

DELFT UNIVERSITY OF TECHNOLOGY

REPORT 13-13

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Pseudo-well-based method for automatic well placement

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ISSN 1389-6520

Reports of the Department of Applied Mathematical Analysis

Delft 2013

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# An extension of the Zandvliet-Handels-van Essen-Brouwer-Jansen Pseudo-well-based method for automatic well placement

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## Abstract

Oil production optimization is one of the main concerns of the reservoir management. In this regard, optimal location of wells can lead to a significant increase in the oil production. The purpose of this research is to investigate a new extension of the automatic well-placement optimization technique maximizing the net present value (NPV) of the oil production. Since finding the optimal location of wells may require expensive and time consuming iterations through the reservoir simulator, we propose a deterministic (gradient-based) algorithm to address this issue. Our approach is an extension of the work in Zandvliet et al [19] and consists of searching the neighbourhood of the initial well which is called the pseudo-wells in order to find the optimal location of wells. Since these pseudo-wells inject or produce at a very low rate, they have a minor effect on the overall flow throughout the reservoir. In this work, we first calculate the gradient of NPV with respect to the flow rate in the pseudo-wells using an adjoint-based method. This helps us to find improving directions on the basis of which the optimal locations of wells can be determined. This searching method continues until no further improvement in the NPV is achieved. The main contribution to the work in [19] is the use of step sizes larger than one which reduces the total convergence time. The method is applied to three waterflooding cases.

## Keywords

Well-placement optimization, adjoint-based, NPV, pseudo-wells, reservoir simulation.

## Introduction

The amount of hydrocarbon recovered can be considerably increased by finding optimal positions of wells. Determining the location of wells is a critical step to be made during an oil field development process, because it can affect many factors including NPV of the project, geological uncertainty and reservoir and fluid properties. The well-placement is often formulated as a nonlinear problem with discrete parameters. In general, two approaches have been used to solve such problems: the current industry practice is based on a manual approach in which they rely on some intuitive engineering judgments to determine well locations. Although such an approach may be reliable for small scale reservoirs with small number of wells, it might be impossible to be used when dealing with large scale reservoirs with large number of wells; the second approach is based on automatic well-placement optimization techniques in which mathematical models and computer algorithms are used to determine well locations. Several automatic techniques have been developed during the past decade (cf. [2,8,12,19]) which can be broadly divided into three general categories. The first category consists of deterministic methods (gradient-based) such as descent methods and direct pattern search. The main challenge in these methods is the calculation of the gradients for which two distinct methods are often used, namely finite difference method and adjoint-based method. The second category consists of stochastic algorithms including Simulated Annealing (SA), Particle Swarm Optimization (PSO), Simultaneous Perturbation Stochastic Algorithm (SPSA) and some Evolutionary Algorithms (EA) such as Genetic Algorithms (GA), Evolutionary Programming (EP) and Evolution Strategies (ES). The third category is called a hybrid method which is a combination of deterministic and stochastic methods. Deterministic methods are generally very efficient; it not only requires few contacts with the reservoir simulation for convergence, but also guarantees increase in the objective function at every iteration. However, when dealing with non-convex problems, it may provide local optimal solutions as it is highly dependent on the initial guess. On the other hand, stochastic algorithms are easy to implement and theoretically able to achieve the global solutions; however they usually incur a large number of runs in the reservoir simulator. Therefore, it might have limited application for large-scale simulations with many wells. Furthermore, they do not guarantee a monotonic increase in the

objective function with successive iterations, implying that increasing the number of iterations may not necessarily lead into a better solution. (cf. [12,16,17,18]).

The well-placement problem is usually formulated as a discrete parameter optimization problem because the well location variables are discrete. Gradients of the objective function (NPV for example) with respect to these discrete variables do not exist. Therefore, gradient-based optimization algorithms have not found much applicability to this problem, and most existing algorithms are gradient-free (stochastic) algorithms. In order to address this issue, Zandvliet et al [19] proposed a continuous approximation to the original discrete-parameter problem such that gradient-based algorithms can then be employed as an indirect method to determine the optimal well locations [17,19]. The basic idea is to optimize the location of wells by searching among the pseudo-wells. Since these pseudo-wells inject or produce at a very low rate, they have a minor effect on the overall flow throughout the reservoir. Therefore, an adjoint model can then be used to calculate the gradient of the objective function (NPV) over the lifespan of the reservoir with respect to the flow rate of each pseudo-well. The pseudo-well with the largest positive gradient determines the direction to which the original well should be moved in order to increase the NPV in the next iteration [12].

The adjoint method in optimal control theory has been applied previously to determine parameters such as well flow rate, bottomhole pressures (BHPs), flowing tubing head pressures and inflow control valves(ICVs) setting to optimize the production and injection rate in a fixed well configuration (cf. [1,3,10,13,18]). Because these are not mixed integer problems, gradient-based methods are used commonly to solve them and the adjoint method efficiently generates the required gradients.

In this research, we present an extension of the pseudo-well-based method proposed in [19], which uses step size equal to one. Our main contribution is the use of step sizes larger than one.

The outline of this paper is as follows: First, problem description and mathematical model is investigated. Then, an adjoint-based optimal location of wells method is implemented. Finally, the advantage of this method is illustrated by three water flooding cases.

## Problem Description

The problem is to maximize the objective function which is the net present value (NPV), defined as the total oil revenue minus the total injection and production cost (water) over a time period  $T$  and the main constraints are the system equations. By introducing Lagrange multipliers, the problem can be transformed into an equivalent unconstrained problem. The optimization variables  $\mathbf{u}$ , indicated as controllable input (also called input variables), could then be locations of the water injection  $\mathbf{w}_{inj}$  or locations of the total production  $\mathbf{w}_{prod}$  in the reservoir. Alternatively we could choose water injection rates  $\mathbf{q}_{w,inj}$ , total production rates  $\mathbf{q}_{t,prod}$ , flowing tubing head pressures  $\mathbf{p}_{tf}$ , or flowing bottom hole pressures  $\mathbf{p}_{wf}$  in the injectors and or the producers, or valve settings  $\boldsymbol{\alpha}$ , with elements  $0 \leq \alpha_i \leq 1$  representing the dimensionless opening of valve. We will start our analysis from an implicitly discrete time version of the system equations as following [7]:

$$\mathbf{g}_k(\mathbf{u}_k, \mathbf{x}_{k-1}, \mathbf{x}_k) = \mathbf{0} \quad , \quad k = 1, 2, \dots, K \quad (1)$$

With the appropriate initial conditions

$$\mathbf{x}_0 = \tilde{\mathbf{x}}_0 \quad . \quad (2)$$

In addition, we consider an output vector  $\mathbf{y}_k$  that is assumed to be a nonlinear function of the input and the states:

$$\mathbf{j}_k(\mathbf{u}_k, \mathbf{x}_k, \mathbf{y}_k) = \mathbf{0} \quad . \quad (3)$$

