#### Mathematics in Waterland

COSSE Workshop Delft University of Technology, Delft, The Netherlands February 7-10, 2011

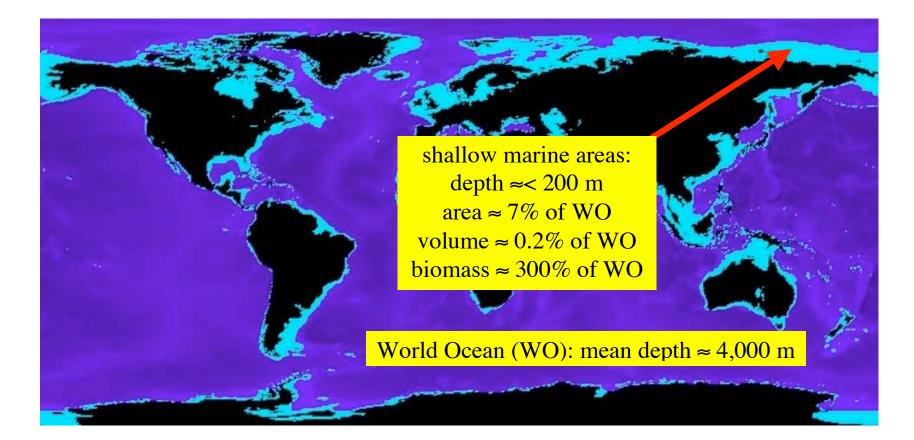
# Beyond Ocean Modelling: Multi-Scale/Physics Numerical Simulation of the Hydrosphere I. Numerics and Preliminary Applications

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# **Motivations** (I)

• Hydrosphere: oceans, shelf seas, estuaries, rivers, groundwater, etc. + sea ice (for the sake of simplicity)



# **Motivations** (II)

• Models exist of each of the component of the hydrosphere, which are generally made up of PDEs derived from fluid mechanics.

• Components of the hydrosphere interact with each other. Therefore, there is a growing need for models dealing with several, if not all, of the components of the hydrosphere. Possible applications are:

- Climate change studies should take into the impact of shallow areas on global biogeochemical cycles, especially that of carbon;

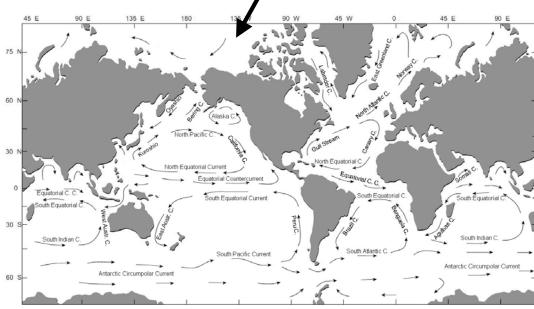
- Environmental impact assessment: pollutants know no borders/boundaries.

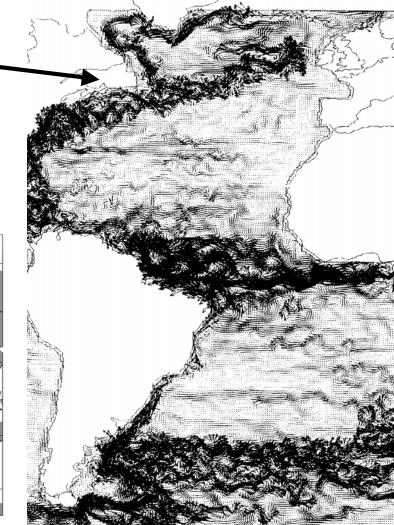
• A daunting multi-scale/physics problem is to be addressed.

#### A flavour of multi-scale (oceanic) dynamics (I)

The real ocean surface circulation is closer to this, with many transient eddies 50-100 km <sup>-</sup> in size.

A common representation of the oceanic surface circulation.

