Description of aero.m

Determination of the aerodynamic forces, moments and power by means of the blade element method; for known mean wind speed, induction factor etc.
Simplifications:

- *uniform flow (i.e. wind speed constant over rotor plane; no yawed flow, windshear or tower shadow)*
- *no wake rotation (i.e. no tangential induction factor)*
- *no blade tip loss factor*

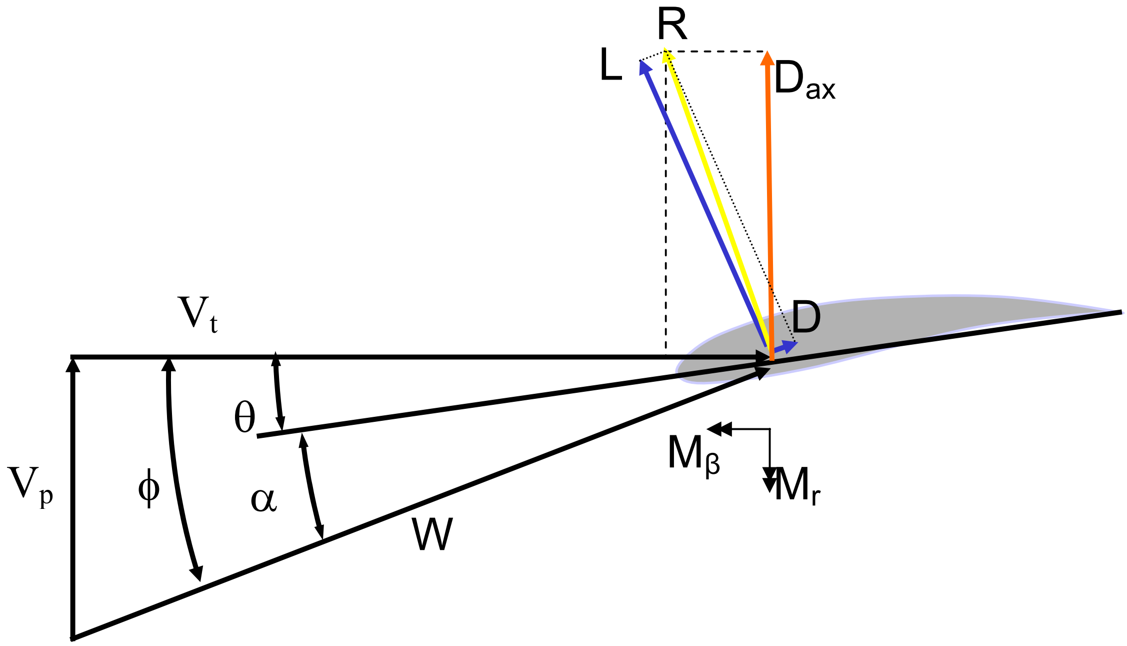


Figure: Aerofoil forces and velocities at a blade section

V_t: tangential velocity component
V_p: perpendicular velocity component
W: resultant velocity
α: angle of attack
θ: pitch angle
φ: angle of inflow

L: lift force (per definition perpendicular to W)
D: drag force (per definition parallel to W)
R: resultant force
D_{ax}: axial force
M_r: rotor torque
M_β: aerodynamic flap moment

For each blade section (with length dr and chord c) the lift- and drag force equal:

$$L = C_l \frac{1}{2} \rho W^2 c\, dr$$

$$D = C_d \frac{1}{2} \rho W^2 c\, dr$$

with ρ the airdensity
and C_l and C_d the lift and drag coefficient resp. of the particular aerofoil; they are both functions of the angle of attack α.

The aerodynamic forces depend on the velocities as seen by the rotating blade, so not only the the wind speed is taken into account but also the blade rotation and the velocities due to flexibility of the wind turbine. In our case (see description of *dynmod*) we have to consider the flapping motion of the rotor blade and the motion of the tower top.

$$V_t = \Omega r$$

with Ω the rotational speed of the wind turbine
and r the radial position of the blade section

$$V_p = V(1-a) - \dot{\beta} r - \dot{x}$$

with V the undisturbed wind velocity
a the induction factor
β the flapping angle of the blade
x the tower top displacement

For the total forces, moments on a rotor blade the forces over all blade sections have to be add. To obtain the forces and moments on the total rotor the forces, moments over all blades are added.

