Mathematics in Waterland

COSSE Workshop

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Beyond Ocean Modelling: Multi-Scale/Physics Numerical Simulation of the Hydrosphere

II. Interpreting the results of complex models

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The need for holistic interpretation methods

• Today's numerical models produce output files that are so huge that the human brain can “read/peruse” only a tiny fraction of them. Thus, making sense of the results is a great challenge.

• Producing graphs based on space-time slices in the output files amounts to ignoring most of the results and relies on the assumption that the slices are well chosen. But, how can we be sure of that as most of the results are ignored?

• Methods are needed that drastically reduce the amount of results submitted to the human brain without leaving data aside. Statistics and timescale analyses fall into this class of interpretation methods. These methods are holistic in that all/most of the results are taken into account.

Water renewal timescales

A tentative definition of **water renewal**:

the processess by which water originally inside the domain of interest is progressively replaced by water originally outside the domain of interest.

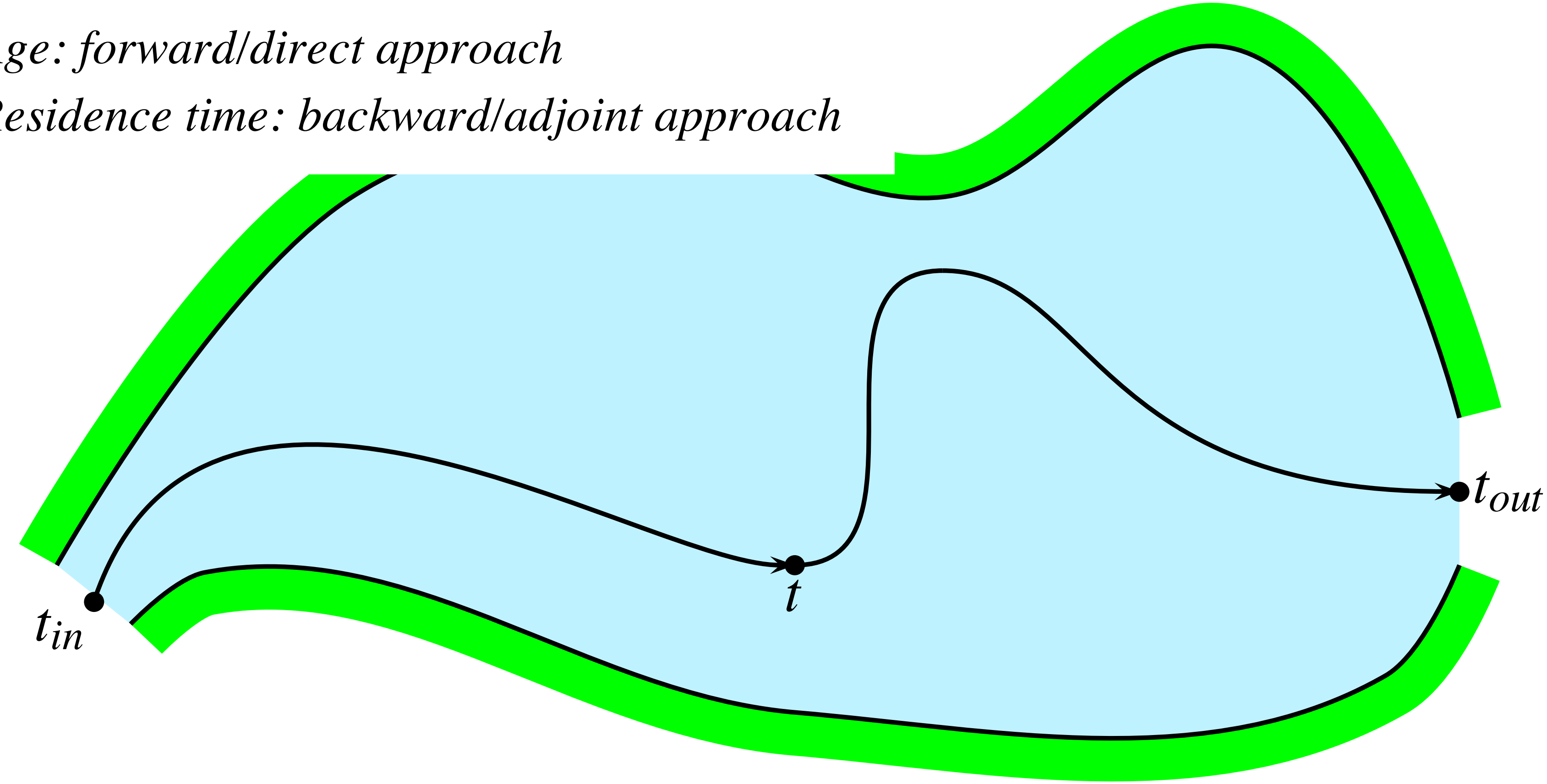
Outline of a method (e.g. Gourgue et al., ECSS, 2007) for assessing the **rate** of **water renewal**:

1. split the water into **original water** and **renewing water**, both of which being regarded as passive tracers;
2. calculate the **time** needed for the **original** water to **leave** the domain of interest;
3. calculate the **time** needed for the **renewing** water to **fill** the domain of interest.

