Published in: Bridge Maintenance, Safety, Management, Life-Cycle Performance and Cost: Proceedings of the Third International Conference on Bridge Maintenance, Safety and Management, Porto, Portugal, 16-19 July, 2006; P.J.S. Cruz, D.M. Frangopol & L.C. Neves (eds.); Taylor & Francis, London, 2006 (CD-ROM).

Statistical inference for Markov deterioration models of bridge conditions in the Netherlands

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ABSTRACT: The focus of this paper is on the statistical estimation of parameters in various types of continuous-time Markov processes using bridge condition data in the Netherlands. The parameters in these processes are transition intensities between discrete condition states. These intensities may depend on the current state and on the age of the structure. The likelihood function for the observed transitions is derived and an outline of the steps to determine the parameters which maximize the likelihood function is given. The results of the fitting procedure are presented in the form of the expected bridge condition over time and the random time to the final state. For each type of Markov process, the quality of fit is compared to that of the other types. The influence of the inspector on the condition process is also discussed.

1 INTRODUCTION

The Civil Engineering Division of the Ministry of Transport, Public Works and Water Management in the Netherlands is responsible for the inspection and maintenance of highway bridges nationwide. For the purpose of maintenance and inspection planning, ongoing research is focused on determining the uncertain rate of deterioration over time. An approach using a Weibull probability distribution, fitted to observed and censored bridge lifetimes, is reported on in van Noortwijk and Klatter (2004). Unfortunately, observations of bridge lifetimes are very scarce so in this paper a different approach is presented.

Bridges in the Netherlands are subject to periodic visual inspections. The duration between inspections ranges from 2 to 14 years with an average of around 6 years. At each inspection, the inspector records individual damages and uses the damage rating scheme in Table 1 to indicate their severity. Also, the overall condition of the structure is indicated using the structure rating scheme in Table 1.

Table 1. Condition rating scheme for infrastructures in the Netherlands.

State	Damage rating	Structure rating
0	none	perfect
1	initiation	very good
2	minor	good
3	multiple/serious	reasonable
4	advanced/grave	mediocre
5	threat to safety/functionality	bad

Stochastic processes are most suited for modelling dynamical systems with uncertainty over time. For modelling the uncertain progress through the discrete condition scale used for bridges, a Markov process is the obvious choice. This approach nowadays is quite common for the assessment of bridge deterioration over time. During the preparations for the application of a Markov process to the bridge condition data in the Netherlands, it became clear that the required statistical methods for the estimation of model parameters were not present in the literature relating to bridge management systems. In the theory of statistics, the type of information that is available is referred to as current status data or panel data. With periodic inspections, only the status at the time of the inspection is known. The times of transitions between inspections, or the exact length of time that a structure is present in a state, is not known and can not be observed. Even continuous monitoring of bridges would not be able to supply this information, because the difference between states like those in Table 1 is rather vague. State duration models like those proposed by Kleiner (2001) or Mishalani and Madanat (2002) are therefore not well suited for this application. The latter reference includes a review of transition probability estimation in the literature and discusses their limitations. Another overview of the estimation of bridge transition probabilities in the literature is given in Frangopol et al. (2004).

The data used in the following analysis has been obtained by querying the condition database for inspection results on concrete structures in the Netherlands. In order to analyse the rate of transitions, the query is defined such that there are at least two observations of the condition for every structure. The query results in 10310 individual inspection events on 6871 concrete structures. Due to their definition in the database, there are more structures than there are actual bridges. A bridge may consist of more than one structure. Of all transitions, 14% are transitions to better states and 17.5% of these can be attributed to maintenance with some degree of certainty. Maintenance activities were registered on a very limited basis, which makes it difficult to relate increases in structure quality to maintenance or just variability in the observations. These transitions to better states were removed, together with incorrect or inconsistent data, to result in a dataset of 8511 transitions.

