Opportunities from Quantum Computing for Net-Zero Power System Optimisation

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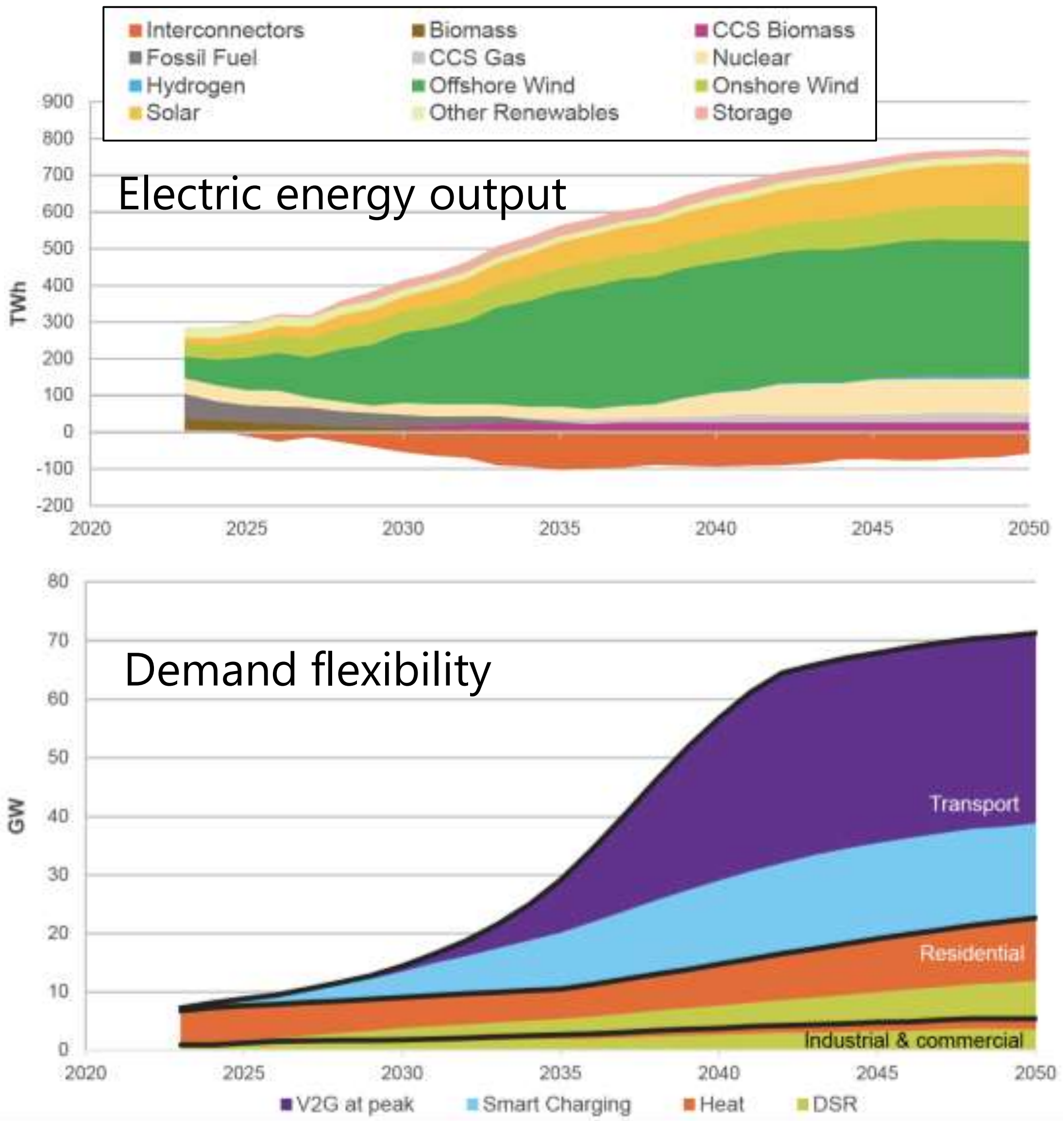
Overview

- Power system optimisation across scales
- Net-zero and new sources of complexity
- Opportunities from quantum computing
- Case study: Quantum annealing for combinatorial optimal power flow
- Conclusions and Future Directions

Transition to a Net-Zero Power System

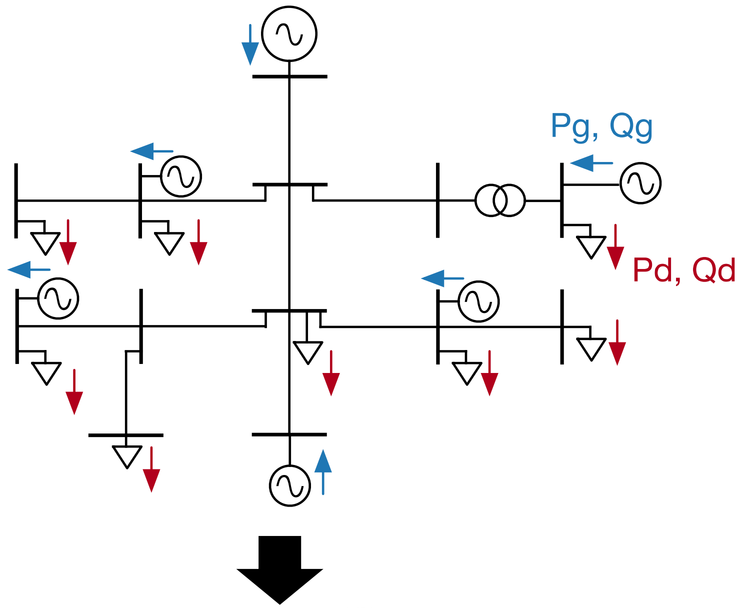
- Major power system trends:
  - Renewable generation
  - Electrification of transport & heating
  - Consumer-level ICT infrastructure
- Unresolved challenges for planning & operation:
  - From thousands to millions of market participants
  - Flexibility in local distribution networks
  - More variable and uncertain supply



UK Future Energy Scenarios (NESO, 2024)

Optimal Power Flow (OPF) Problems

- OPF problem:
  - Schedule generation to meet demand within network constraints at lowest cost
  - Underlies economic dispatch, unit commitment and expansion planning
- Challenges:
  - AC power flows are nonlinear
  - Fast network dynamics
  - Imbalances can lead to blackouts
  - High reliability target (e.g. 99.97%)



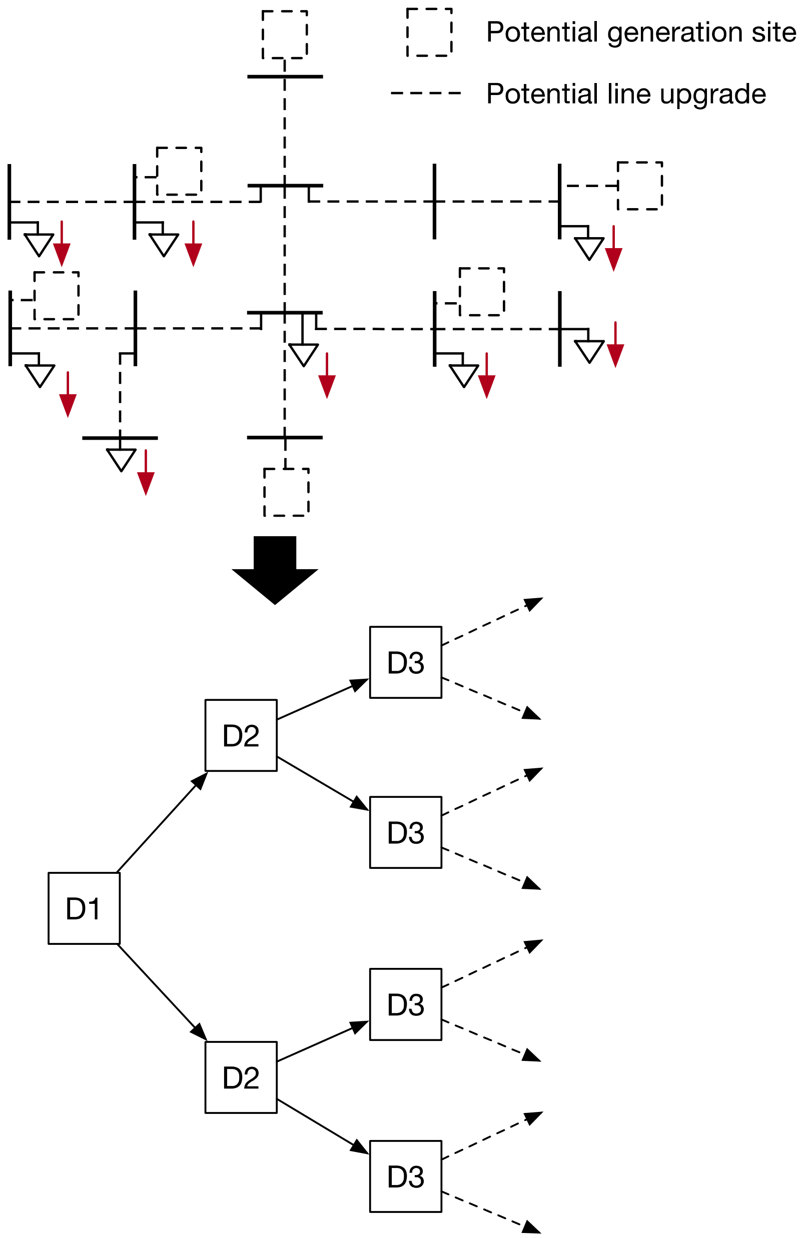
$$\min_{P_{G_i}} \boldsymbol{c}^T \boldsymbol{P}_G$$

