The impact of transaction costs on portfolio optimization

Supervised by:
dr.ir. J. van den Berg

Stephen Chan
Bachelor Thesis Informatics & Economics
Erasmus University

Acknowledgments

I want to thank my supervisor Jan van den Berg for his supervision and helping me write this bachelor thesis. Furthermore, I am grateful to Martin Martens for his input and though on writing my thesis. Without them I would not have finished this thesis.

Content

1. Introduction

1.1 Introduction

In the world of the stock market, people try to put a portfolio of stocks together that generates returns as high as possible. A difficulty in portfolio optimization is that an increase in the expected return will lead to an increase in risk. People try to minimize the risk and maximize the return. Much research has been done to find the optimal solution to success. In the late 90's everybody was very bullish about the stock market. The internet hype was one of the reasons that drove the stock market to new highs. The stock prices of every company that had to do with the internet outperformed the stock market even if the company didn't make any profit. Investors expected the company would make profit in the future and would like to invest in the stock due to the incredible growth rate. Taking a second mortgage on the house was not an uncommon idea to get more money and invest in the stock market. Another phenomenon was day trading that was very popular, because of the high volatility in the stock market. The possibility of trading stocks over the internet was one of the reasons that the volatility in the stock market was rising and currently everybody can trade stocks at home over their computer. Financial institutions like banks and brokers saw their income increase by the high volume of transactions in the stock market where they charged transaction fees to their customers for each transaction. After 2000 the bull market ran out of steam and stock market began to decline. Many people lost their money and the economy growth rate began to decline.

1.2 Goal

Financial institutions charge their customers transaction fees for trading over the stock market. The two most common ways to charge their customers are based on a:

1. Fix transaction fee
2. Variable transaction fee.

Transaction cost will have an effect on the portfolio optimization and the frequency of time rebalancing the portfolio. The questions are what the role of the transaction cost is over the portfolio and over the frequency of rebalancing the portfolio. To find out what is the true optimal frequency of rebalancing is so unrealistic assumption must be made. In the real world analysts calculate the expected returns of each stock before setting up their portfolios. The expected returns are usually different from the real returns and will have an effect on the frequency of rebalancing the portfolio. Different data will change the optimal frequency of rebalancing and the effect of the transaction cost on the portfolio. To really find the optimal rebalancing frequency of a portfolio and capture the impact of the transaction cost real future returns will be used to compute the optimal portfolio in each period. Not imposing this assumption, the true optimal rebalancing frequency cannot be found of a portfolio.

1.3 Methodology

First literature about portfolio optimization will be studied first to understand the theory before applying here to simulate portfolios over different moment of times. A program will be designed and implemented to simulate portfolios and to calculate their performances. Changing the following parameters:

- Frequency changes of the portfolio
- The variable transaction fee.

Finally, conclusions will be made based on the results and suggestions for further research will be mentioned.

1.4 Structure of this paper

The structure of this thesis is as followed: Chapter 1: Introduction of the thesis. Chapter 2: Background literature will be provided to gain some insight in the subject.
Chapter 3: Introduction of the research approach and the assumptions will be explained here. Chapter 4: Experimental setup of the impact of transaction cost on portfolio optimization will be described. The results of the experiment will be shown in chapter 5. Finally, in chapter 6 conclusions and suggestions will be made based on the results of this experiment.

2. Portfolio Optimization

In this chapter information how to optimize the portfolio according to the theory of Markowitz will be explained. Furthermore, existing theory for prediction the stock prices will be provided.

2.1 Portfolio optimization according to Markowitz

In 1952 Markowitz wrote an article about portfolio optimization[1]. The article explained how to maximize the portfolio return and minimize the risk or standard deviation. The theory is the foundation of many modern portfolio theories. A portfolio should consist of multiple investments to reduce risk. Before Markowitz wrote that article the investment though was simply to invest in the stocks with the highest expected returns and ignore all the other stocks with lower expected returns. Markowitz was aware that diversifying the investment in multiple stocks risk can reduced. If one of the stocks in the portfolio goes down, the other stocks that might go up can absorb the negative return. Diversifying the portfolio instead of investing it all in one stock significantly reduced the risk[2]. The key point here is to find the portfolio with a certain expected return, which minimizes the risk. Combinations of all stocks can give the same expected return, but not the same level of risk. The picture here beneath illustrates all different kind of combinations of investments. The portfolio where the risk is minimized for a certain expected return lies in the curve called efficient frontier. All the portfolios in the efficient frontier have minimal risk with a maximal expected return and all portfolios below the efficient frontier are not optimal.

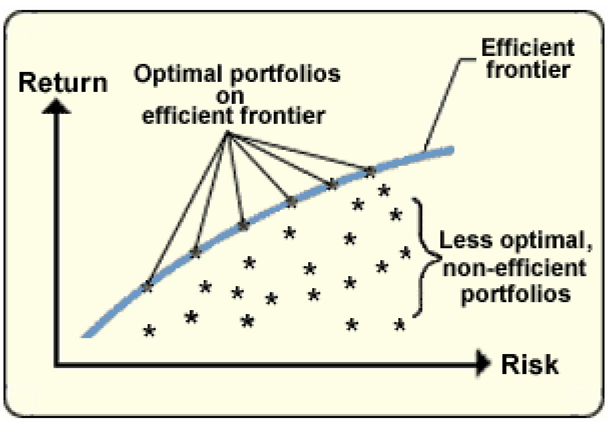


Fig. 1. Here the efficient frontier is shown. All the portfolios below the efficient frontier are not optimal[3].

The key is to calculate the portfolios that lie on the efficient frontier. This is a quadratic problem that can be solved with the computer.