As mentioned above, the objective function  $\mathcal{J}$  is the net present value (NPV) of the water flooding process which can be expressed as:

$$\mathcal{J}(\mathbf{u}_{1:K}, \mathbf{y}_{1:K}(\mathbf{u}_{1:K})) = \sum_{k=1}^K \mathcal{J}_k(\mathbf{u}_k, \mathbf{y}_k) \quad (4)$$

Where  $\mathcal{J}_k$  represents the present value of oil revenues minus the water injection and production costs during the time step  $k$ . We should therefore formally write  $\mathcal{J}_k(\mathbf{u}_k, \mathbf{y}_k(\mathbf{u}_k, \mathbf{x}_k(\mathbf{u}_{1:k})))$ , but to keep the notation tractable we use  $\mathcal{J}_k(\mathbf{u}_k, \mathbf{y}_k)$  instead. In practice, the elements of the input vector  $\mathbf{u}_k$  are often constrained to stay within certain limits. Another form of constraint is when we require the volume of injected water to be equal to the total volume of the fluids produced, a situation known as voidage replacement. Also well rates are usually constrained to some maximum values. These limitations may all be expressed as equality or inequality constraints, which can be represented in a general form as:

$$\mathbf{c}_k(\mathbf{u}_k, \mathbf{y}_k) = \mathbf{0} \quad (5)$$

and

$$\mathbf{d}_k(\mathbf{u}_k, \mathbf{y}_k) \leq \mathbf{0} \quad (6)$$

The control problem can now be formulated as:

$$\max_{\mathbf{u}_{1:K}} \mathcal{J}(\mathbf{u}_{1:K}, \mathbf{y}_{1:K}(\mathbf{u}_{1:K})) \quad (7)$$

Subject to

- System equations (1):  $\mathbf{g}_k(\mathbf{u}_k, \mathbf{x}_{k-1}, \mathbf{x}_k) = \mathbf{0}$ ,  $k = 1, 2, \dots, K$
- Initial conditions (2):  $\mathbf{x}_0 = \tilde{\mathbf{x}}_0$
- Output equations (3):  $\mathbf{j}_k(\mathbf{u}_k, \mathbf{x}_k, \mathbf{y}_k) = \mathbf{0}$
- Equality constraints (5):  $\mathbf{c}_k(\mathbf{u}_k, \mathbf{y}_k) = \mathbf{0}$
- Inequality constraints (6):  $\mathbf{d}_k(\mathbf{u}_k, \mathbf{y}_k) \leq \mathbf{0}$ .

Where:

- $\mathbf{u}_k \in \mathbb{R}^n$  where  $n$  is the number of inputs,
- $\mathbf{x}_k \in \mathbb{R}^m$  where  $m$  is the number of states,
- $\mathbf{y}_k \in \mathbb{R}^p$  where  $p$  is the number of outputs.

Starting from optimization problem (7), we compute the optimal control  $\mathbf{u}_{1:K}$  using a gradient-based algorithm, which requires the derivatives of  $\mathcal{J}(\mathbf{u}_{1:K}, \mathbf{y}_{1:K}(\mathbf{u}_{1:K}))$  with respect to  $\mathbf{u}_{1:K}$ . An efficient way to compute these derivatives is with the adjoint method. The problem in determining the derivatives is the indirect dependence of the variation  $\delta \mathcal{J}$  in the objective function on a variation  $\delta \mathbf{u}_{ik}$  of the input. Here,  $\delta \mathbf{u}_{ik}$  means the variation of element  $i$  of vector  $\mathbf{u}_k$  at time  $k$ . Note that a variation  $\delta \mathbf{u}_{ik}$ , at an arbitrary time  $k$ , does not only directly influence  $\mathcal{J}_k$ , but also the states  $\mathbf{x}_{k:K}$  in equation (1) which in turn, through equation (3), influence the outputs  $\mathbf{y}_{k:K}$  and thus  $\mathcal{J}$  at later times. The effect of a single variation  $\delta \mathbf{u}_{ik}$  should therefore be computed, using the chain rule for differentiation, as following:

$$\frac{d\mathcal{J}}{du_{ik}} = \left[ \frac{\partial \mathcal{J}_k}{\partial \mathbf{u}_k} + \sum_{j=k}^K \frac{\partial \mathcal{J}_j}{\partial \mathbf{y}_j} \left( \frac{\partial \mathbf{y}_j}{\partial \mathbf{u}_k} + \frac{\partial \mathbf{y}_j}{\partial \mathbf{x}_j} \frac{\partial \mathbf{x}_j}{\partial \mathbf{u}_k} \right) \right] \frac{\partial \mathbf{u}_k}{\partial u_{ik}} \quad (8)$$

Where we used the ordinary differential  $d$  instead of the variational symbol  $\delta$ .

**Objective function for optimal well-placement.** The general objective function  $\mathcal{J}$  is the NPV, defined as the discounted

total oil revenue minus the discounted total injection and production cost (water) over a time period  $T$  with  $K$  time steps. We denote by  $r_o$  the oil revenue per unit volume (bbl/m<sup>3</sup>), and  $r_{w,inj}$  denotes the water injection cost per unit volume (bbl/m<sup>3</sup>),  $r_{w,prod}$  denotes the water production cost per unit volume (bbl/m<sup>3</sup>). The objective function can then be written as:

$$\mathcal{J}[\mathbf{q}_{o,1:K}(\mathbf{w}), \mathbf{q}_{w,prod,1:K}(\mathbf{w}), \mathbf{q}_{w,inj,1:K}(\mathbf{w})] = \sum_{k=0}^{K-1} \left[ \underbrace{-\sum_{j=1}^N r_o q_{o,j,k}(\mathbf{w})}_{\text{Oil revenue}} + \underbrace{\sum_{j=1}^N r_{w,bij} q_{w,inj,j,k}(\mathbf{w})}_{\text{Water disposal cost}} - \underbrace{\sum_{j=1}^N r_{w,prod} q_{w,prod,j,k}(\mathbf{w})}_{\text{Injection cost}} \right] \underbrace{\frac{\Delta t_k}{(1+b)^{\frac{t_k}{\tau}}}}_{\text{Discount factor}} \quad (9)$$

Where  $\mathbf{y}_{1:K} = \mathbf{q}_{o,j,k}$ ,  $\mathbf{u}_{1:K} = \mathbf{q}_{w,inj,j,k}$  and  $\mathbf{y}_{1:K} = \mathbf{q}_{w,prod,j,k}$  are representation oil production rates, water injection rates and water production rates, respectively, in well  $j$  during time step  $k$ ;  $N$  is the total number of wells;  $t_k$  and  $\Delta t_k = t_{k+1} - t_k$  are the time and the time interval corresponding to time step  $k$ . We use the convention that injection rates are positive and production rates negative. In addition, it is assumed that all wells are controlled on BHP such that well rates are affected by  $\mathbf{W}$  which is a vector of spatial coordinates of the to be optimized well. Also, the production rates  $\mathbf{q}_o$ ,  $\mathbf{q}_{w,inj}$  will always be a function of well position  $\mathbf{W}$  because it is not possible to directly control the flow rates in a producer. The term  $b$  is the discount rate for a reference time  $\tau$ .