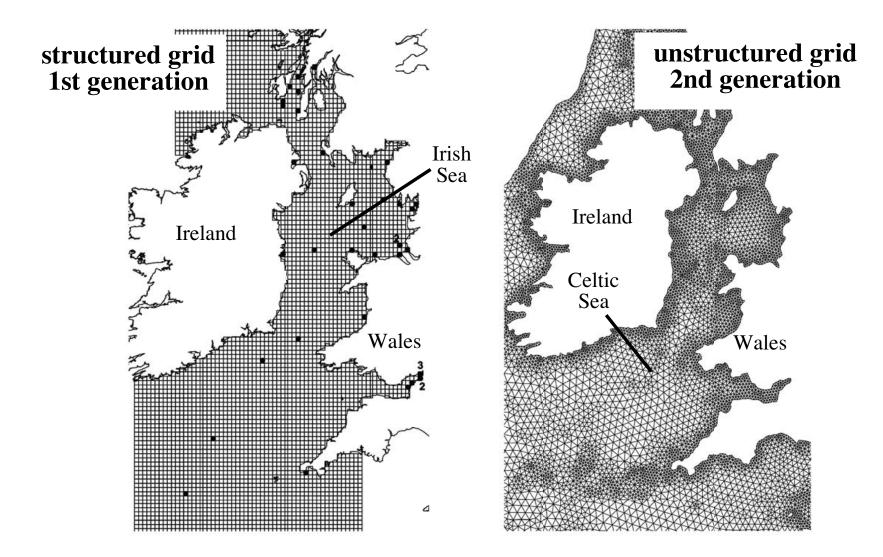




#### A flavour of multi-scale (oceanic) dynamics (II)

108 Characteristio length scale [m] thermotides haline circulation A wide range of timecirculation 10<sup>6</sup> - 1000 km cells space scales of motion. geostrophic eddies Only the largest ones can be resolved in the 104 inertial fronts waves models, the smallest-ones internal 1 km gravity having to be waves 10<sup>2</sup> = swell parameterised — to sound waves boundary account for their impact layer wind turbulence sea on larger scales of 10<sup>0</sup> 🚽 1 m motion. micro turbulence 10<sup>-2</sup> 1 cm 1 second 1 minute 1 hour 1 day 10 1 year 10 100 1000 1010 10-2 100 10<sup>2</sup> 104 106 108 Characteristic time scale [s]

#### Structured vs unstructured grids



# **Structured grids**

### Advantages

- Finite-difference methods are easy to implement.
- Programming is easy.
- Well known in the realm of oceanography/meteorology.

# Disadvantages

- Representation of the coastlines (staircase-like).
- Difficult to enhance resolution (even with curvilinear coordinates or embedded grid systems).
- Singularity of the pole(s) in global models.

# **Unstructured grids**

# Disadvantages

- Numerical methods are uneasy to implement.
- Programming is uneasy.
- Not well known in the realm of oceanography/meteorology.

# Advantages

- Representation of the coastlines.
- Easy to enhance resolution (including adaptive strategies).
- No singular points in global models.

# Are adaptive- and unstructured-mesh models coming of age?

- Classical structured-mesh eco-hydrodynamic models with (almost) constant resolution may be getting obsolete.
- Time may be ripe for developing models in which resolution may be enhanced **where** and **when** needed, facilitating multi-scale approaches.

• Encouraging example: reduced-gravity simulation of a baroclinic eddy\* in the Gulf of Mexico by means of a Discontinuous Galerkin (DG), finite-element, adaptive-mesh model (Bernard et al., *Ocean Dynamics*, 2007).  $\Rightarrow$  movie1

This simulation is one order of magnitude cheaper than a constantresolution one of the same accuracy!

<sup>\*</sup> Features of the eddy inspired by Lewis and Kirwan (JGR, 1987).



The Second-generation, Louvain-la-Neuve, Ice-ocean Model (SLIM) http://www.climate.be/SLIM htp://www.climate.be/timothy



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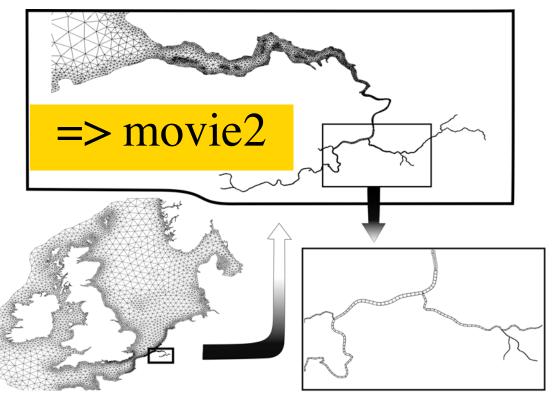


# Multi-scale modelling: the Scheldt basin (I)

The main advantage of unstructured meshes probably is that multiscale modelling is rendered easier. Example: the Scheldt tributaries, River, Estuary and ROFI (www.climate.be/timothy).

