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1 Book review

3 **Elicitation of expert opinions for uncertainty and risks**

3 Bilal M. Ayyub, CRC, Boca Raton, 2001, 302pp., ISBN 0-8493-1087-3

5 This book is divided into seven chapters. The first chapter “Knowledge and ignorance”
7 is a review of theories of knowledge starting with the first Greek philosopher Thales (585 BC)
9 and going up to the present, with many tables and block diagrams. The second chapter “Information-
11 based system definition” deals with systems engineering in relation to knowledge, uncertainty
13 and ignorance. Chapter 3 “Experts, opinions and elicitation methods” will be very familiar to
15 readers of *Experts in Uncertainty* (Cooke, Oxford University Press, 1991). Chapter 4 “Expressing
17 and modeling expert opinions” summarizes the many representations of uncertainty,
19 including fuzzy sets, rough sets, evidence theory, probability and possibility theory. Chapter 5,
21 entitled “Consensus and aggregating expert opinions” reviews the many ways of combining the
23 many types of uncertainty identified in Chapter 4. Chapter 6 contains “Guidance on expert-opinion
25 elicitation”, and the final chapter treats some applications. Each chapter contains exercises.

15 The book has a very wide sweep. The reader is presented a very wide range of possibilities
17 for dealing with uncertainty and does not come away with a clear recommendation how to choose
19 among them. Nor does the extended excursion into the theory of knowledge yield a framework for
21 evaluating methods for dealing with uncertainty. While the book contains much useful encyclopedic
23 material, there is, in my view, a significant selection bias at work. Thus, “interest in analyzing and
25 modeling uncertainty and ignorance” is said to be “started by the works of Zadeh (1965 and 1978),
27 Dempster (1976a, b), Shafer (1976), Sugeno (1974 and 1977), Klir and Folger (1988), Pawlak and
29 Smithson (1989).” (p. 85). The reader is not told about the foundational work of Keynes, Borel,
31 Von Mises, Ramsey, Von Neumann and Morgenstern, Popper, De Finetti, Savage, and many, many
33 others. Modern philosophy of science is wholly absent. Semantic analysis, formulated by Mach and
35 applied by Einstein and Bohr to enable the revolutions of relativity and quantum mechanics, has
37 been a central theme in modern philosophy of science. Its absence is particularly unfortunate, as this
39 broad tradition would supply ample conceptual tools for a critical evaluation of the many putative
41 representations of uncertainty.

29 The technical body of the book is contained in Chapters 4 and 5. The exposition in Chapter 4
31 will not please the mathematicians. The “Fundamentals of classical set theory” (pp. 127–129) is
33 very far from that. Thus, elements of sets are said to be either “discrete or continuous”, which is
35 meaningless without a topology (which one?). Sets are also said to be “convex or non-convex” which
37 is meaningless without addition and scalar multiplication. The reader does not come away knowing
39 what a set, or set theory, is. Much more attention is given to fuzzy sets and fuzzy arithmetic.

35 The exposition of fuzzy set theory leaves me totally confused. The fuzzy membership function
41 $\mu_A(x): X \rightarrow [0, 1]$ is said to represent the “degree of compatibility” of element x with set A . Elsewhere

1 this is described as “membership uncertainty” or “degree of belief that x belongs to A ” (p. 214).
 2 A is “crisp” if μ_A takes only values zero and one, otherwise A is fuzzy. I get an email from an
 3 unknown “Quincy”. My degree of belief is $\frac{1}{2}$ that Quincy is a man, and $\frac{1}{2}$ that Quincy is a woman.
 4 So $\mu_{\text{MAN}}(\text{Quincy}) = \frac{1}{2}$; $\mu_{\text{WOMAN}}(\text{Quincy}) = \frac{1}{2}$. Ayyub says that my degree of belief that Quincy is a
 5 man OR a woman is the maximum of these two degrees of belief, i.e. $\frac{1}{2}$; and my degree of belief that
 6 Quincy is a man AND a woman is the minimum, also $\frac{1}{2}$. Does this represent someone’s uncertainty?

7 Similar problems run throughout Ayyub’s exposition. For example, fuzzy set A is said to be equal
 8 to fuzzy set B if and only if $\mu_A(x) = \mu_B(x)$ for all $x \in X$. Presumably every element is certain to
 9 belong to the “universal set” X , thus $\mu_X(x) = 1$ for all $x \in X$. The complement A' of fuzzy set
 10 A is defined by the membership function $\mu_{A'}(x) = 1 - \mu_A(x)$. The union “which corresponds to
 11 the connective ‘or’” is defined $\mu_{A \cup A'}(x) = \max\{\mu_A(x), \mu_{A'}(x)\} \neq \mu_X(x)$. So $X \neq A \cup A'$. This does not
 12 correspond to the connective ‘or’. Indeed, if $\mu_A(x) = \frac{1}{2}$ for all x , then $A \cup A' = A \cap A' = A$, which is a
 13 contradiction in classical set theory. The difference $B - A$ of fuzzy sets B, A , is defined as $\mu_{B-A}(x)$ if
 14 $\mu_A(x) = 0$, and 0 otherwise. Since for any A , $\mu_{A \cup A'}(x) \neq 0$, it follows that $X - (A \cup A') = (A \cup A') -$
 15 $X = \emptyset$. Translating all this into the natural language: “ A or not- A ’ is not equal to the universal set
 16 X , but there is also no difference between them.”

17 These examples betray a more serious problem, the lack of interest in foundations. Here, the neglect
 18 of modern philosophy of science exacts a heavy toll. In the context of representing uncertainty,
 19 foundations mean articulating a formal system and giving operational definitions for the primitive
 20 terms in the formal system. Without foundations there is no coherent interpretation of a theory;
 21 just continual shifting from one position to another. A rich tradition describes the ways in which
 22 foundations can be laid, and the foundations of probability provide many examples by people who
 23 thought about this long before Zadeh. In essence, operational definitions come down to this: If Sam
 24 says “the fuzzy membership of ‘John’ in the set ‘tall’ is 0.7”, what does that mean? What sentences
 25 in the natural language not containing “fuzzy” is Sam committed to? What observable behavior of
 26 Sam does that entail? The avid interest among fuzzy theorists in new definitions and generalizations,
 27 combined with a total lack of interest in foundations, is, for someone versed in the philosophy of
 28 science, a source of profound bewilderment.

29 In sum, this book will be appreciated for its encyclopedic value by those who already understand
 30 fuzzy sets. However, the unsuspecting engineer who tries to apply the methods in this book will
 31 run into problems like those described above, and will find no guidance how to proceed. Meanwhile
 32 quantitative expert judgment in the form of subjective probability is opening many new areas or
 33 research for uncertainty analysts. Dependence modeling, quasi random numbers, Markov Chain Monte
 34 Carlo, copulae, sensitivity measures; these are some of the active areas for which readers will have
 35 to consult other sources.

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