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LOAD FLOW COMPUTATIONS IN HYBRID TRANSMISSION -
DISTRIBUTION POWER SYSTEMS

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Abstract

We interconnect transmission and distribution power systems and perform load flow computations in the hybrid network. In the largest example we managed to build, fifty copies of a distribution network consisting of fifteen nodes is connected to the UCTE study model, resulting in a system consisting of 5003 nodes. For this model, the Newton load flow model converges in seven iterations. Future work in this direction should allow to build even larger networks.

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1 Introduction

In this report we describe load flow computations in hybrid transmission - distribution power systems. We aim at building such hybrid systems by assembling the two types of networks and construct in this way networks that are large in size. Such large scale systems are indispensable in the study of future generation power systems. The approach to build the networks we study in this report provides an alternative for the one considered in [3]. In this reference copies of a transmission network are interconnected. The transmission network that we employ in this report are those publicly available in the MATPOWER software environment [5] on one hand and the UCTE[1] study model on the other. The transmission network we employ resembles the study model of an existing small residential area described in [1] and was provided by ENDINET.

We solve the load flow equations by Newton's method. The interconnecting power system networks we persue requires due care in order to assure the convergence of the non-linear iterations. This motivated the study of a two-bus and a single-branch network into factors that hamper the convergence. A model of a phase-shifting transformer is introduced in order to be able to interconnect subsystems at different power levels.

In the largest hybrid network we managed to build in this report, fifty copies of the distribution network considered in connected to the UCTE model resulting in a network with 5003 nodes. The load flow computation in this network converges in seven iterations. The model can in principle be extended to an even larger scale.

This report is structured as follows. In Section 2 the load flow equations are introduced. In Section 3 the MATPOWER package is described. In Section 4 the transmission and distribution networks considered in this report are described. In Section 5 numerical results for load flow computations in hybrid systems are presented. The conclusions of this work finally are presented in Section 6.

2 Load Flow Equations

In this section, the theory of load flow computations in power system networks is briefly reviewed [4]. The goal of such computations is to determine the voltage magnitude and phasor (or angle) for each bus in the network. They require the topology and physical parameters of the network as input. Examples of load flow computations will be given in forthcoming sections.

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[1]UCTE is a former association of transmission system operators in Europe. As of July 2009, the European Network of Transmission System Operators for Electricity (ENTSO-E), a newly formed association of 42 TSOs from 34 countries in Europe, has taken over all operational tasks of existing European TSO associations, including UCTE. See http://www.entsoe.eu/

Any bus in a network is fully described by the following four parameters: the voltage phasor magnitude $|V|$, the voltage angle δ, the injected active power P and the passive (or reactive) power Q. There are three types of busses in a network: a load (or PQ) bus, a generator (or PV) bus, and a slack bus. Loads are modeled as constant power sinks. The injected active and passive power of a load bus must be specified, while the voltage magnitude and the angle remain to be determined. A generator bus has known P and $|V|$ values while the Q and δ values remain to be determined. Models for power systems have to comprise a generator to supply the occurring losses. This generator is called the slack bus. The slack bus determines the reference voltage. Only for this node both the voltage phasor magnitude and the angle are defined as input, while its other parameter values result from the load flow calculation. This information is summarized in Table 1. In this table, i, N_g and N denote the index of the bus, the number of generator busses, an the total number of busses, respectively.

type of bus	specified parameters	unspecified parameters
slack bus $i = 1$	$|V_i|, \delta_i$	P_i, Q_i
generator bus $i = 2, ..., N_g + 1$	$P_i, |V_i|$	Q_i, δ_i
load bus $i = N_g + 2, ..., N$	P_i, Q_i	$|V_i|, \delta_i$

Table 1: Network bus types with associated specified and unspecified parameters.

Each branch in a network has electrical resistance R and reactance X. The impedance of a branch is given as

$$Z = R + jX.$$

The admittance is defined as the inverse of the impedance, $Y = \frac{1}{Z}$. The real and imaginary part of Y are called the conductance G and the susceptance B, respectively. They are given by

$$Y = \frac{1}{Z} = \frac{1}{R + jX} = \frac{R}{R^2 + X^2} - j\frac{X}{R^2 + X^2} = G + jB.$$

The elements of the admittance matrix $\mathbf{Y} \in \mathbb{C}^{N \times N}$ can be written as

$$Y_{ik} = |Y_{ik}|\angle\theta_{ik} = |Y_{ik}|(\cos\theta_{ik} + j\sin\theta_{ik}) = G_{ik} + jB_{ik}.$$

The voltage at bus i for $1 \leq i \leq N$ can be expressed as

$$V_i = |V_i|\angle\delta_i = |V_i|(\cos\delta_i + j\sin\delta_i). \tag{1}$$

According to Kirchoff Current Law the admittance describes the relation between the current injected at the network and the bus voltages

$$\mathbf{I} = \mathbf{YV}, \tag{2}$$

implying that the injected current at bus i equals

$$I_i = Y_{i1}V_1 + Y_{i2}V_2 + \ldots + Y_{iN}V_N = \sum_{k=1}^{N} Y_{ik}V_k. \tag{3}$$

The complex power injected into bus i is defined as

$$S_i = V_i I_i^*, \tag{4}$$

where I_i^* is the complex conjugate of I_i. The real and imaginary part of the power S_i is referred to as the active power P and the reactive power Q, respectively, i.e.,

$$S_i = P_i + jQ_i.$$

Combining the equations for the injected current at bus i with the voltage at bus i gives

$$
\begin{aligned}
S_i &= |V_i| \angle \delta_i \sum_{k=1}^{N} Y_{ik}^* V_k^* = \sum_{k=1}^{N} |V_i V_k Y_{ik}| \angle(-\theta_{ik} - \delta_k + \delta_i) \tag{5} \\
&= \sum_{k=1}^{N} |V_i V_k V_{ik}| [\cos(-\theta_{ik} - \delta_k + \delta_i) + j \sin(-\theta_{ik} - \delta_k + \delta_i)] \\
&= \sum_{k=1}^{N} |V_i V_k V_{ik}| [\cos(\theta_{ik} + \delta_k - \delta_i) - j \sin(\theta_{ik} + \delta_k - \delta_i)].
\end{aligned}
$$

Thus, the injected active and reactive power at bus i can be rewritten as

$$
\begin{aligned}
P_i &= \sum_{k=1}^{N} |V_i V_k Y_{ik}| \cos(\theta_{ik} + \delta_k - \delta_i) \tag{6} \\
&= |V_i|^2 G_{ii} + \sum_{k=1, k \neq i}^{N} |V_i V_k Y_{ik}| \cos(\theta_{ik} + \delta_k - \delta_i),
\end{aligned}
$$

and

$$
\begin{aligned}
Q_i &= -\sum_{k=1}^{N} |V_i V_k Y_{ik}| \sin(\theta_{ik} + \delta_k - \delta_i) \tag{7} \\
&= -|V_i|^2 B_{ii} + \sum_{k=1, k \neq i}^{N} |V_i V_k Y_{ik}| \sin(\theta_{ik} + \delta_k - \delta_i), \tag{8}
\end{aligned}
$$

respectively. These $2N$ equations (for P_i and Q_i as $1 \leq i \leq N$) for $2N$ unknowns (V_i and δ_i) are the so-called load flow equations. For the load bus i both the equation for P_i and for Q_i are employed to determine V_i and δ_i (see Table 1). For a generator bus i only the equation for P_i is employed to find δ_i as the value for Q_i for such a bus is not specified and $|V_i|$ is known. In the slack bus both the voltage angle and magnitude are specified and the corresponding equations for P_i and Q_i can be eliminated from the model.

3   The MATPOWER Package

For most power flow problems an analytical solution is either difficult or impossible to find. In these cases one can resort to numerical techniques to find a sufficiently accurate approximations. The most popular numerical method for solving the power flow problems is Newton's (or Newton-Raphson's) method. The application of this method on the load flow equations is extensively described in [2].

MATPOWER [5] is a package implemented in MATLAB allowing to employ Newton's method for solving power flow problems. The Jacobian system at each Newton iteration is solved by a direct solver. A flat start is typically employed. The required input is specified as a set of matrices in a so-called case-file. This set consists of the bus-matrix, the generator-matrix and the branch-matrix. To generate these input matrices, the apparent power has to be specified and all the busses in the network have to be numbered. In the following we will describe the bus-matrix, the generator-matrix and the branch-matrix in more details.