2 MATHEMATICAL MODEL

First, the likelihood function of the observed bridge conditions is derived under the assumption that the Markov property holds. The likelihood function is maximized to determine the parameters which assign the highest probability of occurrence to the observations. Next, various continuous-time Markov processes are presented which will be used to fit the data.

2.1 Likelihood of observed bridge conditions

Assume that the database contains inspection results on a total of n bridges. Each bridge is observed at a finite number of successive moments in time: $t_1 < t_2 < \ldots < t_{m_j}$, where m_j is the number of registered inspections for the j-th bridge ($j = 1, 2, \ldots, n$). Let $\{X(t), t \geq 0\}$ be a stochastic process which models the state of a bridge as a function of time t. In this paper, time represents the age of a bridge. At each inspection, the bridge is observed to be in one of a finite number of discrete states: $X(t) \in \{0, 1, 2, \ldots, 5\}$. Let x_i ($i = 1, 2, \ldots, m_j$) be the observed bridge condition at time t_i, therefore x_i is a realization of the deterioration process at time t_i: $X(t_i) = x_i$. For the j-th bridge, there are m_j observations: $X(t_1) = x_1, X(t_2) = x_2, \ldots, X(t_{m_j}) = x_{m_j}$.

The likelihood of the observations is the probability that the chosen model results in the realizations $x_1, x_2, \ldots, x_{m_j}$. Let $\boldsymbol{\theta}$ denote the vector with all the model parameters, then the probability of the observations for the j-th bridge is

$$f_j(\boldsymbol{x}; \boldsymbol{\theta}) = \Pr\left\{X(t_1) = x_1, X(t_2) = x_2, \ldots, X(t_{m_j}) = x_{m_j}\right\}.$$

The notation $f(\boldsymbol{x}; \boldsymbol{\theta})$ means that f depends on $\boldsymbol{x}$ and $\boldsymbol{\theta}$, but $\boldsymbol{\theta}$ is assumed to be fixed. This probability can be rewritten using conditional probabilities as

$$\begin{aligned} f_j(\boldsymbol{x}; \boldsymbol{\theta}) = {} & \Pr\left\{X(t_{m_j}) = x_{m_j} \mid X(t_0) = x_0, X(t_1) = x_1, \ldots, X(t_{m_j-1}) = x_{m_j-1}\right\} \cdot \\ & \cdot \Pr\left\{X(t_{m_j-1}) = x_{m_j-1} \mid X(t_0) = x_0, X(t_1) = x_1, \ldots, X(t_{m_j-2}) = x_{m_j-2}\right\} \\ & \cdots \Pr\left\{X(t_1) = x_1 \mid X(t_0) = x_0\right\} \Pr\left\{X(t_0) = x_0\right\}. \end{aligned}$$

Note that the state at time (or age) $t_0 = 0$ has been included. In general, it is assumed that $\Pr\{X(t_0) = 0\} = 1$, which means that it is assumed that each object always starts in the initial (perfect) state 0. The Markov property simplifies the previous probability to

$$f_j(\boldsymbol{x};\boldsymbol{\theta}) = \Pr\left\{X(t_{m_j}) = x_{m_j} \mid X(t_{m_j-1}) = x_{m_j-1}\right\} \cdot \\ \cdot \Pr\left\{X(t_{m_j-1}) = x_{m_j-1} \mid X(t_{m_j-2}) = x_{m_j-2}\right\} \cdots \Pr\left\{X(t_1) = x_1 \mid X(t_0) = x_0\right\}. \quad (1)$$