$$\begin{aligned}
\text{subject to:} \quad & \text{AC flow} && \mathbf{S}_G - \mathbf{S}_L = diag(\overline{\mathbf{V}})\overline{\mathbf{Y}}^*_{\text{bus}}\overline{\mathbf{V}}^* \\
& \text{Line Current} && |\overline{\mathbf{Y}}_{\text{line},i\to j}\overline{\mathbf{V}}| \leq \mathbf{I}_{line,max} \\
& && |\overline{\mathbf{Y}}_{\text{line},j\to i}\overline{\mathbf{V}}| \leq \mathbf{I}_{line,max} \\
& \textit{or}\ \text{Apparent Flow} && |\overline{V}_i \overline{\mathbf{Y}}^*_{\text{line},i\to j,\text{row-i}}\overline{\mathbf{V}}^*| \leq S_{i\to j,max} \\
& && |\overline{V}_j \overline{\mathbf{Y}}^*_{\text{line},j\to i,\text{row-j}}\overline{\mathbf{V}}^*| \leq S_{j\to i,max} \\
& \text{Gen. Active Power} && \mathbf{0} \leq \mathbf{P}_G \leq \mathbf{P}_{G,max} \\
& \text{Gen. Reactive Power} && -\mathbf{Q}_{G,max} \leq \mathbf{Q}_G \leq \mathbf{Q}_{G,max} \\
& \text{Voltage Magnituge} && \mathbf{V}_{min} \leq \mathbf{V} \leq \mathbf{V}_{max} \\
& \text{Voltage Angle} && \boldsymbol{\delta}_{min} \leq \boldsymbol{\delta} \leq \boldsymbol{\delta}_{max}
\end{aligned}$$

(Chatzivasileiadis, DTU, 2018)

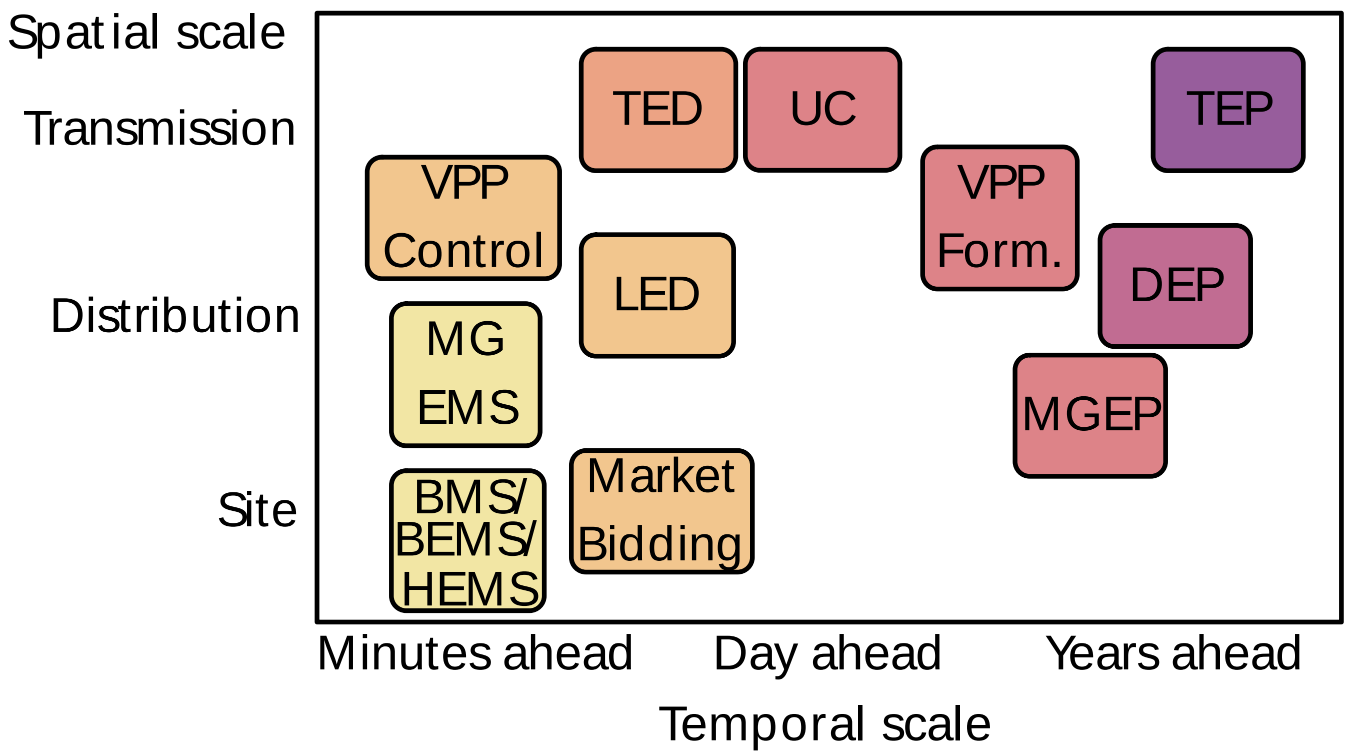
Combinatorial Power System Optimisation Problems

- Discrete variables introduced into OPF problems by:
  - Unit commitment decisions (e.g. startup costs, min on/off times)
  - Discrete sources of flexibility (e.g. on/off EV charging)
  - Network upgrade decisions
  - Resource placement & sizing
- Decision space grows exponentially



Power System Optimisation Problems Across Scales

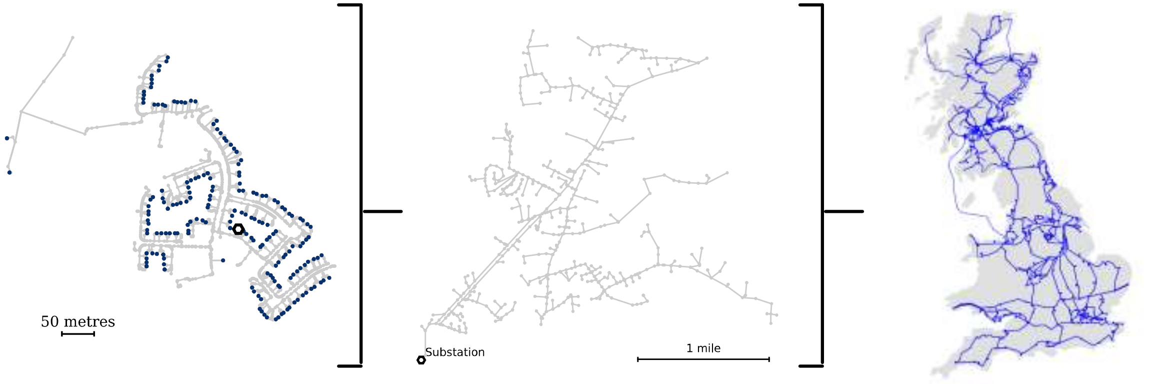
- Expansion planning:
  - Transmission (TEP)
  - Distribution (DEP)
  - Microgrid (MGEP)
- Economic dispatch:
  - Transmission (TED)
  - Local (LED)
- Unit Commitment (UC)
- VPP formation and control
- Energy management systems:
  - Microgrid (MGEMS)
  - Building (BEMS)
  - Home (HEMS)
- Battery Management System (BMS)



