Stakeholder Preference with Probabilistic Inversion: Application to Competitiveness Indices

R.E.J Neslo, S. Tarantola, R.M. Cooke

Abstract

An index over countries is a numerical scale used to compare countries with one another or with some reference number based on one or more specified aspects. An index is used by policymakers, because it provides information about the monitoring of countries progress in a given policy field. There are several institutions (UNDP, WEF, World Bank, etc.) that construct indices. The index must be a single unambiguous number which captures the facts of the specified aspects and or the different opinions of stakeholders about the specified aspects. These aspects may be *Competitiveness*, *Quality of Life*, or *Environmental Performance Index.*

The methods usually applied for constructing an index make use of surveys and statistical data. In the surveys respondents are asked to score the countries with respect to specified aspects on a scale defined by the institutions. The scores over the specified aspects together with the statistical data are first transformed to an interval $[0, 1]$ and then aggregated using a non transparent set of weights to create an index. The weights reflect the preference of the institution rather then the preferences of the respondents, even though the preferences of respondents may be captured by the use of surveys.

The statistical data used may also be irrelevant in the construction of an index if either the data do not affect the specified aspects or if the data only affect the specified aspects for a subset of countries.

These considerations motivated a search for alternative methods for constructing indices over countries. In this research the Global Competitiveness Index constructed by the World Economic Forum (WEF) is considered as a test case. Every year the WEF constructs a Global Competitiveness Index (GCI).This index is meant to measure the amount of competitiveness of each country and to shed some light on why some countries grow and others do not in terms of macroeconomics, institution, and technology. Competitiveness is defined as the set of institutions, policies, and factors that determine the level of productivity of a country.

The main objective of this research is to construct an index for competitiveness for a set of countries based on preferences of respondents. Respondents are presented with a pair of countries and asked which country out of the pair they would invest their money. This is repeated for each pair of countries, for each respondent.The respondents were experts at the JRC.

In the second phase of this research, the index obtained is compared with the index from the World Economic Forum to determine if there is any correlation between the two indices. Finally regression is used to select statistical data that are relevant to competitiveness of the set of countries. The coefficients obtained from the regression analysis can be compared the weights selected by the World Economic Forum. Finally, new techniques of probabilistic inversion will be used to quantify regression coefficients directly, without recourse to the Law of Comparative Judgment.

1 Introduction

The objective of this paper is to demonstrate a new method for constructing a competitiveness index. One might ask what factors determine the competitiveness of a country. According to the World Economic Forum (WEF) competitiveness is defined as the set of institutions, policies, and factors that determine the level of productivity of a country. The definition of the WEF is used as the starting point of the paper.

The WEF method can be formulated as a bottom-up approach in constrast to the top-down approach that will be used here. The bottom-up approach of the WEF gathers a set of data that capture different aspects of competitiveness of a country and then aggregates the data into a competitiveness index. The top-down approach constructs scores of the countries out of the preferences of the respondents and then tries to explain the scores by means of regression.

1.1 Overview of Paper

First paired comparison is used to process the preferences of the respondents into scale values. The method of Thurstone[5] is contrasted with a new method based on probabilistic inversion techniques proposed by Cooke[2].

Second the scale values are regressed onto quantities that putatively determine the competitiveness of a country. The regression coefficients reflect the linear dependence of competitiveness scale values on the explanatory variables. They are not weights in the sence of the WEF, as they are not all positive nor normalized.

Finally a direct fitting method is applied to find coefficients of a linear model of competitiveness without first deriving scale values. These results are compared with those of the WEF.

2 Methods

The Law of Comparative Judgment[1] was introduced to define the concept of *psychological continuum*. Thurstone[5] argued that a series of stimuli can be ordered on the *psychological continuum* by pairwise comparing them. He later stated that the Law of Comparative Judgment can be applied to any type of stimuli including attitudes and values.