Age and residence time

Age: forward/direct approach

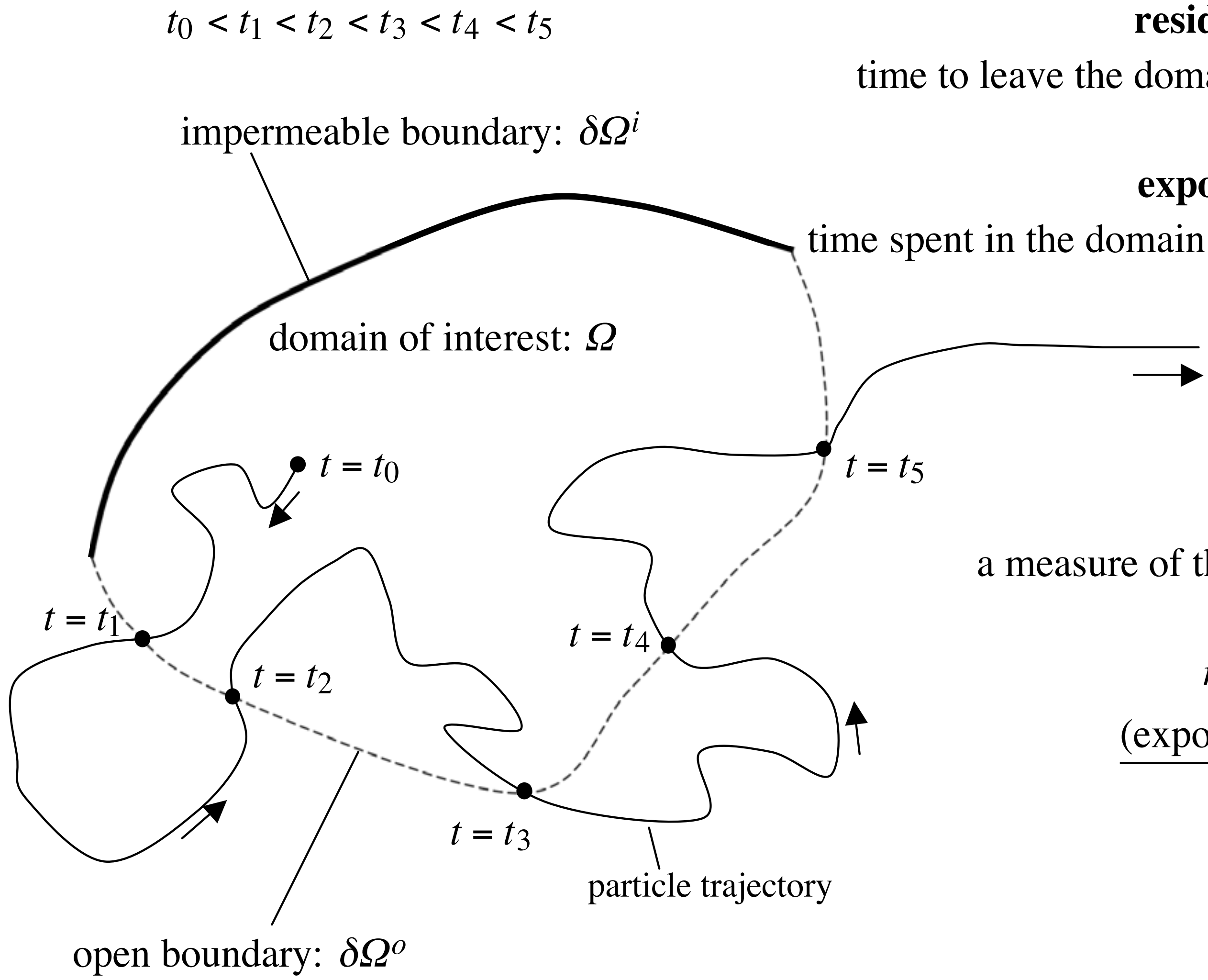
Residence time: backward/adjoint approach



$$age = t - t_{in}\ , \quad residence\ time = t_{out} - t$$

$$transit\ time = age + residence\ time$$

The exposure time, an alternative to the residence time



residence time:

time to leave the domain for the first time = $t_1 - t_0$

exposure time:

time spent in the domain = $(t_1 - t_0) + (t_3 - t_2) + (t_5 - t_4)$

return coefficient:

a measure of the propensity to return into the domain

$$r = \text{return coefficient} = \frac{(\text{exposure time}) - (\text{residence time})}{(\text{exposure time})}$$

$$0 \le r \le 1$$

$r = 0$: no return

$r = 1$: frequent reentries

Age: basic variables and equations

- $\rho c_i(t,\mathbf{x},\tau)\delta V\delta\tau$: mass of the i-th constituent in δV, whose age lies in the interval $[\tau-\delta\tau/2,\ \tau+\delta\tau/2]$ $(\delta\tau \rightarrow 0)$, where $c_i(t,\mathbf{x},\tau)$ is the **concentration distribution function**.

- Concentration: $C_i(t,\mathbf{x}) = \int_0^\infty c_i(t,\mathbf{x},\tau)\ d\tau$

- Age concentration: $\alpha_i(t,\mathbf{x}) = \int_0^\infty \tau c_i(t,\mathbf{x},\tau)\ d\tau$

- Mean age: $a_i(t,\mathbf{x}) = \dfrac{\alpha_i(t,\mathbf{x})}{C_i(t,\mathbf{x})}$

(Delhez et al., OM, 1999;
Deleersnijder et al., JMS, 2001)

Age: basic variables and equations (continued)

- Simple mass budget considerations yield:

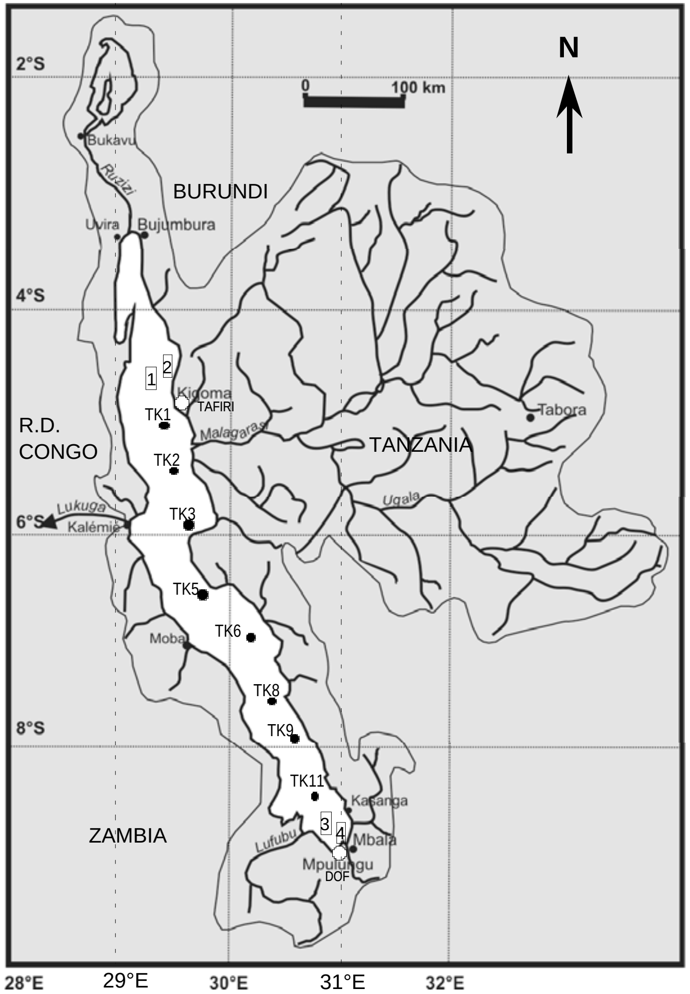
$$\frac{\partial c_i}{\partial t} = \underbrace{p_i - d_i}_{\text{source - sink}} - \nabla \bullet \underbrace{(\mathbf{u}c_i - \mathbf{K} \bullet \nabla c_i)}_{\text{advection + diffusion}} - \underbrace{\frac{\partial c_i}{\partial \tau}}_{\text{ageing}}$$

$$\frac{\partial C_i}{\partial t} = \underbrace{P_i - D_i}_{\text{source - sink}} - \nabla \bullet \underbrace{(\mathbf{u}C_i - \mathbf{K} \bullet \nabla C_i)}_{\text{advection + diffusion}}$$

$$\frac{\partial \alpha_i}{\partial t} = \underbrace{C_i}_{\text{ageing}} + \underbrace{\pi_i - \delta_i}_{\text{source - sink}} - \nabla \bullet \underbrace{(\mathbf{u}\alpha_i - \mathbf{K} \bullet \nabla \alpha_i)}_{\text{advection + diffusion}}$$

- All advection-diffusion operators are of the same form.

Renewal of Lake Tanganyika's epilimnion water (I)



Meromictic lake that lies between Congo, Burundi, Tanzania and Zambia. Its hypolimnion is the second largest anoxic water body in the world.

The water fluxes (entrainment) through the permanent thermocline are the main source of “new” water and nutrients for the epilimnion.

Finite-element, reduced-gravity model of the epilimnion that distinguishes between original epilimnion water and renewing water — from lower layer.

Renewal of Lake Tanganyika's epilimnion water (II)

- Epilimnion water concentration: $C_e(t,\mathbf{x})$, with $0 \le C_e(t,\mathbf{x}) \le 1$
  Hypolimnion water concentration: $C_h(t,\mathbf{x})$, with $0 \le C_h(t,\mathbf{x}) \le 1$

with
$$\begin{cases} C_e(0,\mathbf{x}) = 1 \quad C_e(\infty,\mathbf{x}) = 0 \\ C_h(0,\mathbf{x}) = 0 \quad C_h(\infty,\mathbf{x}) = 1 \\ C_e(t,\mathbf{x}) + C_h(t,\mathbf{x}) = 1 \end{cases}$$

⇒ movie1

- Two seasons:
  dry season (April-August) with strong winds from south-east;
  wet season (September-March) with weak winds.
  Dry season: thermocline is deeper in the north and oscillates;
  Wet season: thermocline oscillations get progressively damped.

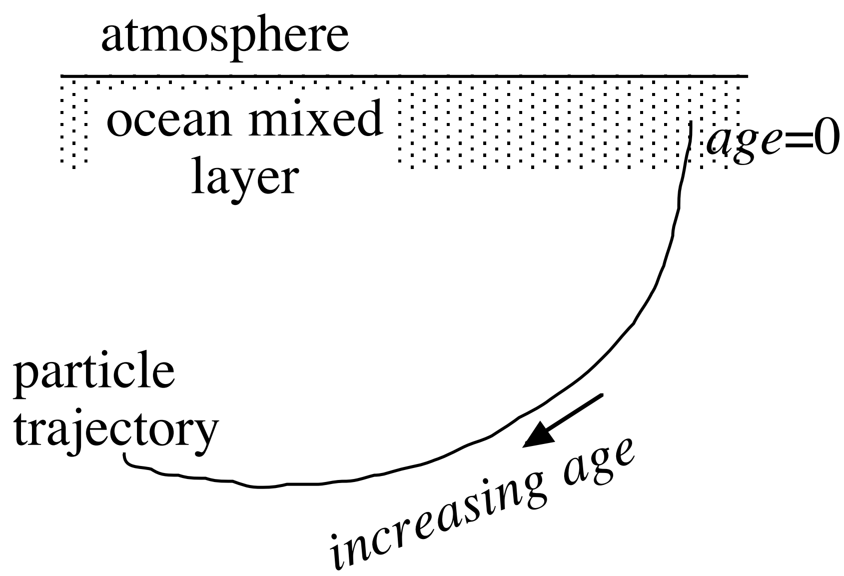
(Gourgue et al., ECSS, 2007)

Ventilation of the World Ocean (I)

According to England (JPO, 1995), the "World Ocean circulation at its largest scale can be thought of as a gradual renewal or ventilation of the deep ocean by water that was once at the sea surface."

Therefore, the age, a measure of the time since leaving the ocean upper mixed layer, is a popular diagnostic tool in the World Ocean.

estimating ocean ventilation rate



age = time elapsed since leaving surface mixed layer

Ventilation of the World Ocean (II)

- At a steady state, the *water age distribution* $c(\mathbf{x},\tau)$ is satisfies

$$\frac{\partial c}{\partial \tau} = -\nabla \bullet (\mathbf{u}c - \mathbf{K} \bullet \nabla c) \ , \ \left[c(\mathbf{x},\tau)\right]_{\Gamma} = \delta(\tau - 0) \ , \ \left[c(\mathbf{x},0)\right]_{\Omega} = 0$$

with τ = the age, Γ = the ocean surface and Ω = the ocean interior.

