach (during arrivals)) depends – besides operational feasibility – on the probability of failure of the I-Wake system components. For the assessment of the on-board I-Wake failure probability, a causal model has been constructed using discrete Bayesian Belief Networks (BBNs) (See figure 3.1).

3.1.1 Application of Discrete BBNs for the On-board Wake Vortex Detection, Warning and Avoidance Maneuver Probability

In this Section, we aim to model a system which represents the on-board wake vortex detection, warning and avoidance maneuver failure. Moreover, we intend to quantify such a model. A discrete BBN will be used since the problem initially for two reasons:

- To obtain insight into the setting of requirements for the I-Wake operation
- To support the I-Wake system design and development

From the problem statement, we will be able to constrain the variables to take two states. The assumptions taken in to account to represent the BBN of Section 2.2 as a Fault Tree, namely, a failure of a parent-node occurs if any of its children-nodes fails, does not hold here. Hence, an appropriate approach which may be applied to model the failure of the aircraft/pilot in performing the I-Wake WV DWA maneuver amounts to discrete BBNs. Throughout Section 3.1, we will give a detailed description of the application of the methodology to this particular case.

3.1.2 Discrete BBNs

The problem above can be modelled using discrete BBNs as shown in Figure 3.1. This BBN has 11 nodes and 10 edges. The nodes correspond to binary variables with states 0 (not failure) and 1 (failure) specified in the problem and edges that represent relationships between variables. Each BBN node is labelled with a lowercase literal and the full name of the variable that is of interest. Names or labels are used in the text to refer to the variables. The BBN in Figure 3.1 shows the graphical representation of dependences and (conditional) independences in this problem. One reads that d, e, f are independent and they influence g (g is a child of d, e, f and d, e, f are called the parents of g; denoted as $pa(g)$). As we said before, a graphical representation of BBN does not, however, give us all the information we need about joint distribution. To completely describe the joint distribution represented by BBN, we need to specify conditional probability tables of all variables in the BBN given their parents. Then the joint distribution can be given by formula 27 in Chapter 2.

Due to the statement of the problem, we will need the specification of the conditional probability tables of $P(T \mid j, i)$, $P(i \mid h, g, c)$, $P(g \mid f, e, d)$ and $P(c \mid b, a)$.

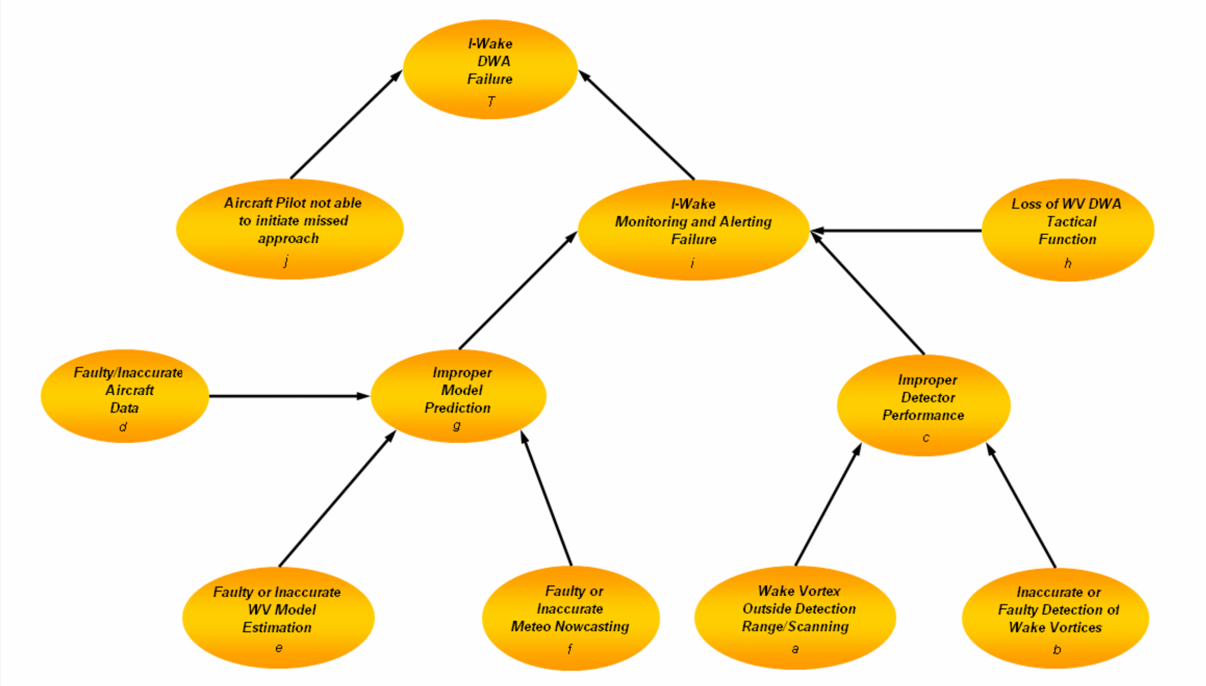


Figure 3.1: BBN for the on-board WV DWA maneuver probability.

The nodes in the Bayesian Belief Network have the following explanation:

- ***I-Wake DWA Failure (T):*** represents the probability distribution of aircraft/pilot not able to perform the I-Wake Detection, Warning and Avoidance Maneuver as required.
- ***Aircraft/Pilot not able to initiate missed approach (j):*** represents the probability of an aircraft/pilot not able to initiate an evasive action (missed approach) as required.
- ***I-Wake Monitoring and Alerting Failure (i):*** represents the probability of not providing a timely warning to the flight crew when one should be given. As a result, the crew might NOT be able to initiate/instruct the pilot to perform an evasive action.
- ***Loss of WV DWA Tactical Function (h):*** represents the probability of an undetected loss of the WV DWA function. In case of a Detected Loss, the crew is aware that NO cautions/warnings will be given and a transition will be made to the ICAO Mode (the separation will increase, and the DWA maneuver will not be necessary).
- ***Improper Model Prediction (g):*** represents the probability that the predictions of Wake Vortex locations and strength are inaccurate/wrong.
- ***Faulty/Inaccurate Aircraft Data (d):*** represents the probability that the predictions of wake vortex locations and/or strengths made by the WV Model, on the basis of other aircraft data, are inaccurate/wrong. As a result, incorrect information is passed to the WV DWA tactical function, causing improper functioning.
- ***Inaccurate or Faulty WV Model Estimation (e):*** represents the probability that the predictions of wake vortex locations and/or strengths made by the WV Model, on the basis of aircraft data and meteo data, are inaccurate/wrong. As a result, incorrect

information is passed to the WV DWA tactical function, causing improper functioning.

- ***Inaccurate or Faulty Meteo Nowcasting (f):*** represents the probability that the meteorological conditions (i.e. nowcasting data) provided by the meteo systems are inaccurate or wrong. As a result, incorrect information is passed to the WV DWA tactical function, causing improper functioning.
- ***Improper Detector Performance (c):*** represents the probability that the tactical WV DWA (e.g. on-board LiDAR) performs significantly less than the crew expect (while they are not aware of the inaccuracies) (i.e. inaccurate/wrong alerts are given).
- ***Wake Vortex Outside Detection Range/Scanning Volume (a):*** represents the probability that the on-board LiDAR does not detect the wake vortices of the leading aircraft, because these are outside the scanning volume of air ahead of the aircraft.
- ***Inaccurate or Faulty Detection of Wake Vortices (b):*** represents the probability that the on-board LiDAR does not detect wake vortices of the leading aircraft accurately, when these are inside the planned scanning volume of air ahead of the aircraft.

Figure 3.2 contains the lower right-most fragment of the BBN. The nodes are Wake Vortex Outside Detection Range/ Scanning Volume (a) and Inaccurate or Faulty Detection of Wake Vortices (b).

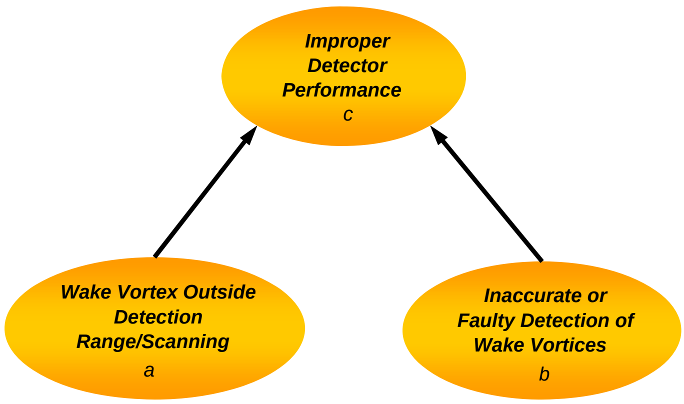


Figure 3.2: Discrete Bayesian belief net for the Improper Detector Performance Variable

The probabilities of a, b should be specified. We assume that $P(a=1)=P(b=1)=0.001$. From Figure 3.2 one knows that c is influenced by a and b but the nature of this dependence cannot be concluded from the graphical representation. How the state of c is affected by combinations of states of a and b must be read from conditional probability table of c given a, b. Such a conditional probability table may be very general and may describe many different types of dependencies between variables. Different probability tables will lead to different probability of failure of c. The conditional probability table will be as shown in Table 3-1.

Conditional Probability Table for $P(c \mid a, b)$				
a	1		0	
b	1	0	1	0
c=1	0.9999	0.001	0.001	0.0001

Table 3-1: Conditional Probability Table for $P(c \mid a, b)$ in the discrete BBN in Figure 3.2

The marginal distributions of *a* and *b* have been specified as:

a	0	1
	0.999	0.001

b	0	1
	0.999	0.001

Table 3-2: Marginal distributions for nodes *a* and *b* from the discrete BBN shown in Figure 3.2[18]

Using formula 27 in Chapter 2 we can calculate the joint distribution of (*a*, *b*, *c*).

$$P(a, b, c) = P(a)\, P(b)\, P(c \mid a, b)\,.$$

For the BBN model we get

$$P(a=1, b=1, c=1) = P(a=1)\,P(b=1)\;P(c=1 \mid a=1, b=1) = \left(10^{-3}\right)\cdot\left(10^{-3}\right)\cdot(0.9999) = 9.999\times 10^{-7}$$
$$P(a=1, b=0, c=1) = P(a=1)\,P(b=0)\;P(c=1 \mid a=1, b=0) = \left(10^{-3}\right)\cdot(0.999)\cdot\left(10^{-3}\right) = 9.99\times 10^{-7}$$
$$P(a=0, b=1, c=1) = P(a=0)\,P(b=1)\;P(c=1 \mid a=0, b=1) = (0.999)\cdot\left(10^{-3}\right)\cdot\left(10^{-3}\right) = 9.99\times 10^{-7}$$
$$P(a=0, b=0, c=1) = P(a=0)\,P(b=0)\;P(c=1 \mid a=0, b=0) = (0.999)\cdot(0.999)\cdot\left(10^{-4}\right) = 9.98\times 10^{-5}$$

Summing these probabilities we get that in this case the probability of failure of *c* is equal to 1.02798×10^{-4}. Taking different conditional probability tables in the BBN model one can obtain a whole variety of distributions for *c*.