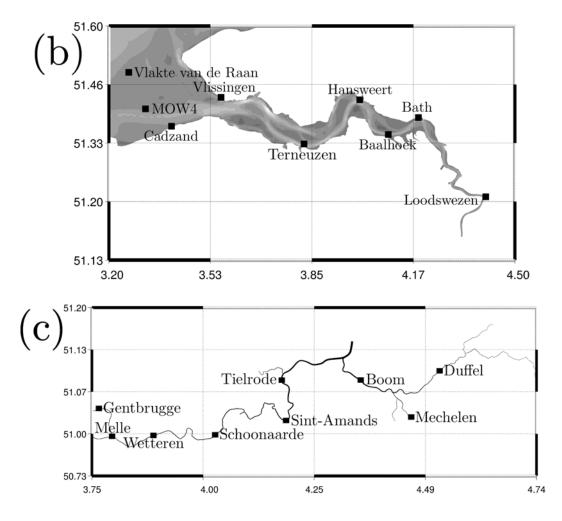
• 40% of the meshes in the estuary, which represents 0.3% of the computational domain.

• No major problem with open boundary conditions (for tides, storms, river discharge).

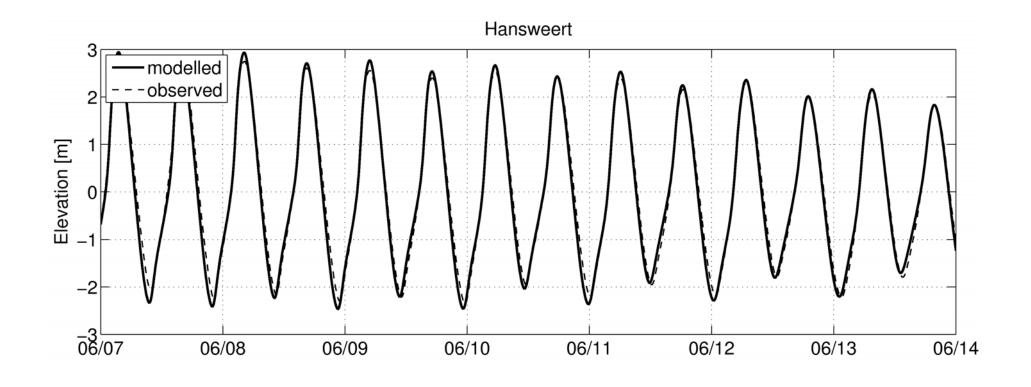


(de Brye et al., *Coastal Engineering*, 2010)

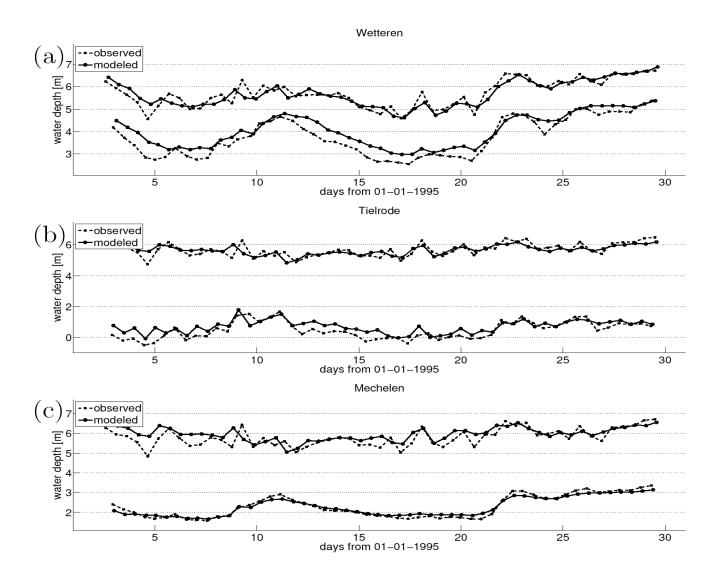
#### Multi-scale modelling: the Scheldt basin (II)