- *Global water age distribution* $\mu(\tau)$: the volume of the water whose age lies in the interval $[\tau, \tau + \Delta\tau]$ $(\Delta\tau \to 0)$ is $\Omega\mu(\tau)\Delta\tau$, with

$$\mu(\tau) = \frac{1}{\Omega}\int_{\Omega} c(\mathbf{x},\tau)\, d\mathbf{x} \qquad \Rightarrow \qquad \int_0^{\infty} \mu(\tau)\, d\tau = 1$$

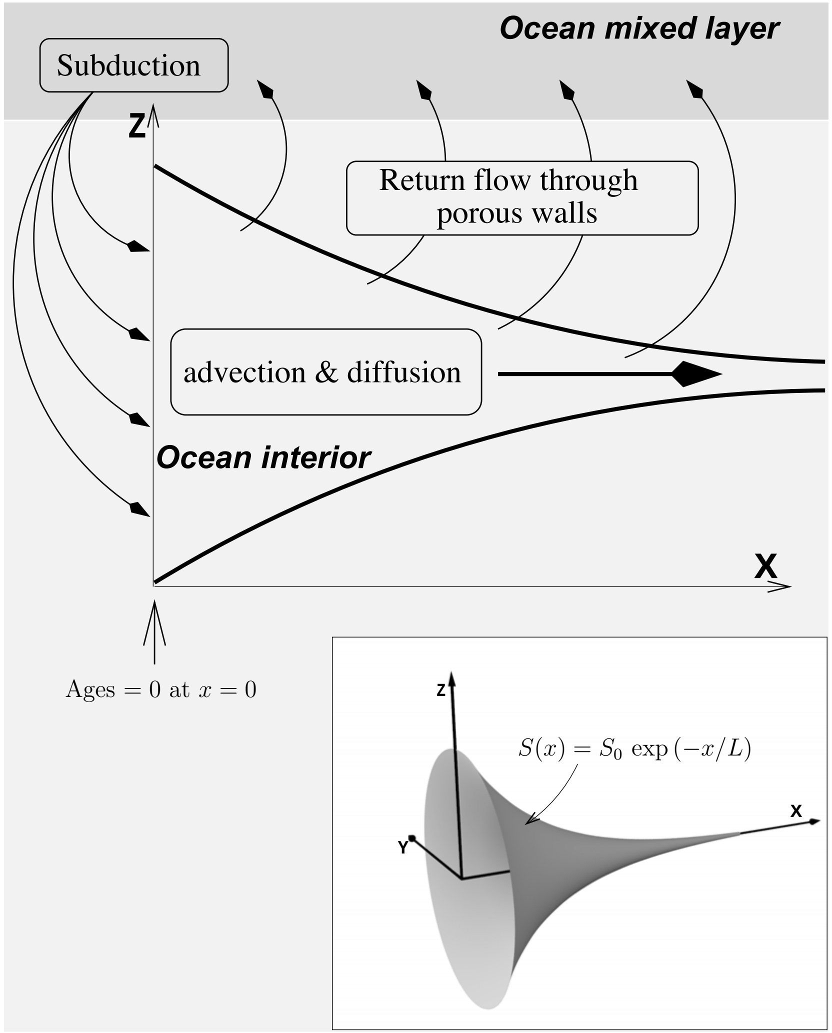
(Bolin and Rodhe, Tellus, 1973)

- *Global mean water age*: $\bar{a} = \int_0^{\infty} \tau\, \mu(\tau)\, d\tau = \frac{1}{\Omega}\int_0^{\infty}\int_{\Omega} \tau\, c(\mathbf{x},\tau)\, d\mathbf{x}\, d\tau.$

Ventilation of the World Ocean (III)

The leaky-funnel model
a World Ocean idealization,
is based on the following
key assumption:

The horizontal circulation in the actual ocean may be thought to be a consequence of ***localized sinking*** *and* ***generalized upwelling***.
(Warren, 1981)



(Mouchet and Deleersnijder, Tellus, 2008)

Ventilation of the World Ocean (IV)

- Parameters of the leaky funnel model:

U = water velocity, K = diffusivity

L = e-folding length scale for the section: $S(x) = S_0\, e^{-x/L}$

L is also the mean length of the water parcel trajectories in the funnel

- The leaky funnel water age distribution is

$$\mu(\tau) \;=\; \sqrt{\frac{K}{\pi L^2 \tau}}\exp\left(-\frac{U'^2\tau}{4K}\right) \;+\; \frac{1}{\theta}\left[1+\mathrm{erf}\left(\frac{1}{\theta}\sqrt{\frac{L^2\tau}{K}}\right)\right]\exp\left(-\frac{U\tau}{L}\right)$$

with $Pe = UL/K$, $\dfrac{1}{\theta} = \dfrac{U'}{2L}\left(1-\dfrac{2}{Pe'}\right)$, $Pe' = U'L/K$ and $U' = U + K/L$.

- The leaky funnel mean age is $\bar{a} = \dfrac{L}{U + K/L}$.

Ventilation of the World Ocean (V)