For every stimulus i and j in the set of stimuli, the probability that i is preferred to j is given by $P(i > j) = \frac{number of times i > j}{number of individuals}$. Thurstone assumed that these probabilities are equal to $\phi\left(\frac{\mu_i - \mu_j}{\sqrt{\sigma_i^2 + \sigma_i^2 - 2\rho_{ij}\sigma_i\sigma_j}}\right)$ with ϕ the standard normal cumulative distribution function from which he obtained the mathematical formulation of the Law of Comparative Judgment.

$$\mu_i - \mu_j = x_{ij}\sqrt{\sigma_i^2 + \sigma_i^2 - 2\rho_{ij}\sigma_i\sigma_j} \qquad (1)$$

With

- μ_i is the mean psychological scale value of stimuli i
- x_{ij} is the inverse cumulative of the normal distribution corresponding with the proportion of occasions on which the magnitude of stimulus i is preferred to of stimulus j
- σ_i is the dispersion of stimulus i
- ρ_{ij} is the correlation between the scale values of stimuli i and j

In the standard version, all standard deviations are assumed to be one, and all correlations are assumed to be zero (this is sometimes called Thurstone's model C). Equations (1) are then solved for

[1] Conceived by L. L. Thurstone, the law of comparative judgment (LCJ) is a general mathematical representation of a discriminal process, which is any process in which a comparison is made between pairs of a collection of entities with respect to magnitudes of an attribute, trait, attitude, and so on. Examples of such processes are the comparison of perceived intensity of physical stimuli, and comparisons of the level of extremity of an attitude expressed within statements

the μ_i's, which are taken as the scale values, unique up to a positive affine transformation.

2.1 Scale Values Using Probabilistic Inversion

The problem can also be formulated as a probabilistic inversion problem as follows. Given a random vector $\mathbf{Y} \in \mathbb{R}^M$ and a measurable function $G : \mathbb{R}^N \rightarrow \mathbb{R}^M$, find a vector $\mathbf{X}$ such that $G(\mathbf{X}) \sim \mathbf{Y}$, where $\sim$ means that $G(\mathbf{X})$ and $\mathbf{Y}$ have the same distribution [1].

In the paired comparison method there is data available of the form p_{ij}, where p_{ij} is the probability that country i is preferred to country j. Competitiveness is interpreted as utility. Utility is affine unique, so it may be assumed that the competitiveness of each of the countries is represented, for each respondent by a number in the interval $[0, 1]$. It is also assumed that the 0-1 values can be chosen to be the same for all respondent. With n countries the set of utility functions is $[0, 1]^n$. The problem is to find a distribution over the set $[0, 1]^n$ such that the probability of drawing a utility function for which $u_i > u_j$ equals the probability of drawing a respondent preferring country i to country j.

The probabilistic inversion problem takes the following mathematical form. Vector $\mathbf{X}$ is a random vector in $[0, 1]^n$, where n is the number of countries. Vector $\mathbf{Y}$ is a random vector with entries $\mathbf{Y}_{\{ij\}}$ taking values one with probability p_{ij} and takes value zero otherwise. Let $G_{\{ij\}}(\mathbf{X}) : \mathbb{R}^n \rightarrow \{0, 1\}$ taking value one if $x_i > x_j$ and zero otherwise.

The problem of finding an $\mathbf{X}$ such that $G(\mathbf{X}) \sim \mathbf{Y}$ can be solved using either IPF(Iterative Proportional Fitting)[3] or PARFUM (Parameter Fitting for Uncertainty Models)[4]. If the problem is feasible IPF is known to converge faster than PARFUM to a solution, but if the problem is infeasible PARFUM is known to converge to a minimally infeasible solution, whereas IPF fails to converges.

IPF and PARFUM are resampling methods. Starting with a large sample from $[0, 1]^n$, these methods iteratively find a set of weights for this sample such that, when the sample is re-sampled with these weights, $G(\mathbf{X})$ is as close as possible to $\mathbf{Y}$. Scale values are obtained by taking the mean of each resampled sample of each x_i after applying either IPF or PARFUM depending on the feasibility of the problem. The entire joint distribution of $\mathbf{X}$ is available as well. After this exercise linear regression is applied to identify quantities that explain the level of competitiveness.

2.2 Direct Fitting Using Paired Comparison Data

According to the definition of the WEF the competitiveness of a country depends on a combinations of factors, policies, and institutions that determine the level of competitiveness. These factors, policies and institutions are grouped into nine pillars:

1. Basic Requirements - Macroeconomy (BR-M)
2. Efficiency Enhancers - Market Efficiency (EE-ME)
3. Innovation And Sophistication Factors - Innovation(IASF-I)
4. Efficiency Enhancers - Technological Readiness(EE-TR)
5. Basic Requirements - Health And Primary Education(BR-HAPE)
6. Efficiency Enhancers - Higher Education and Training(EE-HET)
7. Basic Requirements - Infrastructure(BR-If)
8. Basic Requirements - Institutions(BR-Is)
9. Innovation And Sophistication Factors - Business Sophistication(IASF-BS)

Before obtaining the GCI for a country the WEF groups the nine pillars into three basic groups namely: basic requirements, efficiency enhancers and innovation factors. The GCI then becomes

$$\begin{aligned} GCI &= \alpha_1 * Basic\ Requirements \\ &+ \alpha_2 * Efficiency\ Enhancers \\ &+ \alpha_3 * Innovation\ Factors \end{aligned} \quad (2)$$

where $\alpha_1,\ \alpha_2,\ \alpha_3$ are weights that sum up to one.