Net-Zero Computational Challenges

New computational challenges:

- Grid-edge flexibility
  → Millions of controllable devices
  → Transmission–distribution interaction
- Energy storage
  → Time-coupling
  → Nonlinear battery degradation & efficiency
- Uncertainty
  → Chance constrained optimisation
- Net-Zero market reform
  → Strategic multi-level problems
  → Local–national market interaction



Residential Distribution

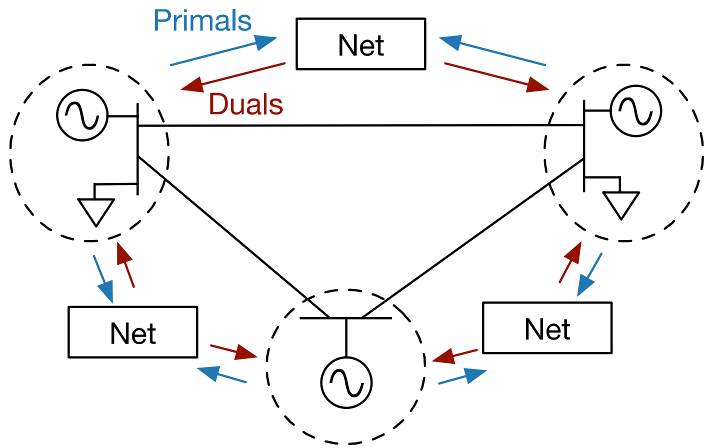
Primary Distribution

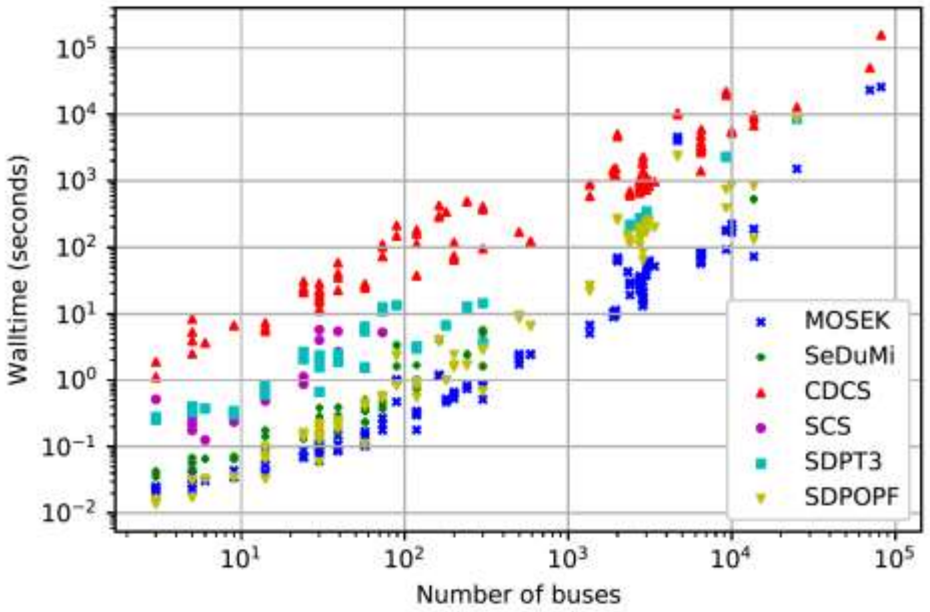
National Transmission

(Timera, 2022)

Limits to Scaling Up Classical Optimisation

- Linear approximations:
  - Transmission: losses and reactive power flows can be neglected
  - Distribution is more nonlinear and linearisations depend on operating-point
- Convex relaxations:
  - SOCP and SDP (conditionally) exact
  - Scalability still an issue
    e.g. interior point methods are $O(n^3)$
- Distributed Optimisation:
  - Solve via iterations of parallel sub-problems
  - Trade-off between accuracy and convergence time

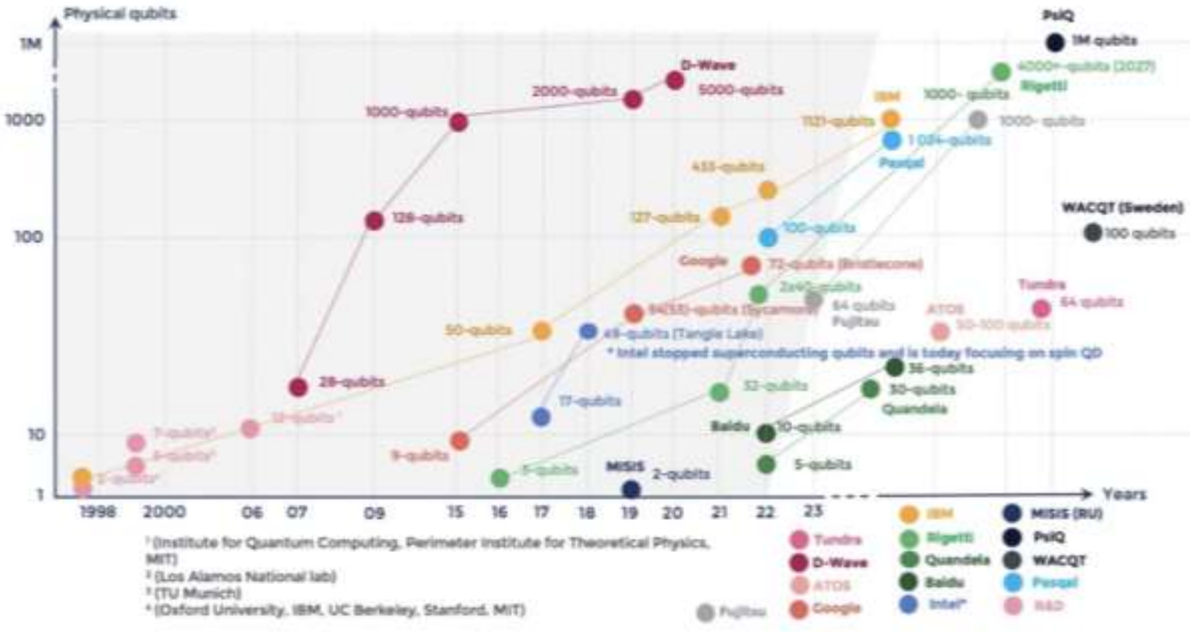




Number of buses vs. solution time for SDP OPF with distributed optimisation. (Fan et al., 2018)

Quantum Computing

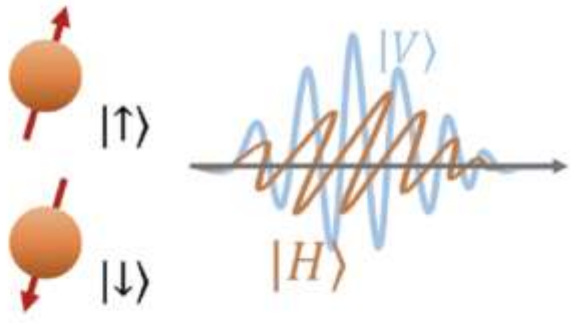
- Potential speed-ups (polynomial or exponential) for many algorithms
- Quantum supremacy in 2019 (Google)
- Important caveats:
  - Still too small for most industrial applications
  - Speed-ups often depend on noiseless computation (or error correction)
  - Noisy Intermediate-Scale Quantum (NISQ) era expected to last 10+ years
  - Hidden overheads (e.g. loading classical data due to low clock rates)



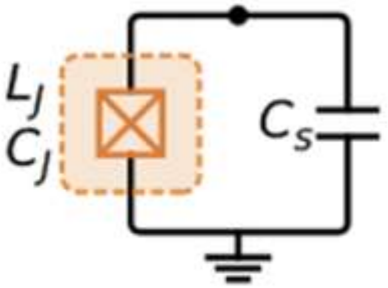
(Yole Development, 2023)

Qubit (Quantum Bit)