The formulation of this quadratic problem is[4]:

Objective function:

$$\underset{w_i}{Min.}\ \sigma^2 = \sum_{i=1}^{N} w_i \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} w_i w_j \sigma_{ij} \ \textbf{ or } \ Min.\ \sigma^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i \operatorname{cov}(x_i, x_j)$$

$\operatorname{cov}(x_i, x_j) =$ Covariance between x_i and x_j

w_i = Weight of stock i

Subjected to

$$\sum_{i=1}^{N} w_i = 1 \ , \ \sum_{i=1}^{N} w_i E(r_i) = E(r_a) \ , \ \forall_i \ w_i >= 0$$

$E(r_i)$ = Expected return of stock i

$E(r_a)$ = Expected portfolio return

The last restriction where all weights must be greater or equal to zero prevents to go short in stocks (negative weights). The goal function is to find the portfolio with the minimum variance or risk at a given expected return of the portfolio. The expected return of the portfolio is the interval between the stock with the lowest expected return and the stock with the highest expected return. By changing the expected portfolio return a complete efficient frontier can be seen.

For calculating the return over a certain period:

$$r_i = \ln\left(\frac{s_{i,t}}{s_{i,t-1}}\right)$$

$s_{i,t}$ = Stock price of stock i at time t

r_i = Return of stock i

One way to calculate the variance (σ^2) of each stock is described by the following formula:

$$\sigma_i^2 = \frac{\sum_{i=1}^{M} (r_i - \bar{r})^2}{M - 1}$$

r_i = Return of stock i
$\bar{r}$ = Average return of stock i
M = Number of samples
σ_i^2 = variance of stock i

The risk of the portfolio can be calculated with covariance. Covariance described the degree to which two variables vary together.

The formula for covariance is[5]:

$$\operatorname{cov}(r_x, r_y) = \frac{\sum_{i=1}^{M} (r_x - \bar{r}_x)(r_y - \bar{r}_y)}{M - 1}$$

where $\bar{r}$ is the average return. Symbol n stands for the number of observations. If the value of cov(*x*,*y*) is positive, it means when r_i goes up (down), r_y goes up (down) too.

The efficient frontier contains many optimal portfolios, but the question is which one is actually the best. The Sharpe Ratio finds the best portfolio in the optimal curve using the following optimization criterion[6].

Max: $Sharpe\,Ratio_i = \dfrac{r_i - r_f}{\sigma_i}$

r_i = Expected return of portfolio i
r_f = Risk free rate
σ_i = standard deviation of portfolio i

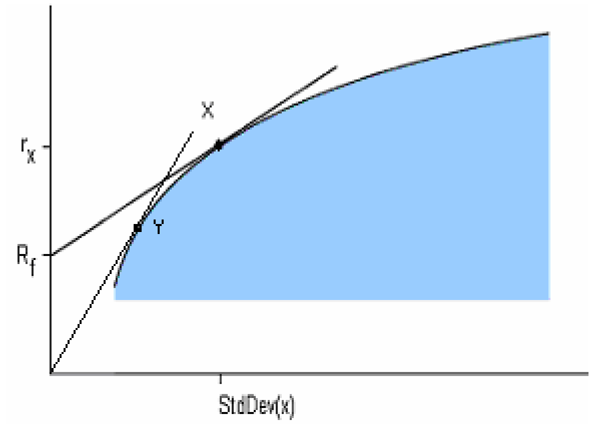


Fig. 2 The efficient frontier is displayed here. When the risk free rate is not equal to zero the Sharpe Ratio identifies point X of the curve as the optimal portfolio. If risk free rate is zero point Y is in that case the optimal portfolio in the curve[7].

The Sharpe Ratio measures the degree of reward-to-risk. The best portfolio to select is the one where the reward-to-risk ratio is as high as possible.

2.2 CAPM

Three economists William Sharpe, John Lintner and Jack Treynor in the 60's came up with the Capital Asset Pricing Model, or CAPM[8]. The CAPM concludes that the expected risk premium on each investment is proportional to its β. The relationship can be written as:

Expected risk premium on stock = beta times expected risk premium on market

$$r_i - r_j = \beta(r_m - r_f)$$

r_i = Expected return on stock i

r_f = Risk free interest rate

r_m = Return on the market

The difference between the return of the market and the risk free interest rate is called the market risk premium $(r_m - r_f)$. The β described the proportion of the expected risk premium on the market. For example β is 0.5, means half the expected risk premium on the market. The CAPM predicts that stocks with high β's generated higher average returns than stocks with lower β's. This is confirmed by rewriting the formula above.

$$r_i = r_f + \beta(r_m - r_f)$$

Higher β will increase the value on the right side and will leads to a higher expected stock return. All investment returns should lie on the linear sloping line between the risk free rate and the market portfolio. This is called the Security Market Line (SML).

2.3 The Arbitrage Pricing Theory

An interesting alternative to the CAPM is the Arbitrage Pricing Theory (APT)[9] for forecasting expected returns. The APT states that the stock return is determined by the macroeconomics' factors exposures and partly "noise" – event that are unique to that stock.

$$r_n = \sum_{k=1}^{K} X_{n,k} b_k + \mu_n$$

r_n = Return of stock n

$X_{n,k}$ = The exposure or weight of stock n to factor k

b_k = The factor return for factor k

μ_n = "noise" or event that cannot be explained or unique to stock n

So the expected risk premium of stock n is determined by the sum of the factors times the expected risk premium plus "noise".

$$r_n - r_f = \sum_{k=1}^{K} X_{n,k} (r_n - r_f) + \mu_n$$

$r_n - r_f$ = The expected risk premium of stock n

$X_{n,k}$ = The exposure of stock n to factor k

r_f = Risk free interest rate

μ_n = "noise" or event that cannot be explained or unique to stock n

2.4 The Wiener Process

The Wiener process[10] also known as Brownian motion described the behavior of stock prices. It describes the present value with the chance that the value can change based on the probability distribution.

$$\delta z = \varepsilon\sqrt{\delta t}$$

δz = Change of the stock price during a small period of time δt
ε = Random value drawn from a standardized distribution, $\phi(0,1)$
δt = Period of time

The Wiener process has a drift rate of zero, which means that the expected value in the future is equal to the present value. A generalized Wiener process can be defined as the previous formula and adding a constant term a and b.

$$\delta x = a\delta t + b\varepsilon\sqrt{\delta t}$$

a = constant drift term
b = constant variance term

This model failed to capture a key aspect of stock prices. The expected stock percentage return is independent of the stock's price. The expected drift rate and the variance rate should be replaced respectively by μS and σS.

$$\frac{dS}{S} = \mu S dt + \sigma S dz$$

$\frac{dS}{S}$ = The return of stock in period δt
μS = Expected drift rate in S
σS = Volatility in S

3. Research Approach

This section the process of calculating the portfolio, rebalancing the portfolio will be described. Furthermore, the effect of transaction costs and the pseudo code of the program will be discussed.