Turn to the left-most part of the BBN in Figure 3.1. The children nodes are Faulty or Inaccurate Aircraft Data (*d*), Faulty or Inaccurate WV Model Estimation (*e*) and Faulty or Inaccurate Meteo Nowcasting (*f*). The probabilities of *d*, *e* and *f* are specified, namely, $P(d=1)=P(e=1)=P(f=1)=0.001$.

[18] Another possibility to specify the marginal distributions of each node for the discrete BBN for the on-board WV DWA manoeuvre probability would be the elicitation of quantiles from experts. The questionnaire is shown in Appendix A.5. This will allows taking into account the information from not only an expert and this information could be analyzed by the method described in Section 2.1.3.

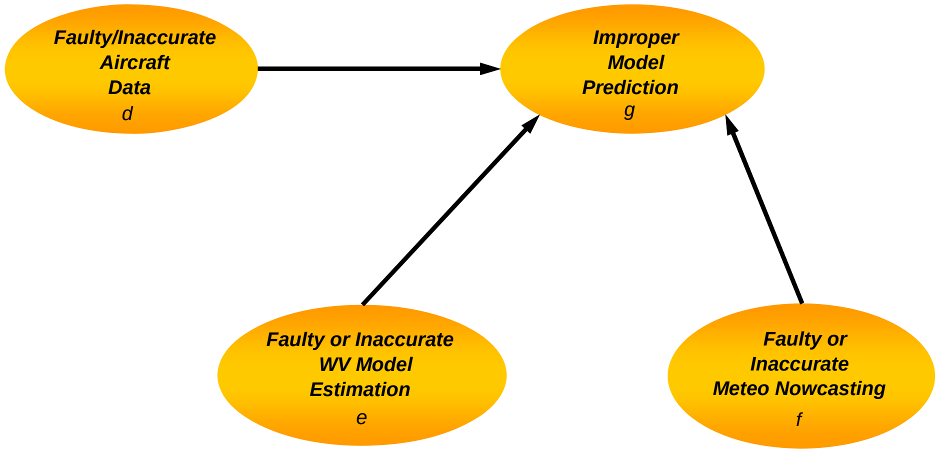


Figure 3.3: Discrete Bayesian belief net for an Improper Model Prediction Variable

From Figure 3.3 one knows that *g* is influenced by *d*, *e* and *f* but as we said before the nature of this dependence cannot be deduced from this visual representation. How the state of *g* is affected by the combinations of states of *d*, *e* and *f* must be read from the conditional probability table of *g* given *d*, *e* and *f*. The conditional probability table will be as shown in Table 3-3.

Conditional Probability Table for *P (g I f, e, d)*								
d	1				0			
e	1		0		1		0	
f	1	0	1	0	1	0	1	0
g=1	0.999999	0.001	0.001	0.0001	0.001	0.0001	0.0001	0.000001

Table 3-3: Conditional probability table for $P(g \mid f, e, d)$

The marginal distributions of *d*, *e* and *f* have been specified as follows:

d	0	1
	0.999	0.001

e	0	1
	0.999	0.001

f	0	1
	0.999	0.001

Table 3-4: Marginal distributions for nodes *d*, *e* and *f* from the discrete BBN shown in Figure 3.3

Using formula 27 Chapter 2 we can calculate the joint distribution of (*d*, *e*, *f*, *g*).

$$P(d, e, f, g) = P(d)\, P(e)\, P(f)\, P(g \mid d, e, f)\,.$$

For the BBN model we get the probability of the failure of *g* given the variables *d*, *e*, and *f* by summing all the probabilities in (*d*, *e*, *f*, *g*) for which *g*=1:

$$P(g = 1) = \sum_{c,\, g,\, h} P(d, e, f, g = 1) = 1.3004 \times 10^{-6}$$

The upper part of the BBN structure for the on-board wake vortex detection, warning and avoidance maneuver is shown in Figure 3.4. The probabilities of failures for nodes *c* and *g* from the discrete BBN were already obtained.

Since a failure of the I-Wake Monitoring and Alerting is caused by the failure of *c*, *g* and *h* we can get the probability of the I-Wake Monitoring and Alerting Failure using BBN in Figure 3.4.

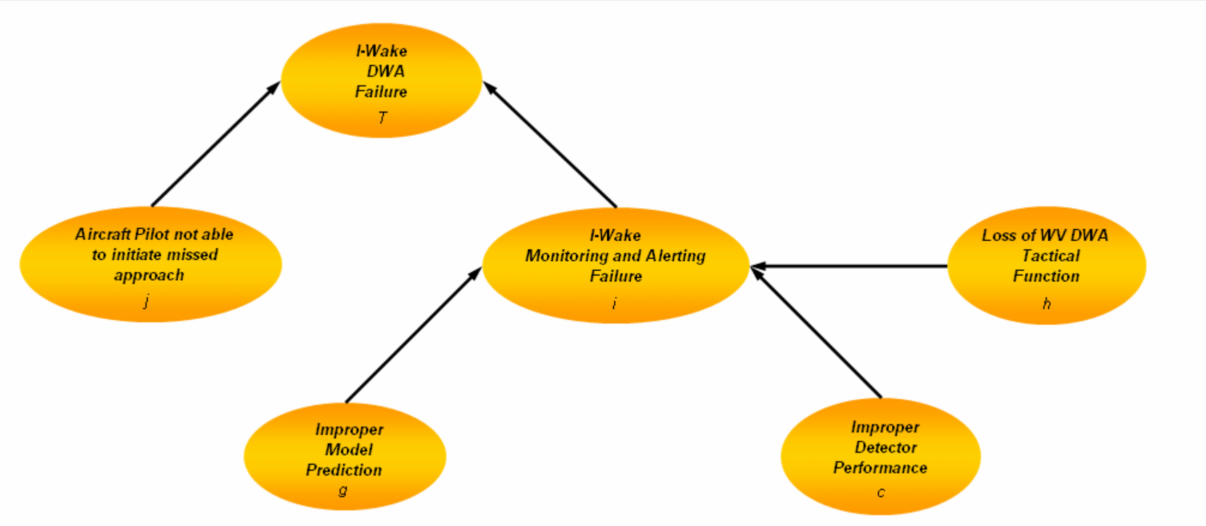


Figure 3.4: Upper part of the BBN structure

To calculate the probability of the I-Wake Monitoring and Alerting Failure given the states of the variables *c*, *g* and *h*, we specify the following conditional probability table and an additional marginal distribution for *h*:

Conditional Probability Table for *P* (*i* I *c, g, h)*								
c	1				0			
g	1		0		1		0	
h	1	0	1	0	1	0	1	0
i=1	0.999999	0.001	0.001	0.0001	0.001	0.0001	0.0001	0.000001

h	
0	1
0.999	0.001

Table 3-5: Conditional Probability Table for $P(i \mid c, g, h)$ and marginal distribution for node h

Then, to calculate the probability of failure of *i* given events *c*, *g*, and *h*, we first use formula 27 Chapter 2 to get the joint distribution of (*c*, *g*, *h*, *i*)

$$P(c, g, h, i) = P(c)\, P(g)\, P(h)\, P(i \mid c, g, h)$$

After that we can calculate the probability of the I-Wake Monitoring and Alerting Failure given the variables *c*, *g*, and *h* by summing all the probabilities in (*c*, *g*, *h*, *i*) for which *i*=1:

$$P(i=1) = \sum_{c,\,g,\,h} P(c, g, h, i=1) = 1.10939 \times 10^{-6}$$

To calculate the probability of the I-Wake DWA Failure, we take into account that i and j influence T and we specify the following conditional probability table:

Conditional Probability Table for $P(T \mid i, j)$				
i	1		0	
j	1	0	1	0
$T=1$	0.9999	0.001	0.001	0.0001

j	
0	1
0.999	0.001

Table 3-6: Conditional Probability Table for $P(T \mid j, i)$

And since $P(j=1) = 0.001$, we can calculate:

$$P(i=1,\ j=1, T=1) = P(i=1)P(j=1)P(T=1|i=1,\ j=1) =$$
$$= (1.10939 \times 10^{-6})(0.001)(0.9999) = 1.10928 \times 10^{-9}$$
$$P(i=1,\ j=0, T=1) = P(i=1)P(j=0)P(T=1|i=1,\ j=0) =$$
$$= (1.10939 \times 10^{-6})(0.999)(0.001) = 1.10828 \times 10^{-9}$$
$$P(i=0,\ j=1, T=1) = P(i=0)P(j=1)P(T=1|i=0,\ j=1) =$$
$$= (0.999999)(0.001)(0.001) = 9.99999 \times 10^{-7}$$
$$P(i=0,\ j=0, T=1) = P(i=0)P(j=0)P(T=1|i=0,\ j=0) =$$
$$= (0.999999)(0.999)(0.0001) = 9.98999 \times 10^{-5}$$

And, finally the probability of the I-Wake DWA Failure is:

$$P(T=1) = \sum_{i,\,j} P(i,\ j, T=1) = 1.00902 \times 10^{-4}$$

3.1.3 Models Calculations

Main ideas behind BBNs and probability tables necessary to quantify the BBN in Figure 3.1 were presented in Section 3.1.2. The data required to quantify such a model are not real and are only used to show the use of the methodology (Two questionnaires are proposed to elicit data for the input of this model in Appendix A.4). The joint distribution represented by this structure can be calculated with formula 27 Chapter 2. There exist many BBN software packages supporting fast calculations and specification of BBNs. In Figure 3.5, the BBN of the on-board WV DWA maneuver in Netica is shown. Marginal probabilities of all variables (in percentages) are given. We can see that the probability of the I-Wake DWA Failure is about 0.0100902 in the BBN (probabilities are given in percentages).