The probability of the successive observations $x_1, x_2, \ldots, x_{m_j}$ on the j-th bridge is now reduced to a product of transition probabilities:

$$f_j(\boldsymbol{x};\boldsymbol{\theta}) = \prod_{i=1}^{m_j} P_{x_{i-1},x_i}(t_{i-1}, t_i), \quad (2)$$

where $P_{x_{i-1},x_i}(t_{i-1}, t_i) = \Pr\{X(t_i) = x_i \mid X(t_{i-1}) = x_{i-1}\}$ is the general notation for the transition probabilities. If we have more than one bridge, the likelihood function is defined as the product of the probabilities of observations for the individual bridges:

$$L(\boldsymbol{\theta};\boldsymbol{x}) = \prod_{j=1}^{n} f_j(\boldsymbol{x};\boldsymbol{\theta}), \quad (3)$$

which means that we assume the inspections between any two bridges to be independent. As with $f(\boldsymbol{x};\boldsymbol{\theta})$, the notation $L(\boldsymbol{\theta};\boldsymbol{x})$ now means that $\boldsymbol{x}$ is fixed. Eqs. (2) and (3) can now be easily combined to obtain

$$L(\boldsymbol{\theta};\boldsymbol{x}) = \prod_{j=1}^{n}\prod_{i=1}^{m_j} P^{(j)}_{x_{i-1},x_i}(t_{i-1}, t_i), \quad (4)$$

where $P^{(j)}$ is used to distinguish the transition probabilities for each bridge.

The transition probability $\Pr\{X(t_1) = x_1 \mid X(t_0) = x_0\}$ in Eq. (1) assumes that no changes have been made to the object between the start of its service life at time t_0 and the first (registered) inspection at time t_1. The bridge condition database in the Netherlands, has been put into use in late 1985, which means that for all bridges constructed before this time, there is an information gap. If any maintenance was performed before 1985, we will not know about it. In these cases, we have set $\Pr\{X(t_1) = x_1 \mid X(t_0) = x_0\} = \Pr\{X(t_1) = x_1\} = 1$ in order to avoid a serious underestimation of the rate of deterioration. This assumes that the Markov process starts in state x_1 at time t_1 with probability one.

2.2 *Continuous-time Markov process*

The stochastic process which describes the uncertain bridge condition over time, $\{X(t), t \geq 0\}$, is modelled by a continuous-time Markov process (CTMP). Contrary to a discrete-time Markov process, a CTMP allows transitions to occur on a continuous timescale. A discrete-time process is often chosen with the argument that subsequent calculations are more simple compared to those when using a continuous-time process. Although this is true, the difference in complexity is small and does not warrant the loss of generality resulting from the restriction to a discrete timescale.

It is assumed that the reader is familiar with the basic theory of Markov processes. Only the required notation is defined here. Let the transition probability function be given by $P_{i,j}(s,t) = \Pr\{X(t) = j \mid X(s) = i\}$ and the corresponding matrix by $\boldsymbol{P}(s,t) = \|P_{i,j}(s,t)\|$. The transition probability function $P_{i,j}(s,t)$ gives the probability of moving from state i at age s to state j at age t. This matrix is the solution of the differential equation

$$\frac{\partial}{\partial t}\boldsymbol{P}(s,t) = \boldsymbol{P}(s,t)\boldsymbol{Q}(t), \quad (5)$$

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which is known as the forward Kolmogorov equation and $\boldsymbol{Q}(t)$ is the transition intensity matrix. The transition intensities may depend on the age of the bridge, but generally it is assumed to be constant, i.e. $\boldsymbol{Q}(t) = \boldsymbol{Q}$. It should be stressed that t is the time since the start of the process $X(t)$ and not the time since entering the last state. If the transition intensities depend on t, it means that they depend on the age of the bridge, not on how long the bridge has been in the last state. Table 2 shows samples of the four types of transition intensity matrices which will be applied to the data. These sample matrices only contain 4 states, whereas the analysis considers all 6 states defined in Table 1. A distinction is made based on the dependence on time (age) and on the dependence of

Table 2. Samples of the four types of transition intensity matrices applied in the analysis.