### Adjoint Method

Adjoint-based optimization techniques have been used in many applications in the reservoir engineering. They were first introduced in this field during the 1970s for computer-assisted history matching. About a decade later they also have been applied for the optimization of tertiary recovery processes such as surfactant, polymer, CO<sub>2</sub> or steam flooding; see e.g. [4-6, 9-11]. The first paper on gradient-based control of water flooding is the one by [1], followed by, among others, [13, 15 and 18]. However, the real applications of adjoint method were limited until the advent of “smart well” and “smart field” technology which was a breakthrough for the industry uptake of these methods; see [3]. Since that time a series of studies have been conducted covering different aspects of adjoint-based optimization of reservoir flooding while several large reservoir simulation packages have been equipped with the adjoint functionality. In the adjoint method, we find the optimal value of a set of vectors  $\mathbf{u}_k$  of the control variables to maximize the objective function  $\mathcal{J}_k(\mathbf{u}_k, \mathbf{y}_k)$  (Eq.4). The control variables could be well-flow rate, BHPs, or ICVs. The water injection flow rate  $\mathbf{q}_{w,inj}$  is part of control variables depending on whether the injectors are rate-controlled or BHP controlled. On the other hand, the oil- and water production rate  $\mathbf{q}_o$  and  $\mathbf{q}_{w,prod}$  are excluded from our control variables. The adjoint method is a robust method to calculate the derivative vectors of equation (8).

This method work as follows: Starting from a set of initial control vectors  $\mathbf{u}_k^i$ , it uses a gradient-based technique (steepest-ascent technique) to determine the improved control vector in every iteration.

$$\mathbf{u}_k^{i+1} = \mathbf{u}_k^i + \alpha^i \left( \frac{d\mathcal{J}}{d\mathbf{u}_k^i} \right)^T \quad k=1,2,3,...,K \quad (10)$$

As discussed above, the well-placement is a discrete-parameter problem. Therefore, a gradient-based approach cannot be directly used in order to solve such problem. However, an alteration of this approach can help us to use adjoint method to identify the optimal well locations, called  $\mathbf{W}$ , that maximize NPV. In that alteration, the uncontrolled well rates ( $\mathbf{u}$ ) are defined as a function of the well location  $\mathbf{w}$  such that  $\mathcal{J} = \mathcal{J}_k(\mathbf{u}_k, \mathbf{y}_k(\mathbf{w}))$ . Starting from the initial well locations  $\mathbf{w}^i$ , improved locations are found using the steep-ascent technique where  $d^i$  denotes the improving direction.

Where  $\beta^i$  is an integer representing the step size as depicted in **Figure 1**. It is worth mentioning that, the gradient of  $\frac{\partial \mathcal{J}}{\partial \mathbf{w}}$  is not defined because  $\mathbf{w}$  is a discrete parameter [19].

The basic idea is to optimize the location of wells (injectors or producers) by searching among the pseudo-wells. Since these pseudo-wells inject or produce at a very low rate (e.g., less than one percent of the flow rate of the to be placed well), they have a minor effect on the overall flow throughout the reservoir. In this research, we have formulated the single well location problem in a 2D reservoir. Well locations are represented as a vector of two control variables, namely  $w^i = [w_x^i \ w_y^i]^T$ , where  $w_x^i$  and  $w_y^i$  are the well's  $x$ - and  $y$ -coordinates, and the superscript  $i$  is the iteration counter. Each to be optimized wells are surrounded by eight pseudo-wells provided that the well is not located along the margin of the reservoir. In iteration  $i$ , we defined the  $x$ - and  $y$ -coordinates of pseudo-well  $j$  by  $w_{x,j}^i$  and  $w_{y,j}^i$  respectively, and total flow rate in pseudo-well  $j$  during time step  $k$  by  $q_{j,k}^i$ . Therefore, an adjoint model can then be used to calculate the gradient of the objective function ( $\mathcal{J}$ ) over the life-cycle of the reservoir with respect to the prescribed small flow rates  $q_{j,k}^i$  of each pseudo-well and at each reservoir simulation time step. The pseudo-well with the largest positive sum of gradients over all time steps determines the direction ( $d^i$ ) to which the original well should be moved in order to increase the NPV in the next iteration. This procedure is depicted in **Figure 1**. for different step sizes.

$$j_{\max} = \arg \max_{j=1,2,\dots,8} \sum_{k=0}^{K-1} \frac{d\mathcal{J}}{dq_{j,k}^i} [u_{1:K}, y_{1:K}(w^i)] \quad (11)$$

$$d^i = [w_{x,j_{\max}}^i \ w_{y,j_{\max}}^i]^T - w^i \quad (12)$$

Our approach is based on several iterations on Eqs. (11) and (12) with step size greater than one until the optimal well location have been found. In other words, this searching method continues until no further improvement in the NPV is achieved. In the next section, we provide three examples to demonstrate the applicability of the approach.