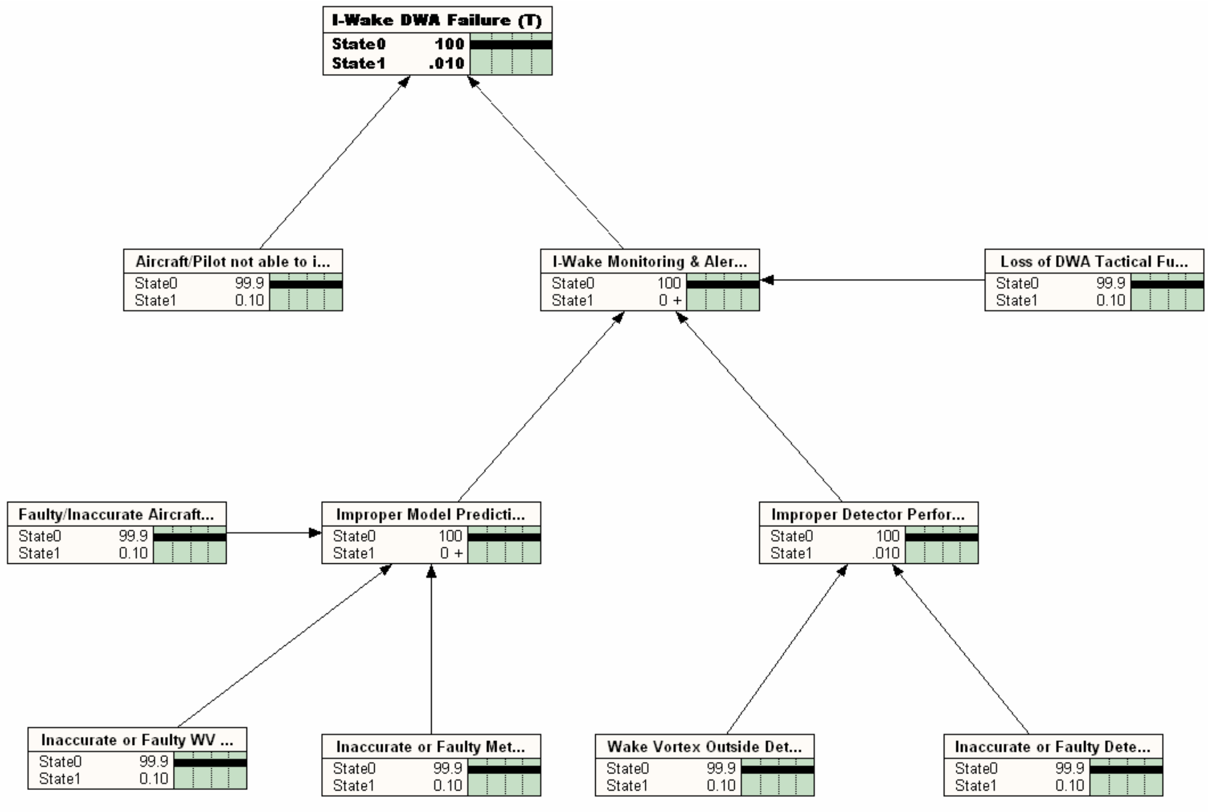


Figure 3.5: BBN for the on-board WV DWA maneuver probability.

We have shown that this problem can be modelled by discrete BBNs. The graphical representation of the directed acyclic graph for BBN does not contain much information about the underlying joint distribution. However, it is much rich in dependence structures that can be described. BBN allows their nodes to have more than two states. It does not restrict 'basic' variables to be independent and allows much richer set of dependence structures to be used. It comes with the price of course. The quantification of discrete BBNs is very cumbersome.

Finally, we can point out that the choice of model always depends on the statement of the problem to solve. In this case the appropriate model was: Discrete BBNs.

4. BAYESIAN BELIEF NETS

This Chapter is just an overview of the main definitions about BBNs and related concepts. It is based on [Kurowicka D., Cooke R.M., Charitos T., Speijker L.J.P. 2001; Kurowicka D., Cooke R.M. 2006].

4.1 Definitions

A Bayesian Belief Network is a probabilistic model based on the notion of conditional independencies and dependencies (CIDs).

Definition 4.1.1. Conditionally Independent Variables. Two variables X_1, X_2 are independent conditional on some other variables contained in the set *S*, if the conditional probability factors like $p(X_1, X_2|S) = p(X_1|S)\,p(X_2|S)$ given $p(S) > 0$. This is equivalent to $p(X_1|X_2, S) = p(X_1|S)$ or $p(X_2|X_1, S) = p(X_2|S)$ provided that $p(X_1, S), p(X_2, S), p(S) > 0$. This means that X_2 is irrelevant to X_1 if the joint state of the variables in the set S is known.

Definition 4.1.2. Independent Events. *The events A and B are independent if and only if $p(A \cap B) = p(A)p(B)$.*

Definition 4.1.3. Conditionally Independent Events
Weak definition: *The events A and B are conditionally independent given the event C if and only if $p(A \cap B\,|\,C) = p(A\,|\,C)p(B\,|\,C)$. This is written as $A \perp B|\ C$. It is assumed that $p(C) > 0$.*

If C has probability one, then the weak definition is just a restatement of the definition of independence. It is a symmetric relation between A and B because $A \perp B|\ C$ implies $B \perp A|\ C$. It also implies that $A \perp B^c|\ C$. However it neither implies nor is implied by $A \perp B|\ C^c$. To assume that $A \perp B|\ C$ means that A and B are independent if C occurs and does not say anything about the relation of between A and B if C does not occur.

Strong definition: *The events A and B are independent given any event in the partition of the sample space generated by the events C and D: $C \cap D, C^c \cap D, C \cap D^c, C^c \cap D^c$. That is*

$$A \perp B\,|\,[C, D] \Leftrightarrow A \perp B\,|\,C \cap D,\ A \perp B\,|\,C^c \cap D,\ A \perp B|C \cap D^c,\ A \perp B\,|\,C^c \cap D^c.$$

Note that $A \perp B\,|\,[C, D]$ does not imply that $A \perp B\,|\,C$ or that $A \perp B\,|\,C \cup D$.

4.2 Discrete Bayesian Belief Nets

A finite valued BBN is a directed acyclic graph, together with an associated set of probability tables. The graph consists of nodes and arcs. The nodes represent variables, which can be discrete or continuous. The arcs represent causal/influential or functional relationships between variables.

Throughout this Section, let the set of all variables in a domain be denoted as V, and the variables as $X_1, X_2, \ldots \in V$. The conditional independencies and dependencies (CIDs) underlying a multivariate probability distribution for the variables in V are reflected by the graphical structure of a Bayesian Belief Network, the so-called *directed acyclic graph* (DAG).

A Bayesian Belief Network (BBN) consists of a set of nodes and a set of directed edges between nodes. The nodes represent variables, which can be discrete or continuous. The edges or arcs represent causal/ influential relationships between variables.

The parents $pa(X_1)$ of a variable X_1 in the DAG, is the set of variables $U \in V$, such that there exists a directed edge $U \to X_1$. The variable X_1 is called a child of U. To each variable W with parents $X_1, \ldots, X_n$ there is attached a conditional probability table $P(W \mid X_1, \ldots, X_n)$. In case W has no parents, the associated probability table reduces to unconditional probabilities $P(W)$.

A definition for Bayesian Belief Networks can be the following.

Definition 4.2.1: *A Bayesian Belief Network for a set of variables $V = \{X_1, \ldots, X_n\}$ consists of (1) a network structure S that encodes a set of conditional independence assertions about variables in V, and (2) a set P of local probability distributions associated with each variable. Together, these components define the joint probability distribution for V. The network structure S is a directed acyclic graph. The nodes in S are in one-to-one correspondence with the variables in V.*

We use X_i to denote both the variable and its corresponding node, and $pa(X_i)$ to denote the parents of node X_i in S as well as the variables corresponding to those parents. The absence of possible arcs in S implicitly means conditional independence.

Theorem 4-2-1: **(The Chain Rule)**. *Given the structure S, the joint probability distribution for $V = \{X_1, \ldots, X_n\}$ is given by*

$$P(V) = \prod_{i=1}^{n} p\left(X_i \middle| pa(X_i)\right) \qquad (1)$$

The aforementioned theorem illustrates the importance of exploiting conditional independence in order to calculate the joint probability table $P(V)$. The set of parameters θ

of a Bayesian network model is the set of conditional probabilities $p(X_i|pa(X_i))$, where $pa(X_i)$ denotes the parents of a variable X_i in the DAG. This sort of factorization has two consequences regarding learning Bayesian Belief Networks. Firstly, since each of the conditional probabilities typically involves only a small number of variables, i.e. $|pa(X_i)| << |V|$ for all $i \in V$, the parameters of a Bayesian Belief Network can be estimated from finite data. Secondly, the parameters of a Bayesian Belief Network, since they are conditional probabilities, can be calculated from the probability distribution implied by the data.

The structure S of a Bayesian Belief Network has two components: the global structure G and a set of local structures M. G is, as we mention previously, a directed acyclic graph, while the set of local structures $M = \{M_1, ..., M_n\}$ is a set of n mappings, one for each variable X_i, such that M_i maps each value of $\{X_i, pa(X_i)\}$ to a parameter in θ.

Evidence: *There are two kinds of evidence that can be entered in a Bayesian Belief Network and change the probability distributions of variables, hard and soft evidence. If the evidence on a variable or otherwise a statement of the certainties of its states gives the exact state of the variable, we call this hard evidence or instantiation; otherwise we call it soft evidence.*

4.3 Continuous Bayesian Belief Nets

The Vine-copula method presented in this section is another tool that enables the user to construct a multivariate distribution. In this method we need one dimensional marginal distributions and information about the dependence structure. Marginal distributions and dependence can be obtained using expert opinion. In this section we present general description of this method. We introduce only basic definitions and procedures, which will be used during this project. For more information about vines and copulas we refer to [Bedford T.J., Cooke R.M. 2002; Kurowicka D., Cooke R.M. 2004; Hanea A., Kurowicka D., Cooke R.M. 2005; Kurowicka D., Cooke R.M. 2006].