	State-independent	State-dependent
Time-independent	$\begin{bmatrix} -\lambda & \lambda & 0 & 0 \\ 0 & -\lambda & \lambda & 0 \\ 0 & 0 & -\lambda & \lambda \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Type A	$\begin{bmatrix} -\lambda_0 & \lambda_0 & 0 & 0 \\ 0 & -\lambda_1 & \lambda_1 & 0 \\ 0 & 0 & -\lambda_2 & \lambda_2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Type B
Time-dependent	$\begin{bmatrix} -\lambda(t) & \lambda(t) & 0 & 0 \\ 0 & -\lambda(t) & \lambda(t) & 0 \\ 0 & 0 & -\lambda(t) & \lambda(t) \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Type C	$\begin{bmatrix} -\lambda_0(t) & \lambda_0(t) & 0 & 0 \\ 0 & -\lambda_1(t) & \lambda_1(t) & 0 \\ 0 & 0 & -\lambda_2(t) & \lambda_2(t) \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Type D

state. As this is a condition model, the process starts in the initial (0) perfect state and successively moves through each state until the final absorbing (5) state.

The model with intensity matrix of type A has only one parameter, namely the transition rate $\lambda > 0$. The condition process represented by this model has the same exponential waiting time, $F(w) = 1 - \exp\{-\lambda w\}$, for each state. Model B has five parameters, $\boldsymbol{\theta} = \{\lambda_i; i = 1, 2, \ldots, 5\}$, such that the waiting time in each state can be different, i.e. $F_i(w) = 1 - \exp\{-\lambda_i w\}$. The type C and D matrices in Table 2 both have intensity functions which depend on the process age. One is essentially free to choose these functions and here $\lambda(t) = \alpha\beta t^{\beta-1}$ is used with $\alpha > 0$, such that the integrated intensity is $\Lambda(t) = \alpha t^{\beta}$. The model with type C intensity matrix has two parameters: $\boldsymbol{\theta} = \{\alpha, \beta\}$ and the model with type D matrix has ten parameters: $\boldsymbol{\theta} = \{\alpha_i, \beta_i; i = 1, 2, \ldots, 5\}$ and $\lambda_i(t) = \alpha_i\beta_i t^{\beta_i - 1}$. The rate of transitions decreases with increasing bridge age if $\beta < 1$ and increases if $\beta > 1$. If $\beta = 1$ in model C, the transition intensities become independent of time. Model A can be obtained from model C by taking $\beta = 1$ such that $\lambda(t) = \alpha$. Model D is the most general formulation in Table 2 and all three other models can be derived from model D by taking suitable values for the parameters.

If $\boldsymbol{Q}$ is constant, as in types A and B of the matrices in Table 2, the Markov process is stationary and the transition probability function only depends on the length of time between inspections: $P_{i,j}(s, t) = P_{i,j}(0, t - s)$. For this case, the notation $P_{i,j}(t - s)$ will be used. The transition probability matrix is the solution of Eq. (5) and is given by

$$\boldsymbol{P}(t - s) = \boldsymbol{P}(0) \exp\{(t - s)\boldsymbol{Q}\}, \tag{6}$$

where $\boldsymbol{P}(0) = \boldsymbol{I}$ is the identity matrix. If $\boldsymbol{Q}$ is not constant, as is the case for matrix types C and D, the solution can easily be numerically calculated by an iterative Euler scheme based on the following relationship:

$$\boldsymbol{P}(s, t + \Delta t) = \boldsymbol{P}(s, t) \left[\boldsymbol{Q}(t)\Delta t + \boldsymbol{I}\right], \tag{7}$$

where Δt is a small timestep.

2.3 Parameter estimation

The transition probability matrices given by Eqs. (6) and (7) are used to calculate the value of the likelihood function (4) for each of the models represented by the intensity matrices in Table 2. The

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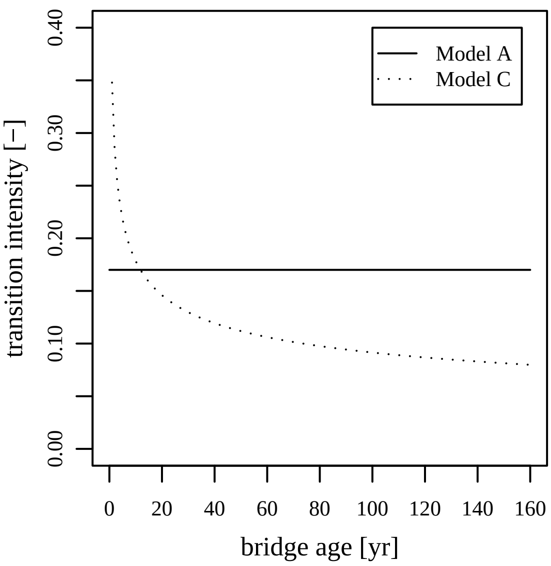