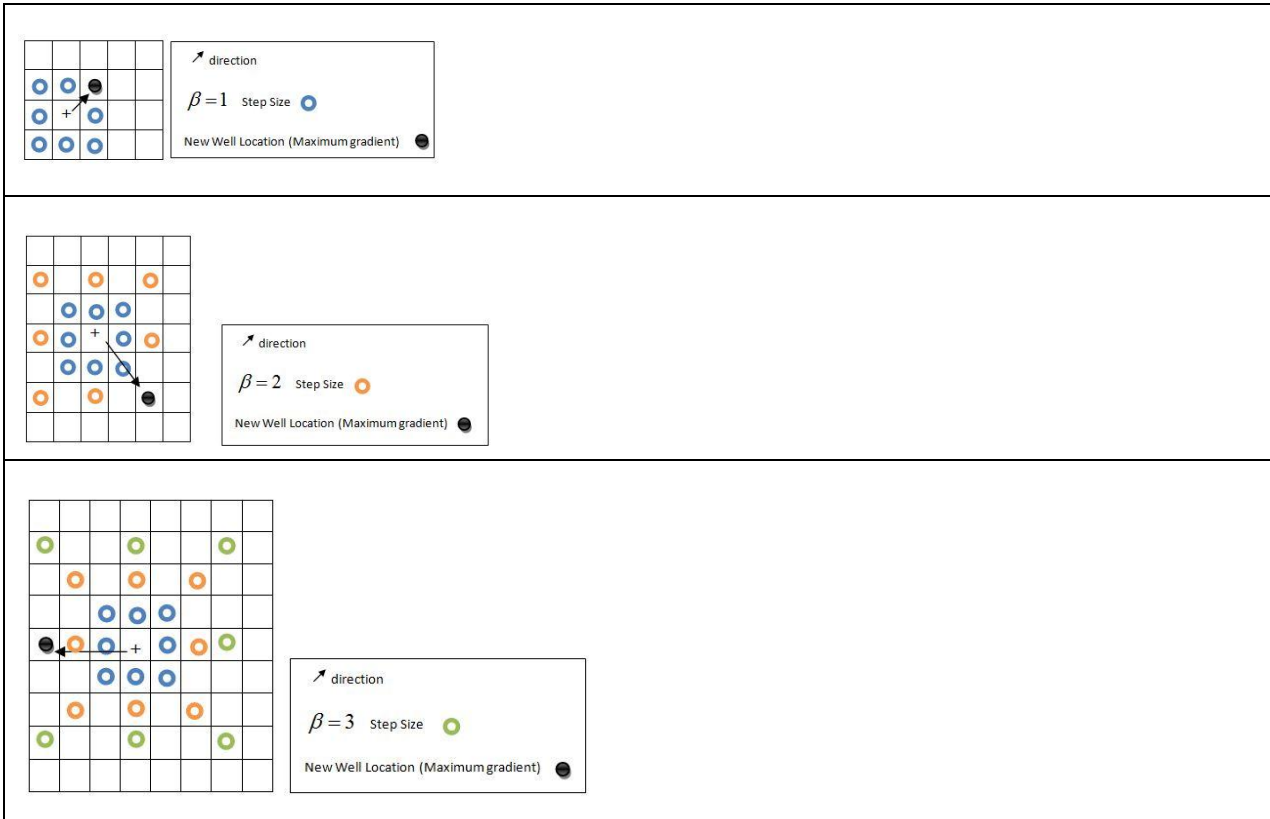


Figure 1: Step sizes greater than one.

## Examples

**Reservoir Model.** The reservoir under consideration is a horizontal, 2D, oil/water reservoir with homogeneous porosity distribution throughout the reservoir. The geological and fluid properties are given in **Table 1** and the economics values for computing the objective function are given in **Table 2**. At a depth of 4000m, the initial reservoir pressure is 40.0 MPa and the initial water saturation is 0.20. the wells are vertical (producer locations are fixed in each of the four corner of the grid blocks and injector locations are the parameter to be optimized), and their well indices are computed using a Peaceman model with a wellbore radius of 0.1 m and skin factor of 0. Simulations are run for two finite periods (T=2.5 and 10 years) and the production constraint is: The producers operate at the constant BHP of 39.7 MPa and the injector operates at a constant BHP of 40.3 MPa with an additional water-cut constraint of 95%.

$\emptyset$	0.20	--
K	$5 \times 10^{-13}$	$m^2$
$\rho_o, \rho_w$ (at 0.1 MPa)	1.000	$Kg/m^3$
$c_o, c_w, c_r$	$1.0 \times 10^{-9}$	$1/Pa$
$\mu_o, \mu_w$	$1.0 \times 10^{-3}$	$Pa \cdot s$
$\rho_{cow}$	0	$Pa$
$n_o, n_w$	2	--
$k_{ro}^0$	0.90	--
$k_{rw}^0$	0.60	--
$S_{or}, S_{wc}$	0.20	--

Table 1: Rock and Fluid properties.

$r_o$	300	$\$/m^3$
$r_{w, inj}$	15	$\$/m^3$
$r_{w, prod}$	5	$\$/m^3$
$b$	0	--

Table 2: Values for computing NPV.

In Table 1,  $\rho_o, \rho_w$  are oil and water density, respectively;  $\mu_o, \mu_w$  are oil and water viscosity, respectively;  $c_o, c_w, c_r$  are oil, water and rock compressibility, respectively;  $\rho_{cow}$  is oil/water capillary of density;  $\emptyset$  is porosity;  $k_{ro}^0, k_{rw}^0$  are end point relative oil and water permeability, respectively; and  $S_{or}, S_{wc}$  are residual oil and water saturation, respectively.

**Example 1.** The reservoir in this example is  $210 \times 210 \times 10$  m in size and has a pore volume (PV) of  $8.82 \times 10^4 m^3$ , and is modelled by  $21 \times 21$  grid blocks that are  $10 \times 10 \times 10$  m in size using a black oil formulation. The reservoir has four producers with fixed locations and one injector whose location is to be optimized. The location of one injector is optimized for T=2.5 years with the reactive constraint mentioned above. The permeability distribution is homogenous throughout the reservoir with  $5 \times 10^{-13} m^2$ . The pseudo-wells rates are set to  $0.01 m^3/d$ , which is less than 0.1% of the injection flow rates of the to be optimized well. The algorithm is initialized from the northwest of the corner (the method converge to almost the same local optimum starting from each of the corners) and the step sizes of  $\beta^i=1$ ,  $\beta^i=2$  and  $\beta^i=3$  were used. The producer and injector BHPs are 39.7 and 40.3 MPa, respectively.

In this example, following the descriptions mentioned in the adjoint method section, we use a step size larger than one to find the optimal location of well. We then compare our results with work in [19] which has been done in the same reservoir.

In our strategy, we have run the simulation with three different step sizes and a same initial location of well. As it is shown in **Figure 2**, in the first simulation ( $\beta^i=1$ ), the optimal location is reached in 248 sec with 11 iterations. In the second simulation ( $\beta^i=2$ ), the same NPV has been found with less time (184 sec) and fewer iterations (7 iterations). Moreover, in the third simulation ( $\beta^i=3$ ), the same NPV has been reached in even shorter time (136 sec) and only 5 iterations. Therefore, we propose to use the step size 3 because it enables us to find the same NPV with less iteration which means fewer contacts with the reservoir simulation.

Comparing the results with Zandvliet's works [19], the speed of convergence to the optimal location is much faster than his work. While in Zandvliet' technique it converges in 12 iterations and takes 286 sec, our approach converges to the same



optimal solution only in 5 iterations with 136 sec. Moreover, as shown in **Figure 2**, our approach guarantees a monotonic increase in NPV in every successive iteration, whereas in work in [19] it is not monotonic in the first three iterations.