4.3.1 Vines

A graphical model called vines was introduced in [Cooke R.M. 1997]. A *vine* on *n* variables is a nested set of trees, where the edges of tree j are the nodes of tree $j+1$, and each tree has the maximum number of edges. A *regular* vine on *n* variables is a vine in which two edges in tree j are joined by an edge in tree $j+1$ only if these edges share a common node. A regular vine is called a *canonical* vine if each tree T_i has a unique node of degree $n-i$, hence has maximum degree. A regular vine is called *a* D*-vine* if all nodes in T_1 have degree not higher than 2 (see Figure 4.1). There are $n(n-1)/2$ edges in a regular vine on *n* variables. Each edge in a regular vine may be associated with a constant conditional rank correlation (for $j=1$ the conditions are vacuous) and, using a copula (bivariate distribution on a unite square with uniform margins), a joint distribution satisfying the vine-copula specification can be constructed and sampled on the fly [Cooke R.M. 1997]. The conditional rank correlations associated with each edge are determined as follows: the variables reachable from a given edge are called the constraint set of that edge. When two edges are joined by an edge of the next tree, the intersection of the respective constraint sets are the conditioning variables, and the symmetric differences of the constraint sets are the conditioned variables. The regularity condition insures that the symmetric difference of the constraint sets always contains two variables. Each pair of variables occurs once as conditioned variables. For the precise definitions and all properties of a regular vine we refer to [Bedford T.J., Cooke R.M. 2002].

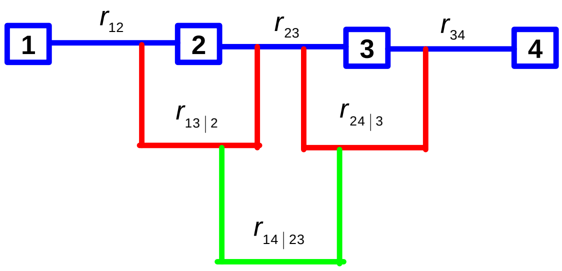


Figure 4.1: D-vine on four variables with (conditional) rank correlations assigned to the edges.

The rank correlation specification on regular vine plus copula determines the whole joint distribution. To sample a distribution specified by the D-vine in Figure 4.1, $D(1,2,3,4)$ the following algorithm can be used: Sample four independent variables distributed uniformly on interval $[0,1]$, U_1, U_2, U_3, U_4 and calculate values of correlated variables X_1, X_2, X_3, X_4 as follows

$$
\begin{aligned}
x_1 &= u_1; \\
x_2 &= F^{-1}\left(u_2 \mid x_1; r_{12}\right); \\
x_3 &= F^{-1}\left(F^{-1}\left(u_3 \mid F\left(x_1 \mid x_2; r_{12}\right); r_{13|2}\right) \mid x_2; r_{23}\right); \\
x_4 &= F^{-1}\left(F^{-1}\left(F^{-1}\left(u_4 \mid F\left(F\left(x_1 \mid x_2; r_{12}\right) \mid F\left(x_3 \mid x_2; r_{23}\right); r_{13|2}\right); r_{14|23}\right) \mid F\left(x_2 \mid x_3; r_{23}\right); r_{24|3}\right) \mid x_3; r_{34}\right)
\end{aligned}
$$

(2)

where $F\left(X_j \mid X_j; r_{ijkl}\right)$ denotes the cumulative distribution function for X_j given X_i under the conditional copula with correlation r_{ijkl}.

To shorten the notation that will be used in describing the general sampling procedure for D-vine the above algorithm can be stated as:

$$
\begin{aligned}
x_1 &= u_1; \\
x_2 &= F^{-1}_{2|1;x_1}\left(u_2\right); \\
x_3 &= F^{-1}_{3|2;x_2}\left(F^{-1}_{3|12;F_{1|2}(x_1)}\left(u_3\right)\right); \\
x_4 &= F^{-1}_{4|3;x_3}\left(F^{-1}_{4|23;F_{2|3}(x_2)}\left(F^{-1}_{4|123;F_{1|23}(x_1)}\left(u_4\right)\right)\right).
\end{aligned}
$$

Figure 4.2 shows the procedure of sampling value of X_4 graphically. Notice that for the D-vine values of $F_{2|3}$ and $F_{1|23}$ that are used to conditionalize copulae with correlations $r_{24|3}$ and $r_{14|23}$ to obtain $F_{4|23}$ and $F_{4|123}$, respectively have to be calculated. In Figure 4.2 the diagonal band copula [Cooke R.M., Waij R. 1986] is used.

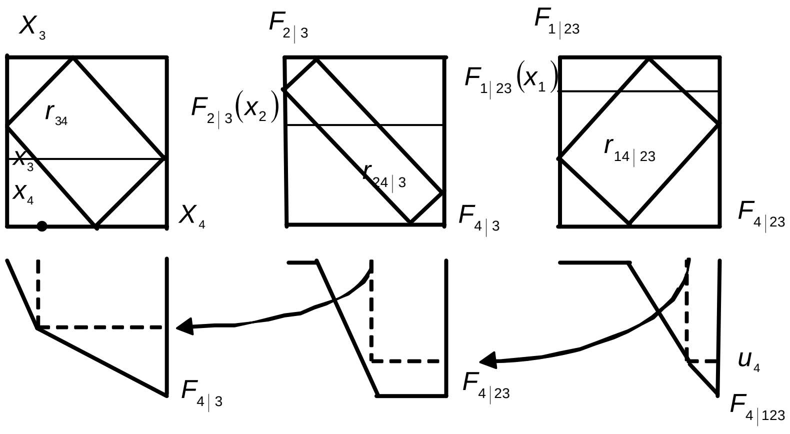


Figure 4.2: Procedure of sampling value of X_4 in D-vine.

In general we can sample an n-dimensional distribution represented graphically by the D-vine on n variables with (conditional) rank correlations

$$
\begin{array}{cccccc}
r_{12}, & r_{13|2} & r_{14|23} & \cdots & r_{1,n-1|n-2,\ldots,2} & r_{1,n|2,\ldots,n-1} \\
 & r_{23} & r_{24|3} & \cdots & r_{2,n-1|n-2,\ldots,3} & r_{2,n|3,\ldots,n-1} \\
 & & & & \cdots & \cdots \\
 & & & & r_{n-2,n-1} & r_{n-2,n|n-1}
\end{array}
$$

assigned to the edges of the vine as follows

$$
\begin{aligned}
x_1 &= u_1; \\
x_2 &= F^{-1}_{2|1;x_1}(u_2); \\
x_3 &= F^{-1}_{3|2;x_2}\left(F^{-1}_{3|12;F_{1|2}(x_1)}(u_3)\right); \\
x_4 &= F^{-1}_{4|3;x_3}\left(F^{-1}_{4|23;F_{2|3}(x_2)}\left(F^{-1}_{4|123;F_{1|23}(x_1)}(u_4)\right)\right); \\
&\cdots \\
x_n &= F^{-1}_{n|n-1;x_{n-1}}\left(F^{-1}_{n|n-2,n-1;F_{n-2|n-1}(x_{n-2})}\left(\ldots F^{-1}_{n|123,\ldots,n-1;F_{1|23,\ldots,n-1}(x_1)}(u_n)\ldots\right)\right)
\end{aligned}
\tag{3}
$$

The rank correlation is actually a measure of the dependence between two random variables joint by the copula. The rank correlation specification on regular vine plus copula determines the whole joint distribution. The procedure of sampling such a distribution can be written for any regular vine.

4.3.2 Copula

Definition (Copula) *A* **copula** C *is a distribution on the unit square with uniform margins.*

Copulas are then functions that join or "couple" bivariate distribution functions to their marginal distribution functions.

Definition *Random variables* X *and* Y *with distributions* F_X, F_Y *respectively are joined by copula* C *if their joint distribution can be written*

$$F_{XY}(x, y) = C(F_X(x), F_Y(y)).$$

For rich exposition of copulae we refer to [Joe H. 1997; Doruet Mari D. and Kotz S. 2001; Nelsen R. 1999; Bedford T.J., Cooke R.M. 2002]. In this document we mainly use Frank's copula [Frank M.J. 1979].

4.3.2.1 Diagonal Band Copula

The diagonal band copula is a simple bivariate distribution on the unit square with uniform margins. For positive correlations its mass is concentrated on the diagonal band with vertical bandwidth (denoted β). Mass is distributed uniformly on the rectangle and is uniform but twice as thick in the triangular corners. For negative correlation the band is drawn between the other corners. The correlation value depends on the bandwidth. For positive correlations the density of the diagonal band distribution is given by

$$f_\alpha(u,v) = \frac{1}{2(1-\alpha)}(1_{\alpha-1\le u-v\le 1-\alpha} + 1_{1-u-v\ge\alpha} + 1_{1-u-v\le-\alpha}) \qquad (4)$$

where $0 \le \alpha \le 1$, $0 \le u,v \le 1$ and 1_A denotes indicator function of A.

The density of the diagonal band copula with correlation 0.8 is shown in Figure 4.3.

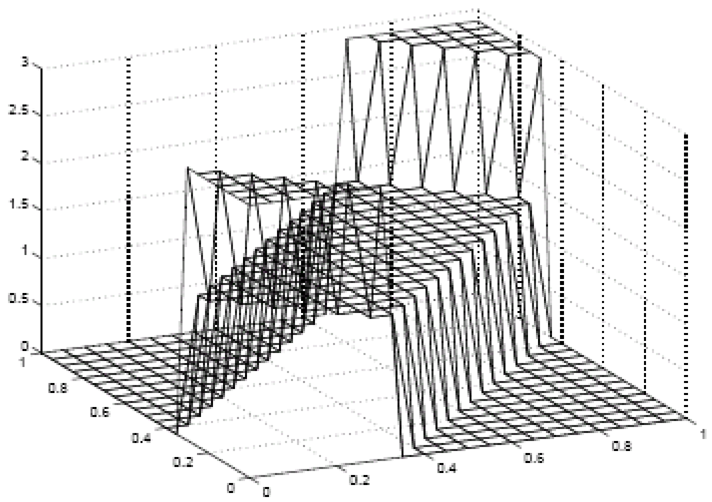


Figure 4.3: A density function of the diagonal band copula with correlation 0.8.