The bus-matrix has 17 columns and a number of rows equal to the number of busses in the network. The first column contains the bus number. In the other columns, the type, the power demand, the voltage level, the voltage or the initial guess of the voltage, and some other parameters of each bus is specified. The voltage is given in the slack bus. For the generator buses, an initial guess for the voltage angle is supplied. For the load buses, an initial guess for both the voltage magnitude and the angle is supplied.

The generator-matrix contains the information about the generators.

The branch-matrix contains the specifications of the transmissions lines between the busses. Here the resistance, the reactance, the charging susceptance and some of the other line parameters are defined. Moreover, if a network includes a transformer (see Section 4.1.2), all the specifications of the transformer are coded in the branch-matrix.

It is important to note that some of the parameters in the matrices given above are defined in the per-unit system (see Section 4.1.1), while other parameters ought to be given in SI-units.

4   Networks Considered

In this section all networks used as building blocks for the hybrid networks in this work are collected. In Subsection 4.1, simplified network are analyzed. In Subsection 4.2 and in Subsection 4.3, the networks used as the distribution and the transmission part of the hybrid network, respectively, are described.

4.1 Simplified Networks

In this subsection we first study a two-bus and subsequently a single-branch network. In the former a load is connected to a generator that necessarily is the slack bus. In Subsubsection 4.1.1 we study this generator-load configuration without transformer both analytically and numerically. In Subsubsection 4.1.2 we study the same configuration with transformer. In Subsubsection 4.1.3 finally we study a single-branch network.

4.1.1 Two-Bus Network

Despite its simplicity, the study of a two-bus network does provide valuable information on building hybrid transmission-distribution power systems. It provides insight on factors limiting the convergence of the Newton process, in a strategy to speed-up the convergence and shows the relationship between the voltage magnitude and the reactive power on one hand and the relation between the voltage angle and the active power on the other.

Analytical Solution We will associate the generator and the load as Bus1 and Bus2, respectively. Bus1 is thus a the slack node. The voltage magnitude and the voltage angle of this node are therefore equal to $|V_1| = 1.0$ p.u. and $\delta_1 = 0$ degrees. To find the voltage amplitude and angle at Bus2, a load flow computation has to be performed. In the following we will first provide an analytical solution that we will subsequently illustrate with a numerical example.

To derive the analytical solution, we start from the load flow equations at Bus2 that are given by

$$P_2 = |V_2|^2 G_{22} + |V_2 V_1 Y_{21}| \cos(\theta_{21} + \delta_1 - \delta_2), \tag{9}$$

$$Q_2 = -|V_2| B_{22} - |V_2 V_1 Y_{12}| \sin(\theta_{21} + \delta_1 - \delta_2). \tag{10}$$

These equations can alternatively be written as

$$\cos(\theta_{21} + \delta_1 - \delta_2) = \frac{P_2 - |V_2|^2 G_{22}}{|V_2 V_1 Y_{21}|}, \tag{11}$$

$$\sin(\theta_{21} + \delta_1 - \delta_2) = \frac{Q_2 - |V_2|^2 B_{22}}{|V_2 V_1 Y_{21}|}. \tag{12}$$

Given that $\cos^2 \alpha + \sin^2 \alpha = 1$, we have that

$$|V_1 Y_{21}|^2 |V_2|^2 = P_2 - 2P_2 G_{22}|V_2|^2 + G_{22}^2|V_2|^4 + Q_2^2 + 2Q_2 B_{22}|V_2|^2 + B_{22}^2|V_2|^4,$$

yielding the following quadratic equation in $|V_2|^2$

$$(G_{22}^2 + B_{22}^2)|V_2|^4 + (2Q_2 B_{22} - 2P_2 G_{22} - |V_1|^2|Y_{21}|^2)|V_2|^2 + P_2^2 + Q_2^2 = 0. \tag{13}$$

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The discriminant of this equation is equal to

$$D = (2Q_2 B_{22} - 2P_2 G_{22} - |V_1|^2 |Y_{21}|^2)^2 - 4(G_{22}^2 + B_{22}^2)(P_2^2 + Q_2^2).$$

The solution for $|V_2|^2$ is therefore given by

$$|V_2|^2 = \frac{1}{2(G_{22}^2 + B_{22}^2)}(-2Q_2 B_{22} + 2P_2 G_{22} + |V_1|^2 |Y_{21}|^2 \pm \sqrt{D}). \qquad (14)$$

Substituting

$$B_{22} = \frac{-X}{R^2 + X^2} \qquad (15)$$

$$G_{22} = \frac{R}{R^2 + X^2}$$

$$Y_{21} = \frac{-R}{R^2 + X^2} + j\frac{X}{R^2 + X^2} \text{ (thus } |Y_{21}| = \frac{1}{\sqrt{R^2 + X^2}})$$

into Equation (14), $|V_2|^2$ can be expressed in terms of V_1, P_2, Q_2, R and X to obtain that

$$|V_2|^2 = Q_2 X + P_2 R + \frac{1}{2}|V_1|^2 \qquad (16)$$

$$\pm \frac{1}{2}\sqrt{(-2Q_2 X - 2P_2 R - |V_1|^2)^2 - 4(P_2^2 + Q_2^2)(R^2 + X^2)}.$$

Taking the square root of Equation (16), gives the value for V_2. The voltage angle at Bus2 is found by substituting V_2 into Equation (11) or Equation (12)

$$\delta_2 = -\arccos(\frac{P_2 - |V_2|^2 G_{22}}{|V_1 V_2 Y_{21}|}) + \delta_1 + \theta_{21}. \qquad (17)$$

Using the expression for G_{22} and Y_{21} in terms of R and X, and denoting θ_{21} as $\arccos(\frac{-R}{\sqrt{R^2+X^2}})$, the angle can be expressed as

$$\delta_2 = -\arccos(\frac{P_2 - |V_2|^2 \frac{R}{R^2+X^2}}{|V_1 V_2| \frac{1}{\sqrt{R^2+X^2}}}) + \delta_1 + \arccos(-\frac{R}{\sqrt{R^2 + X^2}}). \qquad (18)$$

Using Equation (16) and Equation (18) the voltage amplitude and angle can be computed from the other model parameters. The problem is thus solved.

To illustrate the usage of Equation (16) and Equation (18), we consider the network shown in Figure 1 that was taken from [4]. In this example we have Bus1 is the slack bus so that $|V_1| = 0$ and $\delta_1 = 0$. We also have that $P_2 = -0.6$ MW and $Q_2 = -0.3$ MVAr. In Figure 1 both P_2 and Q_2 have positive sign as it the power consumed at the load bus is shown. In the calculation however the minus sign will be used to be consistent with the convention of a load given earlier. The busses are connected by a short transmission line $R = 0.025$ p.u.,

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$X = 0.25$ p.u. and zero shunt capacitance. The admittance matrix is thus equal to

$$Y = \begin{bmatrix} 0.396 - 3.96i & -0.396 + 3.96i \\ -0.396 + 3.96i & 0.396 - 3.96i \end{bmatrix}.$$

Substituting the given values for $|V_1|, \delta_1, P_2, Q_2, R$ and X in Equation (16) and Equation (18), gives two possible values for $|V_2|$ in a per-unit system and two corresponding values for δ_2: $|V_2| = 0.8853$ p.u. with $\delta_2 = -0.1617$ rad and $|V_2| = 0.1914$ p.u. with $\delta_2 = -0.8458$ rad. Only the first solution is feasible, because the second one gives a very low value of voltage and, therefore, a very high current.

To transform the solutions from a per-unit system to the actual voltage, the base voltage level has to be chosen, because

$$\text{Quantity (per unit)} = \frac{\text{Quantity (normal units)}}{\text{Base value of quantity (normal units)}}. \qquad (19)$$

So, if the base voltage is 138kV, then the realistic voltage magnitude at Bus2 is approximately equal to 122kV, while the voltage magnitude at Bus1 is 138kV.
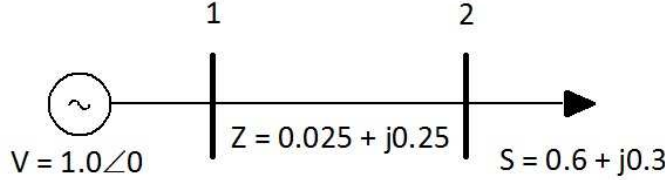

Figure 1: The two-bus network considered.