3.1 Calculating the optimal portfolio

In this chapter, the model of the simulation will be explained. The literature of portfolio optimization theory of Markowitz will be used to manage the portfolio. The portfolio can be made of up to 10 stocks where historical stock prices are used here. Over a period of time a new portfolio will be calculated with the latest historical data. First determine what the interval of time for updating the portfolio.

Calculate the optimal portfolio curve according to the model of chapter 2. The following formulas will be used here.

Return of stocks: $r_i = \ln\left(\frac{s_{i,t}}{s_{i,t-1}}\right)$

Covariance-matrix: $\operatorname{cov}(r_x, r_y) = \frac{\sum_{i=1}^{M}(r_{x,i} - \bar{r}_x)(r_{y,i} - \bar{r}_y)}{M-1}$

To calculate the covariance-matrix historical data from a year ago will be used.

One of the key assumptions is perfect information. As mentioned before we have chosen to use the real future return of stocks as the expected return of stocks.

$$E(r_{i,t}) = r_{i,t+1}$$

$$Sharpe\,Ratio_i = \frac{r_i - r_f}{\sigma_i}$$

The Sharpe Ratio will be used to find a point in the curve. Here the risk free rate will be zero. The Sharpe Ratio will be expected return of portfolio i divided by the standard deviation of portfolio i. This is done to find the optimal portfolio described in Fig. 2(in the previous chapter).

$$Sharpe\,Ratio_i = \frac{r_i}{\sigma_i}$$

3.2 Rebalancing the portfolio

For each transaction commission fee must be paid. The transaction cost consists of a certain percentage over the transaction value. The first time a portfolio is put together no transaction cost will be charged to make it possible to invest 100% in stocks.

$$1 = \sum_{i=1}^{10} w_i$$

In the beginning the sum of all weight is 1.
Over a certain period the portfolio will be updated to meet the current market situation.

The new weight of the stock i over the interval of time $t+1$:

$$w_{i,t+1} = (1 + r_i) w_{i,t}$$

Now a new optimal portfolio is calculated. The sum of the new portfolio calculated by the computer is equal to 1. This should be adjusted to the current portfolio performance.

$$r_p = \sum_{i=1}^{10} w_i r_i$$

The formula here above calculated the return of the portfolio.

$$w_{i,new} = w_i (1 + r_p)$$

Check to current weight of the stock with the new weight adjusted to the portfolio performance to see if stocks must be sold or not. If the weight must be reduced stock should be sold.

$$w_{i,sell} = w_{i,t} - w_{i,new}$$

$w_{i,sell}$ = weight that must be sold

The proportion of the portfolio received from selling stocks must be reduced with the transaction cost.

$$p_{sell} = \sum (1 - t) w_i$$

t = percentage of the value as transaction cost
w_i = weight of stock i
p_{sell} = percentage of the portfolio sold after cost

Before investing in stocks the budget, it must be reduced with the transaction cost that will take place when buying stocks.

$$p_{cash} = (1-t)\,p_{sell}$$

The value of the stocks that must be bought is:

$$p_{buy} = \sum w_{i,new} - w_{i,t}$$

Now p_{buy} is the budget that can be invested in stocks. The budget isn't enough to invest in stocks to the desired level of the new portfolio. The weight will be slightly under the desired level of the new optimal portfolio. The new weight of the stocks that must be bought will be calculated with the following formula.

$$w_{i,buy} = \frac{w_{i,new}}{p_{buy}}\,p_{cash} + w_i$$

Now this process can be repeated numerous times.

An example will describe the problem here above.

Imagine that the portfolio can only consist of 3 stocks.

At time t=1 the portfolio contains these positions:

	Stock 1	Stock 2	Stock 3
t=1	0.4	0.3	0.3

40% of the portfolio invested in stock 1, 30% invested in stock 2 and the rest in stock 3. At t=2 the returns of the stocks are:

	Stock 1	Stock 2	Stock 3
t=2	0.04	0.02	0.01

The new weights of the portfolio are:

$$w_{i,t} = (1+r_i)\,w_i$$

	Stock 1	Stock 2	Stock 3	Total Value
t=2	0.416	0.306	0.303	1.025

The total value of the portfolio is the sum of all the weights.

Now a new optimal portfolio is determined.

Stock 1	Stock 2	Stock 3
0.25	0.4	0.35

Because the sum is equal to 1 and the current portfolio is not equal to 1 it has to be adjusted with the following formula:

$$w_{i,new} = w_i(1+r_t)$$

After adjusted the optimal portfolio it will be:

Stock 1	Stock 2	Stock 3
0.25625	0.41	0.35875

	Stock 1	Stock 2	Stock 3	Total Value
Desired portfolio	0.25625	0.41	0.35875	1.025
Current portfolio	0.416	0.306	0.303	1.025
Differences	-0.15975	0.104	0.05575	
	sell	buy	buy	

By comparing the desired portfolio and the current holding of the portfolio some positions of the stocks must be lowered and some stocks must be bought more. Some of stock 1 must be sold and the other two positions must be increased.

Over the value of 0.15975 (=0.416 – 0.25625) sold from stock 1 fee must be paid.
The transaction fee is 1% of the transaction value here.
The value after deducting the fee here is 0.15975 x (1-0.01) = 0 .1581525
This is the value that can be invested in the other 2 stocks.
Since transaction fee is known if it is invested in those 2 stocks it will be deducted to calculate the value after the cost.
0.1581525 x (1-0.01) = 0.156570975
That is the value that can be invested in the 2 stocks.