- Qubit state: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
  - Probability amplitudes: $\alpha, \beta \in \mathbb{C}$ and $\alpha^2 + \beta^2 = 1$,
  - Measurement: $\mathbb{P}[0] = \alpha^2$ and $\mathbb{P}[1] = \beta^2$
- 2-qubits: superposition of 4 states $|00\rangle, |01\rangle, |10\rangle, |11\rangle$
- n-qubits: superposition of 2^n states
- Implementation:
  - Controllable 2-level quantum systems
  - Examples:
    - Superconducting circuits (phase, charge or flux)
    - Trapped ions
    - Neutral atoms



Electrons, Nuclei, Photons

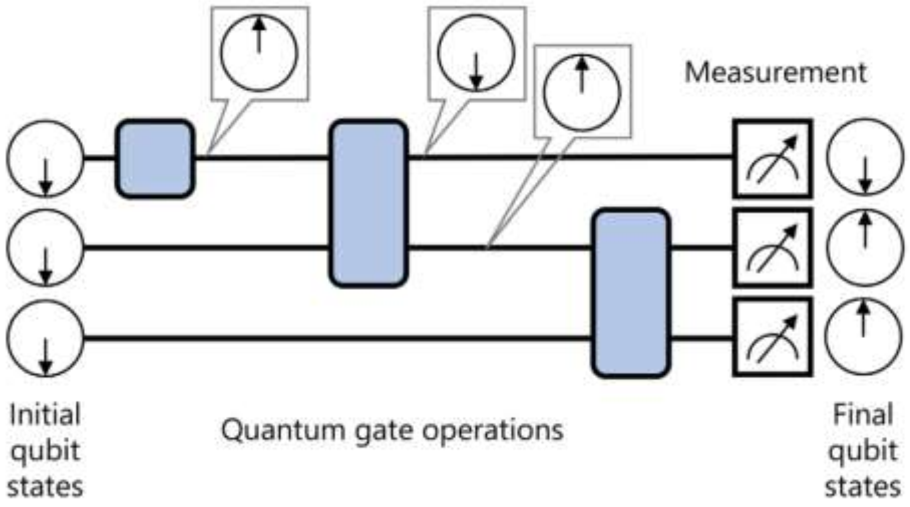


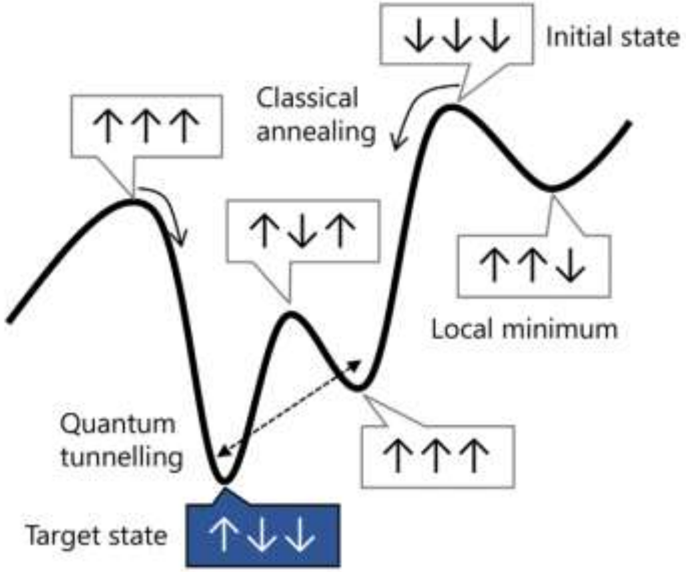
Superconducting circuits

(Chae et al., *Nano Convergence*, 2018)

Models of Quantum Computing

- Gate-based:
  - Gates manipulate and entangle qubit states
  - Theoretically can implement any quantum algorithm
  - Circuit depth limited by noise
- Quantum annealing (QA):
  - Uses the quantum adiabatic theorem for analogue computation
  - Relevant for a limited set of calculations (including quadratic unconstrained binary optimisation (QUBO) problems)
  - Solutions may be approximate due to noise and limited annealing time

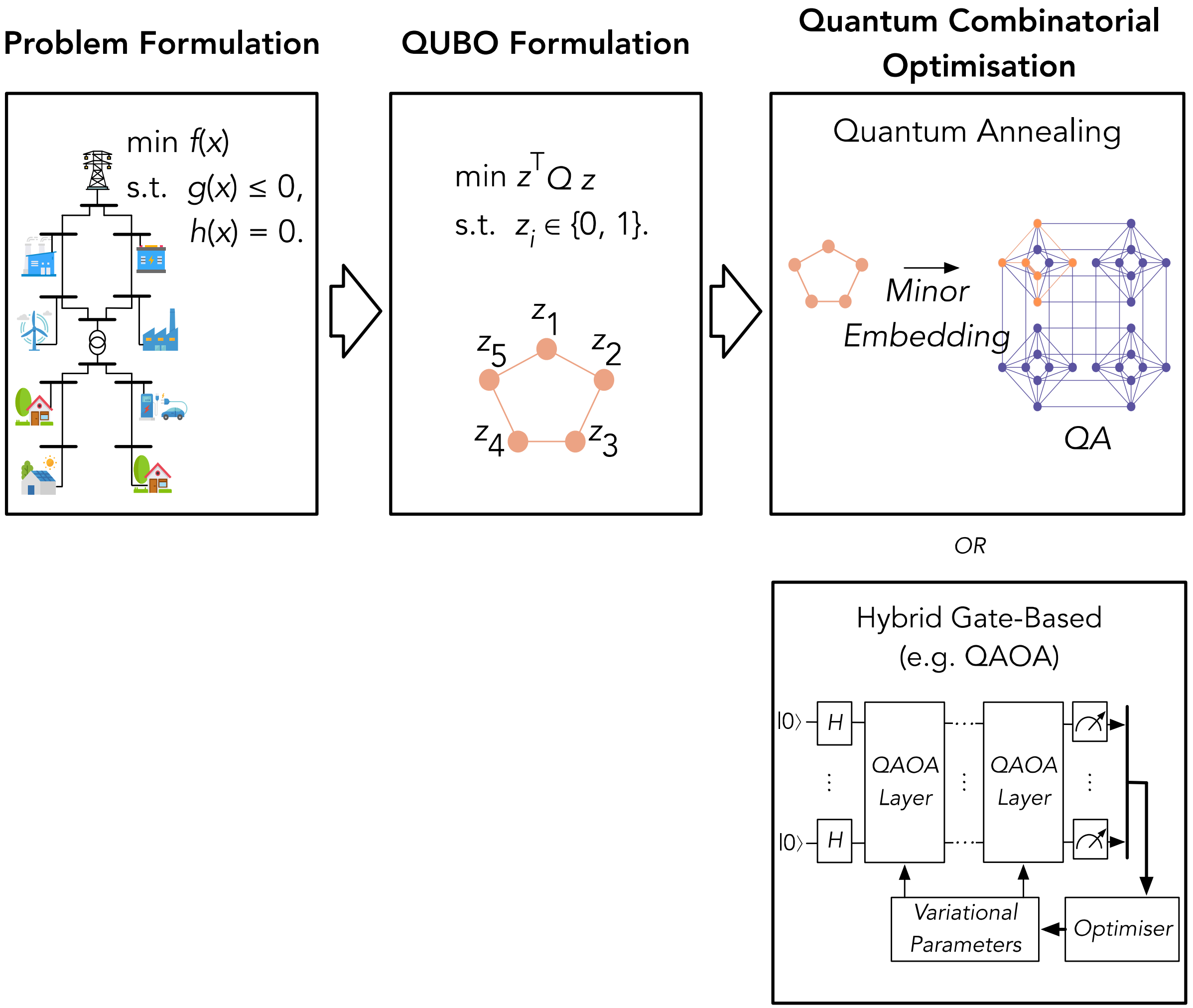




(Chia et al., *Adv. Quantum Tech.*, 2024)