Assume for the moment that competitiveness can be written as a weighted linear sum of the values of these nine pillars plus an unexplained term which is normally distributed with mean μ and standard deviation σ. A new probabilistic inversion can formulated as follows;

Define a new random vector $\mathbf{C}$, whose entries $c_1, c_2, \cdots, c_n$ determine the level of competitivenes of each selected country. $\mathbf{C}$ can be modelled as

$$\begin{aligned} c_1 &= \sum_{j=1}^{m} \omega_j \varphi_{1j} + \epsilon_1 \\ \cdots &= \cdots \\ c_n &= \sum_{j=1}^{m} \omega_j \varphi_{nj} + \epsilon_n \end{aligned}$$

$$\sum_{j=1}^{m} \omega_j = 1$$

$$\omega_1, \cdots, \omega_m \geq 0 \quad (3)$$

where n is the number of countries, m the number of pillars, φ_{ij} is the value of pillar j for country i, and $\epsilon_i \sim \mathcal{N}(\mu_i, \sigma_i)$.

Paired comparison data is available and will be used in the same manner as before. The problem is to find a distribution over the coefficients ω_j and ϵ_i such that, sampling $(\omega, \epsilon) = (\omega_1, \cdots, \omega_m, \epsilon_1, \cdots, \epsilon_n)$ from this distribution, the probability that $c_i > c_j$ equals the probability of drawing a respondent for whom the competitiveness of country i is greater than that of country j, while keeping σ_i as small as possible. The starting distribution for the ϵ_i is the standard normal distribution.

Before applying the direct method it is useful to convert the values of the pillars to Z-scores by substracting the mean and dividing by the standard deviation.

2.3 Remarks

Currently there is no goodness of fit test avialable for the direct fitting method. The only way of knowing whether the direct fitting method is a good fit is by looking at the probabilities after sample re-weighting. If the probabilities after sample re-weighting do not differ much from the probabilities that the respondents provided then the direct method is a good fit. The following heuristic can be applied to measure goodness of fit.

$$\delta = \sqrt{\frac{2}{n(n-1)} \sum_{i=1}^{n} \sum_{j>i}^{n} \frac{\left(p'_{ij} - p_{ij}\right)^2}{p_{ij}^2}} \quad (4)$$

where p'_{ij} is the probability that country i is preferred over country j after applying sample reweighting and δ the relative error. If δ is close to zero then it may be concluded that the direct method is a good fit.

3 Results

Eight countries were selected for comparisons:

1. Finland
2. USA
3. UK
4. Italy
5. Hungary
6. China
7. Mexico
8. Korea

Twenty seven respondents were ask to pairwise compare these eight countries. Values for these countries for the nine pillars are retrieved from the WEF. The values are shown below.

Country	(BR-M)	(EE-ME)	(IASF-I)
Finland	0.548	0.73	1.077
USA	-0.488	1.41	1.505
UK	-0.298	1.164	0.471
Italy	-1.092	-1.057	-0.856
Hungary	-1.489	-0.092	-0.631
China	1.273	-0.837	-0.699
Mexico	0.496	-1.122	-1.258
South-Korea	1.048	-0.196	0.39

Tab. 1: The values of the pillars for each of the countries

Country	(EE-TR)	(BR-HAPE)
Finland	0.794	0.92
USA	1.057	-0.056
UK	0.84	0.369
Italy	-0.517	1.146
Hungary	-0.331	0.645
China	-1.419	-1.859
Mexico	-1.228	-0.307
South-Korea	0.804	-0.858

Tab. 2: The values of the pillars for each of the countries

Country	(EE-HET)	(BR-If)	(BR-Is)	(IASF-BS)
Finland	1.246	1.138	1.537	0.778
USA	0.988	1.172	0.831	1.3
UK	0.866	0.687	0.912	0.881
Italy	-0.573	-0.52	-0.81	0.068
Hungary	-0.3	-0.651	-0.2	-0.991
China	-1.207	-1.155	-0.89	-1.217
Mexico	-1.324	-1.232	-1.271	-1.16
South-Korea	0.304	0.561	-0.109	0.34

Tab. 3: The values of the pillars for each of the countries

3.1 Scale Values

The following table gives the scale values of each of the countries after applying the IPF method with 40000 samples and 100 iterations.