The value of stocks that must be bought is:

Here,
0.104 + 0.05575 = 0.15975

The new position of stock 1 and stock 2 are:

Stock 2: $\frac{0.104}{0.15975} \cdot 0.156570975 + 0.306 = 0.4079304$

Stock 3: $\frac{0.05575}{0.15975} \cdot 0.156570975 = 0.357640575$

	Stock 1	Stock 2	Stock 3	Total Value
Desired portfolio	0.25625	0.41	0.35875	1.025
New portfolio	0.25625	0.40793	0.357641	1.021820975

Here the new portfolio is slightly different from the desired portfolio, but the error is assumed to be acceptable.

3.3 Effect of transaction costs

As mentioned in the beginning financial institutions charges their customers fees for trading stocks. Active portfolio managers try to rebalance their portfolio to adapt it to the current information available. The more the portfolio is rebalanced the higher the performances, but it will reach a point where the performances will decline if frequency is too high. Each time changing the portfolio will lead to cost and to cover the cost the portfolio return must outperform the cost, but in reality it is not the case.
The idea is drawn in a chart where the curve represents the performance of the portfolio. Higher frequency will lead to a higher return, but it will reach a point where it will hurt the portfolio return due to transaction costs. Not only the transaction costs will hurt the performance, but also it will increase the exposure of the portfolio to all the local ups and downs of the stocks. An example of a stock return is displayed in Fig. 4. Here the best case is always to rebalance when the stock is at its local high, but it is very difficult to predict the expected returns over a short period of time. Predicting the wrong returns will hurt the performance, so the best solution is not to rebalance the portfolio too often. Rebalancing the portfolio over a longer period will counter the problems with local ups and downs of the stocks. This is the first assumption of the impact of the transaction costs on portfolio optimization.

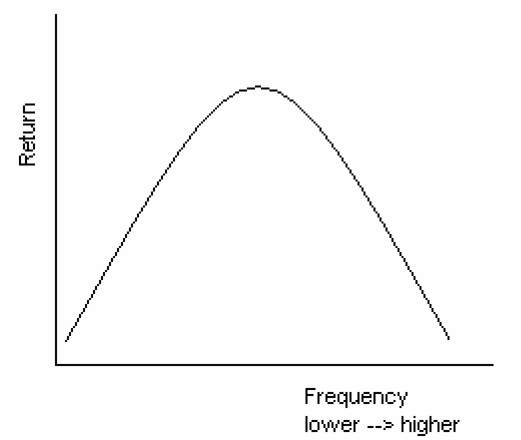


Fig. 3 Return of the portfolios based on the frequency changes.

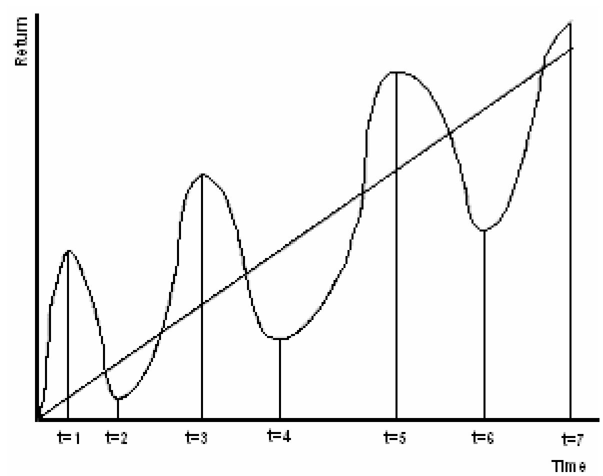


Fig. 4 Example of a stock return over time. Best times to rebalance the portfolio are at time 1,3,5 and 7. In all the other cases it is best to hold the portfolio.

The second assumption is that changes in the covariance matrix will have an effect on the result. Big changes in the covariance matrix will lead to an almost different portfolio. Changing the current portfolio almost completely will lead to higher transaction costs and that will affect the performance of the portfolio. Big changes in the covariance matrix occur when the stocks behave different than the past. It will lead to a different graph than the one displayed in Fig. 3. Another assumption is that the portfolio manager will update his portfolio every fixed interval. This is not realistic compared to the real world due to the fact that portfolio will only be rebalanced if the new portfolio will outperform the current portfolio including the transaction costs. The fourth assumption here is that there is perfect information. The expected return is the actual return of the future. This assumption is important, because the expected return of the stocks have a major impact rebalancing the portfolio. Faulty expected returns will lead to different stock holdings and the performance will be quit different. It will also have an impact on finding the optimal rebalancing frequency. Each expected returns have their own optimal frequency. The last assumption is that each time it is fully invested in stocks. All the return of the portfolio will directly be invested in stocks.

3.4 Pseudo Code

Set the begin date and end date.
Define the rebalancing frequencies and put the data in a list.
Set transaction costs

Find the position of the begin date and end date in the database. Check if the positions are correct. That is if the end date position is not before the begin date position.

Start the frequency list

- Calculate the number of times the portfolio must be rebalanced between the end and begin date.
- Compute the returns of the stocks every day.
- Calculate the covariance-matrix with the returns of the stocks every day as input.
- Retrieve the future returns of the stocks for the coming rebalancing period.
- The covariance-matrix and the future returns are used to compute the efficient curve.
- Find the Sharpe ratio point of the efficient curve and store the portfolio as desired portfolio in the system.
- Check if this is the first time the portfolio is rebalanced.
  If this is the first time then:
    Set the desired portfolio as portfolio. First time, no costs involved.
  Else
    For stock is 1 to stock 10
      Go through every stock if the weight of the current portfolio is smaller or larger than the desired portfolio stock weight. First all stocks are sold than after paying the costs stocks can be bought.
        If the current weight of the portfolio is smaller than then desired weight then:
          the different between the two weights must be sold
          Add this up in the variable named cash.
        Else
          The different between the two weights should be bought. This amount can be added up in the buy variable. Count the number of stocks where the weight must be increased.
    Next stock

    Calculate the cash after subtracting the transaction costs twice. One for selling the stocks and the other for buying the stocks.

    For x=1 to number of stocks where stocks should be bought
      Portfolio of stock x =

$$\frac{weight\ that\ should\ be\ bought}{total\ \ amount\ \ of\ \ buy\ weight} \cdot cash + current\ weight$$

    Next stock x

Calculate the performance of the portfolio by multiplying the returns with the weights of the stocks.