The (aerodynamic) power equals the product of rotational speed and rotor torque:

$$P = \Omega M_r$$

By definition the thrust and power coefficient equal:

$$C_{d_{ax}} = \frac{D_{ax}}{\frac{1}{2} \rho \pi R^2 V^2}$$

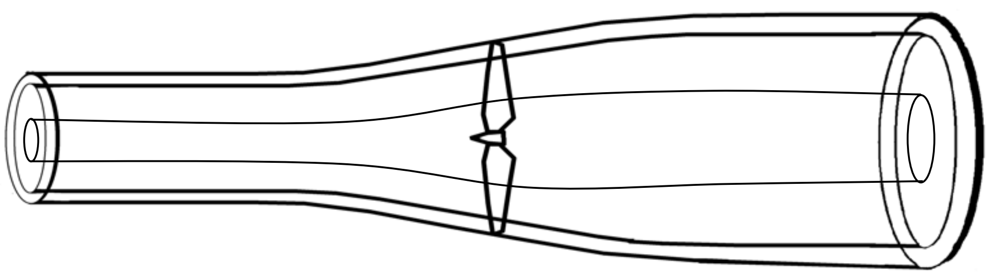
$$C_P = \frac{P}{\frac{1}{2} \rho \pi R^2 V^3}$$

Description of bem.m / fun_bem

Determintion of the aerodynamic forces, moments and power by means of the blade element - momentum method (BEM); for known wind speed, pitch angle, etc.
Simplifications:

- *uniform flow (i.e. wind speed constant over rotor plane; no yawed flow, windshear or tower shadow)*
- *no wake rotation (i.e. no tangential induction factor)*
- *no blade tip loss factor*
- *just one annular section (the total rotor plane)*

The main problem of rotor aerodynamics is that on one hand the aerodynamic forces depend on the induction factor and on the other hand the induction factor depend on the aerodynamic forces. In order to overcome this problem a combination of two methods are used: blade element method (see *aero.m*) and momentum theory (see *fun_bem.m*). According to momentum theory the thrust coefficient equals:
$C_{D_{ax}} = 4\,a\,(1-a)$ with a the induction factor
For values values of the induction factor larger than 0.5 (partial) flow reversal occur so momentum theory no longer can be applied; instead some empiral relation is used. Generally the rotor disc is divided into several angular sections (annuli; in the figure below 3 angular sections are shown), each with its own induction factor.



For simplicity in *fun_bem* the rotor disc is treated as just 1 angular section.

In text books BEM is usually explained by an iteration loop for the calculation of the induction factor:

- Choose an initial value for a
- Calculate C_{Dax} with the aid of blade element theory
- From C_{Dax} follows a new value for a, by application of momentum theory (see equation above)
- Continue until a reaches a constant value

In *bem.m* and *fun_bem.m* this iteration loop is not directly visible since use is made of the standard Matlab routine *fzero* in order to determine the induction factor, for which the thrust coefficient according to blade element theory equals the thrust coefficient according to momentum theory. The difference in thrust coefficient is calculated in *fun_bem*, so when *fzero* determines the value for the induction factor which makes the output of *fun_bem* equal to zero, the required induction factor is obtained.

Description of dynmod.m

Equations of motion of wind turbine (dynamic model): time derivatives of the states and outputs of the wind turbine as function of the states and inputs.

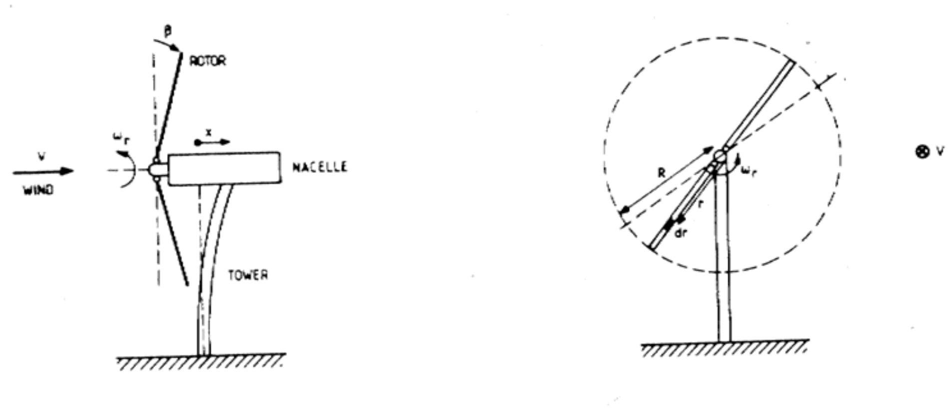


Fig.: Dynamic model wind turbine

The following degrees of freedom are considered (see also the figures).
Flap angle β
Tower top displacement x
Rotor angular velocity Ω (ω_r)
Torsion angle transmission ε

