MSc Project on

Internal waves propagating over a current

supervised by Anna Geyer
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Background

This project is about internal waves in stratified water. Density stratification of water occurs due to temperature differences in the water, for instance in equatorial oceans, or due to differences in salinity, in regions such as river estuaries where salt and fresh water meet. This density stratification gives rise to internal waves. The situation is further complicated by the presence of background currents (such as tidal currents). The goal of this project is to understand the combined effect of density stratification and underlying currents on the propagation of internal waves by deriving and analyzing suitable mathematical model equations. A potential application of this model is the study of internal waves observed in a river estuary such as the Rotterdam Water Way, for which research has been carried out recently at the department of Civil Engineering at TU Delft [3].

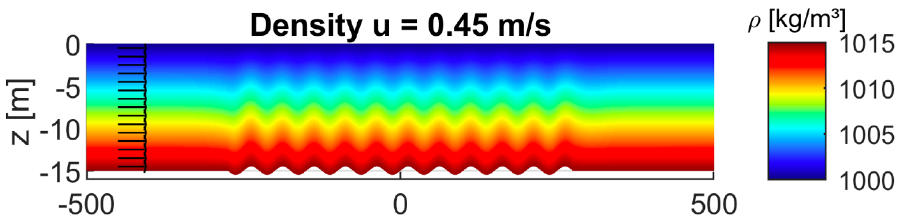

Figure 1: Internal waves in a linearly stratified fluid (MSc thesis of L. Deen, 2024).

Project description

Starting from the two-dimensional Euler equations for inviscible incompressible fluids, one can derive simplified model equations using asymptotic expansions of the unknowns in small physical parameters (such as the wave amplitude). A KdV type equation modeling the propagation of internal waves in a fluid with continuous density stratification was recently derived in [2]. This project is about extending that approach to derive a model equation which also takes into account the presence of an underlying prescribed current [1]. The goal is to describe the effect of this current on the internal waves and to make predictions about the propagation of internal waves in the Rotterdam Water Way.

Outline

- Derive KdV type model equation using depth dependent density and arbitrary background current.
- Reduce the complexity of the general model for a specific density profile and background current[1].
- Derive the speed and direction of the internal waves using the Burns condition.
- Find explicit periodic and solitary solutions of the equation using Jacobi elliptic functions.
- Check shallow-water small amplitude assumptions for Rotterdam Water Way to ensure applicability of the model.
- Compare results (shape of internal waves, propagation speeds, ...) with findings in [3].

Contact

If you are interested in this project do not hesitate to contact me: a.geyer@tudelft.nl

References

[1] R.M. Johnson, *A Modern Introduction to the Mathematical Theory of Water Waves*, Cambridge University Press, Cambridge, UK, 1997.

[2] A. Geyer and R. Quirchmayr, *Shallow water models for stratified equatorial flows*, Discrete Contin. Dyn. Syst. **39** (2019), 4533–4545.

[3] L. Deen, *Mixing of salt by internal waves*, Master thesis at TU Delft, 2024.

[1]In [3] the authors used a constant background current, which is a realistic model for tidal currents. However, it might also be interesting to include a background flow which is purely due to the density gradient in the water. Such a background flow could be approximated by a linear shear flow which changes direction somewhere in the fluid domain (i.e. it would contain a critical layer).