Next frequency in the list.

4. Experimental setup

In the previous chapter the model of the experiment is described. In order to exam the impact of the transaction cost multiple models will be used with each different parameter.

4.1 Hardware and software

The hardware that has been used for experimenting the models consist of a desktop computer made out of the following components:

- AMD XP 1900+ processor
- 512 MB memory
- ATI All-In-Wonder 7500 PCI graphic card
- Western Digital Caviar Special Edition 120 GB hard drive with 8 MB cache
- Epox mainboard 8K3A+ (VIA KT333 chipset)

The following software used in this experiment is:
- Microsoft Windows 2000
- Microsoft Excel XP
- Matlab

4.2 Dataset

For calculating the efficient portfolio curve historical stock prices and the indexes of the stocks are downloaded at http://finance.yahoo.com/. At the website of Yahoo stock quotes can be downloaded in an excel file. Each excel file contains stock quotes from one public listed company or one major index. Historical stock prices dated from June 1st 1994 to April 30th 2004 are used here. The number of stocks where the portfolio can be formed is limited to 10 stocks. The 10 stocks are:

Citigroup	– bank/insurance company
General Electric	– conglomerate that is active in the financial services, household appliance and industrial sector
Exxon Mobil	- The world’s largest energy and petrochemical company
Coca-Cola Company	- One of the biggest beverage manufacturers
Wal-Mart	- The world’s largest retailer
Dell	- One of the major player computer sector
Cisco Systems	- Design and manufacture network- and communication devices
Intel	- A semiconductor company that dominates the computer processor market
PMC-Sierra	- A small semiconductor company that design and manufacture

integrated circuits
Sun Microsystems - Company that is mainly active in the computer server market

These companies are all active in a different kind of sector and represent the broader market. The excel file downloaded from yahoo contain the date, opening price, high and low prices of that day, the closing quote, volume and the adjusted close. Date and adjusted closing prices are the data that will be used in this experiment. All the other data are not relevant here and will be left out. Adjusted close price means that close price is adjusted to stock split and for dividends. In the close price is not adjusted to these events so that the historical prices are not correct.

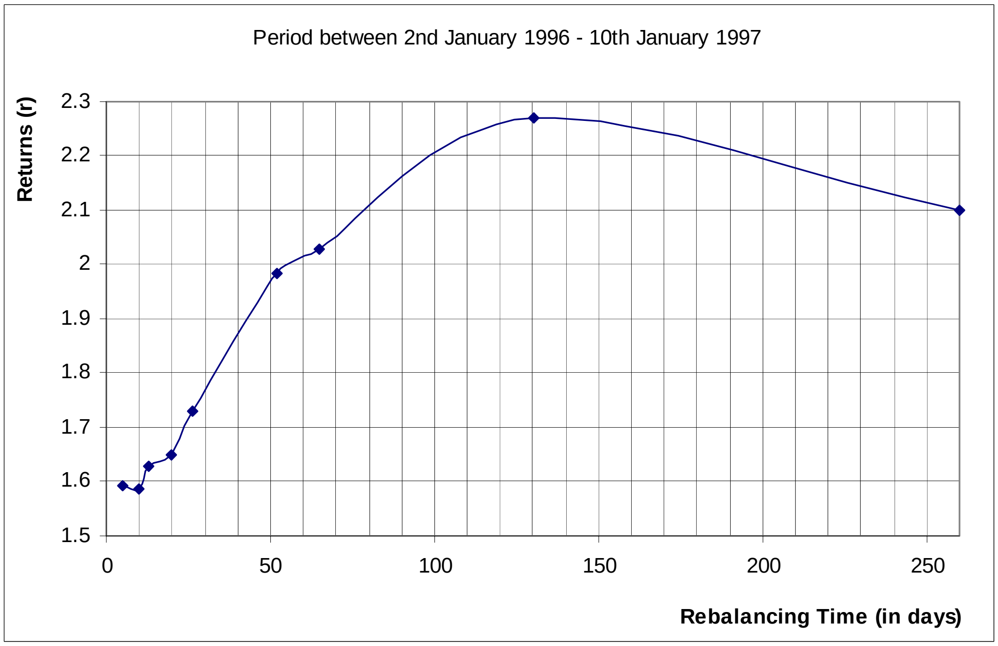
4.3 Processing dataset for use

All excel files must be checked to see the data is complete. To check that the dates of all the excel files are put next together in an excel worksheet. Sometime the numbers of rows in each excel files are not the same. It seems that some trading days are left out. To make sure that all data are the same, the rows of other files will be deleted to compensate for that. The excel file contains the date of the stock price with the most recent date at the top of row and to most historical date at the end of the file. Now all the files are the same stock prices can all be put into one large worksheet. This worksheet contains a column of the date, 10 columns of stock quotes and 3 columns of the indexes. In the program the database will first be rearranged from the most past information on top of the worksheet to the most recent data at the bottom of the worksheet. Now the database is ready to be used of for finding the optimal portfolio according to the theory of Markowitz. Matlab will be used here to find the optimal portfolio and calculate the transaction cost. It will directly extract the data from Excel as input for the program.