Country	Mean value	Dispersion
1	0.62477	0.27041
2	0.57509	0.26013
3	0.62905	0.24685
4	0.35707	0.22641
5	0.46405	0.30304
6	0.62601	0.25808
7	0.27763	0.2339
8	0.45511	0.27586

Tab. 4: The countries's Scale Values using IPF

The scale values of each of the countries after applying the PARFUM method with 40000 samples and 100 iterations are very close to those with IPF.

Country	Mean value	Dispersion
Finland	0.62472	0.26675
US	0.57436	0.26086
UK	0.62794	0.24706
Italy	0.3541	0.23072
Hungary	0.46217	0.29864
China	0.62513	0.25762
Mexico	0.28085	0.23081
South-Korea	0.45157	0.27672

Tab. 5: The countries's Scale Values using PARFUM

As can be noted from the previous two tables both IPF and PARFUM give similar results. The results of Thurstone C, IPF, PARFUM and the GCI can be compared with each other. To do that they are scaled such that the best score has value one and the worst value zero. The result given in the following table.

Country	ThurC	IPF	PARFUM	GCI
Finland	1	1	1	0.9325
UK	0.9551	0.93114	0.9287	0.8090
China	0.8965	0.9010	0.8993	0.1067
US	0.7977	0.7704	0.7673	1
Hungary	0.5697	0.5519	0.5452	0.2416
South-Korea	0.5504	0.5338	0.5284	0.6798
Italy	0.2793	0.2864	0.2846	0.2247
Mexico	0	0	0	0

Tab. 6:

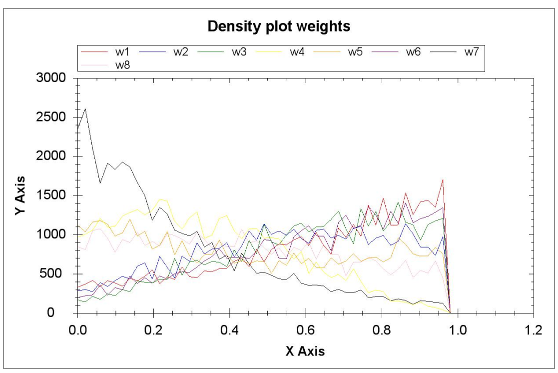

Fig. 1: Densities of Scale values country

3.2 Linear Regression

The nine pillars that form the Global Competitiveness Index (GCI) of the WEF are used in the regression exercise. The product moment correlation matrix of the nine pillars is computed to determine correlation between these nine pillars.

1	2	3	4	5	6	6	8	9
1.00	-0.17	0.07	-0.12	-0.74	-0.12	-0.02	-0.08	-0.12
-0.17	1.00	0.91	0.86	0.26	0.91	0.87	0.92	0.81
0.07	0.91	1.00	0.91	0.15	0.94	0.96	0.91	0.89
-0.12	0.86	0.91	1.00	0.36	0.96	0.97	0.87	0.92
-0.74	0.26	0.15	0.36	1.00	0.42	0.32	0.39	0.39
-0.12	0.91	0.94	0.96	0.42	1.00	0.98	0.97	0.91
-0.02	0.88	0.96	0.97	0.32	0.98	1.00	0.92	0.95
-0.08	0.92	0.91	0.87	0.39	0.97	0.2	1.00	0.82
-0.12	0.81	0.89	0.92	0.39	0.91	0.95	0.82	1.00

Tab. 7: Product moment correlation matrix from the pillars of WEF

¿From the product moment correlation matrix it may be concluded that some of the pillars are stgrongly correlated. Strong correlations are not implausible. For example better **Higher Education** is defined as the quality and quantity of higher education provided within an economy are critical for competitiveness, for preparing qualified staff for more complex roles in production, marketing, management, and R&D. **Innovation** is defined as a creation (a new device or process) resulting from study and experimentation. Thus a country with a high value of the pillar **Higher Education** will most likely have a high value of the pillar **Innovation**. The only correlation coefficient that seems odd is the correlation coefficient of the pillar **Macroeconomy** and the pillar **Health and Primary Education**. The correlation coefficient indicates that if a country becomes macroeconomically more stable the health and primary education of that country will degrade and this of course makes no sense. Of course, it may also reflect a reporting bias

Stepwise regression is used to find a subset of pillars that can best describe one of the scores obtained from one of the models. Stepwise regression can either start with the empty set and add a pillar to the set as long as the probability of seeing a *t-value* from the regression coefficient from pillar is smaller than a given significance level α. Usually the value for α is equal to 0.05 and is also used in this example as the level of significance.