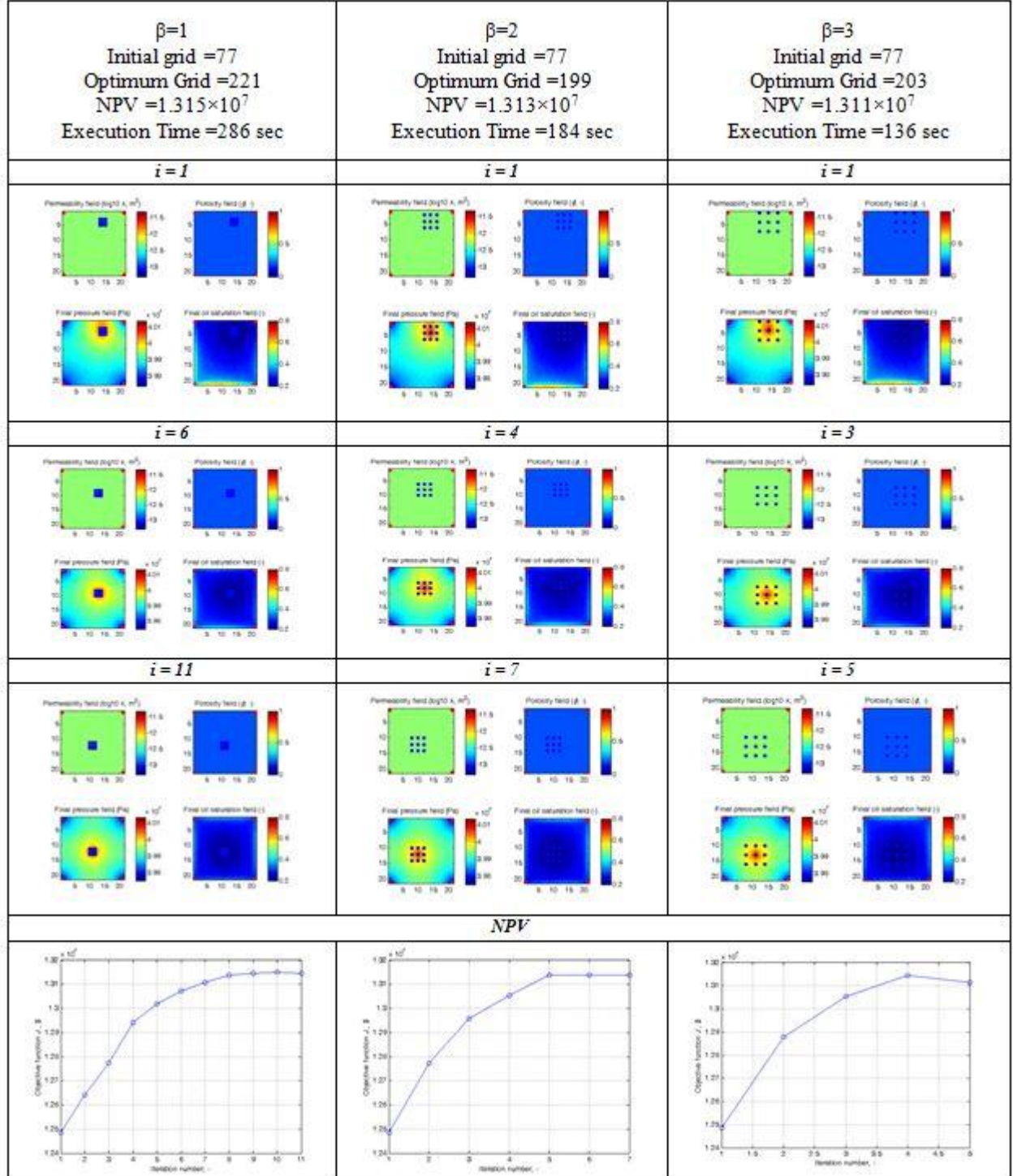


Figure 2: Depict the well locations at the first, middle and final iterations and the NPV at each iteration in Example 1.

**Example 2.** The reservoir in this example is the same as example 1 with  $210 \times 210 \times 10$  m size and has a pore volume (PV) of

$8.82 \times 10^4 \text{ m}^3$ , and is modelled by  $21 \times 21$  grid blocks that are  $10 \times 10 \times 10 \text{ m}$  in size using a black oil formulation. The reservoir has four producers with fixed locations and one injector whose location is to be optimized. The location of two injectors are optimized for  $T=2.5$  years with the reactive constraint mentioned above. The permeability distribution is homogenous throughout the reservoir with  $5 \times 10^{-13} \text{ m}^2$ . The pseudo-wells rates are set to  $0.01 \text{ m}^3/\text{d}$ , which is less than 0.1% of the injection flow rates of the to be optimized well and we use step sizes of  $\beta^i=1$  and  $\beta^i=2$ . The producer and injector BHPs are 39.7 and 40.3 MPa, respectively.

The algorithm is initialized from two initial injection wells that are at the northwest and northeast of the reservoir (The method converges to almost the same local optimum starting from each of the corners) and step sizes  $\beta^i = 1, 2$  are used. In this example, the positions of the wells come close together after a few iterations reaching the optimal location. In such situations, we usually face two problems: first, the two main wells, which are close together, have some pseudo-wells in common. This case happens when the two main wells have only one grid block in between. In order to solve this issue, we remove the common pseudo-wells from pseudo-wells set of one of the main wells and keep them in the others. Second, one of the two main wells acts a pseudo-well for the other and vice versa. This case happens when two main wells become neighbours of each other. As proposed by Zandvliet's works [19], we will merge the two main wells in order to address this issue. This can help us to reduce the simulation time. Besides that, in this way the NPV will be increased through reduction of the drilling costs. While in Zandvliet's works [19], one of the two wells is removed randomly, we propose to keep the main well with the higher NPV and the other will be removed from the simulation. In this example, we have run the simulation with two different step sizes and same initial locations of wells. As it is shown in **Figure 3**, in the first simulation ( $\beta^i=1$ ), the two main wells are merged together after 7 iterations and the optimal locations are reached in 356 sec with 14 iterations. Similarly, in the second simulation ( $\beta^i=2$ ), the main wells are merged together after 4 iterations and the same NPV has been found in shorter time (209 sec) and fewer iterations (8 iterations). Therefore, we propose the step size 2, since it requires less time and fewer iterations to find the optimal solution. It is important to note that, merging of main wells will not always occur. In particular, if we select the initial wells such that their locations are relatively asymmetric, it may be possible that the wells are not merged together in the optimal solution. An example of this case is illustrated in **Figure 4** where we have run the simulation with step size 2. As it can be seen, although after few iterations the two main wells become close together, they do not have any common pseudo-wells and, consequently, they do not merge together in our algorithm.