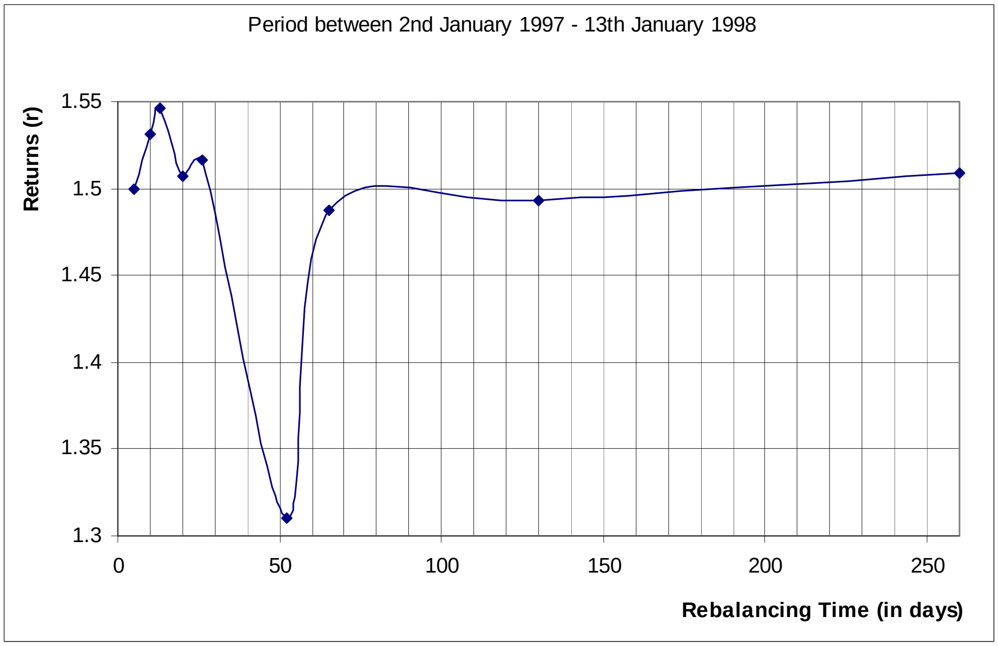
5. Results

The results of the experiment will be put in charts. Period of one year will be used to examine the results. Later on the experiment will be tested over a longer period. The stocks are defined in symbols in the graph. Here beneath the symbols are defined.

Symbol	Company Name
C	Citigroup
GE	General Electric
XOM	Exxon Mobil
KO	Coca-Cola Company
WMT	Wal-Mart
DELL	Dell
CSCO	Cisco Systems
INTC	Intel
PMCS	PMC-Sierra
SUNW	Sun Microsystems

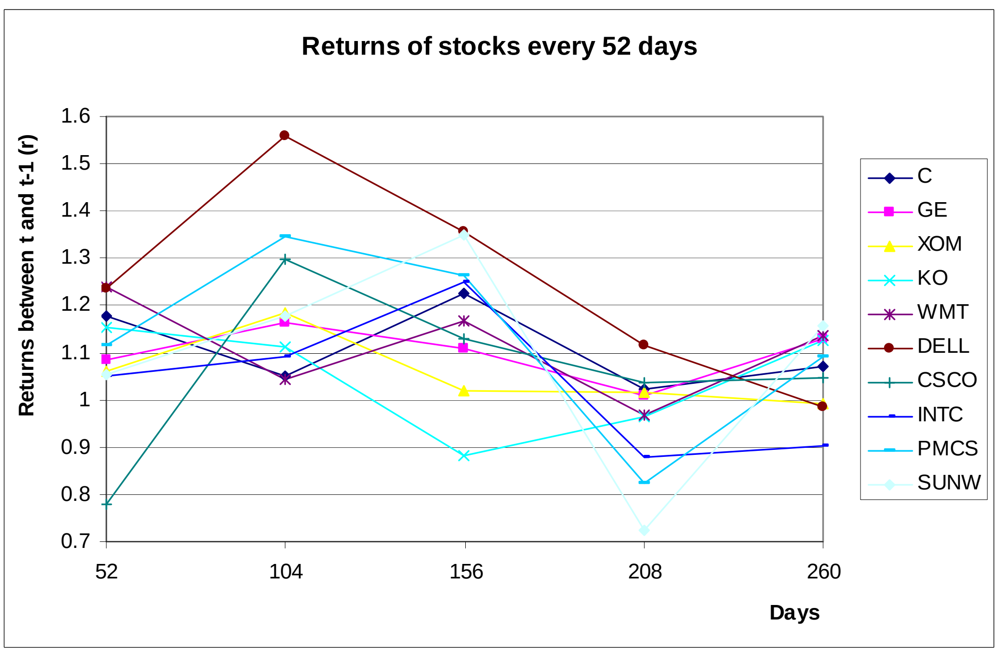


Period between 2nd January 1996 and 13th January 1997, 52 weeks history, transaction costs set at 5%.
The chart here is quit similar to the expected chart.

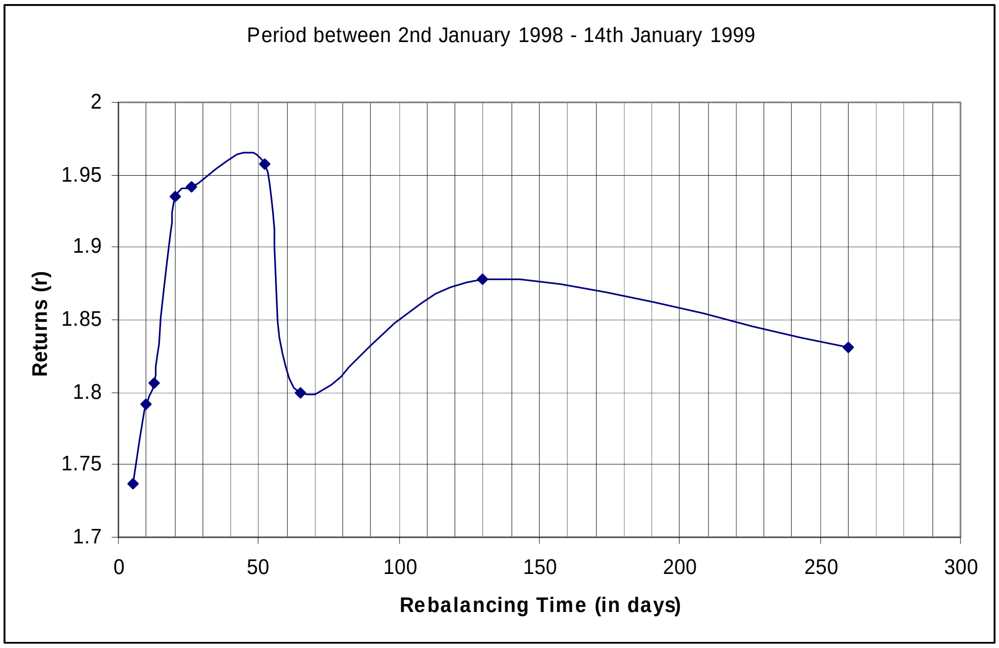


Period between 2nd January 1997 and 13th February 1998, 52 weeks history, transaction costs set at 5%.