Stepwise Regression is done with the mean scale values obtained from the IPF method, because the problem is feasible and converges. The results of the stepwise regression with IPF is given as follows

Coefficient	Value	Std. Error	t-value	$P(> \|t\|)$
β_1	1.1115	0.1748	6.3602	0.0031
β_6	3.5423	0.8763	4.0424	0.0156
β_7	-3.2941	0.8698	-3.7872	0.0193

Tab. 8: Regression results with IPF

Residual standard error: 0.3894 on 4 degrees of freedom
$Multiple\ R - Squared$: 0.9133
$F - statistic$: 14.05 on 3 and 4 degrees of freedom, the $p - value$ is 0.01366

It would appear that competitiveness can best be described with the three pillars **Macroeconomy**, **Higher Education and Training** and **Infrastructure**. The only thing not satisfactory is the fact that the regression coefficient for the pillar **Infrastructure** is negative. Good infrastructure is usually needed for the development of a country. A country with bad infrastructure can most like not transport and facilitate *goods* and *people*, which in return will most likely be less competitive than a country with better infrastructure keeping the pillars **Macroeconomy** and **Higher Education and Training** constant.

The negative regression coefficient for the pillar **Infrastructure** also means that the respondents took other criteria into account while comparing the countries. The respondents queried are proxy experts drawn from the JRC, and this also qualifies the results.

3.3 Direct Fitting

Both iterative resampling methods (IPF and PARFUM) fail to satisfy the quantile constraints when μ_i and σ_i are equal to zero. The reason this is that some countries (eg Finland) strictly dominate others (eg Mexico) across all nine pillars. Setting μ_i and σ_i equal to zero would indicate that there are no unexplained terms and that the model is exactly weighted linear sum of the nine pillars. If respondent used different criteria or a different functional form to compare the countries, then this difference must be captured in the error term.

A good fit without error is found when negative weights are allowed. The starting distribution for each of the nine weights is given by $\mathcal{N}(0,1)$ and μ_i and σ_i are all equal to zero. The problem then is feasible and converges to the solution shown below:

Weight	Mean	Sd
1	-0.105741	0.8608
2	-0.020543	0.89516
3	-0.080717	1.005222
4	-0.515317	0.979088
5	-0.42591	0.811518
6	0.3594	0.938546
7	0.012198	0.984591
8	0.791421	0.854656
9	0.189513	0.976851

Tab. 9: Mean and Standard Deviation of the weights after sampling re-weighting with starting distribution $\mathcal{N}(0,1)$

The direct fitting method can use all the nine pillar to find mean values for the regression coefficients, in contrast to normal regression, which in this exercise can only use at most seven pillars, as there are eight countries.

The relative error defined by equation (4) for the direct fit is equal to $6.283E - 10$ which is approximately zero. From this it may be concluded that the model with negative coefficients is a good fit.

When the coefficients are constrained to be normalized positive weights, the minimal value for the σ_i for which there is a solution is 0.5. The number of samples used is 40000 and the number of iterations is 150. The relative error made is then 1.170E-07. The tables 10 to 13 give respectively the probability before and after sampling re-weighting and the mean and standard deviation of the c's, ω's, and ϵ's .

Probabilty	Before	After
1 < 2	0.37037037	0.370370423
1 < 3	0.444444444	0.444444493
1 < 4	0.148148148	0.148148166
1 < 5	0.407407407	0.407407402
1 < 6	0.481481481	0.481481464
1 < 7	0.185185185	0.185185147
1 < 8	0.333333333	0.333333284
2 < 3	0.518518519	0.518518503
2 < 4	0.185185185	0.185185189
2 < 5	0.444444444	0.44444434
2 < 6	0.555555556	0.555555577
2 < 7	0.111111111	0.11111109
2 < 8	0.407407407	0.407407423
3 < 4	0.074074074	0.074074092
3 < 5	0.333333333	0.333333231
3 < 6	0.481481481	0.481481534
3 < 7	0.148148148	0.148148143
3 < 8	0.296296296	0.296296287
4 < 5	0.481481481	0.481481432
4 < 6	0.814814815	0.814814822
4 < 7	0.296296296	0.296296302
4 < 8	0.592592593	0.592592595
5 < 6	0.703703704	0.703703711
5 < 7	0.333333333	0.333333336
5 < 8	0.481481481	0.481481478
6 < 7	0.148148148	0.148148149
6 < 8	0.259259259	0.259259258
7 < 8	0.703703704	0.703703704