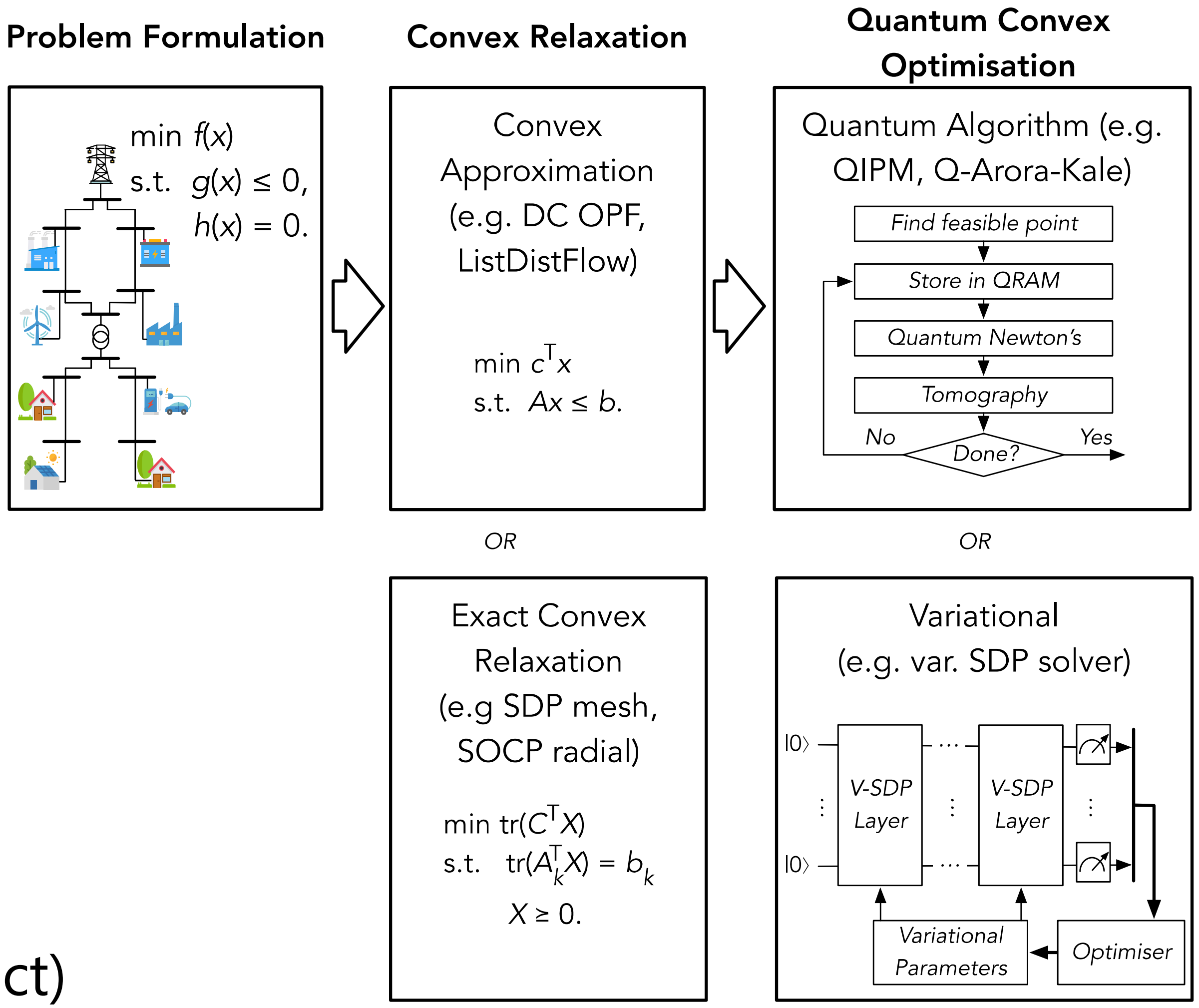
Combinatorial Quantum Optimisation Opportunities

- NISQ approaches for QUBO problems:
  - Quantum annealing (QA)
  - Quantum Approximate Optimisation Algorithm (QAOA)
- Many problems can be reformulated as QUBOs using auxiliary variables
- Speed-ups with noise are not proven
- Power applications:
  - Unit commitment
  - Network expansion planning
  - Coalition formation



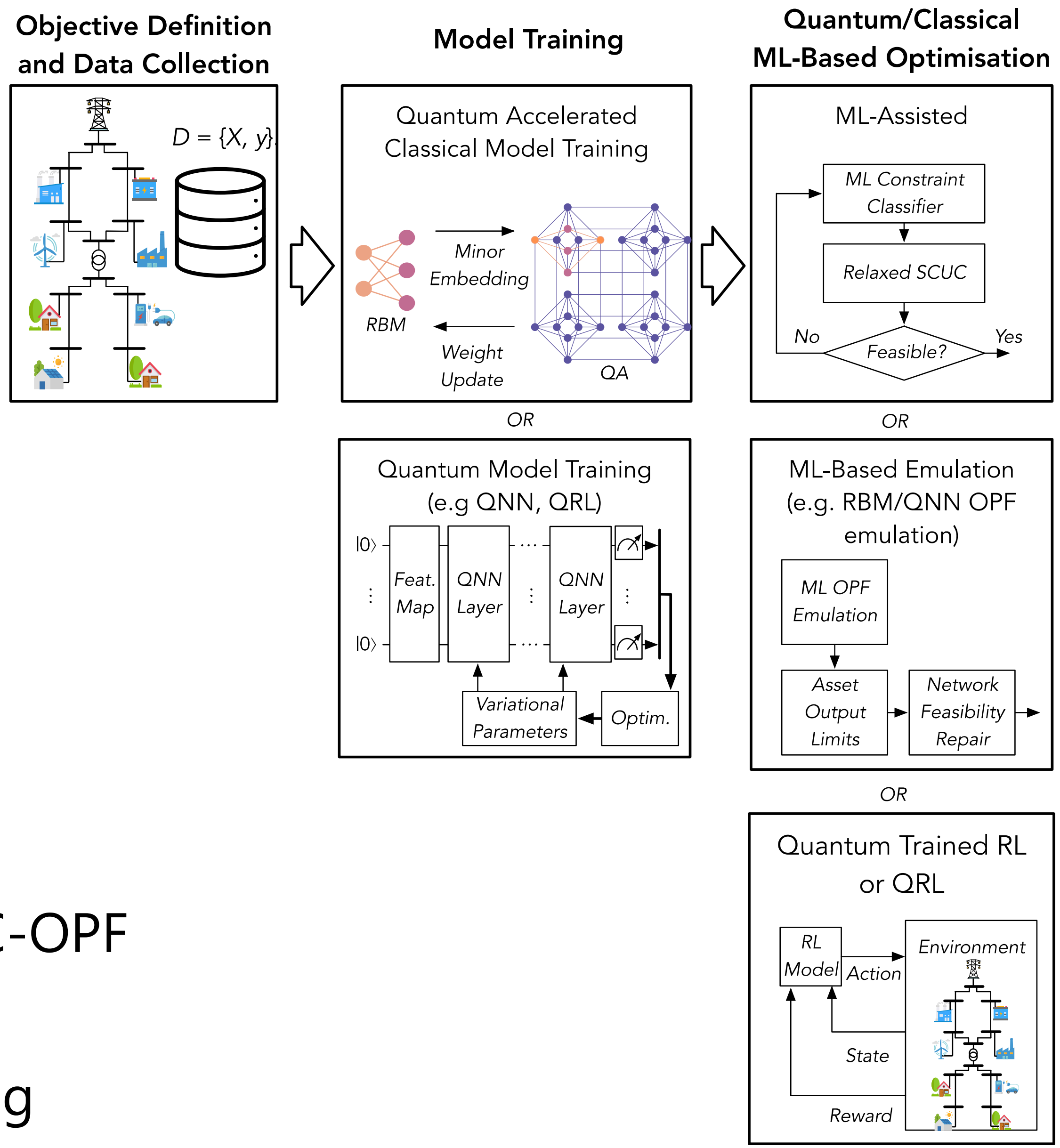
Convex Quantum Optimisation Opportunities

- Proven SDP polynomial speedups:
  - Quantum Interior Point Method
  - Quantum Arora-Kale
- Caveats:
  - Require QRAM (still theoretical)
  - Require large error corrected devices
- NISQ algorithm: Variational SDP solver
- Power applications:
  - Linear OPF (e.g. DC, LinDistFlow)
  - SOCP radial relaxation (sometimes exact)
  - SDP mesh relaxation (sometimes exact)



Machine Learning-Based Quantum Optimisation Opportunities

- Training classical models:
  - Decision trees (DTs)
  - Support vector machines (SVMs)
  - Boltzmann machines (BMs)
- Quantum machine learning models:
  - Variational quantum neural network
  - Quantum reinforcement learning
- Caveats:
  - Training hard without good initialisation
  - Loading input data is a bottleneck
- Power applications:
  - Redundant constraint classification for SC-OPF
  - Neural network OPF emulation
  - Reinforcement learning for market bidding