This chart differs from the expected chart. Further research will be performed here to see why the rebalancing time set at 52 days has a dramatic result. The returns of stocks every 52 days will be displayed here to see if there is an explanation.

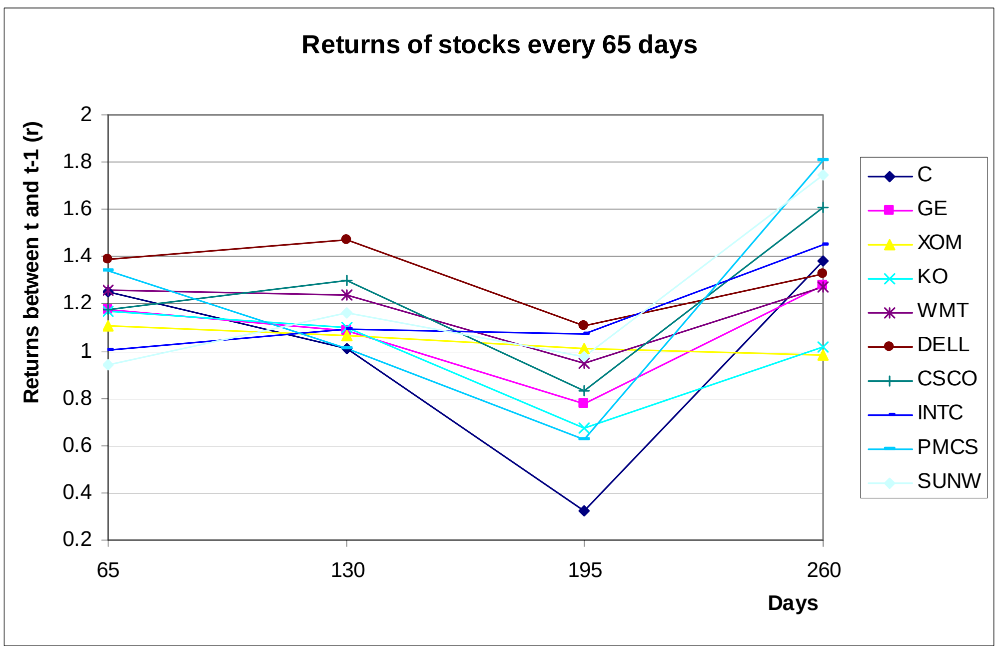


Here the changes of the stock prices have a major impact on the performance. After the first time the portfolio is rebalanced the stocks start to decline. Stocks are not outperforming the costs which will lead to declining portfolio return.

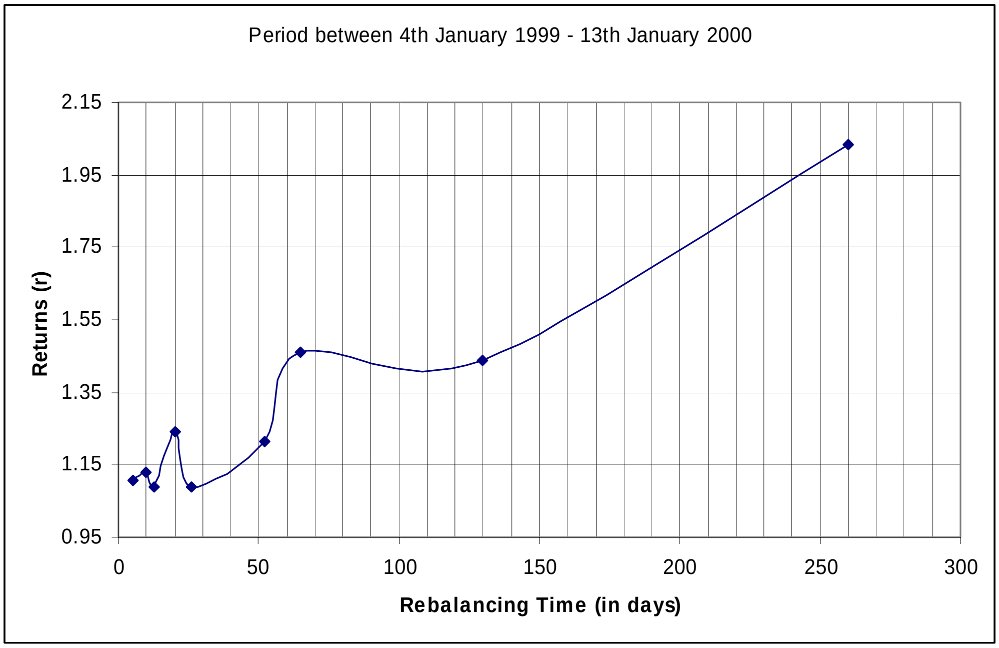


Period between 2nd January 1998 and 14th January 1999, 52 weeks history, transaction costs set at 5%.

