Calculus 1: Lecture 2

Inverse functions and implicit differentiation

Programme

- Inverse trigonometric functions
- Implicit differentiation
- Derivatives of inverse trigonometric functions

Sections 1.5 and 3.5

Responseware TurningPoint: the first steps

1. Go to: responseware.**eu**
2. Use the Session ID SessionID
3. Wait for a question to appear

Attendance

Enter your student number in TurningPoint

Inverse trigonometric functions

Inverse trigonometric functions

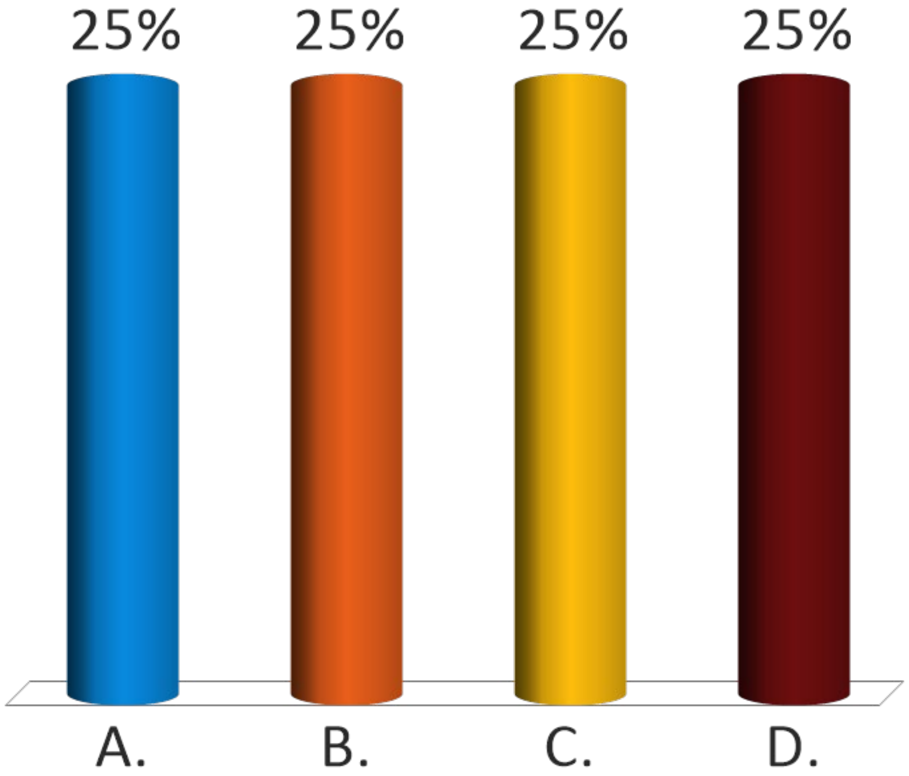
$\arcsin\left(-\frac{\sqrt{3}}{2}\right)$ is equal to:

A. $\frac{4\pi}{3}$

B. $\frac{\pi}{4}$

✓ **C.** $-\frac{\pi}{3}$

D. $-\frac{\pi}{2}$



Inverse trigonometric functions

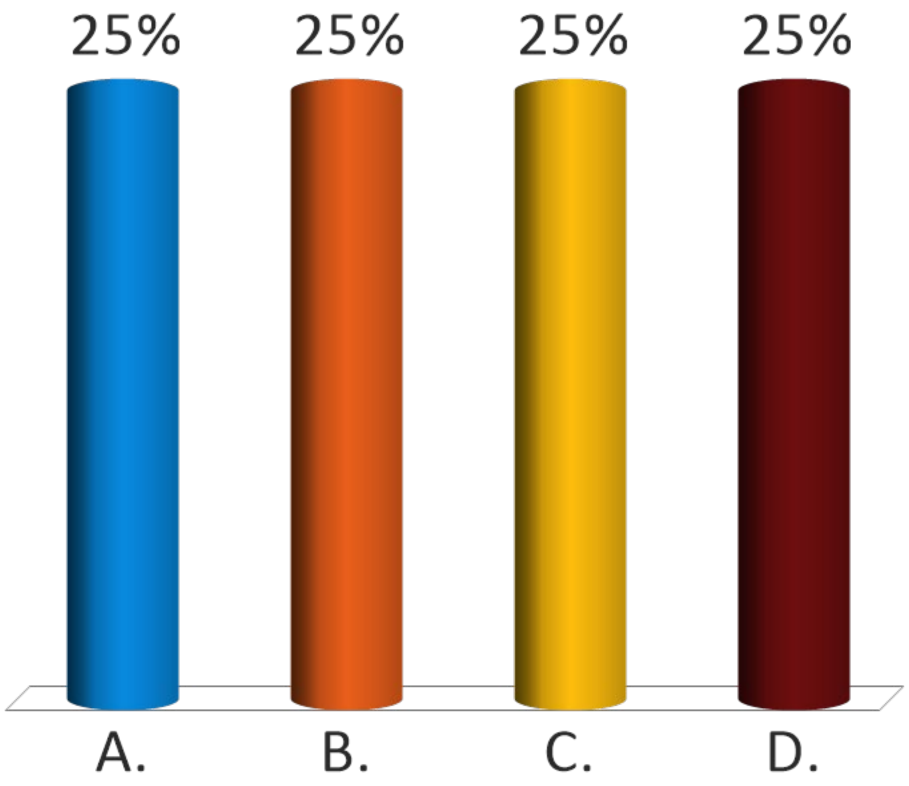
$\arctan \sqrt{3}$ is equal to:

✔ **A.** $\frac{\pi}{3}$

B. $-\frac{2\pi}{3}$

C. $\frac{\pi}{4}$

D. $-\frac{\pi}{3}$

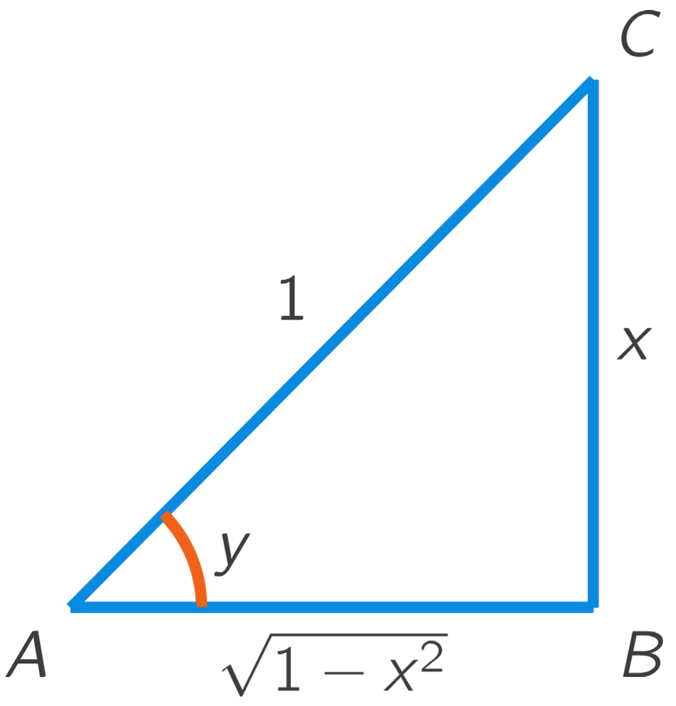


Simplifying expressions

Simplify $\cos(\arcsin x)$ to a formula without (inverse) trigonometric functions.

$$\arcsin(x) = y \Rightarrow \sin y = x = \frac{x}{1} \quad \text{with } -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

$$\Rightarrow \cos y = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$$



Conclusion:

$$\cos(\arcsin x) = \sqrt{1-x^2}$$

Practice

Inverse trigonometric functions

§1.5: 66, 70

You are able to simplify expressions containing inverse trigonometric functions.