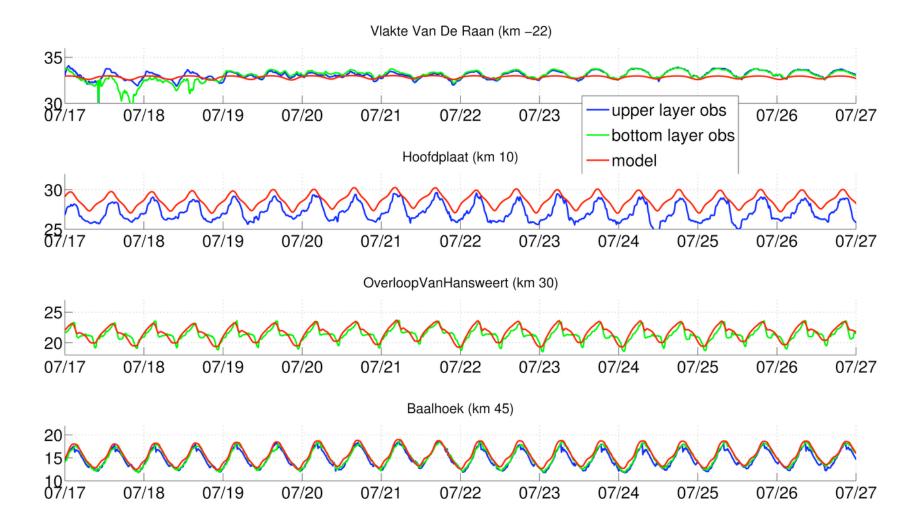
### **Multi-scale modelling: the Scheldt basin (III)**



#### **Multi-scale modelling: the Scheldt basin (IV)**

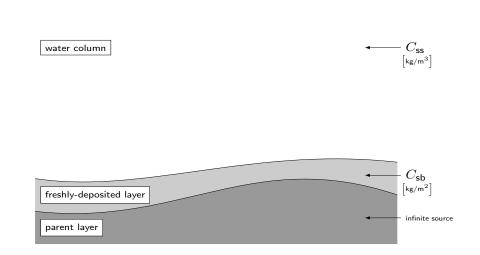


#### Multi-scale modelling: the Scheldt basin (V)

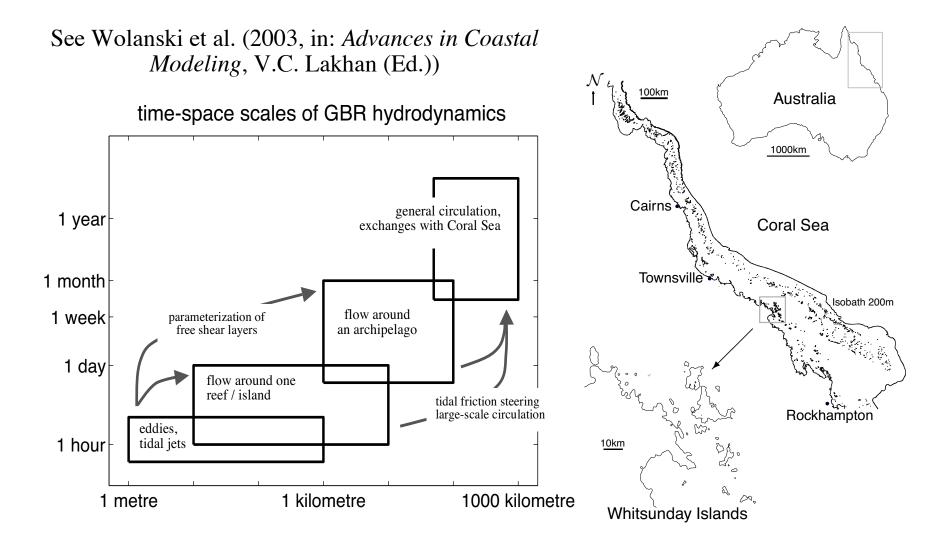


# **Multi-scale modelling: the Scheldt basin (VI)**

- Contaminant modelling in the Scheldt system:
  - Fecal bacteria + experimental design (de Brauwere et al., *Environmental Modelling and Software*, 2009; de Brauwere et al. *Water Research*, 2011);
  - Heavy metals (under development).
- Development of a three-layer sediment module (essential for studying the fate of heavy metals)