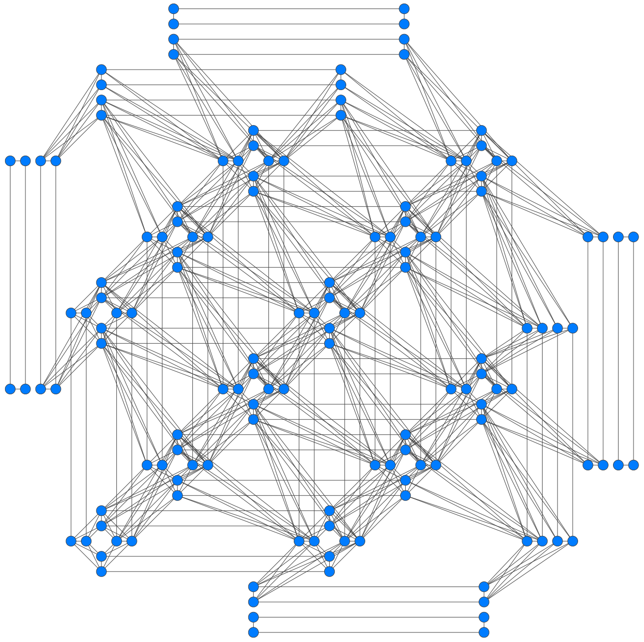
Case Study – Quantum Annealing for Combinatorial Optimal Power Flow

- D-wave LEAP cloud platform provides access to 5,760 qubit annealing processors and hybrid quantum-classical solvers
- Definitive advantage over classical computing unclear in the noisy case
- However, improving in terms of number of qubits and noise level
- QUBO problems can also be solved using the gate-based Quantum Approximate Optimization Algorithm

www.dwavesys.com

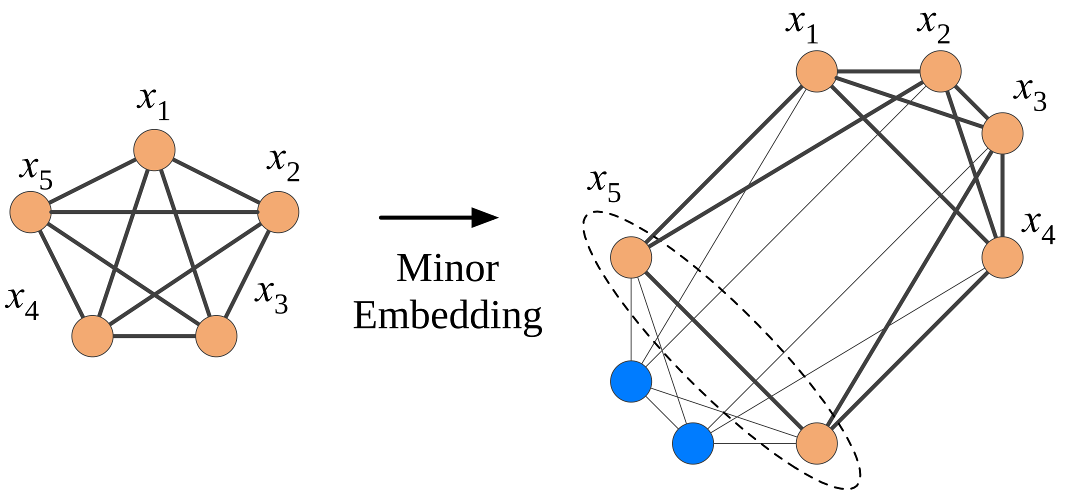
Quantum Annealing for Solving QUBO Problems

1. Formulate QUBO problem
2. Find a 'minor embedding' compatible with the processor's qubit lattice (using chains of coupled qubits)
3. Solution is found based on adiabatic quantum computing (system evolves to problem Hamiltonian ground state)
4. Solution may be approximate due to noise and limited annealing time



3x3 cell (144 qubits) Pegasus topology.

D-wave's Advantage processor is 16x16 (5,760 qubits).



Minor embedding for $\min(x_1 + x_2 + x_3 + x_4 + x_5)^2$

Problem Definition

- Objective: Multi-period cost minimization
  - Net energy import cost
  - EV charging utility
  - Generation investment costs
  - Network reinforcement costs
- Constraints:
  - Voltage limits (linear multiphase power flow model from [1])
  - Electric vehicle energy limits
- Decision variables
  - Electric vehicle on/off charging
  - Generation sizing/placement
  - Network upgrade plan selection

$$
\begin{aligned}
\min \quad & \sum_{i \in \mathcal{V}} \frac{1}{|\Psi_i^{ev}|} \sum_{\psi \in \Psi_i^{ev}} \sum_{t \in \mathcal{T}_i} (A_{\psi t}^{\perp} + \sum_{k \in \mathcal{U}} \Delta_{\psi k t}^{A\perp} x_k^u) \\
& \cdot \tau \lambda_{0t} \eta_i^{ev} \rho_i^{ev} x_{it}^{ev} - \sum_{j \in \mathcal{G}} \frac{1}{|\Psi_j^g|} \sum_{\psi \in \Psi_j^g} \sum_{t \in \mathcal{T}} (A_{\psi t}^{\perp} \\
& + \sum_{k \in \mathcal{U}} \Delta_{\psi k t}^{A\perp} x_k^u) \sum_{s \in \mathcal{S}_j} x_{js}^g \tau \lambda_{0t} \rho_{js}^g \hat{p}_{jt}^g \\
& - \sum_{i \in \mathcal{V}} \sum_{t \in \mathcal{T}_i} \tau \eta_i^{ev} u_i^{ev} \rho_i^{ev} x_{it}^{ev} + \sum_{j \in \mathcal{G}} \sum_{s \in \mathcal{S}_j} c_{js}^g x_{js}^g \\
& + \sum_{k \in \mathcal{U}} c_k^u x_k^u + \sum_{t \in \mathcal{T}} \tau \lambda_{0t} \Delta_{kt}^{po} x_k^u, \\
\text{s.t.} \quad & E_{0i}^{ev} + \sum_{t \in \mathcal{T}_i} \tau \eta_i^{ev} \rho_i^{ev} x_{it}^{ev} \leq \bar{E}_i^{ev} \text{ for } i \in \mathcal{V}, \\
& \underline{v}_\omega \leq \tilde{v}_{\omega t} + \sum_{i \in \mathcal{V}} \frac{1}{|\Psi_i^{ev}|} \sum_{\psi \in \Psi_i^{ev}} \rho_i^{ev} (K_{\psi \omega t}^{\perp} \\
& \quad + \sum_{k \in \mathcal{U}} \Delta_{\psi \omega k t}^{K\perp} x_k^u) x_{it}^{ev} + \sum_{k \in \mathcal{U}} \Delta_{\omega k t}^{v} x_k^u \\
& \quad - \sum_{j \in \mathcal{G}} \frac{1}{|\Psi_j^g|} \sum_{\psi \in \Psi_j^g} \rho_{js}^g (K_{\psi \omega t}^{\perp} + \sum_{k \in \mathcal{U}} \Delta_{\psi \omega k t}^{K\perp} x_k^u) \\
& \quad \cdot \sum_{s \in \mathcal{S}_j} x_{js}^g \hat{p}_{jt}^g \leq \bar{v}_\omega \text{ for } \omega \in \Omega,\ t \in \mathcal{T}, \\
& \sum_{s \in \mathcal{S}} x_{js}^g \leq 1 \text{ for } j \in \mathcal{G}, \quad \sum_{k \in \mathcal{U}} x_k^u \leq 1, \\
& x_{it}^{ev}, x_{js}^g, x_k^u \in \{0, 1\}.
\end{aligned}
$$