Numerical Solution   For the two-bus network given in Figure 1, MATPOWER finds a sufficiently accurate solution that coincides with the first one of the two analytical solutions if a flat start is used as starting guess. The Newton process then converges in 4 iterations as shown Figure 2.

Parametric Study in the Load Values   It is interesting to look at the solution of the two-bus problem for various values of P_2 and Q_2. If Q_2 is fixed at 0.3 MVar and P_2 is allowed to vary between 0 and 0.8 MW, or alternatively, if P_2 is fixed at 0.6 MW and Q_2 is allowed to vary between 0 and 0.8 MVAr, $|V_2|$ and δ_2 change as shown in Figure 3. The figure shows that in the range of values considered, $|V_2|$ is more sensitive to changes in Q_2 and that δ_2 is more sensitive to changes in P_2. This is the basis of the fast-decoupled approach to solving the load flow equations [3].
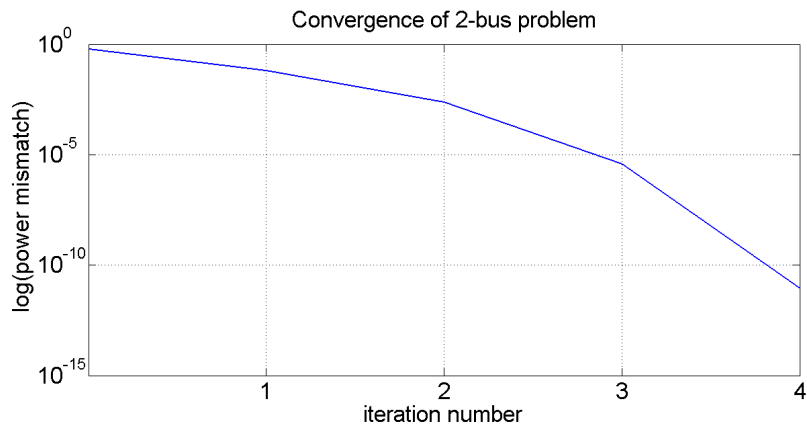
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Figure 2: Convergence of Newton's method in the two-bus network.

Recycling Information in the Newton Process Another important question is how the convergence of Newton's method changes with changes in P_2 and Q_2. If Q_2 is fixed at 0.3 MVAr and P_2 varies between 0 and 1.5 MW, the number of Newton iterations varies as shown in the top part of Figure 4. This figure shows that the number of iterations remains constant and equal to 4 for $P_2 \leq 0.8$ MW and increases in steps to 7 for $P_2 = 1.5$ MW. Equation (16) shows that for values of $P_2 > 1.5$ MW, $|V_2|$ becomes complex-valued and therefore not meaningful. For such values of P_2, Newton's method does not converge. Alternatively, if P_2 is fixed at $.6$ MW and Q_2 varies between 0 and $.8$ MVar, the number of Newton iterations varies as shown in the bottom part of Figure 4. This figure shows that the number of iterations remains constant and equal to 4 for $Q_2 \leq 0.5$ MVar and increases in steps to 6 for $Q_2 = 0.8$ MVar.

In parametric studies as in the examples above, MATPOWER does not automatically reuse information from the previous solves in the current solve. It is however natural to use the solution obtained in the previous solve as starting guess for the non-linear iteration in the current solve. The reduction in the number of Newton iterations this strategy offers is shown in Figure 4.

Parametric Study in Line Reactance In this paragraph we study the influence of the line impedance. It is clear that if the impedance becomes too large, no current will flow from Bus1 to Bus2. In such situations MATPOWER has difficulties to convergence. In our example this happens in case that $R > 0.28$ p.u. and $X > 0.49$ p.u. In our numerical experiments, we will consider the values $0 \leq R \leq 0.28$ p.u. and $0 \leq X \leq 0.28$ p.u. In Figure 5 we plotted the value of $|V_2|$ and δ_2 for various values of R and X. This figure shows that in the range of values considered that $|V_2|$ is more sensitive to changes in R and that δ_2 is more sensitive to changes in X.
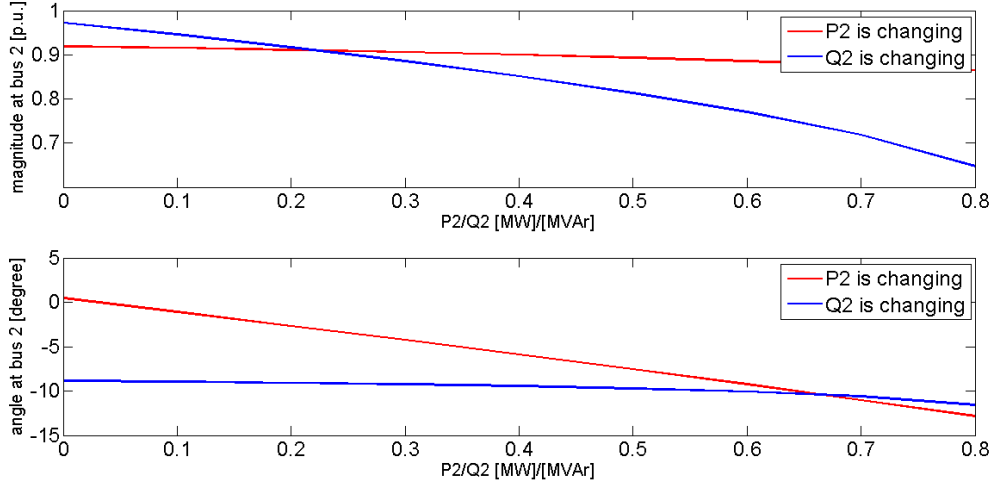
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Figure 3: Dependence of $|V_2|$ (top) and δ_2 (bottom) on P_2 and Q_2 in the two-bus example.

The number of required Newton iterations increases as either R or X increase as shown in Figure 6. In this figure the strategy of recycling information is compared with the more naive approach. The recycling strategy is again shown to be effective in reducing the number of Newton iterations.

4.1.2 Two-Bus Network with Transformer

In the two-bus example considered in the previous susubsection, the voltage level in all busses was considered to be equal. It is often the case however that the nodes are at different voltage levels. To convert the electrical energy from one level to another, a transformer is used.

Transformers consist of two coils wound around a ferromagnetic core. A time-varying current in the primary winding creates a time-varying magnetic flux in the core and therefore a time-varying magnetic flux in the secondary winding. This flux induces a varying electromotive force in the secondary winding, that also can be seen as voltage. The efficiency of a conversion with a transformer is typically more than 99%.

Consider the situation in which a generator that is part of a high-voltage network is connected to a load that is part of a low-voltage network. Assume that the generator and the load are labeled as Bus1 and Bus2, respectively. When the primary coil of the transformer is connected to Bus1 and the secondary coil to Bus2, the electrical energy will be transferred from the primary circuit of the
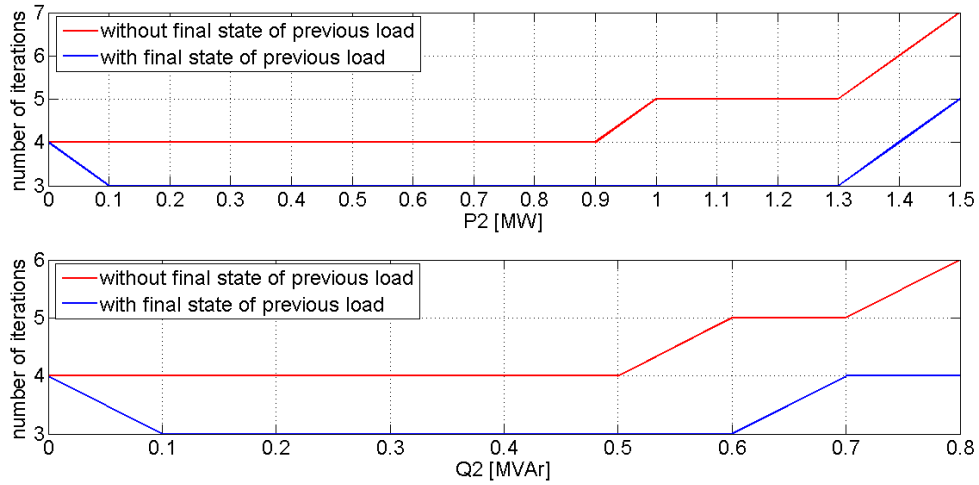
Figure 4: Number of Newton iterations vs. P_2/Q_2 with and without using the previous solution as initial guess in the current solve.
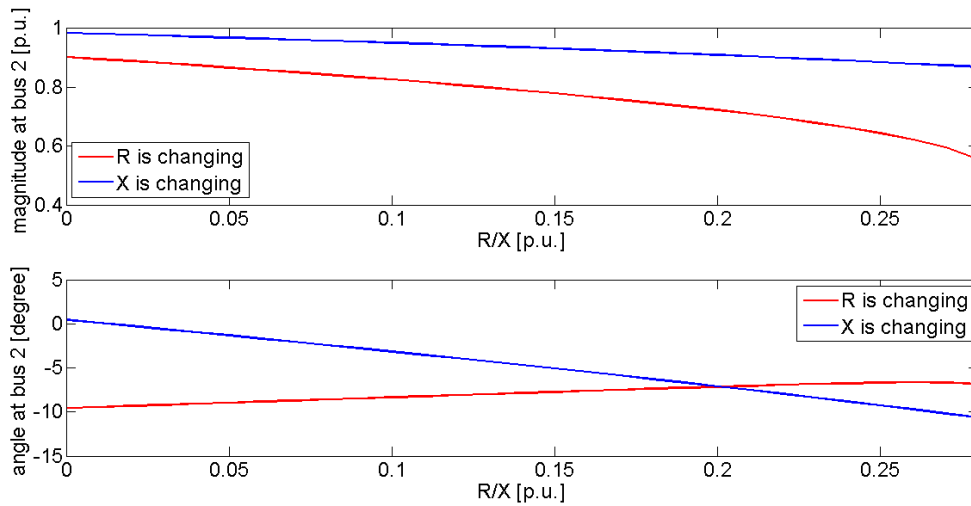