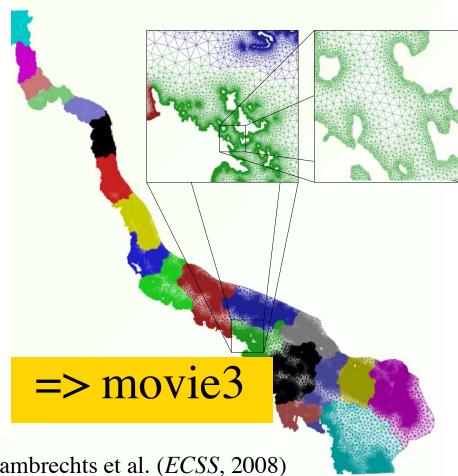
#### **Multi-scale modelling: the Great Barrier Reef (I)**



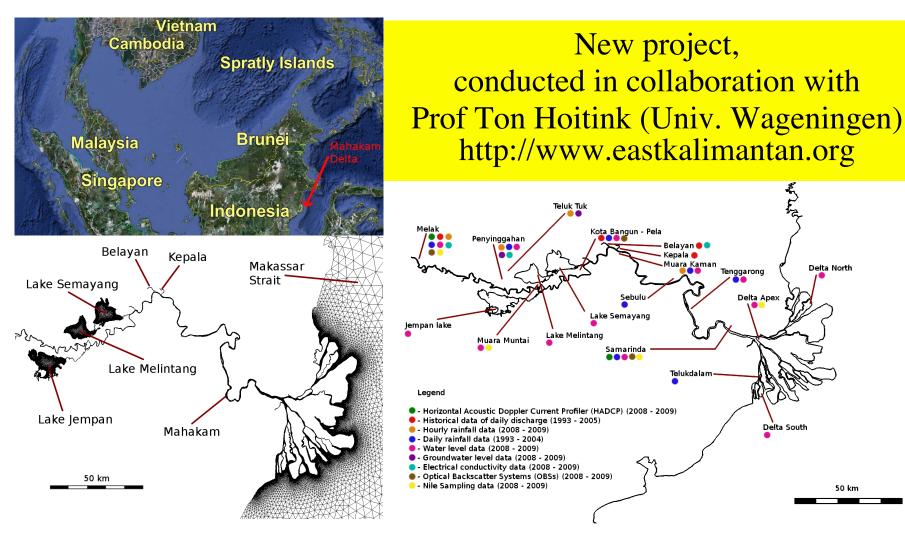
### **Multi-scale modelling: the Great Barrier Reef (II)**

- $O(10^6)$  triangles, with  $\Delta x_{\text{max}} / \Delta x_{\text{min}} \approx 10^2$
- Forcings: wind, tides, Coral Sea inflow
- A wide spectrum of hydrodynamic processes simulated (eddies, tidal jets, "sticky waters", general circulation)

See Legrand et al. (ECSS, 2006) and Lambrechts et al. (ECSS, 2008)