4.3.2.2 Frank's Copula

Frank’s family [Frank M.J., 1979] has a property of reflection symmetry, that is, $c(u,v)=c(1-u,1-v)$. This property is very important from an application point of view. Frank’s copula has one parameter θ:

$$C(u,v;\theta)=-\frac{1}{\theta}\log\left(1+\frac{\left(e^{-\theta u}-1\right)\left(e^{-\theta v}-1\right)}{\left(e^{-\theta}-1\right)}\right)$$

With generating function

$$\varphi(x)=-\ln\frac{e^{-\theta x}-1}{e^{-\theta}-1}.$$

When $\theta\to\infty$ $(\theta\to-\infty)$ then Frank’s copula corresponds to C_U (C_L). $\theta\to 0$ gives independent copula. The density of the Frank's copula with parameter $\theta=7.9026$ and correlation 0.8 is shown in Figure 4.4.

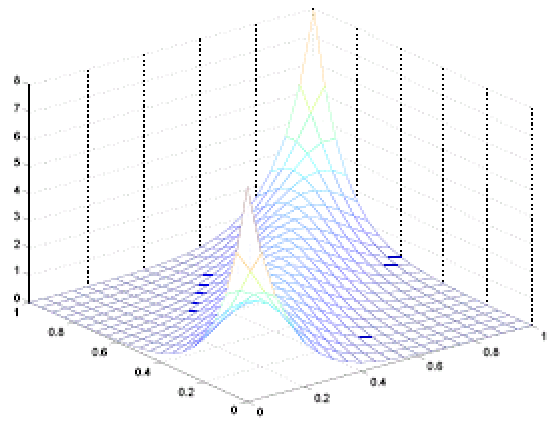


Figure 4.4: Density function of the Frank's copula with parameter $\theta = 7.9026$ and rank correlation 0.8.

4.3.2.3 Normal Copula

If Φ_ρ is the bivariate normal CDF with correlation ρ and Φ^{-1} the inverse of the standard univariate normal distribution function then

$$C_\rho(u, v) = \Phi_\rho\left(\Phi^{-1}(u), \Phi^{-1}(v)\right)$$

$u, v \in [0, 1]$ is called the normal copula.

The density of the normal copula with correlation 0.8 is shown in Figure 4.5.

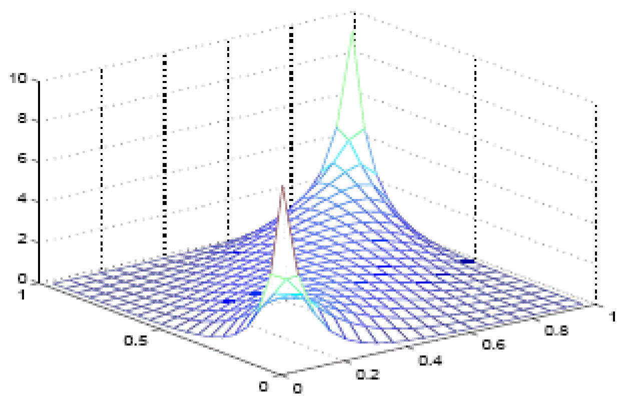


Figure 4.5: A density function of normal copula with correlation 0.8.

5. CONCLUSIONS AND RECOMMENDATIONS

In Europe, wake vortex prediction and detection systems are being introduced in order to increase airport capacity, while maintaining safety. The EC project ATC-Wake aims to develop a ground based system for ATC (Air Traffic Control) that would allow variable aircraft separation distances, as opposed to the fixed distances presently applied at airports. The EC project I-Wake aims to develop an on-board system for pilots in order to minimize the probability of a wake encounter. As part of these projects, quantitative safety assessments were performed. So far, the focus of these safety studies was the assessment of the risk related to the wake encounter itself.

However, for a quantitative safety assessment of the use of wake vortex prediction and detection systems, the following issues should also be considered:

- Humans working with these systems have to react on alerts, so as to ensure that a pilot will be able to initiate a wake vortex avoidance maneuver in time.

- If one or more of the system components provide a wrong or erroneous advice, there will be a higher risk on the presence of (severe) wake vortices. The consequences might be catastrophic, in case reduced separation is applied.

This study therefore has provided insights into the hazards and system failures related to the use of wake vortex prediction and detection systems. Three models, developed by NLR, have been analyzed in detail to support of the setting of requirements for the use of these systems. These models are:

- A stochastic model, based on use of continuous Bayesian Belief Nets, for the initial aircraft separation time between aircraft taking off at a single runway;

- A Fault Tree model for the assessment of the impact of ground based system failures on a wake vortex detection, warning, and avoidance manoeuvre;

- A discrete Bayesian Belief Net for the assessment of the impact of an on-board system failure on a wake vortex avoidance manoeuvre.

Main ideas behind Bayesian Belief Nets and the data requirements for the use of these three models have been presented. The mathematical modeling techniques have been described and discussed in detail. The data required to quantify the models has to be provided by operational experts, and might need to be further validated through expert elicitation sessions and a comparison with wake vortex incident/accident data. Several questionnaires to elicit required data for the use of these models have been proposed.

We should point out that the methodology to use depends on the characteristics of the problem to solve and the assumptions to be taken. If our variables are restricted to take two values or states the modeling approach to follow is discrete BBNs. This approach has been followed for the causal models representing the ground based and on-board detection, warning, and avoidance maneouvre. It has been shown that these discrete BBNs can also be represented as a fault tree (and vice versa). Although, the simplicity of the Fault Trees is appropriate in some cases and its graphical representation contains more information about underlying joint distribution than the directed acyclic graph for BBN; it is much more restrictive in dependence structures that can be described. BBNs allow their nodes to have more than two states. It does not restrict the 'basic' variables to be independent and allows a much richer set of dependence structures to be used. It should be noted that quantification of discrete BBNs can be very cumbersome. If the variables in a mathematical model are continuous, we recommend the use of continuous BBNs. An elicitation procedure, based on (conditional) rank correlations, has been constructed in order to cope with the specification of the joint probability distribution for the continuous BBN for the initial aircraft separation time. Direct quantification of a BBN that is discretised to 10 states for each variable would require the specification of 12,150 probabilities in the conditional probability tables. The quantification with continuous nodes requires only nine algebraically independent (conditional) rank correlations and the specification of the nine marginal distributions for the nodes in the BBN. This demonstrates clearly the reduction of assessment burden once we have quantified influences as (conditional) rank correlations. It is recommended to investigate the quantification of the combination of discrete and continuous nodes in a Bayesian Belief Network.

In the future, when the implementation of the ATC-Wake and the I-Wake system is accomplished, more data will become available to validate the models and to verify the correctness of the setting of requirements for the use of wake vortex prediction and detection systems. In order to assess the risk related to the use of these systems in terms of incident/accident probabilities, a dynamic coupling of the developed fault trees and BBNs with the NLR WAVIR methodology and tool-set is recommended.

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APPENDIX A –QUESTIONNAIRES

A.1 – Expert Distributions for the Nodes of The Aircraft Separation Time Model

The Questionnaire

Please fill in your 5%, 25%, 50%, 75% and 95% quantiles for the following uncertain quantities.

1. What is the wind in meters per second [m/s] predicted by the Meteo/weather forecast system?

5%_______ 25%_______ 50%_______ 75%_______ 95%_______

2. What is the difference in seconds [s] between the separation time advised by the Separation Mode Planner and the separation time that should be advised?

5%_______ 25%_______ 50%_______ 75%_______ 95%_______

3. What is the difference in seconds [s] between the separation time advised by the supervisor and the separation time that should be advised?

5%_______ 25%_______ 50%_______ 75%_______ 95%_______

4. What is the separation time in seconds [s] prescribed by the Air Traffic Control Supervisor for a departing leader and follower aircraft combination?

5%_______ 25%_______ 50%_______ 75%_______ 95%_______

5. What is the difference between actual wind in meters per second [m/s] (measured by the Detector) and predicted wind in meters per second [m/s] (as determined by the Meteo/weather systems)?

5%_______ 25%_______ 50%_______ 75%_______ 95%_______

6. What is the difference in seconds [s] between the actual take off clearance time advised and the time that should be advised?

5%_______ 25%_______ 50%_______ 75%_______ 95%_______

7. What is the time in seconds [s] between start of roll of the leader aircraft and the take off clearance of the controller for the follower aircraft?

5%_______ 25%_______ 50%_______ 75%_______ 95%_______

8. What is the time difference in seconds [s] between the take off clearance and start of roll of the follower aircraft?

5%_______ 25%_______ 50%_______ 75%_______ 95%_______

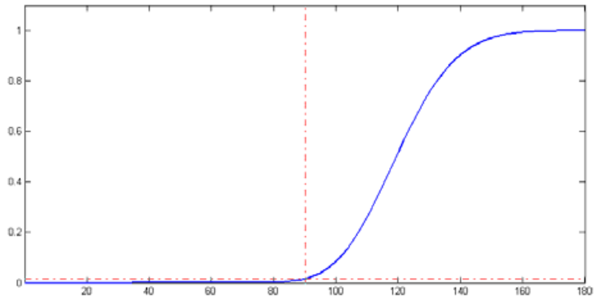
9. What is the time in seconds [s] between start of take off (roll) of leader and follower aircraft (measured from the start of roll of the leader at its Take Off Position)?

5%_______ 25%_______ 50%_______ 75%_______ 95%_______

Distributions and Theoretical States for Each Variable

Initial data used to obtain insight are presented. Therefore, probability distributions for the nodes in the BBN were estimated for given means and standard deviations of the marginal distributions. Those considered adequate for the marginal distributions required are shown below.