Tab. 10: Probabilties experts provided before and after sample re-weighting

Variable	Mean	Sd
w1	0.128193	0.03094
w2	0.108914	0.032686
w3	0.110052	0.032625
w4	0.106927	0.02931
w5	0.116231	0.035212
w6	0.107867	0.04016
w7	0.106991	0.040028
w8	0.109513	0.034559
w9	0.105312	0.039806

Tab. 11: Weights using the model with error terms

The weights that the WEF uses for the three groups (Basic Requirements, Efficiency Enhancers, and Innovation Factors) are equal to $\alpha_1 = 0.4, \alpha_2 = 0.3$, and $\alpha_3 = 0.3$. The weights that are obtained by the model with error terms are equal to $\alpha_1^{'} = 0.46, \alpha_2^{'} = 0.32$, and $\alpha_3^{'} = 0.22$.

The ranking obtained from using the model with error terms (**C**) is almost in line with the ranking of the WEF forum. Transforming both the ranking of the WEF and the ranking of the model with error terms to the 0-1 interval gives table 14.

Variable	Mean	Sd
e1	-0.517382	0.687127
e2	-0.558895	0.504288
e3	-0.312507	0.443142
e4	0.042632	0.551766
e5	0.271453	0.739795
e6	0.963236	0.51758
e7	0.171739	0.627779
e8	-0.377387	0.523269

Tab. 12: Error terms

Variable	Mean	Sd
c1	0.973524	0.500151
c2	0.85927	0.5097
c3	0.652264	0.501791
c4	-0.46481	0.505441
c5	-0.447379	0.506667
c6	-0.890349	0.509587
c7	-0.938001	0.502237
c8	0.252692	0.503093

Tab. 13: Countries score's using the model with error terms's

Country	GCI	**C**
Finland	0.9325	1
USA	1	0.9402
UK	0.8090	0.8319
Italy	0.2247	0.2475
Hungary	0.2416	0.2567
China	0.1067	0.025
Mexico	0	0
South-Korea	0.6798	0.6229

Tab. 14: GCI vs. **C**

4 Discussion

The results presented above show that a top down method for deriving indices is possible, and allows the indices to be explained by regression onto independent variables. Advantages of this method are transparency and direct translation of preferences of experts in paired comparisons. When standard regression is used we are limited in the number of regressors. Moreover, the regression coefficients may be negative, whereas their intuitive interpretation requires a positive dependence. Negative coefficients actually indicates lack of model fit.

The problem of too few regressors is avoided by using probabilistic inversion with direct fitting on the explanatory variables. Without the addition of an error term, the problem of negative coefficients arose. However, this method enables the introduction of an error term to capture lack of fit, while the coefficients are constrained to be positive and sum to one. Of course, such lack of fit should motivate exploration of other independent variables or other functional forms. The results presented above show that a top down method for deriving indices is possible, and allows the indices to be explained by regression onto independent variables. Advantages of this method are transparency and direct translation of preferences of experts in paired comparisons. When standard regression is used we are limited in the number of regressors. Moreover, the regression coefficients may be negative, whereas their intuitive interpretation requires a positive dependence. Negative coefficients actually indicates lack of model fit.

The problem of too few regressors is avoided by using probabilistic inversion with direct fitting on the explanatory variables. Without the addition of an error term, the problem of negative coefficients arose. However, this method enables the introduction of an error term to capture lack of fit, while the coefficients are constrained to be positive and sum to one. Of course, such lack of fit should motivate exploration of other independent variables or other functional forms.

References

[1] R.M. Cooke C. DU, D. Kurowicka. Techniques for generic probabilistic inversion, 2005.

[2] R.M. Cooke. Discrete choice with probabilistic inversion, 2006.

[3] J. Kruithof. Telefoonverkeersrekening, 1937.

[4] R.M.Cooke. Parameter fitting for uncertain models: modelling uncertainty in small models, 1994.

[5] L.L. Thurstone. A law of comparative judgement, 1927.