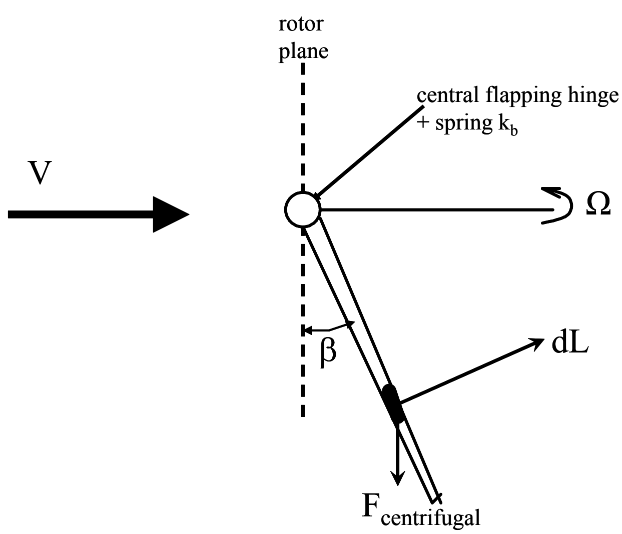


Fig.: Dynamic model rotor blade

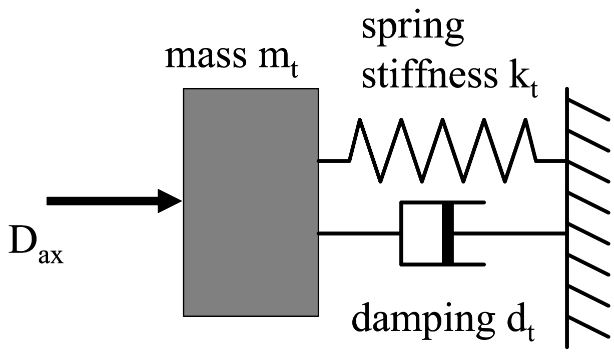


Fig.: Dynamic model tower

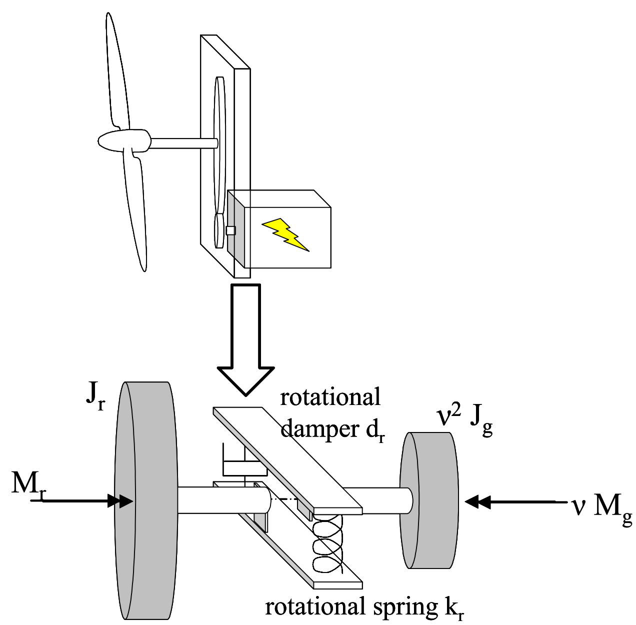


Fig.: Dynamic model transmission; the 2-shaft system is reduced to an equivalent 1-shaft system (low speed shaft).