Figure 5: Dependence of $|V_2|$ (top) and δ_2 (bottom) on R and X in the two-bus example.
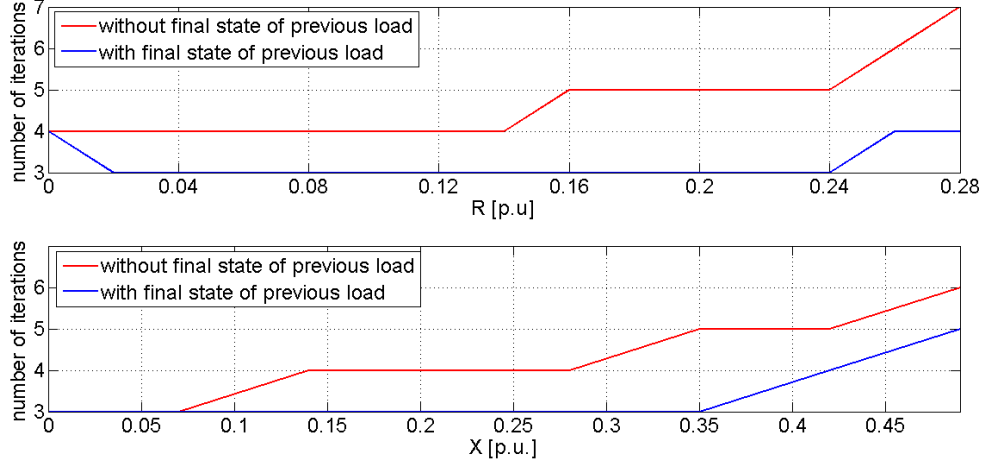
Figure 6: Number of Newton iterations vs. R/X with and without using the final state of the previous load as the begin solution.

generator through the transformer to the load. For an ideal transformer, the turns ratio defined as the ratio of the primary and secondary windings, is equal to the ratio of the primary and secondary voltage, i.e.,

$$\frac{N_2}{N_1} = \frac{V_2}{V_1}.$$

Ideally all the incoming energy from the primary network is transmitted to the secondary network. So, $P_{in} = I_1 V_1 = I_2 V_2 = P_{out}$. From this equality the following ideal transformer equation is obtained

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}.$$

Since $V = Z\,I$, the impedance in one circuit is transformed by the square of the of the turns ratio. Thus, if Z_2 is the impedance of the secondary network, it appears to the primary network as $(\frac{N_1}{N_2})^2 Z_2$. If alternatively the impedance of the primary network is Z_1, the secondary circuit sees this impedance as $(\frac{N_2}{N_1})^2 Z_1$.

In MATPOWER a transformer is modeled as a transmission line, with a series impedance equal to $Z = R + jX$ and a total charging capacitance equal to B_c placed in series with an ideal phase shifting transformer. The transformer with tap ratio magnitude τ and phase shift angle θ_{shift} is located at the *from* end of the connecting line. A tap ratio is needed to produce a transformer with a variable turns ratio. Only if the turns ratio is to stay constant, the tap ratio magnitude is set to 1 and the phase shift angle to zero. The charging
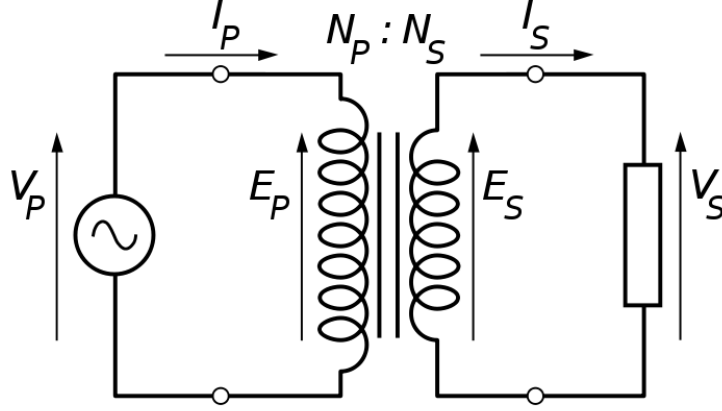
Figure 7: The model of the ideal transformer.

capacitance of the transformer is the ability of the transmission line to store an electric charge. In many cases B_c is set equal to zero.

If the two-bus network from Section 4.1.1 is kept unchanged, except for the voltage level of Bus1, then the solution of the problem will not change in a per-unit system. The actual voltage magnitude however will be different. So, if the voltage level of Bus1 is set equal to 345 kV, then $|V_1| = 345$ kV and $|V_2| = 122.17$ kV. This means that the primary circuit with the generator node sees the impedance of the secondary circuit with the load node as $(\frac{122.17}{345})^2(0.025 + j0.25) = 0.0031 + j0.0313$.

If a transformer with $\tau \neq 1$, nonzero θ_{shift} and B_c is included in the network, the networks admittance matrix is changed. Let us denote the admittance matrix of the original networks by $Y_s = 1/Z_s$. The admittance matrix of the network with transformer can then be written as

$$Y = \begin{bmatrix} (Y_s + j\frac{B_c}{2})\frac{1}{\tau} & -Y_s \frac{1}{\tau \exp(j\theta_{shift})} \\ -Y_s \frac{1}{\tau \exp(-j\theta_{shift})} & Y_s + j\frac{B_c}{2} \end{bmatrix}.$$

Equations (16) and (18) can no longer be used to find $|V_2|$ and δ_2, as B_{22}, Y_{21} and θ_{21} are now dependent on τ, θ_{shift} and B_c. However, the Equation (14) and Equation(17) with $B_{22} = \Im(Y_{22})$, $G_{22} = \Re(Y_{22})$ and $\theta_{21} = \frac{\Re(Y_{21})}{|Y_{21}|}$ remain valid.

So, if the two-bus problem is extended by the transformer with tap ratio magnitude equal to 0.95, phase shift angle of 5° and the total line charging susceptance equal to 0.3 p.u., then the admittance matrix of the problem becomes

$$Y = \begin{bmatrix} 0.4388 - 4.2220i & -0.0520 + 4.1893i \\ -0.7786 + 4.1166i & 0.3960 - 3.8104i \end{bmatrix}.$$

In this case, the two solutions of the problem are $|V_2| = 0.9982$ p.u. and $\delta_2 = -(3.078° + 2 \cdot 5°) = -13.078°$ or $|V_2| = 0.1772$ p.u. and $\delta_2 = -(44.870° + 2 \cdot 5°) = -54.870°$. The former is the realistic one. If the base voltage of the generator bus is 345 kV and the base voltage of the load bus is equal to 138 kV, then the voltage magnitude at Bus1 equals $|V_1| = 345$ kV, and the realistic magnitude at Bus2 is equal to $|V_2| = 136.37$ kV.