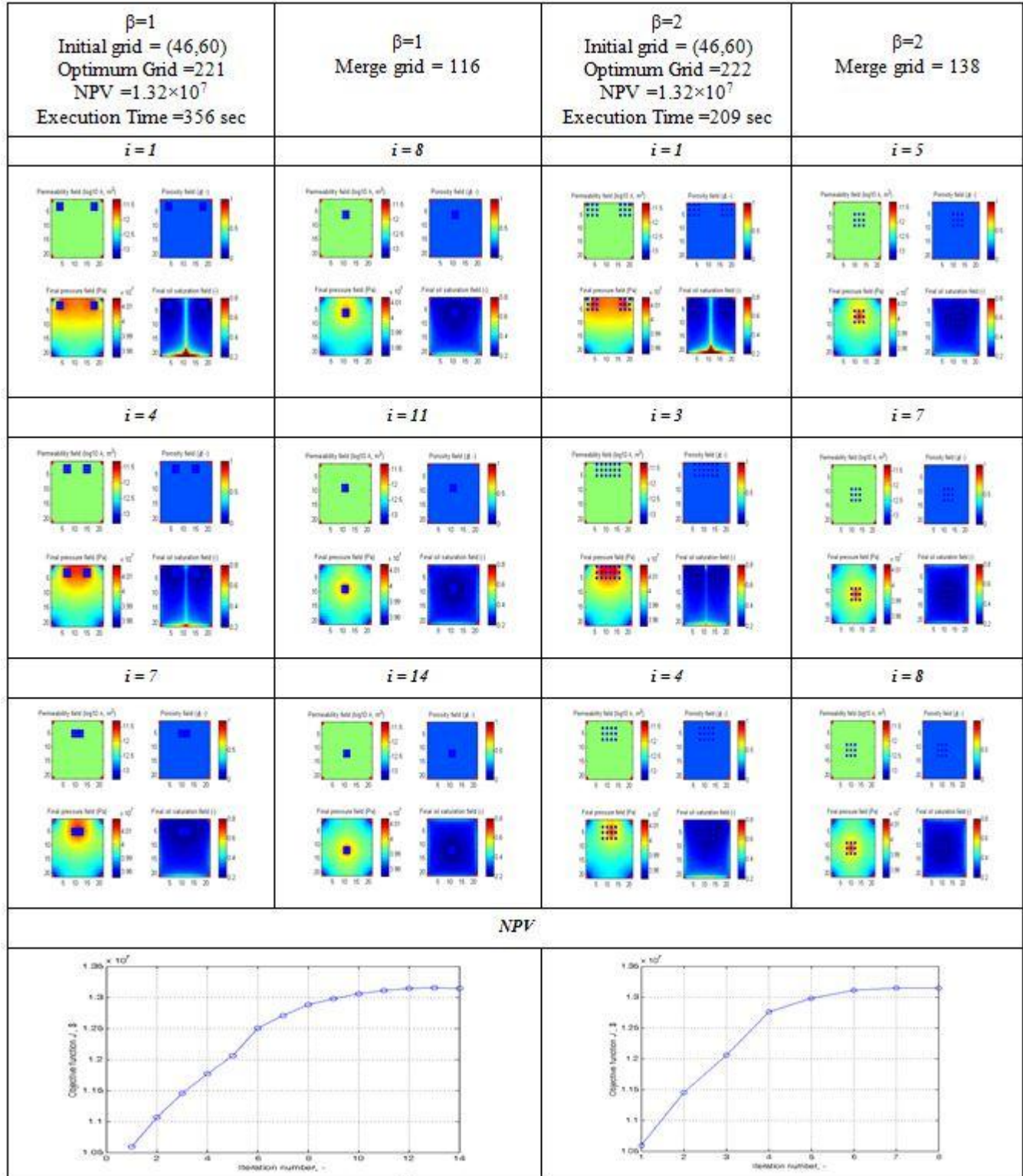


Figure 3: Depict the well locations at the first, middle and final iterations and the NPV at each iteration in Example 2.