Applying several simplifications (e.g. flap angle is small, so sin β≈β and cosβ≈1, gravity is neglected) the dynamics of all subsystem are reduced to 'mass-spring-damper' systems:

$$J_b\,\ddot{\beta} + (k_b + J_b\,\Omega^2)\beta = M_\beta$$

$$m_t\,\ddot{x} + d_t\dot{x} + k_t x = D_{ax}$$

$$J_r\,\dot{\Omega} + d_r\dot{\varepsilon} + k_r\varepsilon = M_r$$

$$J_{tot}\ddot{\varepsilon} + d_r\,\dot{\varepsilon} + k_r\,\varepsilon = \frac{J_{tot}}{J_r}M_r + \frac{J_{tot}}{\nu^2 J_g}\nu M_g \text{ with } J_{tot} = \frac{\nu^2 J_r\,J_g}{J_r + \nu^2 J_g}$$

The equations for the transmission are obtained via reducing the 2-shaft system to an equivalent 1-shaft system (low speed shaft); in doing so the values for the generator angular velocity, generator torque and generator inertia should be properly corrected (depending on the transmission ratio ν).

In *dynmod* the above 2^{nd} order differential equations are rewritten as two 1^{st} order differential equations. In total 7 differential equations are obtained, defining the 7 the states of the wind turbine:

Flap angle of rotor blade β
Flap angular velocity of rotor blade $\dot{\beta}$
Tower top displacement x
Tower top speed $\dot{x}$
Rotor angular velocity Ω
Torsion angle transmission ε
Torsion angular velocity transmission $\dot{\varepsilon}$

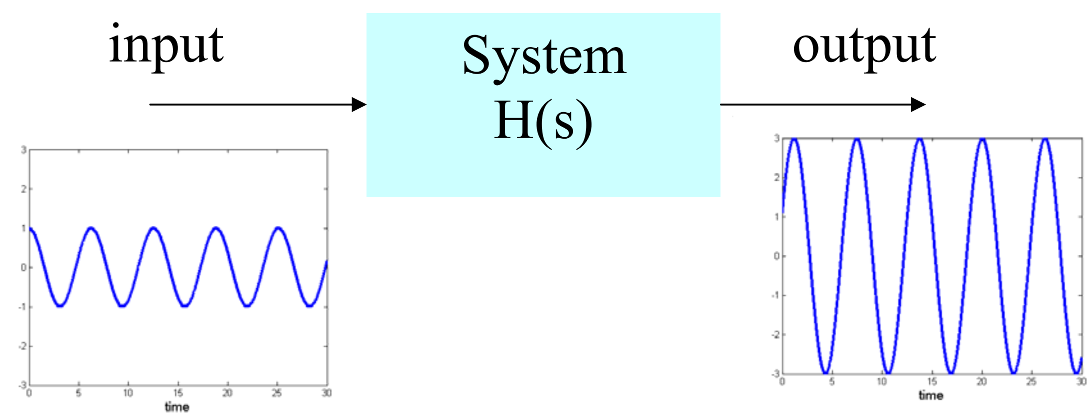
The dynamics of the generator and power electronics is much faster than the dynamics of the mechanical part of the wind turbine. This implies that it is justified to model the generator by means of a static ‘torque-rotational speed’ relation; see *gener.m*.

Description of transfer.m

Determination of the transfer function of the wind turbine. The transfer function H(s) relates all wind turbine inputs to all wind turbine outputs

$$H(s) = \frac{NUM(s)}{DEN(s)}$$

inputs of wind turbine:
1) blade pitch angle theta [degrees]
2) undisturbed wind speed V [m/s]
outputs of wind turbine:
1) axial force Dax [N]
2) aerodynamic flap moment Mbeta [Nm]
3) aerodynamic rotor torque Mr [Nm]
4) generator power Pg [W]
5) blade pitch angle theta [degrees]
6) undisturbed wind speed V [m/s]



The response of a linear system to a harmonic input is again harmonic (with the same frequency). The relation between the output signal and the input signal, for each frequency, is given by H, which is in general a complex number. The absolute value of H is the ratio between the amplitudes of the output and the input; the phase of H is the phase difference between the output and the input. E.g. the transfer function of a second order systeem (eigenfrequency ω_n and critical damping ς) is:

$$H(\omega) = \frac{1}{\omega_n^2 - \omega^2 + j\,2\,\varsigma\,\omega\,\omega_n}$$

Note: generally the transfer function is expressed as function of the Laplace parameter s=jω rather than ω.

