Self-assessment

Below are 15 exercises on vectors, functions, trigonometry, differentiation, and integration, in no specific order. The level of the questions are roughly based on material covered in VWO Wiskunde B. Try the exercises without the help of external material (formula sheets, calculators, textbooks, online material, etc.). You have 2 hours to complete the assessment. We strongly encourage attending the summer school if you find yourself unable to solve a minimum of 10 problems. Good luck!

1. Find the radius and center of the circle described by the equation
$$x^2 + y^2 - 4x - 6y + 1 = 0.$$

2. Convert $9°$ and $36°$ to radians.

3. Solve for θ, where
$$\cos(\theta) \leq \frac{1}{2}, \quad 0 \leq \theta \leq \pi.$$

4. Given that $x = 1$ is a solution, find the other two solutions to the equation
$$x^3 - x^2 - 4x + 4 = 0.$$

5. Find the slope of the curve $y = 4x + e^x$ at $(0, 1)$.

6. Solve the following equation and inequality.
   (a) $|3x - 2| = 4x + 1$
   (b) $\sqrt{x - 1} < x - 7$

7. (from the ijkingstoets) Compute the length of the part of the function $f(x) = |3 + x| - |x - 2|$ that lies between $x = -2$ and $x = 3$.

8. Integrate $\cos(2x + 3)$.

9. Solve the following system of equations.
$$2x + 5y = 10$$
$$4x + 3y = 13$$

10. Find the derivatives of the following functions.
    (a) $f(x) = (2x + 1)(x + 4)$
    (b) $g(x) = \sin(\cos(x))$

11. Find the area between the graph
$$y = \frac{1}{4x + 1}$$
and the x-axis between $x = 1$ and $x = 3$.

12. Let y be a function of x. Find y', if $2y + x = y \sin(x)$.

13. Write the following sum as a single fraction.
$$\frac{x + 4}{2x - 1} + \frac{3x - 1}{x + 1}.$$

14. Let $\mathbf{a} = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$ and let $\mathbf{b} = \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$. Find the vector $\mathbf{a} - \mathbf{b}$ and its magnitude. Compute the angle between the two vectors.

15. Let $y = \ln x$. What is the 100th derivative of y?
    A. $-\frac{99!}{x^{100}}$
    B. $\frac{99!}{x^{100}}$
    C. $-\frac{100!}{x^{100}}$
    D. $\frac{100!}{x^{100}}$
    where $n! = n(n-1)\cdots 1$.