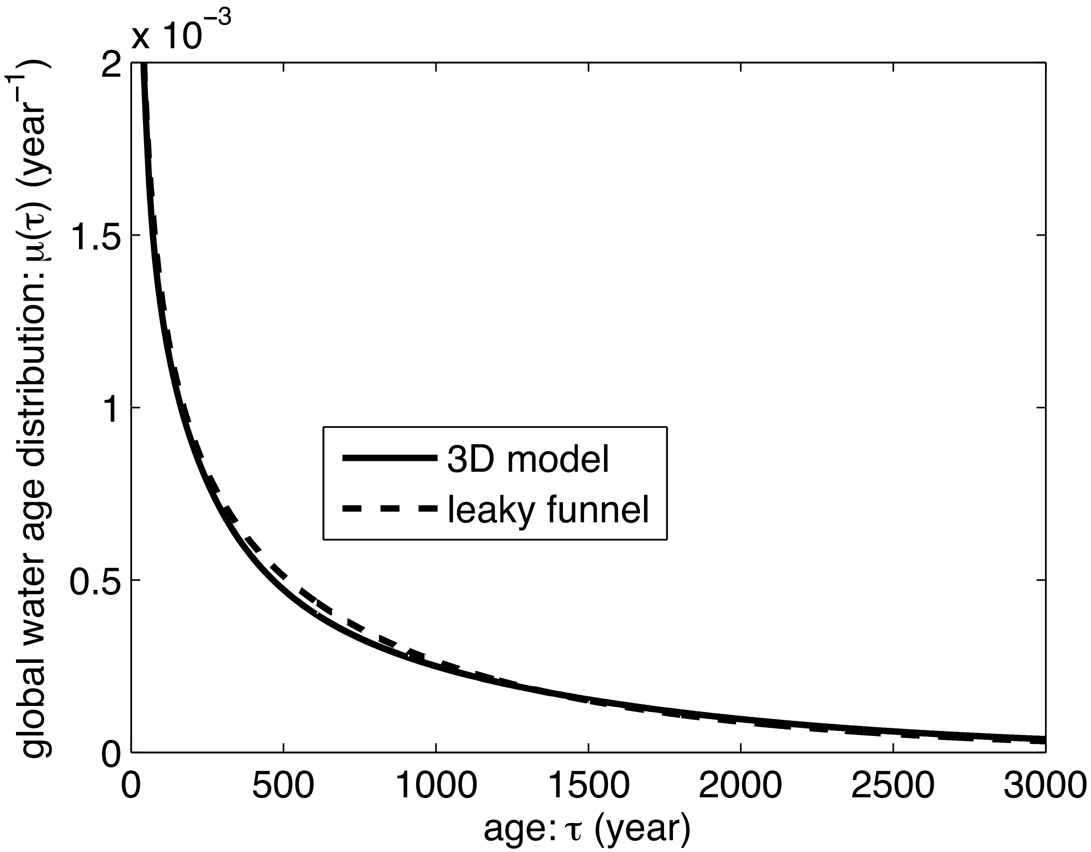
• The leaky funnel age distribution compres very well with that obtained from a 3D OGCM:

The parameters of the leaky funnel are optimised so as to minimise the difference $\left|\mu^{3D}(\tau)-\mu(\tau)\right|$.

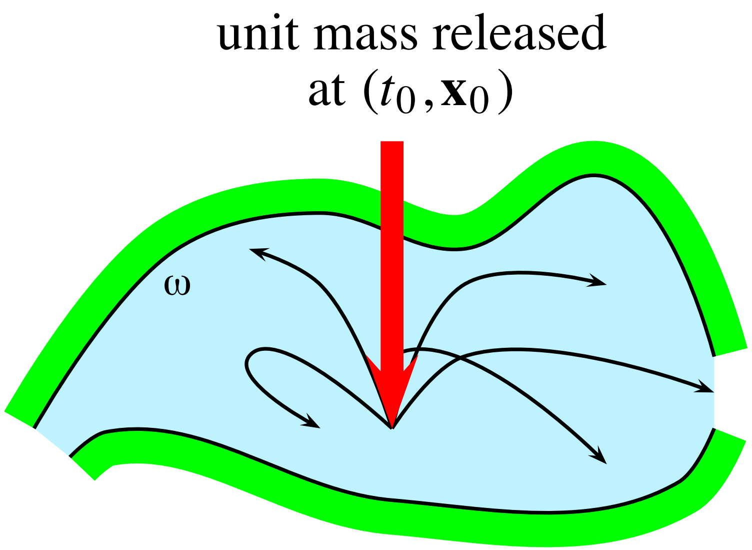
$$U \approx 3\times10^{-3}\,\mathrm{ms^{-1}} \;,\quad L \approx 10^{8}\,\mathrm{m}$$

$$K \approx 1.4\times10^{5}\,\mathrm{m^{2}\,s^{-1}}$$

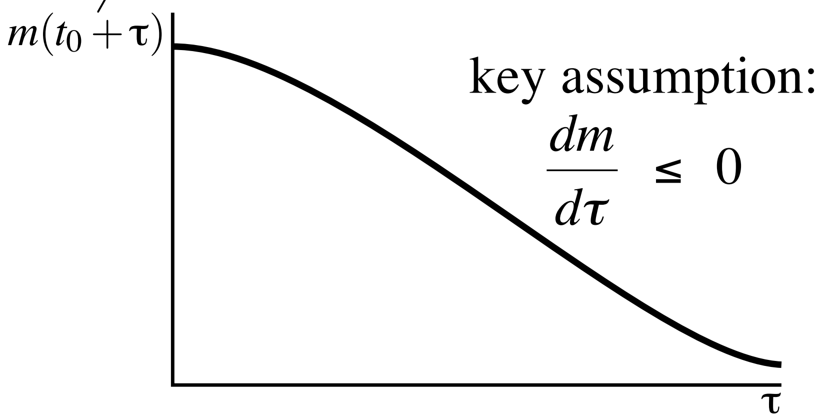
$$\bar{a}^{3D} = 764\,\mathrm{y} \;,\;\; \bar{a} = 721\,\mathrm{y}$$



Residence time: the forward/direct procedure



$$m(t_0+\tau) = \int_\omega C(t_0+\tau, \mathbf{x})\, d\mathbf{x}$$



1. Introduce unit mass of passive tracer at time t_0 and location $\mathbf{x}_0$;
2. Calculate the mass $m(t_0+\tau)$ of the tracer in the domain ω;
3. Residence time: $\theta(t_0, \mathbf{x}_0) = -\int_1^0 \tau\, dm = \int_0^\infty m(t_0+\tau)\, d\tau$.

Residence time: the backward/adjoint procedure (I)

- Using the direct procedure, the number of models runs that are needed is equal to the number of t_0 and $\mathbf{x}_0$ at which the residence time is to be estimated: CPU cost can be prohibitive!

