Adaptive circular deconvolution by model selection under unknown error distribution

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We consider a circular deconvolution problem, where the density f of a circular random variable X has to be estimated nonparametrically based on an iid. sample from a noisy observation Y of X. The additive measurement error is supposed to be independent of X. The objective of this paper is the construction of a fully data-driven estimation procedure when the error density φ is unknown. However, we suppose that in addition to the iid. sample from Y, we have at our disposal an additional iid. sample independently drawn from the error distribution.

First, we develop a minimax theory in terms of both sample sizes. However, the proposed orthogonal series estimator requires an optimal choice of a dimension parameter depending on certain characteristics of f and φ, which are not known in practice. The main issue addressed in our work is the adaptive choice of this dimension parameter using a model selection approach. In a first step, we develop a penalized minimum contrast estimator supposing the degree of ill-posedness of the underlying inverse problem to be known, which amounts to assuming partial knowledge of the error distribution.

We show that this data-driven estimator can attain the lower risk bound in both sample sizes n and m over a wide range of density classes covering in particular ordinary and super smooth densities. Finally, by randomizing the penalty and the collection of models, we modify the estimator such that it does not require any prior knowledge of the error distribution anymore. Even when dispensing with any hypotheses on φ, this fully data-driven estimator still preserves minimax optimality in almost the same cases as the partially adaptive estimator.

References

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