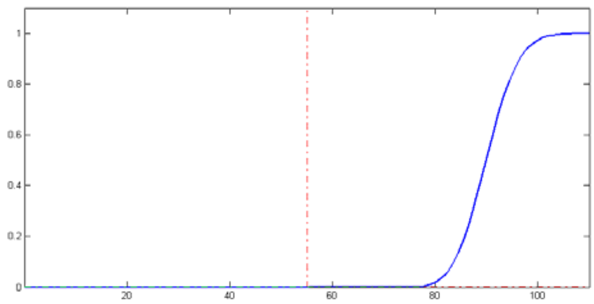
Gamma Distribution for the Aircraft Take Off Separation Time, X_9: $a = 64, b = 1.8750$



By taking the inverse of this distribution we found values corresponding to given probability

Aircraft Take Off Separation Time, X_9	
0.1	101.2228
0.2	107.2282
0.3	111.7043
0.4	115.6269
0.5	119.3756
0.6	123.2044
0.7	127.3904
0.8	132.4079
0.9	139.5799

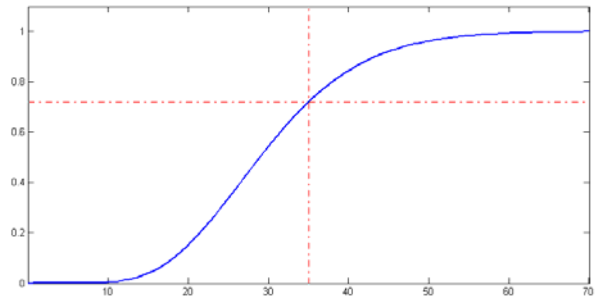
Gamma Distribution for the ATCo Take Off Clearance Time, X_7: $a = 323.9741, b = 0.2778$



By taking the inverse of this distribution we found values corresponding to given probability

ATCo Take Off Clearance Time, X_7	
0.1	83.6544
0.2	85.7670
0.3	87.3123
0.4	88.6473
0.5	89.9074
0.6	91.1794
0.7	92.5535
0.8	94.1790
0.9	96.4646

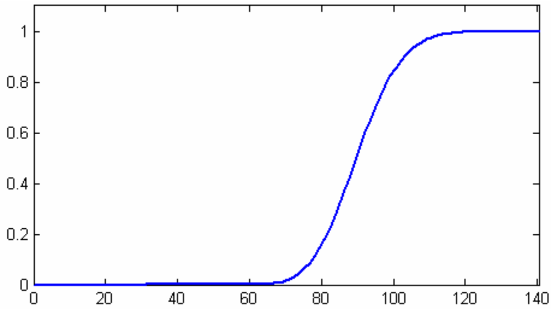
Gamma Distribution for the Pilot Take Off Time, X_8 : $a = 9.0001, b = 3.3333$



By taking the inverse of this distribution we found values corresponding to given probability

Pilot Take Off Time, X_8	
0.1	18.1083
0.2	21.4283
0.3	24.0665
0.4	26.4887
0.5	28.8965
0.6	31.4465
0.7	34.3356
0.8	37.9326
0.9	43.3157

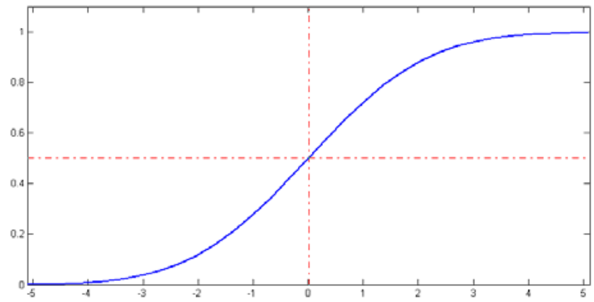
Gamma Distribution for the Prescribed Spacing, X_4 : $a = 81, b = 1.1111$



By taking the inverse of this distribution we found values corresponding to given probability

Prescribed Time Spacing, X_4	
0.1	77.4458
0.2	81.4940
0.3	84.4997
0.4	87.1261
0.5	89.6298
0.6	92.1810
0.7	94.9635
0.8	98.2901
0.9	103.0297

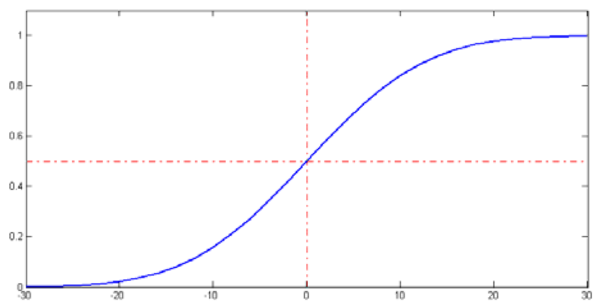
Normal Distribution for Wind Forecast Error, X_1: $\mu = 0, \sigma = 1.7$



By taking the inverse of this distribution we found values corresponding to given probability

Wind Forecast Error, X_1	
0.1	-2.1786
0.2	-1.4308
0.3	-0.8915
0.4	-0.4307
0.5	0.0000
0.6	0.4307
0.7	0.8915
0.8	1.4308
0.9	2.1786

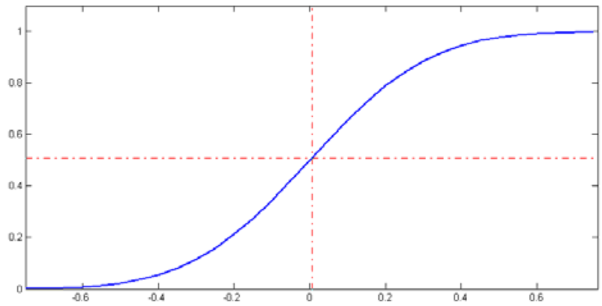
Normal Distribution for Separation Mode Planner Failure, X_2: $\mu = 0, \sigma = 10$



By taking the inverse of this distribution we found values corresponding to given probability

Separation Mode Planner Failure, X_2	
0.1	-12.8155
0.2	-8.4162
0.3	-5.2440
0.4	-2.5335
0.5	0.0000
0.6	2.5335
0.7	5.2440
0.8	8.4162
0.9	12.8155

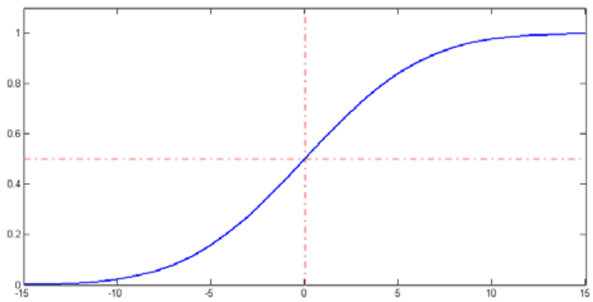
Normal Distribution for Wind Nowcast Error, $X_5: \ \mu = 0, \sigma = 0.25$



By taking the inverse of this distribution we found values corresponding to given probability

Wind Nowcast Error, X_5	
0.1	-0.3204
0.2	-0.2104
0.3	-0.1311
0.4	-0.0633
0.5	0.0000
0.6	0.0633
0.7	0.1311
0.8	0.2104
0.9	0.3204

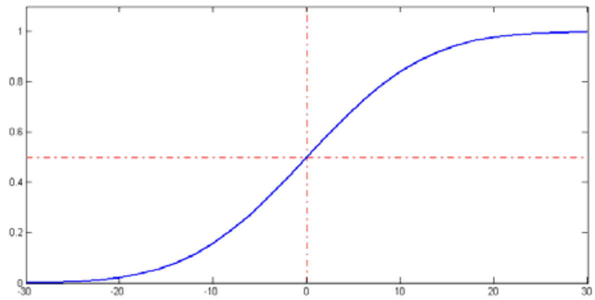
Normal Distribution for Error Runway / Tower Controller, $X_6: \ \mu = 0, \sigma = 5$



By taking the inverse of this distribution we found values corresponding to given probability

Error Runway Tower Controller, X_6	
0.1	-6.4078
0.2	-4.2081
0.3	-2.6220
0.4	-1.2667
0.5	0.0000
0.6	1.2667
0.7	2.6220
0.8	4.2081
0.9	6.4078

Normal Distribution for Error ATC Supervisor, $X_3:\ \mu = 0, \sigma = 10$



By taking the inverse of this distribution we found values corresponding to given probability

Error ATC Supervisor, X_3	
0.1	-12.8155
0.2	-8.4162
0.3	-5.2440
0.4	-2.5335
0.5	0.0000
0.6	2.5335
0.7	5.2440
0.8	8.4162
0.9	12.8155

A.2 – Conditional and Unconditional Rank Correlations

Conditional and Unconditional Rank Correlations Required to quantify the BBN for The Aircraft Separation Time Model Expert Opinion

In the next 8 questions we intend to assess rank correlations between variables of interest. Rank correlation measures monotonic relationship between random variables and can be understood roughly as a degree to which two random variables take high or low values together. In this document, variables are denoted by X_i 's and their median values are denoted by $X_{i_{50}}$'s.

It is obvious that if two variables are independent, then knowing that one of them takes high values, does not give any extra information about the other variable. If variables are completely positively rank correlated then if one is equal e.g. to its 90th percentile the other one is also equal to its 90th percentile. Hence for positively correlated random variables, information that one of them takes high values increases our confidence that the other one will be high as well.

We will ask experts about conditional probability that one variable is above its median given that other variable is above its median. If the variables are independent this probability is equal to ½ if they are positively correlated then this probability is higher than ½ and lower if they are negatively rank correlated.

We also ask experts about the conditional probability that one variable is above its median given that two/three variables are above their medians. The provided number will be between a subinterval of $[0, 1]$, which depends on previously assessed questions.

Consider the relationship between the following two variables:

2. Suppose that the Wind Prediction was observed to be above its median value. What is your probability that the Separation Mode Planner Failure would also lie above its median value?
Probability [0, 1] : 0.25

This can be shortened as

X_1 : Wind Prediction **[m/sec]**	X_2 : Separation Mode Planner Failure **[sec]**
Suppose: $X_1 \geq x_{1_{50}}$; what is $P\left(X_2 \geq x_{2_{50}} \middle| X_1 \geq x_{1_{50}}\right)$?	

Now, consider the relationship between the following three variables:

X_2 : Separation Mode Planner Failure **[sec]**	X_3 : Error ATC Supervisor **[sec]**	X_4 : Prescribed Spacing **[sec]**

First, we consider the relationship between X_3 and X_4, we say

3. Suppose that the Error ATC Supervisor was observed to be above its median value. What is your probability that the Prescribed Spacing would also lie above its median value?

Probability [0, 1] : 0.7

Shortened as:

Suppose: $X_3 \geq x_{3_{50}}$; what is $P\left(X_4 \geq x_{4_{50}} \middle| X_3 \geq x_{3_{50}}\right)$?