Implicit differentiation

Implicit differentiation

We will see that $5x^2 - 7y^2 = C$
is the implicit general solution of the DE $\frac{dy}{dx} = \frac{5x}{7y}$.

$$5x^2 - 7y^2 = C$$

↓ Differentiate w.r.t. x

$$10x - 14y\frac{dy}{dx} = 0$$

↓ Solve for $\frac{dy}{dx}$

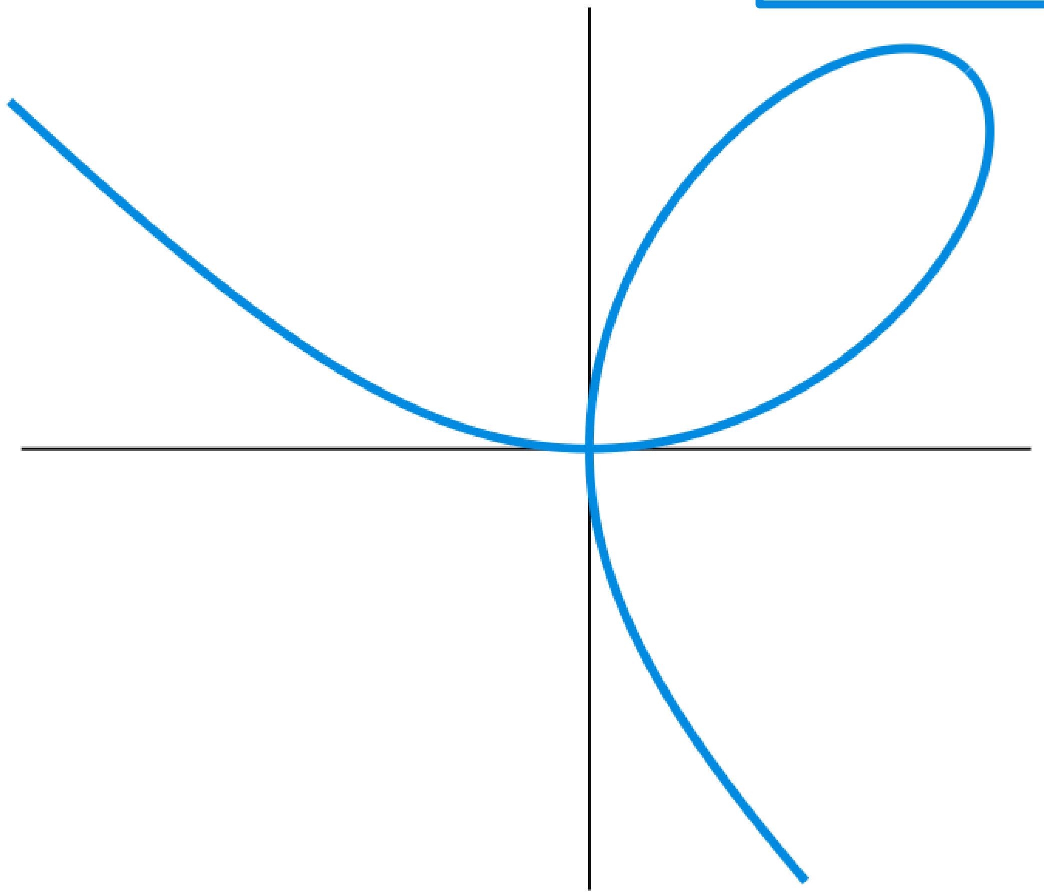
$$\frac{dy}{dx} = \frac{-10x}{-14y} = \frac{5x}{7y}$$

→

- Use the Chain Rule!
- Remember $y = y(x)$
- This is called implicit differentiation.

Folium of Descartes

$$x^3 + y^3 = 6xy$$



Differentiate w.r.t. x under the assumption that y is a function of x (on a part of the curve)

$$3x^2 + 3y^2\frac{dy}{dx} = 6y + 6x\frac{dy}{dx}$$

Formula for $\frac{dy}{dx}$?

Solution curves

Which differential equation has the given solution curves?