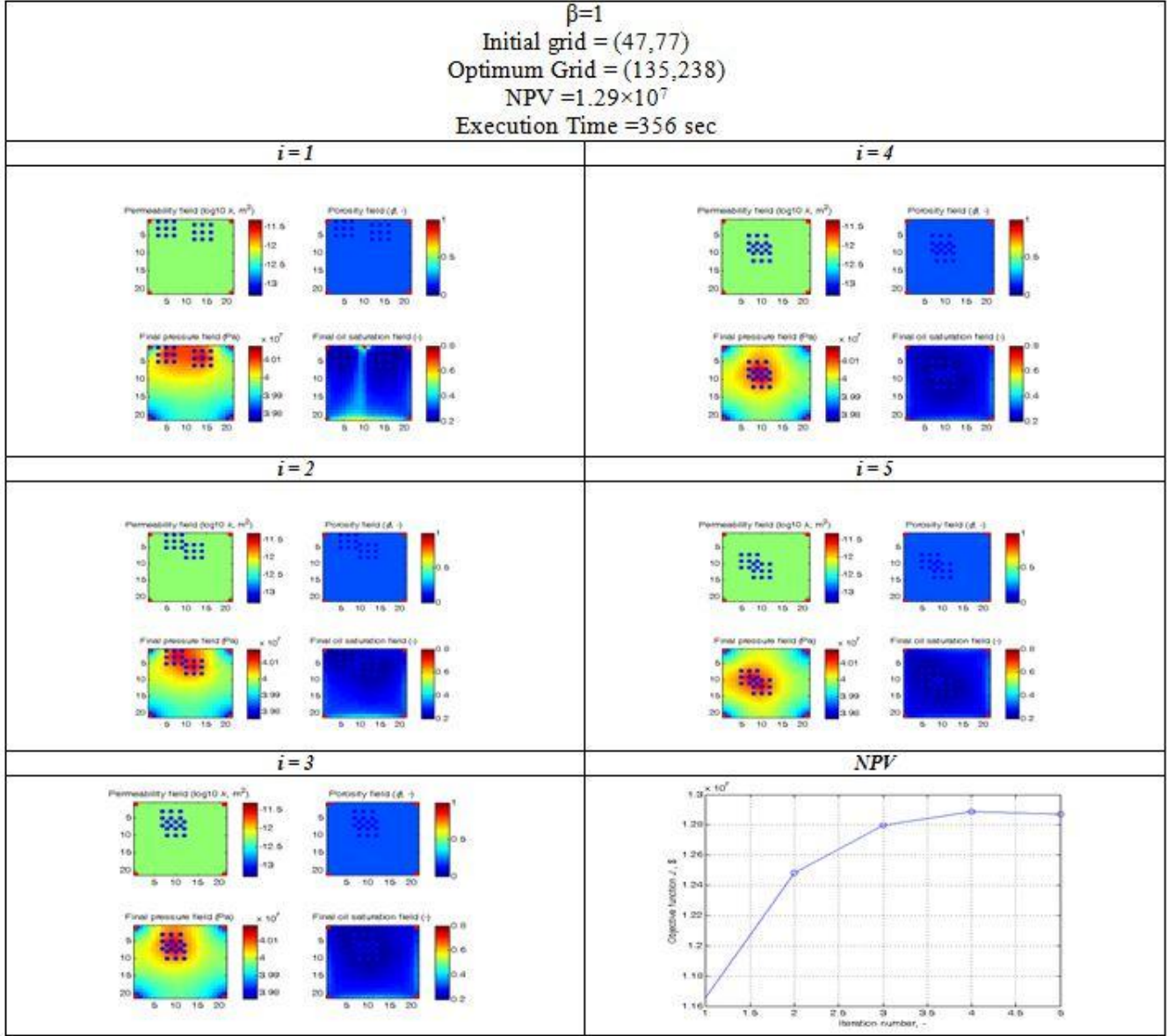


Figure 4: Depict the well locations and the NPV at each iteration do not merge of two main wells when we select the initial wells such that their locations are relatively asymmetric in Example 2.

**Example 3.** The geometrical property of this example is different from the previous examples. The reservoir in this example is  $1010 \times 1010 \times 10$  m in size and has a pore volume (PV) of  $2.0 \times 10^6$  m<sup>3</sup>, and is discretized by  $101 \times 101$  grid blocks that are  $10 \times 10 \times 10$  m in size. Other geological properties are same as Tables 1,2. The reservoir has nine producers with fixed locations and four injectors whose locations are to be optimized. The location of four injectors are optimized for  $T=10$  years with the reactive constraint mentioned above and the permeability distribution is heterogeneous as depicted in **Figure 5**. The pseudo-well rates are set to 0.1 m<sup>3</sup>/d, which is less than 0.1% of the flow of the to be optimized wells, and step sizes of  $\beta^i=1$  and  $\beta^i=2$  are used. The producer and injector BHPs are 39.7 and 40.3 MPa, respectively.



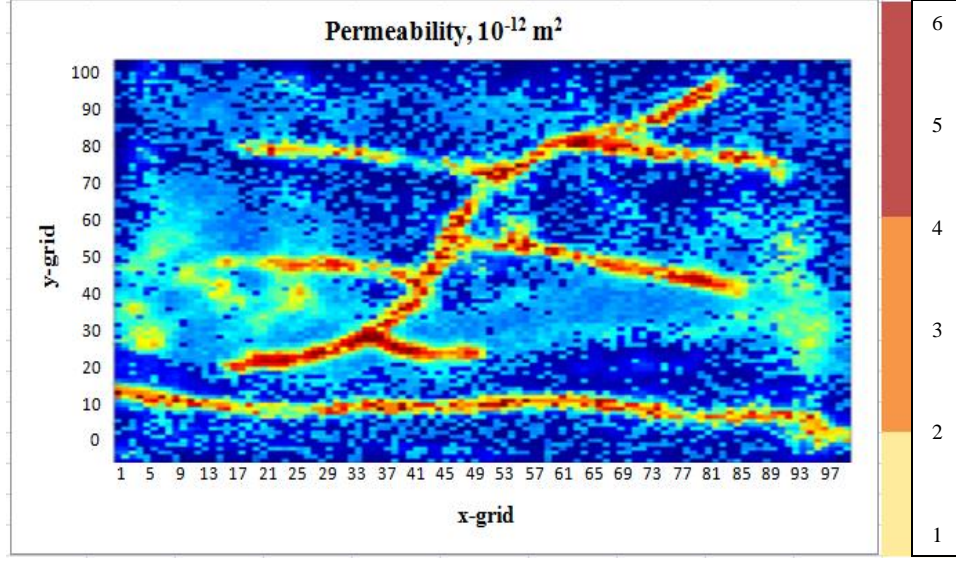


Figure 5: Permeability for Example3.

The algorithm is initialized from four initial injection wells with two different configurations: a standard 13-spot pattern and a mini 13-spot pattern. In the first configuration, unlike the previous examples, the NPV is not increasing in every successive iterations. Yet, although there is minor fluctuation at some point, the overall trend in NPV is still increasing and it finally converges to an optimal value as depicted in **Figure 6**. This convergence occurs after iteration 55 if we use step size  $\beta^i=1$  and iterations 8 if we use step size  $\beta^i=2$ . The same pattern follows in the second configuration as well only with less optimal value for NPV as depicted in **Figure 7**. It is worth mentioning that while we have achieved approximately the same optimal NPV value as Zandvliet's work (19), our method requires less computation than his work. That is because, unlike Zandvliet (19), we assume the location of the production wells to be fixed.