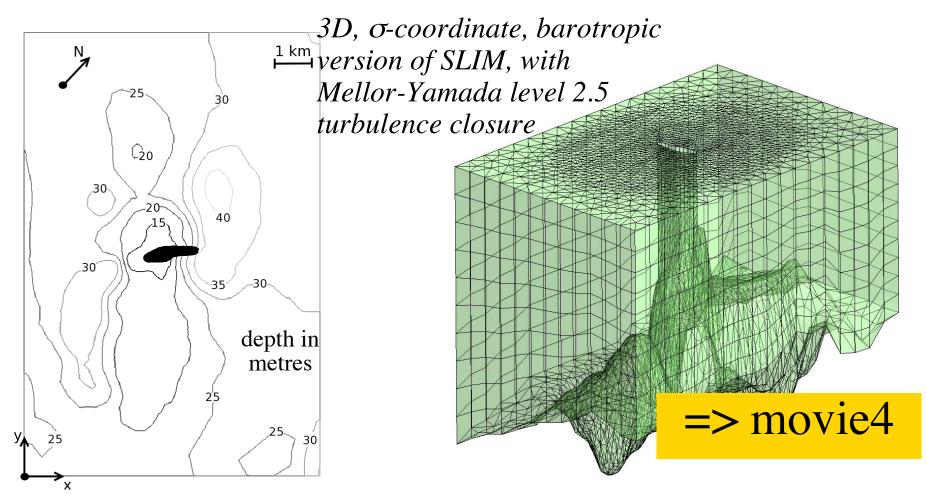


# Mahakam River (Indonesia) land-sea continuum

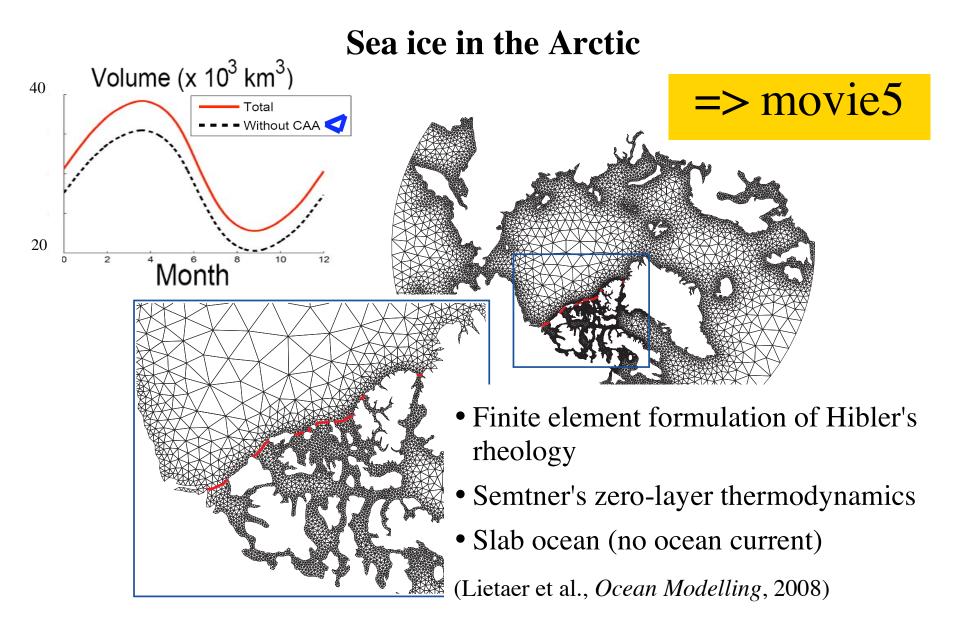


(de Brye et al., Ocean Dynamics, submitted; Sassi et al., Ocean Dynamics, submitted)

#### **Rattray Island eddies and associated upwelling**



See Blaise et al. (CSR, 2007), White et al. (ECSS, 2007), and White and Wolanski (ECSS, 2008)



## **Current developments**

- A underground flow module (Richards equation).
- 3D baroclinic flow module is being tested, and coupled to the sea ice module (Blaise et al. & Comblen et al., *Ocean Dynamics*, 2010)
- Wetting-drying algorithm: speed up of O(200) (Kärnä et al., Computer Methods in Applied Mechanics and Engineering, 2011).
- Speeding up the model by at least one order of magnitude:
- Higher-order implicit schemes (appropriate for a few processors);
- Multi-rate RK (explicit) time steppings (appropriate for a large number of processors) (Constantinescu and Sandu, 2007; Schlegel et al., 2009).

# Conclusion

- SLIM already has some of the building blocks of a multi-scale/physics hydrospheric model.
- The availability of multi-scale data is a key problem.

• Will multi-scale reactive transport models prove superior to those focused on long time and space scales (which often do not represent the most energetic processes, such as floods, tides, etc.)?

• Will pursuing the dream of a model of the whole hydrosphere lead to useful results (even if such a model never comes true)?



You may say I'm a dreamer / But I'm not the only one I hope some day you'll join us (John Lennon, Imagine, 1971)

The author of the present talk is deeply indebted to the SLIM team Colleagues, without whom this presentation could not have come true.