Then, by adding information about X_2 to the same situation, we ask

4. Consider the same situation as in question 2, now with the further information that the Separation Mode Planner Failure is also observed to be above its median value. How does this additional information change your previous estimate of 0.7? According to your previous answer your current estimate should lie in the indicated interval below.

Probability [0.40438, 0.99555] : 0.8

In short form:

Suppose: $X_3 \geq x_{3_{50}}$ and $X_2 \geq x_{2_{50}}$; what is $P\left(X_4 \geq x_{4_{50}} \middle| X_3 \geq x_{3_{50}}, X_2 \geq x_{2_{50}}\right)$?

Similarly, consider the relationship between the following three variables:

X_5 : Wind Error **[sec]**	X_6 : Error Runway/Tower Control **[sec]**	X_7 : ATCo Take Off Clearance Time **[sec]**

This situation first involves X_6 and X_7, we ask

5. Suppose that Error Runway/Tower Controller was observed to be above its median value. What is your probability that ATCo Take Off Clearance Time would also lie above its median value?

Probability [0, 1] : 0.8

This can be shortened as:

Suppose: $X_6 \geq x_{6_{50}}$; what is $P\left(X_7 \geq x_{7_{50}} \middle| X_6 \geq x_{6_{50}}\right)$?

By adding information about X_5 to the same situation, we ask

6. Consider the same situation as in question 4, now with the further information that the Wind Error is also observed to be above its median value. How does this additional information change your previous estimate of 0.8? According to your previous answer your current estimate should lie in the indicated interval below.

Probability [0.60159, 0.99719] : 0.7

Shortened as:

Suppose: $X_6 \geq x_{6_{50}}$ and $X_5 \geq x_{5_{50}}$; what is $P\left(X_7 \geq x_{7_{50}} \middle| X_6 \geq x_{6_{50}}, X_5 \geq x_{5_{50}}\right)$?

Consider the relationship between the following four variables:

X_4 : Prescribed Spacing **[sec]**	X_7 : ATCo Take Off Clearance Time **[sec]**
X_8 : Pilot Take Off Time **[sec]**	X_9 : Aircraft Take Off Separation Time **[sec]**

This situation first involves information of X_8 and X_9, we ask

7. Suppose that Pilot Take Off Time was observed to be above the median value. What is your probability that the Aircraft Take Off Separation Time would also lie above its median value?

Probability [0, 1] : 0.6

Suppose: $X_8 \geq x_{8_{50}}$; what is $P\left(X_9 \geq x_{9_{50}} \middle| X_8 \geq x_{8_{50}}\right)$?

By adding information about X_7 to the same situation, we ask

8. Consider the same situation as in question 6, now with the further information that the ATCo Take Off Clearance Time is also observed to be above its median value. How does this additional information change your previous estimate of 0.6? According to your previous answer your current estimate should lie in the indicated interval below.

Probability [0.20907, 0.99053] : 0.7

Suppose: $X_8 \geq x_{8_{50}}$ and $X_7 \geq x_{7_{50}}$; what is $P\left(X_9 \geq x_{9_{50}} \middle| X_8 \geq x_{8_{50}}, X_7 \geq x_{7_{50}}\right)$?

If we now add information about X_4 to the same situation, we ask

9. Consider the same situation as in question 7, now with the further information that the Prescribed Spacing is also observed to be above its median value. How does this additional information change your previous estimate of 0.7? According to your previous answer your current estimate should lie in the indicated interval below.

Probability [0.40068, 0.99145] : 0.8

Suppose: $X_8 \geq x_{8_{50}}$ and $X_7 \geq x_{7_{50}}$ and $X_4 \geq x_{4_{50}}$; what is

$P\left(X_9 \geq x_{9_{50}} \middle| X_8 \geq x_{8_{50}}, X_7 \geq x_{7_{50}}, X_4 \geq x_{4_{50}}\right)$?

A.3 – Parameter Values for the ATC-WAKE Maneouvre

Table 1 with parameter values for the DWA maneuver (focus on pilot performance/ability)

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
a - Horizontal Scanning Failure	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
b - Vertical Scanning Failure	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
c - Detection Range Error	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
e - Faulty WV Model Estimation	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
f - Faulty/inaccurate Traffic Situation	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
g - Faulty/inaccurate Meteo Nowcasting	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
j - Loss of DWA tactical function	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
k - Controller does not initiate warning	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
m - Pilot not able to perform maneuver	10 -1	10 -2	10 -3	10 -1	10 -2	10 -3

Table 2 with parameter values for the DWA maneuver (focus on ATC performance/ability)

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
a - Horizontal Scanning Failure	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
b - Vertical Scanning Failure	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
c - Detection Range Error	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
e - Faulty WV Model Estimation	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
f - Faulty/inaccurate Traffic Situation	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
g - Faulty/inaccurate Meteo Nowcasting	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
j - Loss of DWA tactical function	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
k - Controller does not initiate warning	10 -1	10 -2	10 -3	10 -1	10 -2	10 -3
m - Pilot not able to perform maneuver	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2

Table 3 with parameter values for the DWA maneuver (focus on Detector performance)

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
a - Horizontal Scanning Failure	10 -6	10 -3	10 -1	10 -6	10 -3	10 -1
b - Vertical Scanning Failure	10 -6	10 -3	10 -1	10 -6	10 -3	10 -1
c - Detection Range Error	10 -6	10 -3	10 -1	10 -6	10 -3	10 -1
e - Faulty WV Model Estimation	10 -3	10 -3	10 -1	10 -2	10 -2	10 -2
f - Faulty/inaccurate Traffic Situation	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
g - Faulty/inaccurate Meteo Nowcasting	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
j - Loss of DWA tactical function	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
k - Controller does not initiate warning	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
m - Pilot not able to perform maneuver	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2

Table 4 with parameter values for the DWA maneuver (focus on Predictor performance)

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
a - Horizontal Scanning Failure	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
b - Vertical Scanning Failure	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
c - Detection Range Error	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
e - Faulty WV Model Estimation	10 -6	10 -3	10 -1	10 -6	10 -3	10 -1
f - Faulty/inaccurate Traffic Situation	10 -6	10 -3	10 -1	10 -6	10 -3	10 -1
g - Faulty/inaccurate Meteo Nowcasting	10 -6	10 -3	10 -1	10 -6	10 -3	10 -1
j - Loss of DWA tactical function	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
k - Controller does not initiate warning	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2
m - Pilot not able to perform maneuver	10 -3	10 -3	10 -3	10 -2	10 -2	10 -2

Table 1 with parameter values for the DWA maneuver (focus on pilot performance/ability)

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
d – Improper Detector Performance	0.002997	0.002997	0.002997	0.029701	0.029701	0.029701
h – Improper Model Prediction	0.002997	0.002997	0.002997	0.029701	0.029701	0.029701
j - Monitoring and Alerting Failure	0.006979	0.006979	0.006979	0.067935	0.067935	0.067935
l - Warning Systems Failure	0.007972	0.007972	0.007972	0.077255	0.077255	0.077255
T – ATC-Wake DWA Failure	0.107175	0.017892	0.008964	0.16953	0.086483	0.078178

Table 2 with parameter values for the DWA maneuver (focus on ATC performance/ability)

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
d – Improper Detector Performance	0.002997	0.002997	0.002997	0.029701	0.029701	0.029701
h – Improper Model Prediction	0.002997	0.002997	0.002997	0.029701	0.029701	0.029701
j - Monitoring and Alerting Failure	0.006979	0.006979	0.006979	0.067935	0.067935	0.067935
l - Warning Systems Failure	0.106281	0.016909	0.007972	0.161141	0.077255	0.068867
T – ATC-Wake DWA Failure	0.107175	0.017892	0.008964	0.16953	0.086483	0.078178

Table 3 with parameter values for the DWA maneuver (focus on Detector performance)

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
d – Improper Detector Performance	0.000003	0.002997	0.271000	0.000003	0.002997	0.271000
h – Improper Model Prediction	0.002997	0.002997	0.101799	0.029701	0.029701	0.029701
j - Monitoring and Alerting Failure	0.003997	0.006979	0.345866	0.039407	0.042283	0.299726
l - Warning Systems Failure	0.004993	0.007972	0.34652	0.049013	0.05186	0.306728
T – ATC-Wake DWA Failure	0.005988	0.008964	0.347174	0.058523	0.061341	0.313661

Table 4 with parameter values for the DWA maneuver (focus on Predictor performance)

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
d – Improper Detector Performance	0.002997	0.002997	0.002997	0.029701	0.029701	0.029701
h – Improper Model Prediction	0.000003	0.002997	0.271000	0.000003	0.002997	0.271000
j - Monitoring and Alerting Failure	0.003997	0.006979	0.273912	0.039407	0.042283	0.299726
l - Warning Systems Failure	0.004993	0.007972	0.274638	0.049013	0.05186	0.306728
T – ATC-Wake DWA Failure	0.005988	0.008964	0.275363	0.058523	0.061341	0.313661

A.4 – Parameter Values for the I-WAKE Maneuvre

Information required for the quantification of the on-board Wake Vortex Detection, Warning and Avoidance Maneuver Probability, and example sub-system requirements are listed in the following Table.

a – Wake Vortex Outside Detection Range/Scanning	0.001
b – Inaccurate or Faulty Detection of Wake Vortices	0.001
d – Faulty or Inaccurate Aircraft Data	0.001
e – Faulty or Inaccurate WV Model Estimation	0.001
f – Faulty or Inaccurate Meteo Nowcasting	0.001
h – Loss of WV DWA Tactical Function	0.001
j – Aircraft Pilot not able to initiate missed approach	0.001

Conditional Probability Table for P(*c* | *a, b*)				
Wake Vortex Detection Range Scanning (*a*)	**Non Outside (0)**		**Outside (1)**	
Detection of Wake Vortices (*b*)	**Non Failure (0)**	**Failure (1)**	**Non Failure (0)**	**Failure (1)**
IMPROPER DETECTOR PERFORMANCE (*c*=1)	0.0001	0.001	0.001	0.9999

P(c=1 | a=0, b=0)
Given 1000000 single runway arrivals from the X airport flights and given Wake Vortex not Outside Detection Range Scanning and Non Failure of the Detection of Wake Vortices; in how many of the arrivals will the Detector Performance still be improper?
P(c=0 | a=0, b=1)=
Given 1000000 single runway arrivals from the X airport flights and given Wake Vortex not Outside Detection Range Scanning, while the Detection of the Wake Vortices fails; in how many of the arrivals will the Detector Performance still be proper?
P(c=0 | a=1, b=0)=
Given 1000000 single runway arrivals from the X airport flights, where even though Wake Vortex Outside Detection Range Scanning, the Detection of the Wake Vortices does not fail; in how many of the arrivals will the Detector Performance still be proper?
P(c=0 | a=1, b=1)=
Given 1000000 single runway arrivals from the X airport flights and given Wake Vortex not Outside Detection Range Scanning and also the Detection of the Wake Vortices fails; in how many of the arrivals the Detector Performance still be proper?