The transfer function of the wind turbine is determined by linearising the equations of motion as given in *dynmod.m*. This linearisation could be done analytically by use of Taylor expansion of the equations. In *transfer.m* it is done numerically by disturbing the inputs and states by a small amount and considering the resulting change in the outputs. The obtained linearised equations can be put in standard form (so-called state space format):

$$\dot{x} = A\,x + B\,u$$
$$y = C\,x + D\,u$$

with x the states of the wind turbine:

1) Flap angle of rotor blade β
2) Flap angular velocity of rotor blade $\dot{\beta}$
3) Tower top displacement x
4) Tower top speed $\dot{x}$
5) Rotor angular velocity Ω
6) Torsion angle transmission ε
7) Torsion angular velocity transmission $\dot{\varepsilon}$

and y, u the outputs and inputs of the wind turbine (see above)

Finally the standard Matlab commands *ss.m* and *tf.m* are used to obtain the transfer function.

Note: numerical linearisation is outside the scope of the wind energy course, so it is not necessary that the listing of *transfer.m* is totally understood.

Description of gener.m

Torque-rpm characteristics of a synchronous generator with AC/DC/AC converter.

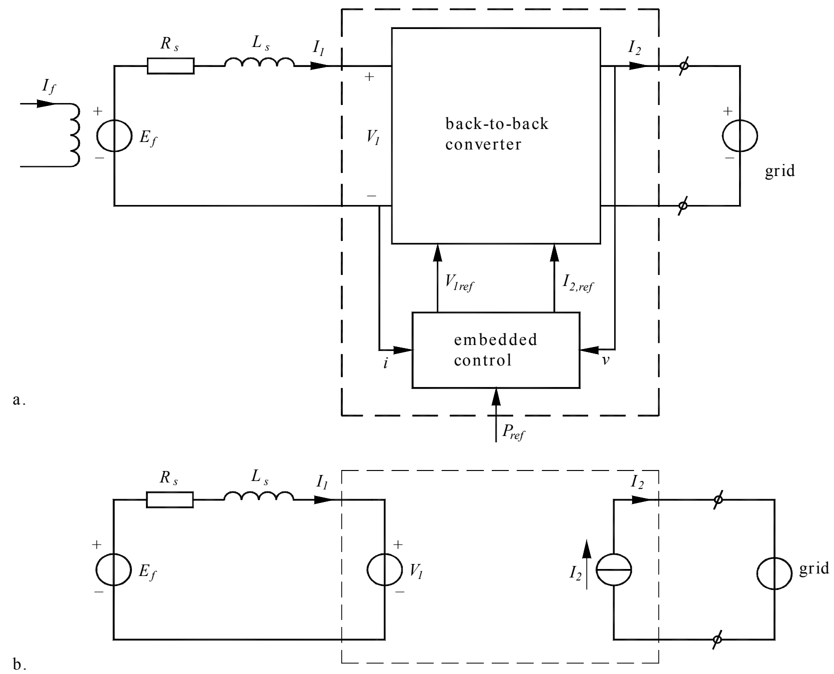


Fig.: Per phase equivalent circuit diagram (bottom) of variable speed conversion system with synchronous generator and back-to-back (AC/DC/AC) converter (top).

In the figure the equivalent circuit is shown of the generator including the power electronics. The values for the resistance and reactances are usually expressed in per unit (pu); these are the dimensionless values based on the nominal condition:

$R_{s_{pu}} = \dfrac{R_s}{3V_{1_n}^{\;2}/P_n}$ stator resistance

$X_{s_{pu}} = \dfrac{X_s}{3V_{1_n}^{\;2}/P_n}$ stator reactance

$X_{m_{pu}} = \dfrac{X_m}{3V_{1_n}^{\;2}/P_n}$ mutual rotor-stator reactance

The inductions equal:

$L_s = \dfrac{X_s}{\omega}$ stator induction

$M = \dfrac{X_m}{\omega}$ stator induction

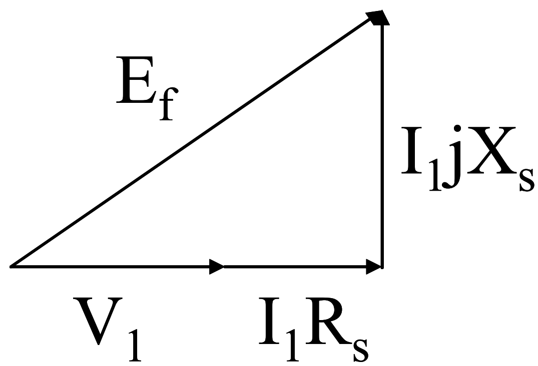
with

$\omega = p\,\omega_g$ the electrical angular velocity, p the number of pole pairs and ω_g the generator shaft angular velocity.