Figure 1. Comparison of the transition intensities as a function of bridge age for models A and C

parameters for each model are fitted to the data by maximum likelihood estimation. An iterative optimization scheme, based on the Euler scheme, known as Fisher's method of scoring is used to determine the values of the parameters which maximize the likelihood function (4) given the set of observations $\boldsymbol{x}$. For general information on this method see e.g. Lindsey (1996) and for a specific application to continuous-time Markov processes see Kalbfleisch and Lawless (1985). A key aspect of this approach is the use of the log-likelihood function $\ell(\boldsymbol{\theta};\boldsymbol{x}) = \log L(\boldsymbol{\theta};\boldsymbol{x})$, which is easier to work with than the likelihood function itself. Note that $\ell(\boldsymbol{\theta};\boldsymbol{x}) \leq 0$ as $0 \leq L(\boldsymbol{\theta};\boldsymbol{x}) \leq 1$.

3 RESULTS

Table 3 summarizes the estimated parameters for each of the four model types. Surprisingly, model

Table 3. Maximum likelihood estimates for the model parameters.

State	Model A	Model B	Model C		Model D	
i	λ	λ_i	α	β	α_i	β_i
0	0.17	0.56	0.49	0.71	0.56	1.02
1	0.17	0.37	0.49	0.71	0.08	1.47
2	0.17	0.10	0.49	0.71	0.09	1.03
3	0.17	0.04	0.49	0.71	0.08	0.81
4	0.17	0.15	0.49	0.71	0.33	0.81

C has a decreasing failure rate ($\beta < 1$). This means that young bridges have a higher transition rate compared to older bridges. This decreasing transition rate is compared with the constant rate of model A in Figure 1. This decreasing transition intensity can be explained by the fact that bridges spend less time in the initial states compared to later states, as can be seen from the parameter values for model B in Table 3. Since there is a positive correlation between bridge age and state, bridges have a higher transition intensity in the earlier stages of life.

The question why bridges transition faster through earlier states now arises. An analysis of the database indicates that this behaviour is a result of how inspectors evaluate the overall condition of a bridge. After each inspection, the inspector records the damages which have been found on the bridge and indicates the severity of each individual damage using the damage rating scheme in Table 1. After these damages have been registered, the database automatically assigns the worst condition of these damages to the overall condition of the bridge. The inspector then has the option

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to overrule this assignment if he thinks the severity of the worst damage is not representative for the whole structure. The data shows that, in more than 95% of the cases, the overall condition of a bridge is left unchanged if the maximum severity of all damages is 1. If the maximum damage has severity 2, a little over 66% of the overall conditions is not changed by the inspectors. For severity 3, this number drops to 33% and for damage severities 4 and 5 it is less than 15%. This indicates that, as the severity of individual damages increases, inspectors are more inclined to override the automatic assignment of the overall structure condition. This explains why the condition of bridges progresses through initial states faster compared to later states.

3.1 *Relative quality of fit*

The Markov models which are represented by their transition intensity matrices in Table 2, all belong to the same family. In fact, as mentioned before, models A, B and C are special cases of model D. Table 4 summarizes the characteristics of the four models and the results for the log-likelihood values. It can immediately be seen that the most general model, D, has the highest log-

Table 4. Log-likelihood results in order of best fit.