Conditional Probability Table for P(*g* | *f, e, d*)								
Aircraft Data	**Non Inaccurate (0)**				**Inaccurate (1)**			
WV Model Estimation	**Non Failure (0)**		**Failure (1)**		**Non Failure (0)**		**Failure (1)**	
Meteo Nowcasting	**Non Failure (0)**	**Failure (1)**	**Non Failure (0)**	**Failure (1)**	**Non Failure (0)**	**Failure (1)**	**Non Failure (0)**	**Failure (1)**
IMPROPER MODEL PREDICTION (*g*=1)	0.000001	0.0001	0.0001	0.001	0.0001	0.001	0.001	0.999999

P(g=1 | d=0, e=0, f=0)
Given 1000000 single runway arrivals from the X airport flights and given Non Inaccurate Aircraft Data, WV Model Estimation and Meteo Nowcasting; in how many of the arrivals will the Model Prediction still be improper?
P(g=0 | d=0, e=0, f=1)
Given 1000000 single runway arrivals from the X airport flights and given Non Inaccurate Aircraft Data and WV Model Estimation but Faulty of the Meteo Nowcasting; in how many of the arrivals will the Detector Performance still be proper?
P(g=0 | d=0, e=1, f=0)
Given 1000000 single runway arrivals from the X airport flights, where although WV Model Estimation fails, the Aircraft Data and Meteo Nowcasting do not fail; in how many of the arrivals will the Detector Performance still be proper?

P(g=0 | d=0, e=1, f=1)
Given 1000000 single runway arrivals from the X airport flights and given Faulty of the WV Model Estimation and Meteo Nowcasting but Non Inaccurate Aircraft Data; in how many of the arrivals will the Detector Performance still be proper?
P(g=0 | d=1, e=0, f=0)
Given 1000000 single runway arrivals from the X airport flights, where though the Aircraft Data fails, WV Model Estimation and Meteo Nowcasting do not fail; in how many of the arrivals will the Detector Performance still be proper?
P(g=0 | d=1, e=0, f=1)
Given 1000000 single runway arrivals from the X airport flights and given Inaccurate Aircraft Data and WV Model Estimation but Non Failure of the Meteo Nowcasting; in how many of the arrivals will the Detector Performance still be proper?
P(g=0 | d=1, e=1, f=0)
Given 1000000 single runway arrivals from the X airport flights and given Faulty of the WV Model Estimation and Meteo Nowcasting but Non Failure of the Aircraft Data; in how many of the arrivals will the Detector Performance still be proper?
P(g=0 | d=1, e=1, f=1)
Given 1000000 single runway arrivals from the X airport flights and given that the Aircraft Data, WV Model Estimation and Meteo Nowcasting fail;in how many of the arrivals will the Detector Performance still be proper?

Conditional Probability Table for P(i | h, g, c)								
Detector Performance	**Non Improper (0)**				**Improper (1)**			
Model Prediction	**Non Improper (0)**		**Improper (1)**		**Non Improper (0)**		**Improper (1)**	
WV DWA Tactical Function	**Non Loss (0)**	**Loss (1)**	**Non Loss (0)**	**Loss (1)**	**Non Loss (0)**	**Loss (1)**	**Non Loss (0)**	**Loss (1)**
I-WAKE MONITORING AND ALERTING FAILURE (i=1)	0.000001	0.0001	0.0001	0.001	0.0001	0.001	0.001	0.999999

P(i=1 | c=0, g=0, h=0)
Given 1000000 single runway arrivals from the X airport flights and given Non Improper Detector Performance and Model Prediction and Non Loss of the WV DWA Tactical Function; in how many of the arrivals will the I-Wake Monitoring and Alerting still fail?
P(i=0 | c=0, g=0, h=1)
Given 1000000 single runway arrivals from the X airport flights and given Non Improper Detector Performance and Model Prediction but Loss of the WV DWA Tactical Function, in how many of the arrivals will the I-Wake Monitoring and Alerting still not fail?
P(i=0 | c=0, g=1, h=0)
Given 1000000 single runway arrivals from the X airport flights, where although Improper Model Prediction, the Detector Performance is not Improper and there is not Loss of the WV DWA Tactical Function; in how many of the arrivals will the I-Wake Monitoring and Alerting still not fail?
P(i=0 | c=0, g=1, h=1)
Given 1000000 single runway arrivals from the X airport flights and given Improper Model Prediction and Loss of the WV DWA Tactical Function but Non Improper Detector Performance; in how many of the arrivals will the I-Wake Monitoring and Alerting still not fail?
P(i=0 | c=1, g=0, h=0)
Given 1000000 single runway arrivals from the X airport flights, where though Improper Detector Performance, there is Non Improper Model Prediction and Non Loss of the WV DWA Tactical Function; in how many of the arrivals will the I-Wake Monitoring and Alerting still not fail?
P(i=0 | c=1, g=0, h=1)
Given 1000000 single runway arrivals from the X airport flights and given Improper Detector Performance and Loss of the WV DWA Tactical Function but Non Improper Model Prediction; in how many of the arrivals will the I-Wake Monitoring and Alerting still not fail?
P(i=0 | c=1, g=1, h=0)
Given 1000000 single runway arrivals from the X airport flights and given Improper Detector Performance and Model Prediction but Non Loss of the WV DWA Tactical Function; in how many of the arrivals will the I-Wake Monitoring and Alerting still not fail?

P(i=0 | c=1, g=1, h=1)
Given 1000000 single runway arrivals from the X airport flights and given Improper Detector Performance and Model Prediction and Loss of the WV DWA Tactical Function; in how many of the arrivals will the I-Wake Monitoring and Alerting still not fail?

Conditional Probability Table for P(T | j, i)				
I-Wake Monitoring and Alerting (*i*)	**Non Failure (0)**		**Failure (1)**	
Aircraft/pilot (*j*)	**Able to initiate missed approach (0)**	**Not able to initiate missed approach (1)**	**Able to initiate missed approach (0)**	**Not able to initiate missed approach (1)**
I-WAKE DWA FAILURE (*T*=1)	0.0001	0.001	0.001	0.9999

P(T=1 | i=0, j=0)
Given 1000000 single runway arrivals from the X airport flights and given that the I-Wake Monitoring and Alerting systems component does not fail and the aircraft pilot is able to initiate an evasive action (missed approach); in how many of the arrivals will the aircraft/pilot will still fail to avoid a wake encounter?
P(T=0 | i=0, j=1)=
Given 1000000 single runway arrivals from the X airport flights and given that the I-Wake Monitoring and Alerting systems component does not fail however the aircraft pilot is not able to initiate an evasive action (missed approach) because he makes a mistake; in how many of the arrivals will the aircraft/pilot still be able to avoid a wake encounter?
P(T=0 | i=1, j=0)=
Given 1000000 single runway arrivals from the X airport flights, where even though the I-Wake Monitoring and Alerting systems component failed, the aircraft pilot is able to initiate an evasive action (missed approach); in how many of the arrivals will the aircraft/pilot still be able to avoid a wake encounter?
P(T=0 | i=1, j=1)=
Given 1000000 single runway arrivals from the X airport flights and given that the I-Wake Monitoring and Alerting systems component failed and also the aircraft pilot is not able to initiate an evasive action (missed approach); in how many of the arrivals will the aircraft/pilot still be able to avoid a wake encounter?

A.5 – Expert Distributions for the Nodes of the On-board Wake Vortex Detection, Warning and Avoidance Manoeuvre Probability Model

The Questionnaire

In this questionnaire, it is assumed that an on-board WV detection system is installed in all aircraft arriving at the destination airport. We would like to obtain your view on the I-Wake operation, in order to obtain insight into the impact of individual subsystem failures on the overall risk of a wake vortex induced incident/accident. This information will be used for the setting of requirements for the individual I-Wake sub-system components and related hazards. Please fill in your 5%, 50% and 95% quantiles for the following uncertain quantities.

Given 100 single runway arrivals from the X airport flights:

1. What is the percentage of flights that the on-board WV detection system (e.g. LiDAR) **does not** detect wake vortices of the leading aircraft, when these are **inside** the planned scanning volume of air ahead of the aircraft?

5%_______ **50%_______** **95%_______**

2. What is the percentage of flights that the on-board WV detection system (e.g. LiDAR) **does not** detect wake vortices of the leading aircraft, because these are **outside** the scanning volume of air ahead of the aircraft?

5%_______ **50%_______** **95%_______**

3. What is the percentage of flights which would register an **improper** detector performance?

5%_______ **50%_______** **95%_______**

4. What is the percentage of flights where the aircraft data, as used in the I-Wake system, is **inaccurate/wrong**?

5%_______ **50%_______** **95%_______**

5. What is the percentage of flights where the WV model locations and/or strengths predictions, **are inaccurate/wrong**?

5%_______ **50%_______** **95%_______**

6. What is the percentage of flights when the meteorological nowcasting data, as used in the I-Wake system, is **inaccurate or wrong**?

5%_______ **50%_______** **95%_______**

7. What is the percentage of flights which would register an **improper** model prediction?

5%_______ **50%_______** **95%_______**

8. What is the percentage of flights which would register a **loss** of the WV DWA tactical function?

5%_______ **50%_______** **95%_______**

9. What is the percentage of flights where a timely warning is **not** provided to the flight crew when one should be given?

5%_______ **50%_______** **95%_______**

10. What is the percentage of flights when an aircraft pilot is **not** able to initiate evasive action (missed approach) when needed?

5%_______ **50%_______** **95%_______**

11. What is the percentage of flights when an aircraft pilot is **not** able to perform the I-Wake Detection, Warning and Avoidance Maneuver when required?

5%_______ **50%_______** **95%_______**