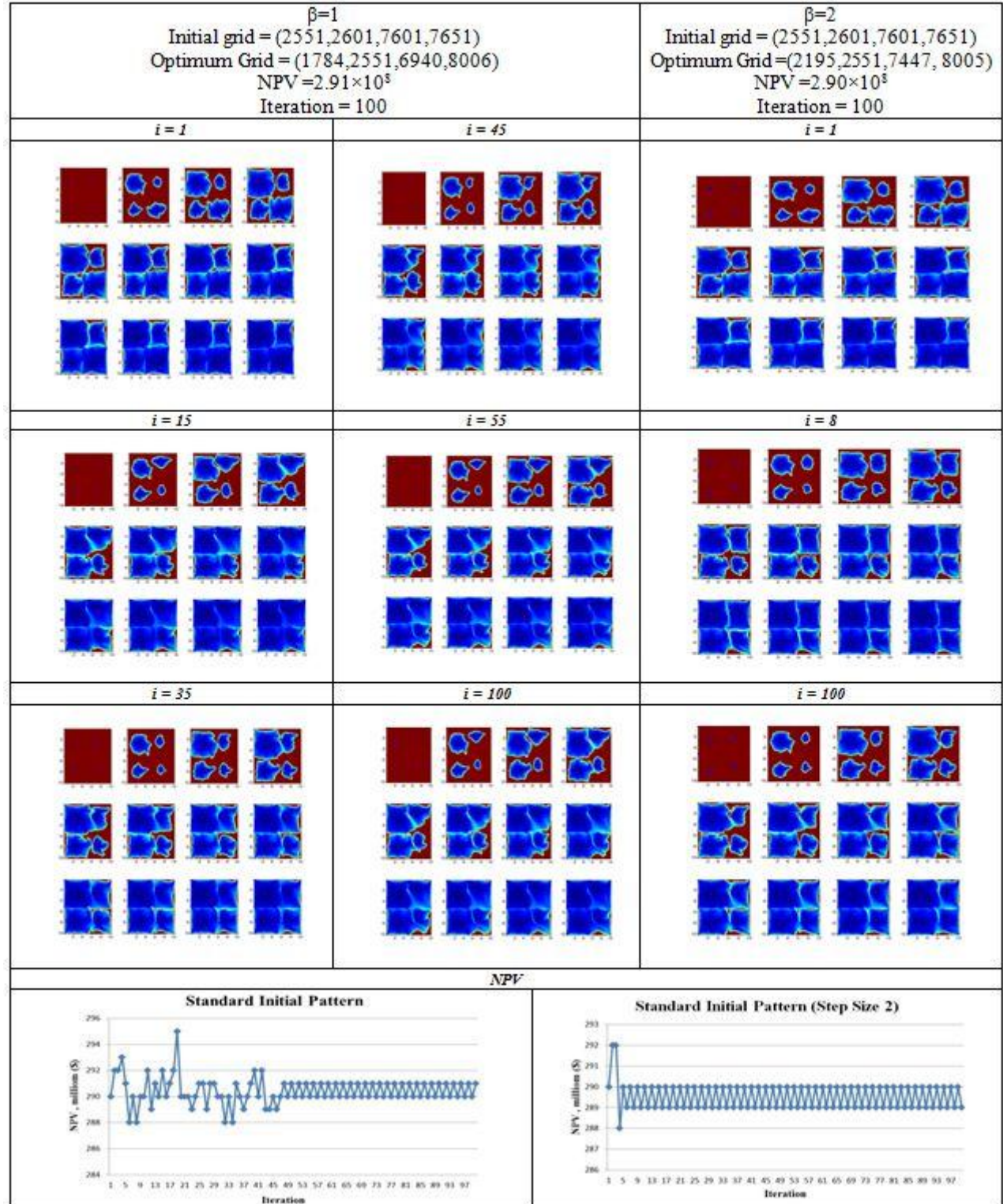


Figure 6: Depict the well locations and End of life water saturation at the first, middle and final iterations and the NPV for  $\beta^i = 1$  and  $\beta^i = 2$  at each iteration with standard initial pattern in Example 3.

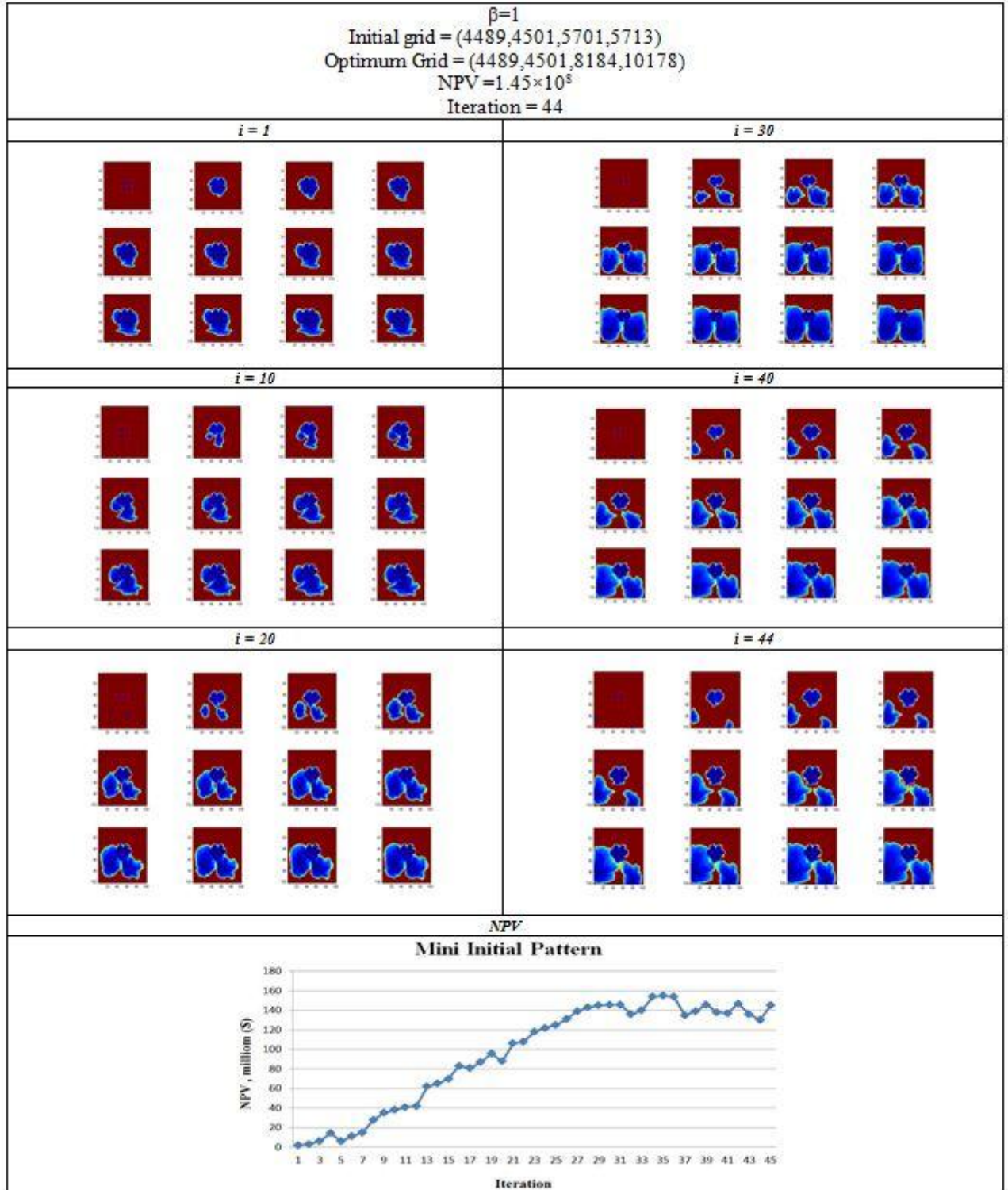


Figure 7: Depict the well locations and End of life water saturation at the first, middle and final iterations and the NPV for  $\beta^i=1$  at each iteration with mini initial pattern in Example 3.

## Conclusions

1. Deterministic gradient-based methods are generally very efficient, not only do they require few contacts with the reservoir simulation for convergence, but they also guarantee increase in the objective function at every iteration. However the NPV is not increasing in every successive iteration such as Example 3. Yet, although there is minor fluctuation at some point, the overall trend in NPV is still increasing and it finally converges to an optimal value.
2. The gradient-based methods provide an overall improve objective function with each iteration, resulting in better well-placement scenario, close to the original selection, within a few iterations.

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