- Delhez et al. (ECSS, 2004) developed an adjoint model that is potentially much more efficient, but requires backward integration in time.

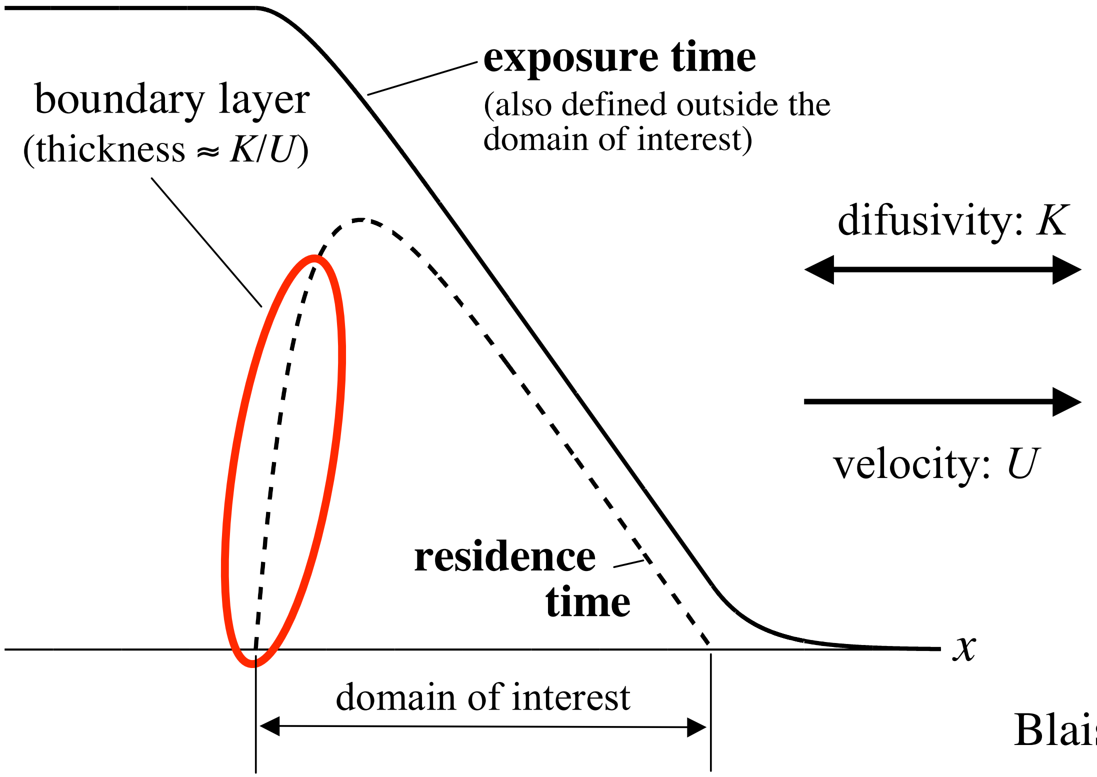
- The residence time $\theta(t,\mathbf{x})$ is the solution of

$$\frac{\partial\theta}{\partial t} = -1 - \nabla\bullet(\mathbf{u}\theta + \mathbf{K}\bullet\nabla\theta)$$

- The boundary conditions depend on those imposed in the forwward problem.

Residence time *vs* exposure time

- Particles that left the domain can enter it again at some later time. This can be taken into account by means of the *exposure time*, i.e. *the time spent in the domain of interest*.

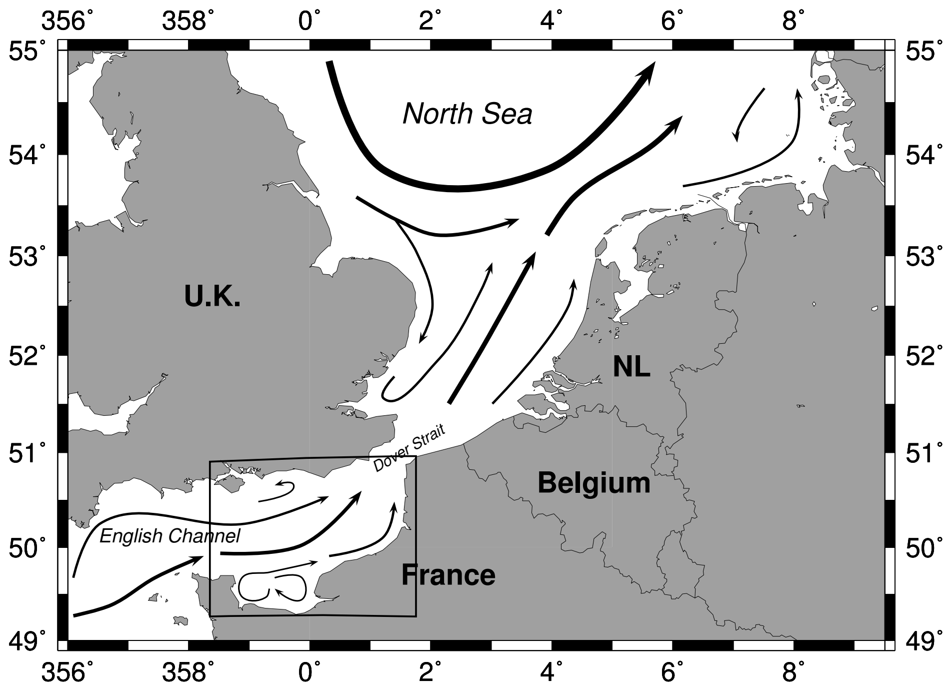


To obtain the exposure time, the same adjoint model equations are to be solved, but in a different domain and with different boundary conditions.