Model	State-dependent	Age-dependent	No. parameters	Log-likelihood
D	yes	yes	10	-8,909
B	yes	no	5	-9,127
C	no	yes	2	-11,122
A	no	no	1	-11,522

likelihood value, which is no surprise given the fact that it has the greatest number of parameters. As in the case of polynomial fitting: the more parameters the model has, the better it fits to the data. To test if the improved fit is statistically significant, a likelihood ratio test can be performed. Let $L_A^* \equiv L(\boldsymbol{\theta}_A^*; \boldsymbol{x})$ and $L_B^* \equiv L(\boldsymbol{\theta}_C^*; \boldsymbol{x})$ be the likelihood functions of models A and C maximized by their respective optimal parameter values $\boldsymbol{\theta}_A^*$ and $\boldsymbol{\theta}_C^*$. According to a well known theorem, see e.g. Mood et al. (1974), $-2\log\{L_A^*/L_C^*\} = -2\{\ell_A^* - \ell_C^*\}$ has approximately a Chi-square distribution with degrees of freedom equal to the difference in the number of parameters between both models. The null hypothesis in the likelihood ratio test is that the difference is small. It is rejected if the likelihood values are significantly different. For models A and C, the p-value is

$$1 - \Pr\left\{\chi^2_{df=1} \leq -2(-11,522 + 11,122)\right\} = 1 - \Pr\left\{\chi^2_{df=1} \leq 800\right\} = 0.$$

The hypothesis that models A and C are not significantly different is therefore rejected at even the smallest significance level. The same holds for the other models: each improvement in the quality of fit is significant from a statistical point of view. Even the smallest improvement, namely model D over model B, is significant. In fact, model D is e^{218} times more likely to generate the observed data than model B.

3.2 *Expected condition and time to final state*

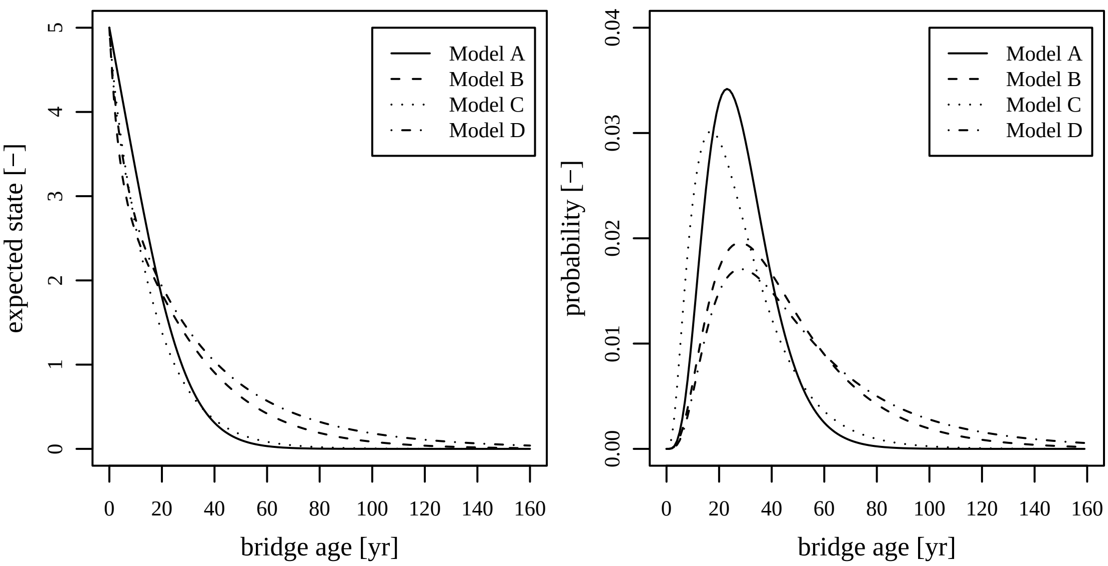
The expected bridge condition and the probability distribution of the random time to the final state can both be calculated by using

$$\Pr\{X(t) = j\} = \sum_{i=0}^{5} \Pr\{X(0) = i\} P_{i,j}(0, t).$$

The expectation of the bridge condition at age t is

$$\mathbb{E}[X(t)] = \sum_{i=1}^{5} i \cdot \Pr\{X(t) = i\}$$

and the (cumulative) probability of reaching state 5 by age t is simply $\Pr\{X(t) = 5\}$. For the Markov processes in this context, the latter probability is also referred to as the first passage time