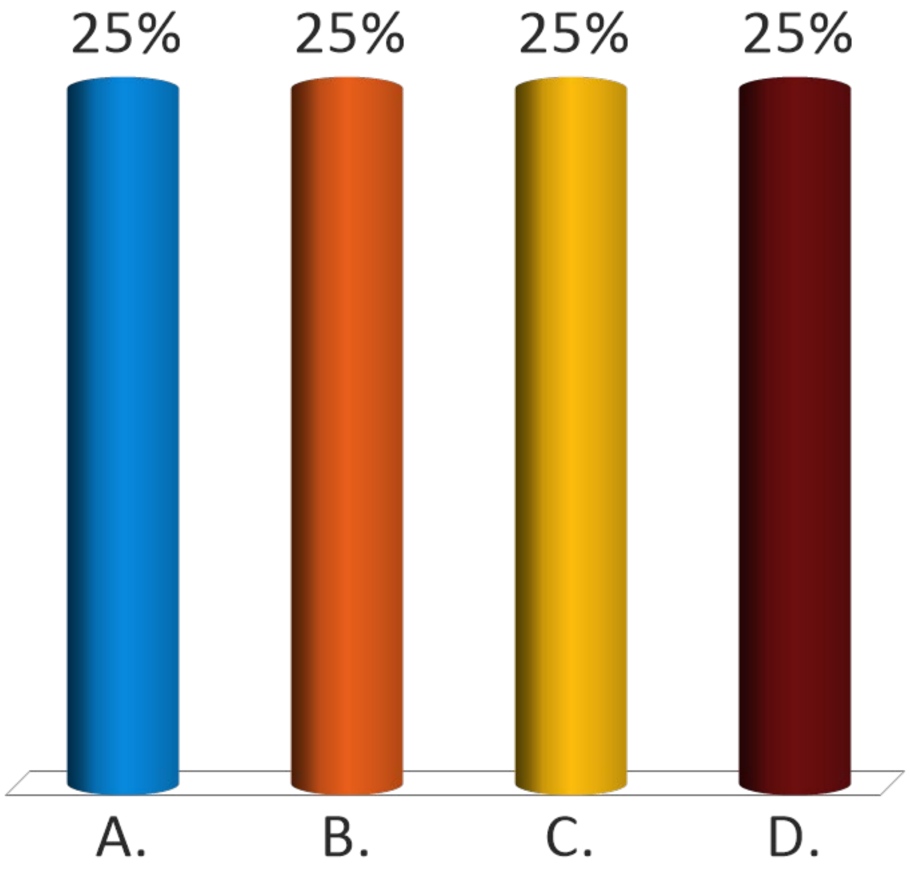
$$y^2 + 2e^y = x^2 + C$$

A. $\frac{dy}{dx} = \frac{y+e^y}{x}$

✓ **B.** $\frac{dy}{dx} = \frac{x}{y+e^y}$

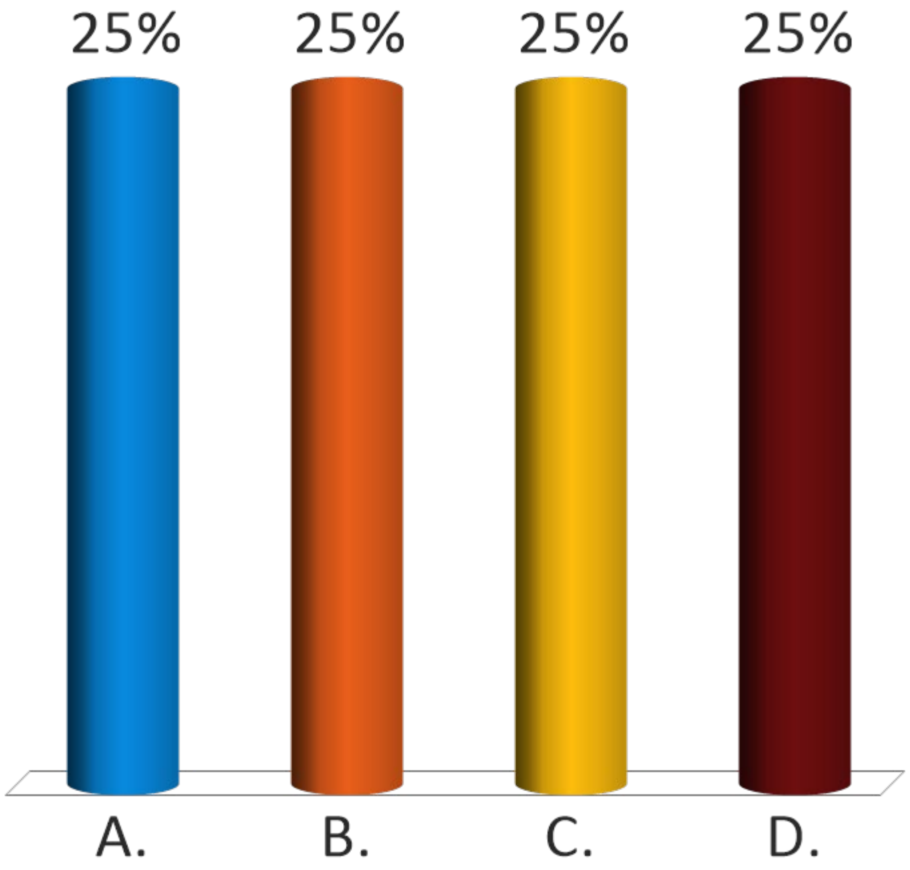
C. $\frac{dy}{dx} = \frac{x^2+C}{y^2+2e^y}$