(Delhez and Deleersnijder, *Ocean Dynamics*, 2006; Blaise et al., *Ocean Dynamics*, 2010)

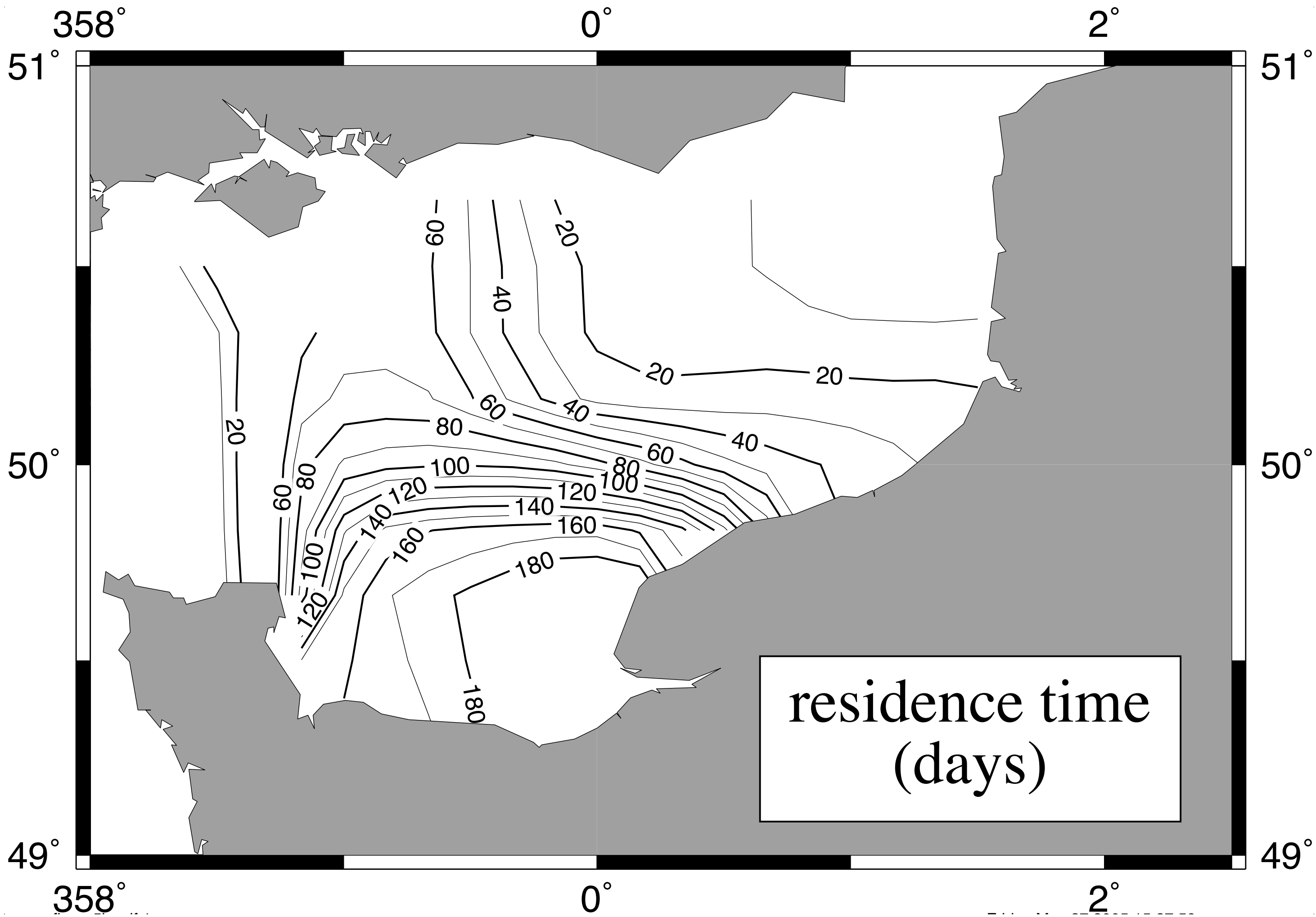
Residence/exposure time in the English Channel (I)

- Horizontal resolution: 10'; 10 σ-levels.
- Free-surface; baroclinic; *k* turbulence closure model.
- Forcings: 10 tidal constituents and NCEP reanalysis met. data.