(a) (b)

Figure 2. Expected condition (a) and probability density of the time to the final state 5 (b) for all four models in Table 2.

into state 5. Figure 2(a) presents the expected condition of a bridge as a function of its age and Figure 2(b) displays the corresponding probability density functions for the time to the final state 5. In Figure 2(a) the effect of state-dependence in model types B and D can clearly be observed: transitions occuring rapidly in initial states and slowing down in later states. From Figure 2(b) it can be seen that the better fitting model types, namely B and D, have longer tails compared to model types A and C. Table 5 shows the expected time to reach the final state. Also, it shows the approximate quantiles of the probability distributions depicted in Figure 2(b). From this table, it

Table 5. The expected time in years to the final state for each model type together with the approximate quantiles of the probability distribution.

Model	Expectation	Approx. quantiles		
		5%	50%	95%
A	28.7	12	28	53
B	45.3	15	40	100
C	27.9	9	25	62
D	51.7	16	45	129

can be concluded that the 90% confidence interval is very wide, therefore the uncertainty is large. Model D, the best fitting model, has a 90% probability of reaching state 5 between roughly 16 and 129 years of age. Note that the concrete bridges in the Netherlands have a design life of around 80 to 100 years. State 5 is not considered as a failure state, but more as an intervention state at which the bridge requires immediate attention. The results presented here can therefore not be compared with the Weibull lifetime distribution for concrete bridges in the Netherlands in van Noortwijk and Klatter (2004), which are for the random time until the end of the functional life of a bridge.

4 CONCLUSIONS

There are three important conclusions to be drawn from the analysis of the bridge condition data in the Netherlands.

First, the database does contain incorrect and incomplete information, but there are sufficient observations for a pattern to arise. This is despite the fact that the visual inspections of bridges are known to be subjective and can vary widely between inspectors (see e.g. Phares et al. (2004)). The random times to reach the final state 5 presented in Figure 2(b) and the corresponding quantiles in Table 5 are in line with expectations from experts.

Second, the uncertainty in the condition process $X(t)$ is very large. This uncertainty is not only due to the natural variability of deterioration, but also due to the measurement variability resulting from the subjectivity of condition evaluations by inspectors. From the analysis presented here, it is concluded that the influence in the model of the inspector's interpretation of the rating scheme in Table 1 is substantial. Future research should therefore include this measurement variability in the model. A review of the application of so-called partially observable Markov processes, which could be used for this purpose, is given in Frangopol et al. (2004)

Third, out of the four model types in Table 2, type D fits best to the data. This model has different transition intensities for each state and these intensities also depend on the age of the structure. Whether this dependence on age is a characteristic of bridge deterioration, or an artefact of the way inspections are performed and registered, can not be determined from this analysis. State-dependence, which is common in most applications of Markov processes to bridge deterioration modelling, is necessary to cope with the larger uncertainty in later condition states.

A general conclusion is that the maximum likelihood estimation procedure outlined in Section 2.3, using the likelihood function derived in Section 2.1, is a suitable statistical model for the specific type of censoring involved with current status data. The transition probability functions for different types of continuous-time Markov processes, presented in 2.2, are not too dificult to calculate, therefore these processes do not result in a substantial increase in the computational effort compared to the more common discrete-time Markov chains.

Current and future research is concerned with improving the quality of the dataset and improving the efficiency and accuracy of the calculations. Also, the aforementioned inclusion of measurement variability will be adressed. Once a suitable model for the degradation over time is selected, it can be used to assess optimal maintenance and inspection times of individual structures. The model could also be used to assess future compliance, given current maintenance practices, of the bridge network with quality demands set forth by the national government in the Netherlands.

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