4.1.3 Single-Branch Network

In this subsubsection we consider networks consisting of several busses interconnected to form a single branch as shown in Figure 8. Such simple networks could for instance model the interconnection of a substation to a group of houses in a residential area. As such they represent basic distribution networks. We study how the number of busses and generators in the network affect convergence of the Newton load flow process and the voltage magnitude at each bus. To assure the convergence of the Newton process in the load flow computation on the networks to be build, one has pay attention to factors such as providing sufficient generation for a given level of load and providing the lines with sufficiently large impedance values.

Figure 8: Single-branch network topology.

In Figure 8 the generator is defined as a slack node with voltage magnitude 230 V and voltage angle 0°. All the load nodes have equal power demand of $P = 8$ GW and $Q = 0$ MVAr. These unusually high values are based on the average power demand of the 15-bus network as will be discussed in Appendix 7.1. The connection lines are made of 70-meters long aluminum wires.

The generator is able to supply at the most $N = 13$ loads. For these networks, the convergence depends on the number of buses. The imposed tolerance is reached in fewer iterations in smaller networks than in larger ones.

The computed voltage magnitude at each node of the network with 13 busses is shown in Figure 10. The voltage magnitude is shown to decrease with the node label.
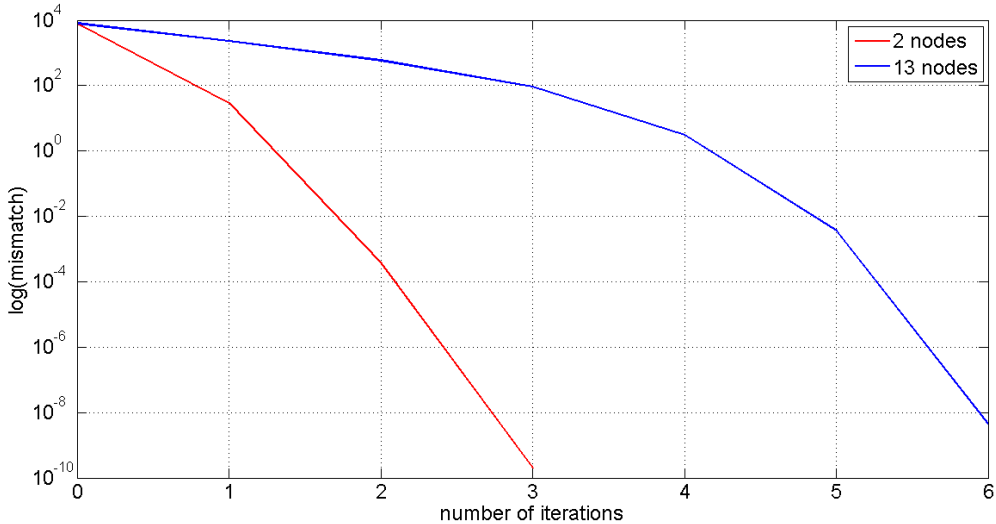
Figure 9: Convergence of a single-branch network for $N = 2$ and $N = 13$.

For total number of busses larger than 13, Newton's method ceases to converge. This problem can be solved by introducing a new generator as Bus14. Doing so, we can enlarge the total number of busses up to 27. If there are still some load nodes that have to be added to the network, the third generator can be connected to the network at Bus28 etc.

Another possibility is to divide the network in the groups of 7 (or less) busses and let one of the busses in each group be a generator. This works, because the maximal distance between the generators is 12 busses. For this solution, the voltage magnitude at every node in the network with 70 busses is given in Figure 11.

4.2   Distribution Network

In this subsection we describe results from load flow computations in a realistic example of a distribution network based upon an existing network in Eindhoven (The Netherlands). This example was provided to us by ENDINET [1]. However, some of the model parameters were adjusted to suit particular needs as detailed in Appendix 7.1.

The topology of the network considered is shown in Figure 12. It consists of an upper and lower branch. The busses in the upper and branch are numbered consecutively from Bus2 to Bus7 and from Bus8 to Bus15, respectively. Bus1 models the interconnection to a higher voltage network and is chosen to be
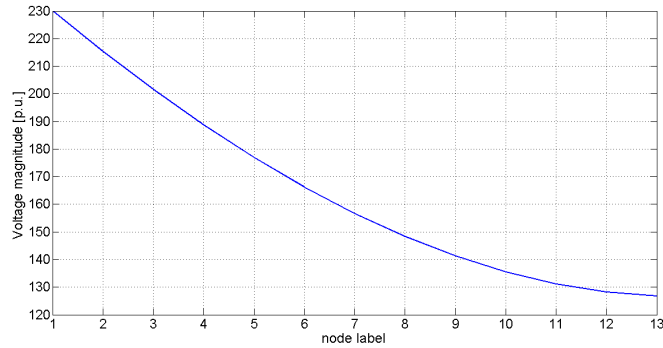
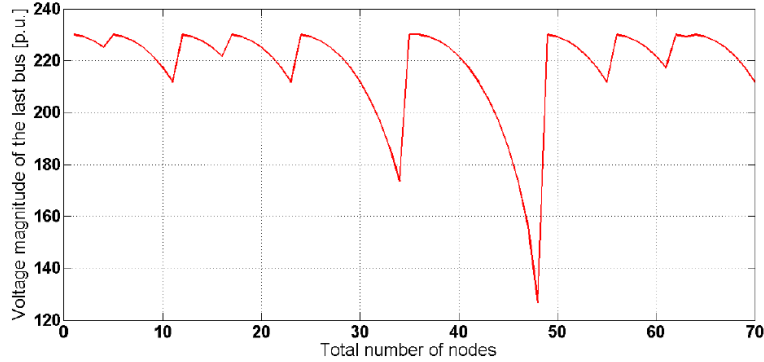Figure 10: Voltage magnitude at each bus of the one-branch network consisting of 13 busses.

Figure 11: Voltage magnitude at each node in a network with generators at random positions in the one-branch network.

the slack bus. We will considered two different scenarios in which Bus15 is either a load or generator. In the latter case a second interconnection to a higher voltage network is taken into account. All other busses in the network represent the interconnection to two or three households and will be modeled as loads. We assume that Bus2, Bus7 and Bus9 have a (unrealistically high) active power demand of 80 GW, 25 GW and 10 GW, respectively, and no reactive power demand. The active and reactive power demand at the other nodes is set to zero, modeling the situation in which the corresponding households do not demand any power (due to holidays for instance). The busses are interconnected by lines of slightly different length made of aluminum or copper resulting in line impedances that slightly one from another. The line between Bus7 and Bus15 is an optional line, which transforms the radial network into a loop network. (In a radial network all the loads are fed by lines connected directly to one central supply. In a network with a loop structure each load is fed from two directions). In this section both network topologies are discussed.
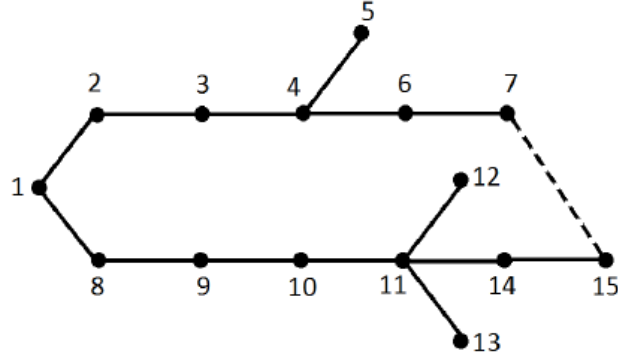
17

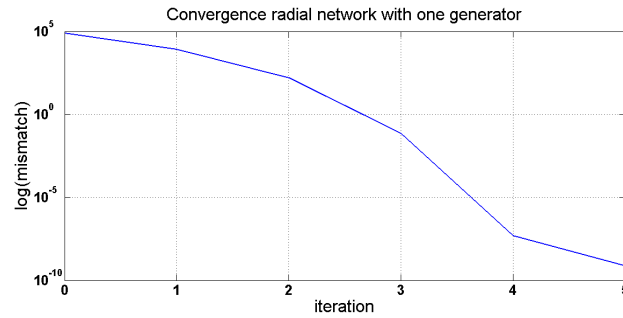Figure 12: Topology of the 15-bus distribution network.

Figure 13: Convergence of Newton's method for the distribution network with radial structure.