D. $\frac{dy}{dx} = \frac{x}{\frac{1}{3}y^3+e^y}$



Derivative

If y is a function of x, what is the derivative of y given $\sin(y) = x$?



A. $y' = \frac{1}{\tan(y)}$

B. $y' = \frac{1}{\sin(y)}$

C. $y' = \frac{-1}{\cos(y)}$

✔**D.** $y' = \frac{1}{\cos(y)}$

Derivatives of trigonometric functions

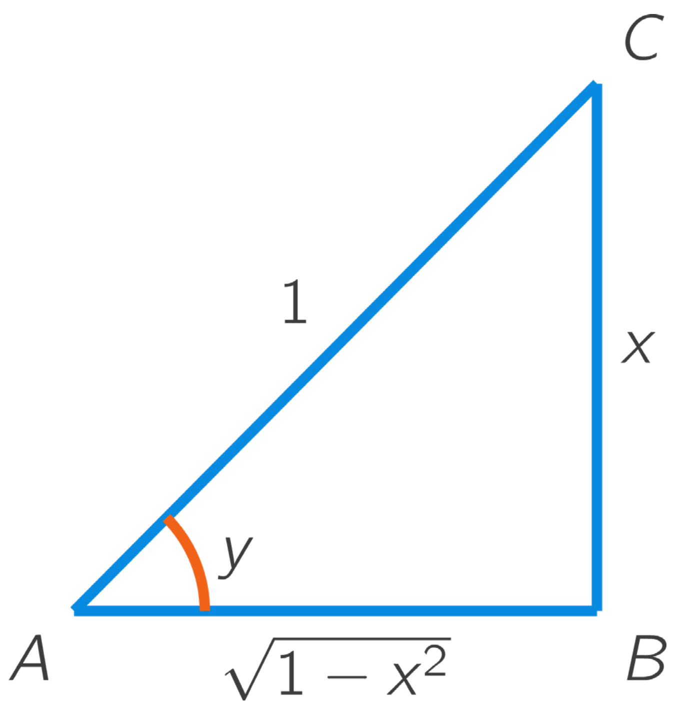
Derivatives of inverse trigonometric functions

$$y = \arcsin(x)$$

$$\sin(y) = x$$

$$\cos(y)\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}$$



Derivatives of inverse trigonometrix functions

$$\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}} \quad \Leftrightarrow \quad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$$

$$\frac{d}{dx}[\cos^{-1} x] = \frac{-1}{\sqrt{1-x^2}} \quad \Leftrightarrow \quad \int \frac{-1}{\sqrt{1-x^2}}\,dx = \cos^{-1} x + C$$

$$\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2} \quad \Leftrightarrow \quad \int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$$

Wrap-up and next lecture

Practice the topics of this lecture to be able to:

- use the inverse trigonometric functions;
- find the derivative using implicit differentiation;
- use implicit differentiation to find the derivative of the inverse of a function.

Next lecture:

The substitution rule

Section 5.5

Practice

Implicit differentiation

Derivatives of implicit trigonometric functions

§3.5: 5, 31, 51

You are now able to:

- find the derivative using implicit differentiation;
- use implicit differentiation to find the derivative of the inverse of a function.

See you next lecture!

TU Delft