From M and the field current I_f the field induced voltage can be determined:

$$E_f = \frac{M}{\sqrt{2}} \omega I_f$$

A main assumption we made is that we assume no phase difference between V_1 and I_1, i.e. cos φ=1 (realised by means of the power electronics); see also the phasor diagram below.



The terminal voltage and stator current are now described by 2 equations:

$E_f^{\,2} = (V_1 + I_1 R_s)^2 + (\omega L_s I_1)^2$ from the phasor diagram, and

$P_g = 3 V_1 I_1$ the electrical power

Elimination of V_1 leads to a quadratic equation in I_1^2 which can thus easily be solved (see listing *gener*).

Note: in order that the equations have a solution the field current should have some minimum value; this is checked in the routine *gener.m*.

The mechanical power of the generator (equal to the aerodynamic rotor power) is:

$$P_{mech} = M_g \, \omega_g$$

The electrical power equals the mechanical power minus the losses dissipated in the resistor:

$$P_g = P_{mech} - 3 I_1^2 R_s$$

The efficiency of the generator can now be expressed as:

$$\eta_g = \frac{P_g}{P_{mech}}$$

The efficiency η_c of the power electronics can not be deduced from the equivalent circuit; instead some value should be assumed. The overall efficiency of the electrical system equals the product η_g and η_c.

Note on P_{ref} (one of the inputs of *gener.m*)

The relation between torque and rotor speed of a synchronous generator depends on the specific configuration of the generator. A generator in combination with an AC/DC/AC converter, however, can practically be given any desired torque – rotor speed relation. One of the inputs of *gener.m* is therefore a reference (set point) generator power; it is assumed that the power electronics are able to adjust the terminal voltage and stator current in such a way that the reference power is realised (i.e.: $P_g = 3 V_1 I_1 = P_{ref}$).

In partial load conditions maximum energy is extracted by the wind turbine in case the tip speed ratio equals the optimal tip speed ratio (for which the power coefficient is maximum).

This implies that the rotor speed and thus also the generator speed should be proportional with the wind speed. Since the power coefficient is constant (and equal to the maximum value) the aerodynamic power is proportional to the 3^{rd} power of the wind speed (and rotor speed). To conclude, for maximum energy extraction at partial load, the reference power should be taken proportional to the 3^{rd} power of generator speed. This is already taken into account in the function call of *gener.m* in *dynmod.m*.

Description of equi.m / fun_equi.m / fun_power.m

Determination of the operating point of the wind turbine for known wind speed; this is the steady state after equilibrium between all acting forces on the wind turbine.
Partial load conditions ($V<=V_n$): blade pitch angle $\theta=\theta_n$ (subscript n stands for nominal conditions). Note: it is not assumed that the wind turbine automatically operates at optimal tip speed ratio.
Full load conditions ($V>V_n$): rotor rotational speed $\Omega=\Omega_n$; blade pitch θ such that power equals nominal power.

During steady wind conditions any transient response will be damped out so the wind turbine will go to some equilibrium condition (specified by the rotor speed, blade flap angle, etc.), also called operating point. The equilibrium condition is determined by equilibrium of all forces and moments acting on the wind turbine; in general this will depend on the mean wind speed. In partial load there will be an equilibrium between the aerodynamic rotor torque and generator torque. So it would be possible to calculate the rotor speed at equilibrium via an iteration loop during which the rotor speed is varied until the aerodynamic rotor torque equals the generator torque. In *equi.m* this is again done using the standard routine *fzero* in combination with *fun_equi.m*; in the latter the difference between the aerodynamic rotor torque and the generator torque is calculated for given rotor speed.
At full load conditions the pitch angle is changed such that the power equals nominal power. It is assumed that the pitch control is such that the blade is rotated towards the wind (i.e. towards zero-lift conditions). This time *fzero* is used in combination with *fun_power.m*; the latter calculates the difference between the aerodynamic power, for given pitch angle, and nominal power.

The equilibrium states of the degrees of freedom (i.e. blade flap angle, tower top displacement, torsion angle transmission) are given by the equations of motion (see dynmod.m) in which all terms which are time dependent (i.e. time derivatives) are omitted.