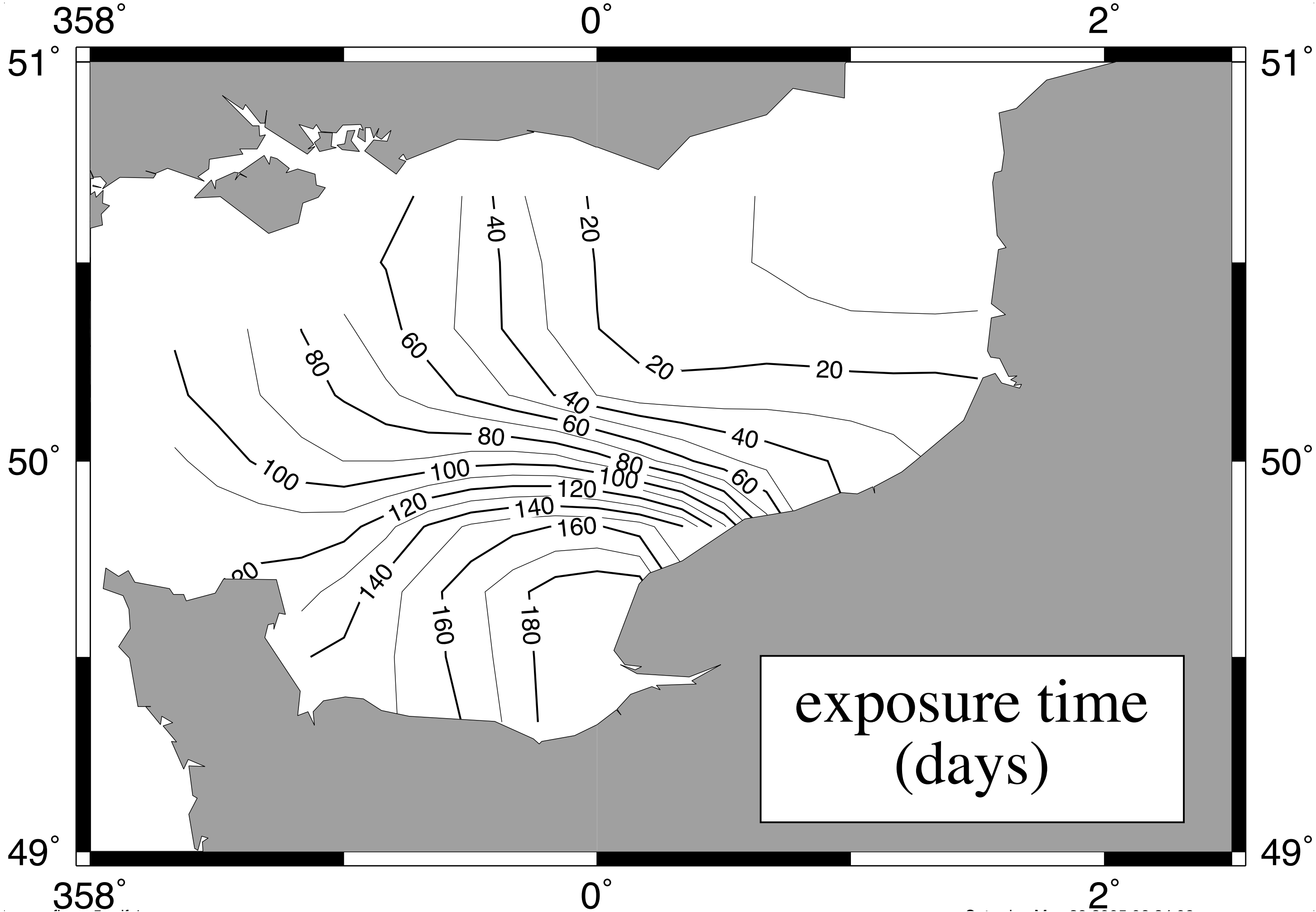


Numerical results from
E.J.M. Delhez's model
(Liège University, Belgium)

Residence/exposure time in the English Channel (II)



Residence/exposure time in the English Channel (III)

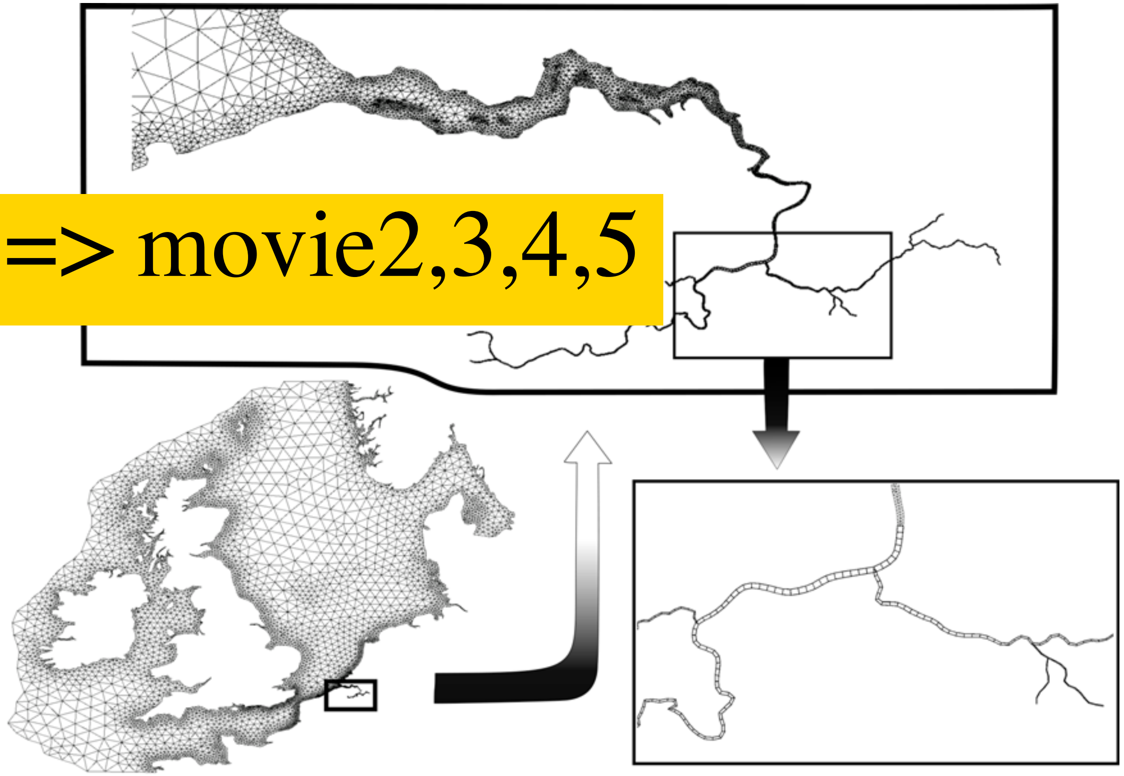


Water renewal in the Scheldt Estuary

A finite-element, unstructured-mesh, depth-averaged model of the Scheldt tributaries, River, Estuary, with 2D elements in the sea and estuary and 1D elements in the riverine part.

- 40% of the meshes in the estuary, which represents 0.3% of the computational domain.
- No major problem with open boundary conditions (for tides, storms, river discharge).

=> movie2,3,4,5



(de Brye et al., *Coastal Engineering*, 2010)

Conclusion

- Using simulated velocity and diffusivity fields, it is possible to compute, at any time and position, the age and residence/exposure time of any type of tracer, including original or renewing water.
- Age and residence/exposure time are of use for assessing the rate at which the water of a wide variety of domains is renewed.
- Timescales paint a picture of the functioning of a system that is different from that obtained by analysing primitive variables, i.e. velocity, surface elevation, temperature, etc.
- Timescales are of use for designing reduced-dimension models, thereby helping in the interpretation of complex flows (e.g. leaky funnel metaphor).

For additional information about:

- timescales, see **www.climate.be/cart**
- the Louvain-la-Neuve, unstructured-mesh, finite-element model, see **www.climate.be/slim**
- the integrated modelling of the Scheldt basin, see **www.climate.be/timothy**