One generator We first consider the situation in which the slack bus is the only generator in the network with either a radial or loop topology. For both topologies, Figure 14 shows the computed voltage magnitude as a function of the bus index. For both topologies, the figure clearly reflects the two-branch nature of the network as the voltage magnitude decreases uniformly from Bus1 to Bus7 and again from Bus8 to Bus15. The sudden drop in voltage in going from Bus1 to Bus2 is due to the large power demand in the latter node. The voltage drops to a lesser extend in the other two load busses being Bus7 and Bus9, due to the lower power demand in these busses. For the loop topology the voltage is more uniform across the network in the sense that in the this topology the voltage drop in Bus2 is less pronounced and that the voltage in the upper (lower) branch is higher (lower) than in the radial topology. We presume that this effect is caused by an energy flow from the generator to the upper branch via the lower branch. For the radial and loop topology the Newton

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process converges in five and four iterations, respectively. The convergence of the Newton iteration for the former topology is shown in Figure 13.

Two generators Next we consider the situation in which both the slack bus and Bus15 are generators in the network with either a radial or loop topology. The corresponding computed voltage is shown in Figure 15. The comparison of this figure with Figure 14 shows that the second generator has a more pronounced impact in the loop topology. In this topology the second generator increases the overall voltage in the first branch and avoids the drop-off of the voltage with the distance from the slack bus in the second. In the radial topology the second generator only affects the branch to which it is connected by slightly increasing the voltage in the busses at largest distance from the slack bus.

Parametric Study in Load requested in Bus3 To conclude this subsection, we perform a parametric study in the load requested in Bus3 in the radial network with one generator. We allow the active power demands in Bus3 (P_3) range from 0 to .5 GW. Figure 16 shows the outcome of this parametric on the magnitude of the computed voltage. This figure shows that the voltage magnitude decreases with an increasing active power demand. This phenomena was already observed in Subsubsection 4.1.1, The radial topology is such that the voltage is the lower branch is hardly affected. Switching to a loop topology would render the voltage more uniform by increasing the voltage in the upper branch and decreasing it in the lower branch.

4.3 Transmission Networks

A transmission network is a high-voltage network that transports electrical energy from the generating plants to substations. The examples of such network used in this report come from two sources. The first source is the MATPOWER package [5]. In this package, the transmission networks are provided as so-called *case*-files. Examples include the case9, case30, case39, case57, case118 and case300 files. As these examples are small in size, we decided to also consider the UCTE network [3]. In the following we describe the transmission network used as example in this report in more detail.

Case9 Network The most simple transmission network available as example within MATPOWER is the case9 network. Its topology is shown in Figure 17. The apparent power and voltage level of the network is 100 MVA and 345 kV, respectively. The network has three generator busses, namely Bus1, Bus2 and Bus3. Bus1 is chosen to be a slack node. Bus4, Bus6 and Bus8 are transmission substations with zero power demand. The power demand at the load
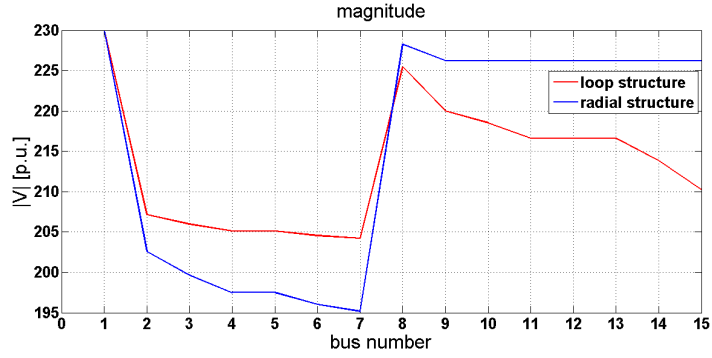
19

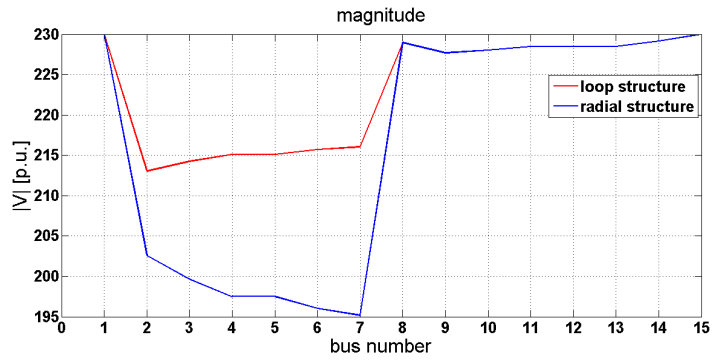Figure 14: Voltage magnitude at every node of the network with one generator.

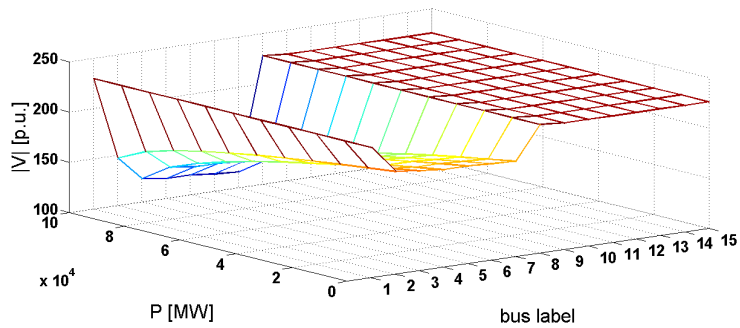Figure 15: Voltage magnitude at every node of the network with two generators.

Figure 16: Voltage magnitude at every node of the radial network with one generator for P_3 ranging for 0 to .5 GW.

busses Bus5, Bus7 and Bus9 is given by $P_5 = 90$ MW, $Q_5 = 30$ MVAr, $P_7 = 100$ MW, $Q_7 = 35$ MVAr, $P_9 = 250$MW and $Q_9 = 50$ MVAr, respectivelty. The resistance of the connecting lines varies between 0 and 0.039 p.u., while the reactance lies between 0 and 0.17 p.u. The total line charging capacitance of the lines takes values between 0 and 0.358 p.u. For this network Newton's method converges in four iterations. The computed voltage magnitude and angle are shown in Figure 18.
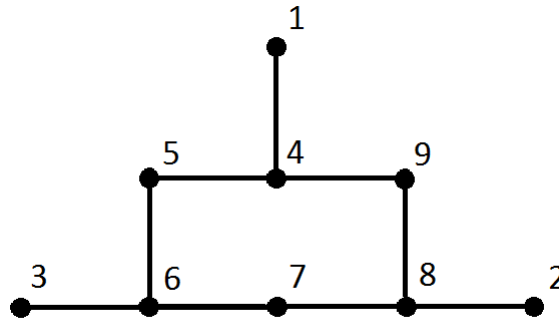

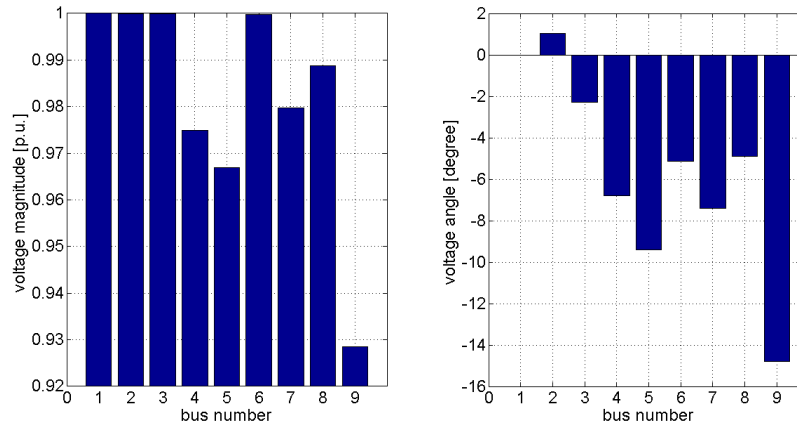Figure 17: The topology of the case9 network.

Figure 18: The computed voltage magnitude (left) and angle (right) at each bus of case9 network.