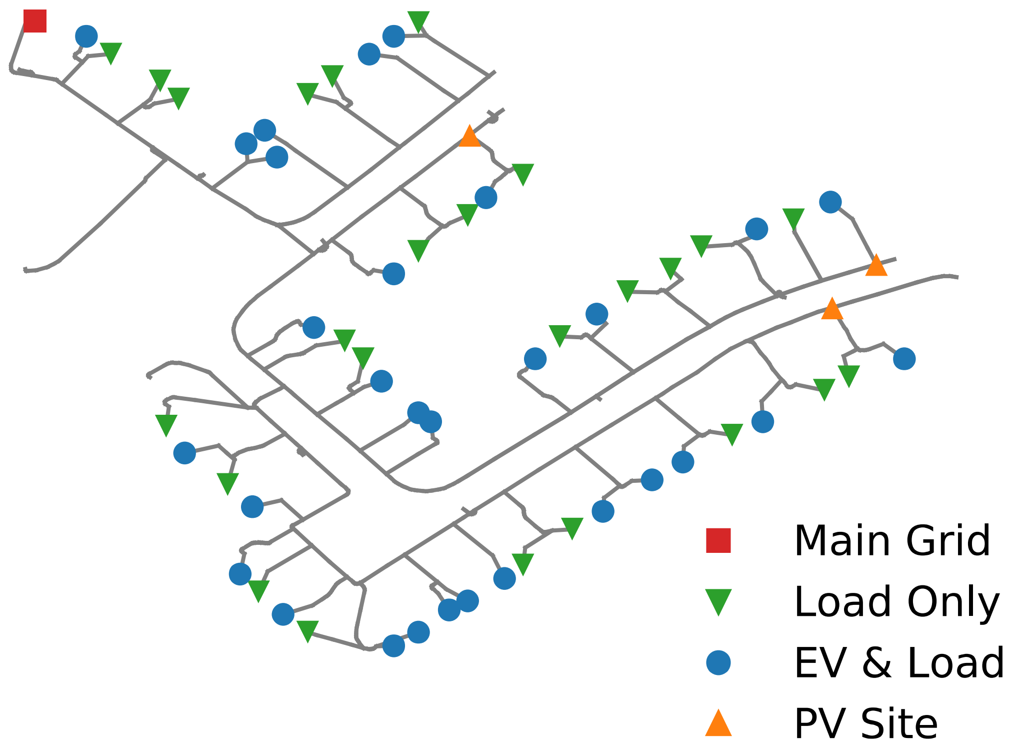
[1] Bernstein, Dall’anese, IEEE ISGT Europe, 2017.

QUBO Equivalent Constraint Penalties

- Constraints enforced within the QUBO problem using equivalent penalty terms
- Linear inequality constraints:
  - Enforce $Ax \leq b$ with $P\left(Ax - b + \delta \sum_{l=0}^{Y-1} 2^l y_l\right)^2$, where y_l are slack auxiliary variables arranged as a binary expansion (with conservativeness less than δ).
- Select one of several discrete options:
  - Enforce $\sum_{i \in N} x_i \leq 1$ with $P \sum_{i,j \in N, i \neq j} x_i x_j$
- Quadratic inequality constraints (due to network upgrades reducing impedances):
  - Enforce auxiliary variable $z = xy$ with $P(xy - 2zx - 2zy + 3z)$
  - Then can enforce inequality constraints on functions of xy using z

Test Case – IEEE European Low Voltage Test Feeder

- Plan network and generation investments and schedule EV smart charging
- Time Horizon: 24h (1-hour resolution)
- Voltage limits: ±5%
- 55 single-phase homes with up to 30 EVs:
  - 75, 85 or 100 kWh batteries at
  - 7.2 kW charging
  - Value energy at 0.50 £/kWh
- Economy7 energy price:
  - 6 am to 11 pm: £0.15/kWh
  - 11 pm to 6 am: £0.07/kWh
- PV investment: 25 or 50 kWp at 3 sites
- Network upgrade plans reduce impedance by 50% or 75%

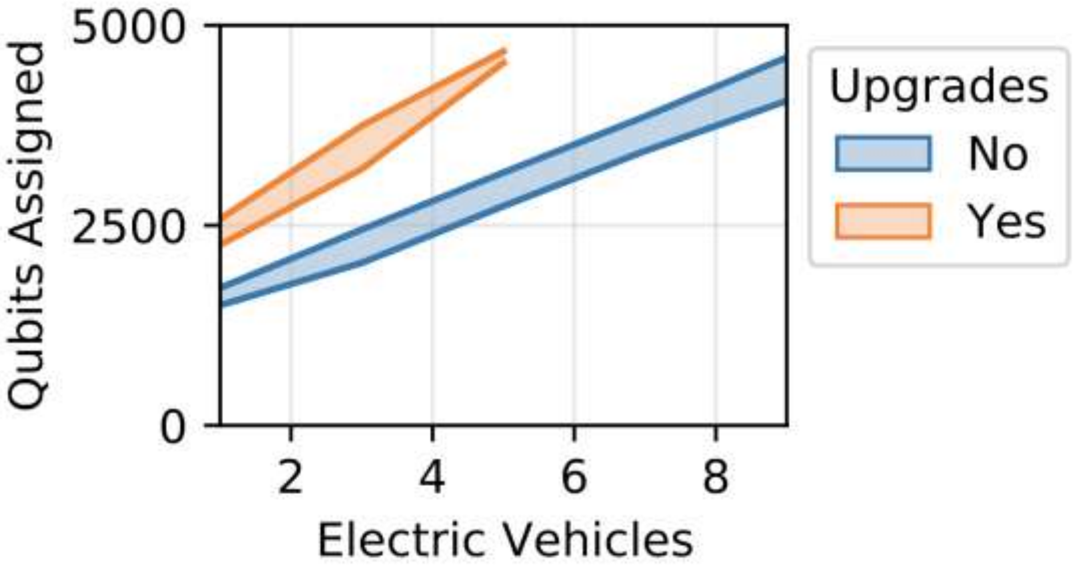


QPU Scaling

No. EVs	Avg. Time (ms) SA	Avg. Time (ms) QA	Avg. Net Utility (£) SA	Avg. Net Utility (£) QA
1	56.4	49.9	−78.9	−70.4
3	84.9	50.0	−47.6	−40.5
5	94.9	51.2	−14.1	−4.6
7	105.7	51.4	−3.2	13.6
9	121.5	52.2	25.5	37.4

Average computation time and net utility when QUBO is solved 10 times with simulated annealing (SA) and quantum annealing (QA).

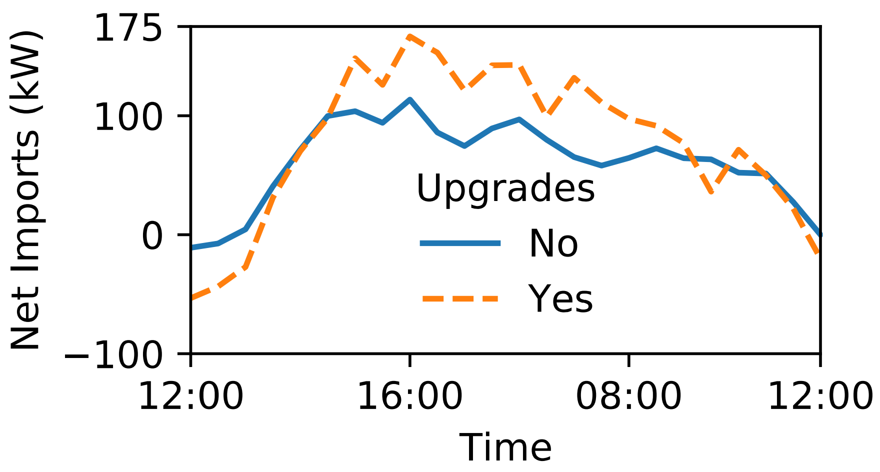
The QUBO was formulated with different numbers of EVs, voltage limits at 3 node–phase pairs, and without network upgrade decisions.