Case300 Network The case300 network has 300 busses of which 69 are generators and 411 transmission lines. The non-zero structure of the admittance matrix reflects how the busses are interconnected as is shown in Figure 19(a). The apparent power is set to 100 MVA. This network includes various voltage

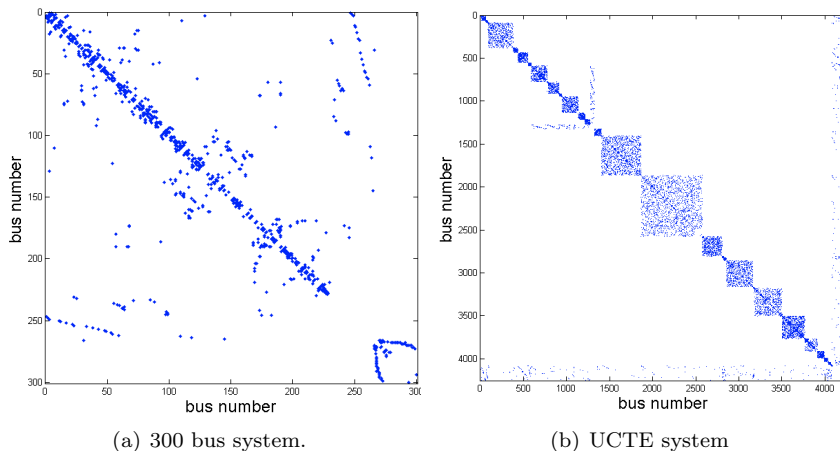| (a) 300 bus system. | (b) UCTE system |

Figure 19: Non-zero structure of the admittance matrix of the case300 and UCTE system.

levels, such as 13.8 kV, 66 kV, 115 kV, 230 kV and 345 kV. Multiple transformers are therefore built into this network. Newton's method converges in only five iterations. The computed voltage magnitude and angle at every bus lies between 0.929 p.u. and 1.073 p.u. and between -37.54° and 35.07°, respectively.

UCTE Network   Apart from the examples provided by MATPOWER, we also looked into the UCTE network having 4253 busses of which 517 are generators. The non-zero structure of the admittance matrix is shown in Figure 19(b). The apparent power is set equal to 100 MVA and the busses are at several voltage levels. Newton's method converges in seven iterations. The voltage magnitude varies between 0.915 p.u. and 1.269 p.u., while the voltage angle takes values between -67.59° and 42.57°.

5   Hybrid Networks

In this section a high voltage transmission network is connected to a low voltage distribution network to form a hybrid network. This interconnection requires putting in a transformer between the networks. Such transformer is modeled as a transmission line inbetween the networks as described in Section 4.1.2. Since there is no need to use a variable turns ratio, the tap ratio magnitude is set to 1 and the shift angle is set to zero. The total charging capacitance of the transformer is set to zero as well. The transformer thus has to convert the voltage levels only. The transmission networks used in this study were already described in Section 4.3. The 15-bus distribution network described in

Section 4.2 with the second generator and the original non-zero load connected to Bus15 is used as the low voltage distribution network. This modified 15-bus network was successfully connected to case30, case39, case57, case118, case300 of MATPOWER and the UCTE network. In some cases the resistance of the interconnecting line had to be adjusted. In the following the connection with case9, case300 and UCTE network is discussed.

5.1   Interconnection of the 15-bus and case9 Networks

We first consider the case9 network with some modifications described in Appendix 7.2 as the transmission network. A transmission line has to be added to connect the high voltage network and the low voltage network. A transformer is placed on the from end of the line as described in Section 4.1.2. The R and X of the connecting line have to be similar to those of the 15-bus distribution network. We therefore choose to set these R and X values equal to those of the line between Bus1 and Bus8 of the 15-bus network.

The transformer is connected to Bus1 of the 15-bus network and to Bus6 of the case9 network as shown in Figure 20 . Bus6 is chosen as it is the load bus with the highest voltage magnitude. Other choices for the interconnecting bus in the case9 network to connect resulted in lower voltage magnitudes in the 15-bus network.

Furthermore, the 15-bus network has two slack busses and the transmission network has one slack bus. In the hybrid network only the slack node of the transmission network is kept. The slack busses of the distribution network are replaced by two generators.

Newton's method does not converge for the original values of power demand in the 15-bus network. It does converge if the demand is divided by at least 1000. For the power demand values of 80 MW, 25MW, 10 MW and 90 MW at Bus2, Bus7, Bus9 and Bus15 respectively, Newton's method converges in 5 iterations as shown in Figure 21. The voltage magnitude at the node varies between 0.907 p.u. and 1 p.u, while the angle varies between -35.45° and 0°.

5.2   Interconnection of the 15-bus and case300 Networks

In interconnecting the 15-bus and the case300 networks, Bus1 of the former is connected to Bus17 of the latter. For Newton's method to converge, the resistance of the transformer transmission line has to be enlarged by at least twenty times. Without this scaling, the interconnection is likely to be insufficiently large to carry the given voltage. After this scaling, Newton's method converges in eight iterations as shown in Figure 22. After scaling the resistance with a factor 30 or 40, for example, Newton's method converges faster.

5.3 Interconnection of the 15-bus and UCTE Networks

In interconnecting the 15-bus network to the UCTE network, Bus1 of the former is connected to Bus4075 of the latter. The resistance of the interconnecting line is equal to the one of the 15-bus - case9 hybrid network. Newton's method converges in seven iterations which is faster than in the previously described 15-bus - case300 hybrid network. The highest voltage magnitude is now 1.243 p.u. and the lowest one equals 0.915 p.u. The nodal voltage angle ranges between -69.49° and 41.98°.

Since the connection of the distribution network has a very small influence on the convergence of the UCTE network, we decided to connect fifty copies of the 15-bus networks to the fifty nodes of the UCTE network with the highest voltage magnitudes. This is evidently done through fifty transformers. All the 15-bus networks are identical and are not interconnected. The hybrid network consists of 5003 nodes. For this network, Newton's method still converges in 7 iterations as shown in Figure 23. The voltage magnitude varies between 0.901 p.u. and 1.246 p.u, while the angle lies between -134.11° and zero.

6   Conclusions

In this report load flow computations in hybrid transmission - distribution networks were performed. A study of a simplified two-bus and single-branch networks allowed to identify factors hampering the convergence of the Newton iterative process inside the load flow computations. This knowledge was subsequently used in coupling existing transmission and distribution networks into a hybrid entity. Transformers were employed to convert power levels between the different parts of the network. In the largest hybrid network we managed to build in this report, fifty copies on the distribution network considered in connected to the UCTE model resulting in a network with 5003 nodes. The load flow computation in this network converges in seven iterations. Future work in this direction should allow to build even larger networks.

Acknowledgement

References

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7 Appendices

7.1 Appendix A

The network provided by ENDINET is shown in Figure 24.

But the original network has a couple important differences with the case files written for this network. First of all, the load values are all given in kV not in GW. The real power demands of Bus2, Bus7, Bus9 and Bus15 from the case file are suspiciously large.

Moreover, the authors of the case files for ENDINET network set the apparent power of the network at 1 MW, but do not modify the impedance of the transmission lines (see Section 7.2 for the details of the modification).

Another little difference can be found in the topology of the networks. From the Figure 24 one can easily see that in the upper branch of Figure 12 Bus3 has to be connected with Bus4, Bus5 and Bus6, which is connected to Bus7.

Of course, this differences can be removed. But after that modification MATPOWER has difficulties with convergence.

Therefore, the original case file stays without any modifications, except for the case of disconnecting the load and generator from bus 15 in Section 4.2 or dividing the load values by 1000 in Section 5.

7.2 Appendix B

In Section 5, we already mentioned that some modifications have to be done before connecting the 15-bus to the case9 network.

Let us start with the apparent power, baseMVA value in MATPOWER. This value is equal to 100 MVA in case9, while baseMVA of the 15-bus network is 1 MVA. Since there are many high voltage networks in MATPOWER and they all have baseMVA=100 MVA, it can be useful to replace the original apparent power of the 15-bus network by 100 MVA. However, one can always transform the transmission network instead of the distribution network in the analogous way.

Furthermore, the input voltage magnitude in per-unit system of the 15-bus network is equal to 10^{-3} instead of 1 as in the transmission networks. For MATPOWER to work properly this value has to be (almost) the same. This is thus another modification of the 15-bus network.

These transformations influence all per-unit values in the 15-bus network. So, the base voltage and the maximum/minimum voltage magnitude have to be modified in such a way that the actual voltage level stays equal to 230 V. This is easily done using Equation (19).

The values of shunt conductance, shunt susceptance and charging capacitance are all equal to zero. Thus, these parameters do not have to be modified.

The resistance and the reactance of the transmission lines however are non-zero. So, these per-unit values have to be changed. The base impedance can be written as

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V_{base}^2}{S_{base}},$$

where S_{base} is the base apparent power and V_{base} is the base voltage. From this follows that

$$R_{new} + jX_{new} = (R_{given} + jX_{given}) \left(\frac{\text{basekV}_{given}}{\text{basekV}_{new}} \right)^2 \left(\frac{\text{baseMVA}_{new}}{\text{baseMVA}_{given}} \right).$$

Figure 20:   Topology of the hybrid case9 - 15-bus network.
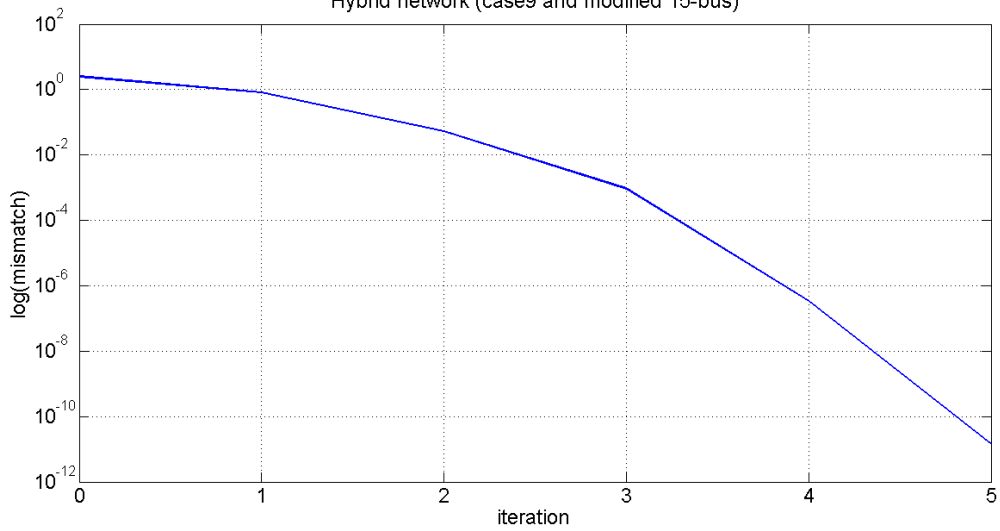

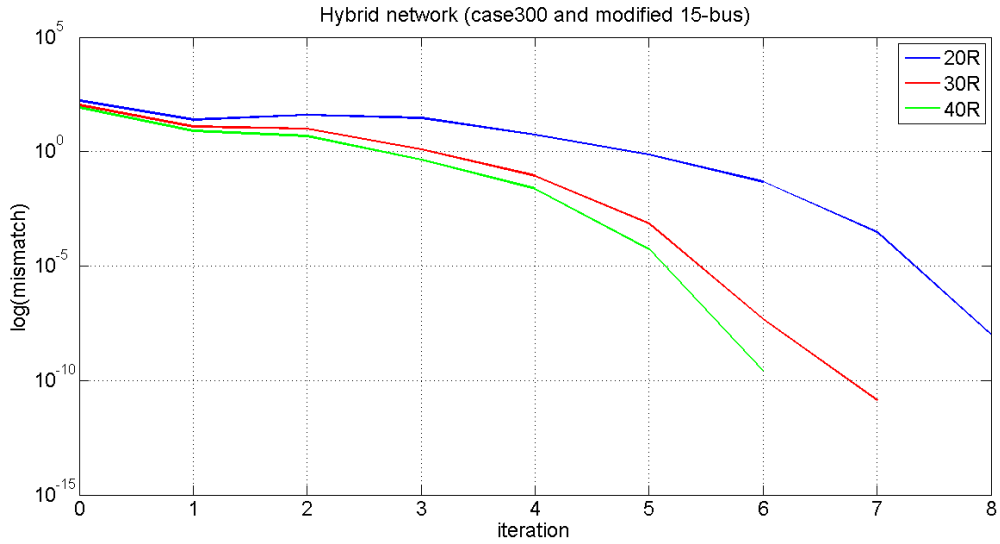Figure 21: Convergence of Newton's method for the hybrid case9 - 15-bus network.

Figure 22: Convergence of Newton's load flow method for the hybrid case300 - 15-bus network for three values of the resistance of the transformer.
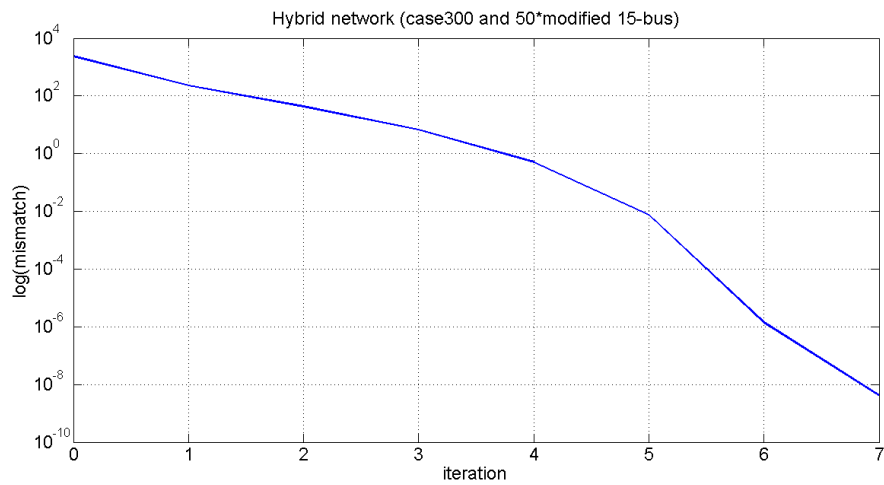

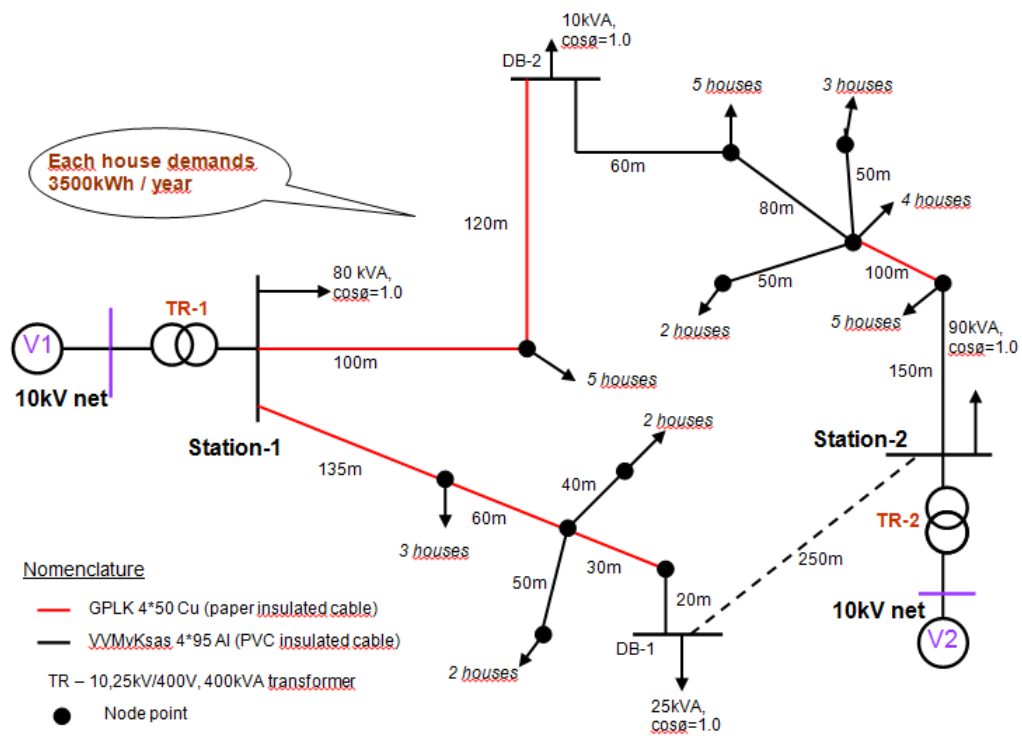Figure 23: Convergence of Newton's load flow method for the hybrid UCTE network - 50 times 15-bus networks.

Figure 24: The distribution network in Eindhoven provided by ENDINET.