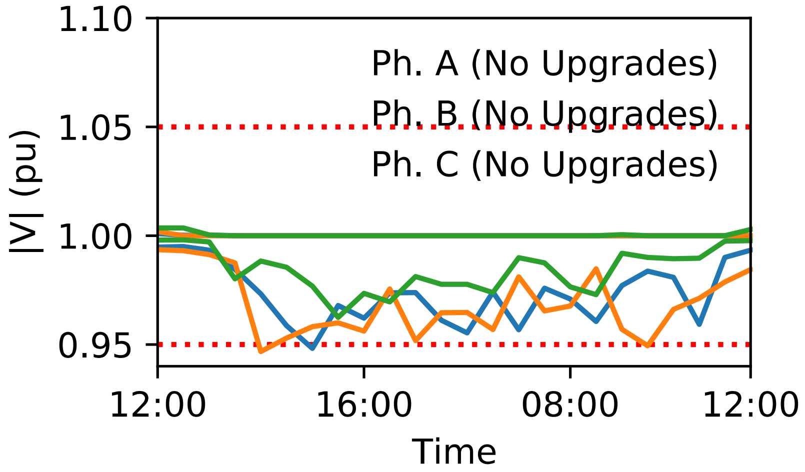


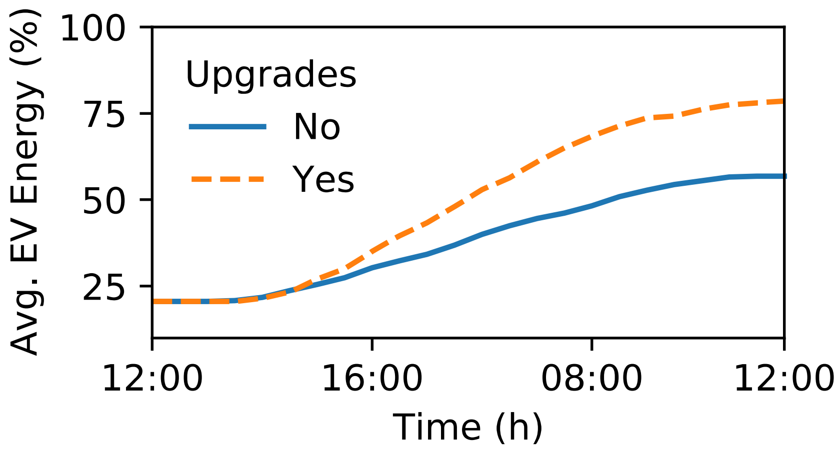
The number of assigned qubits on D-Wave's Pegasus topology (20 minor embeddings each step) for different numbers of EVs, given voltage limits at 3 node-phase pairs,

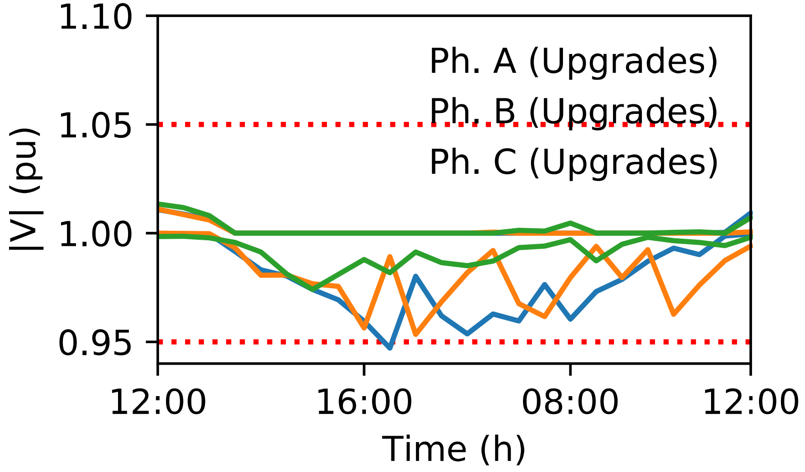
Hybrid Quantum Computing Results

- 30 EVs and 12 bus-phase pair voltage constraints with D-Wave's hybrid solver (120s limit)
- With network upgrades, 50% reduction in impedances and 100 kWp PV built (only 50 kWp PV built without network upgrades)







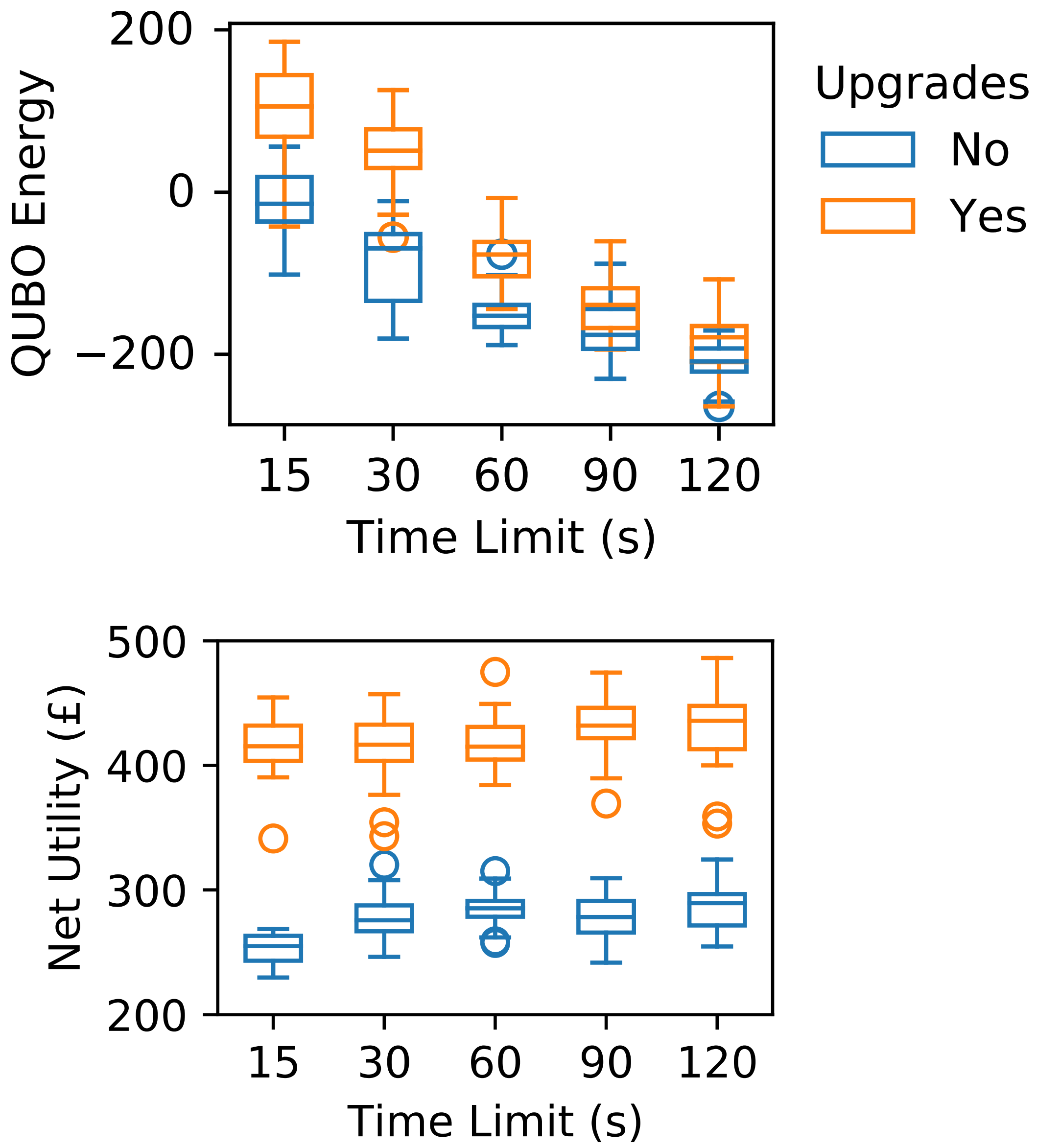


Hybrid Quantum Computing Results 2

- Results solving the QUBO 30 times for a range hybrid solver time limits, with and without network upgrade decisions.
- Distributions are shown for:

  (a) QUBO energy values

  (b) Net utility values
- The box plots show the median (centre line), interquartile range (box), 1.5 times the interquartile range above/below the box (whiskers), and outliers (circles).



Conclusions

- Paper: T. Morstyn and X. Wang "Opportunities for Quantum Computing Within Net-zero Power System Optimization", *Joule*, 2024.
- Net-zero makes optimisation increasingly important (and challenging) for power system operation and planning
- Quantum computing can offer value for a wide range of problem types
- Areas for future work:
  - Opportunity for near-term impact with hybrid quantum–classical computing
  - Benchmarks are important, particularly for NISQ algorithms
  - System operators need holistic computing R&D strategies (e.g. quantum computing, GPU-computing, cloud-to-edge computing)
- Co-chairing new IEEE Power & Energy Society Task Force on quantum computing with Prof. Yan Li (Penn State) – please get in contact if interested

Thanks!

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