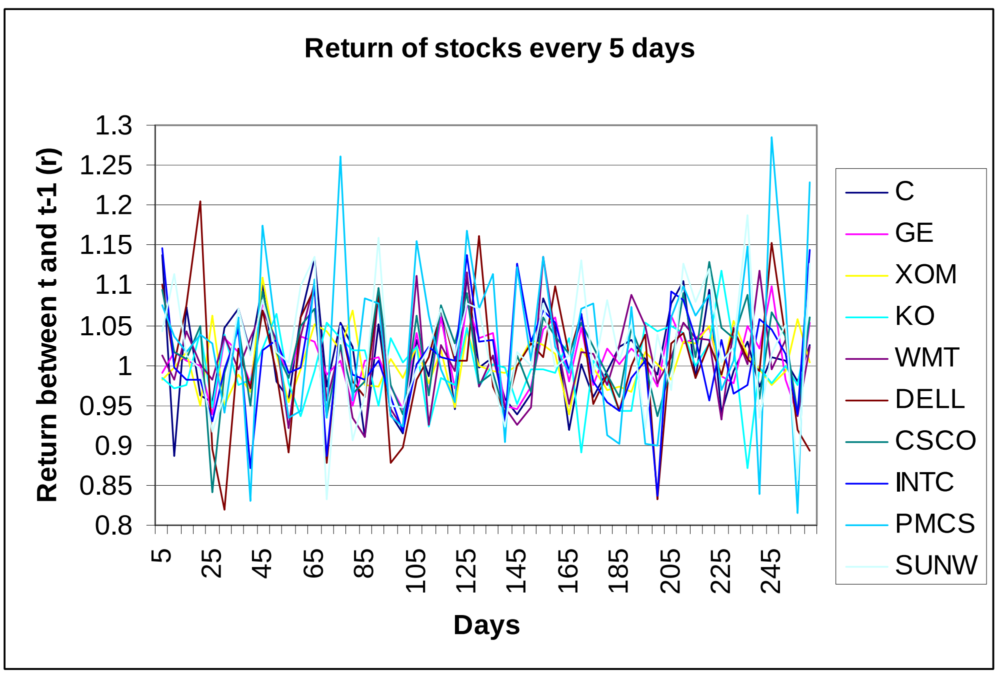
The performance of the portfolio rebalanced every 65 days will further be examined.



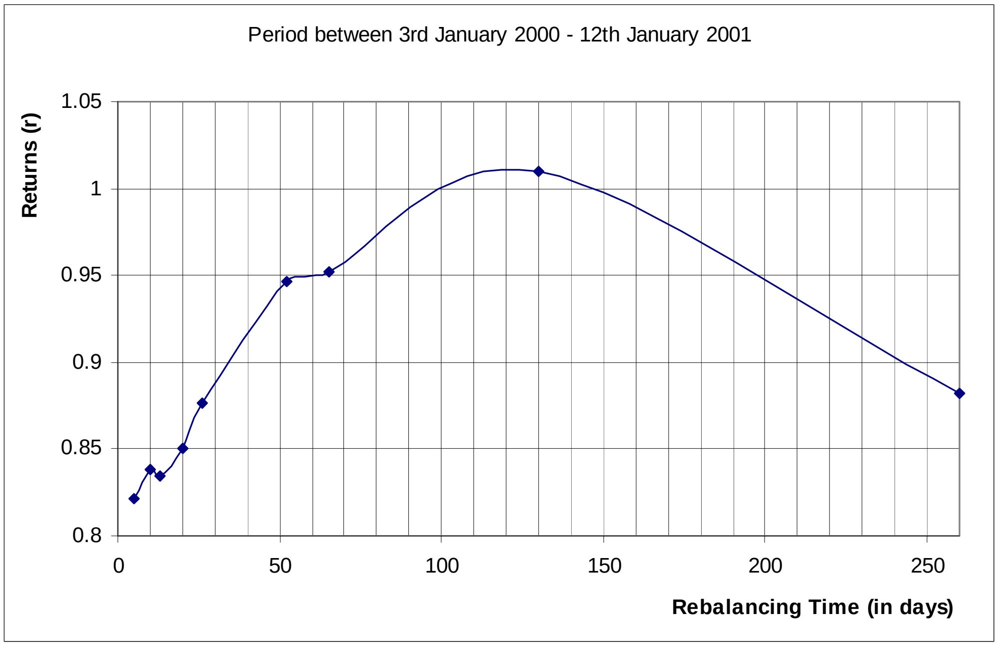
The third time rebalancing the portfolio in a down trend hurts the performance.



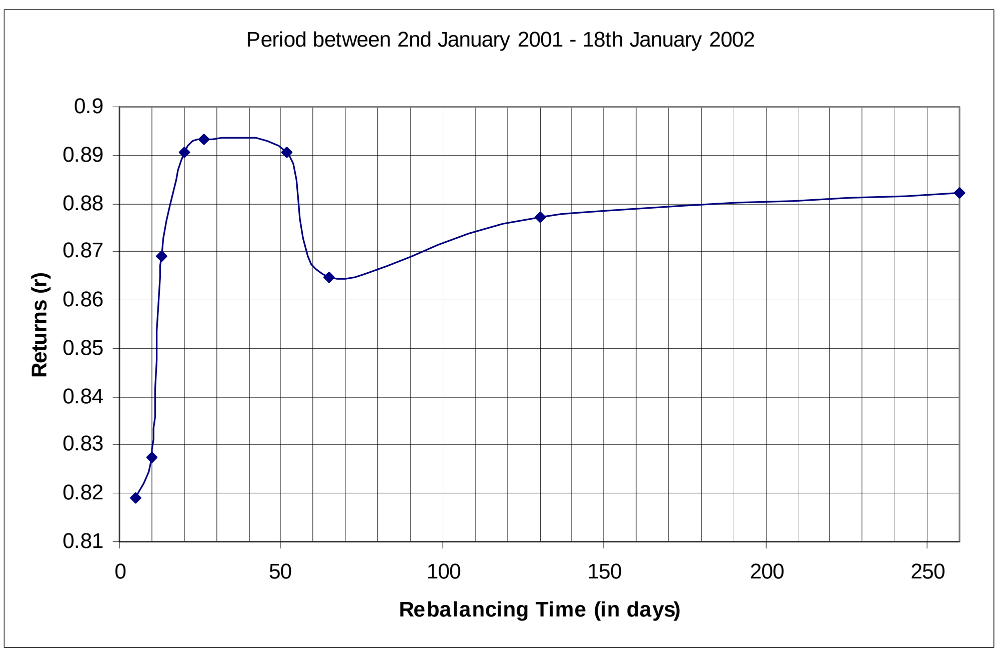
Period between 4th January 1999 and 13th January 2000, 52 weeks history, transaction costs set at 5%.



The stocks of this period are fairly stable. The stocks will not outperform the transaction costs and that will lead to a declining portfolio return especially with high rebalancing frequency. Rebalancing the portfolio once a year is the best solution and that can be reflected in the performance compared to the others.

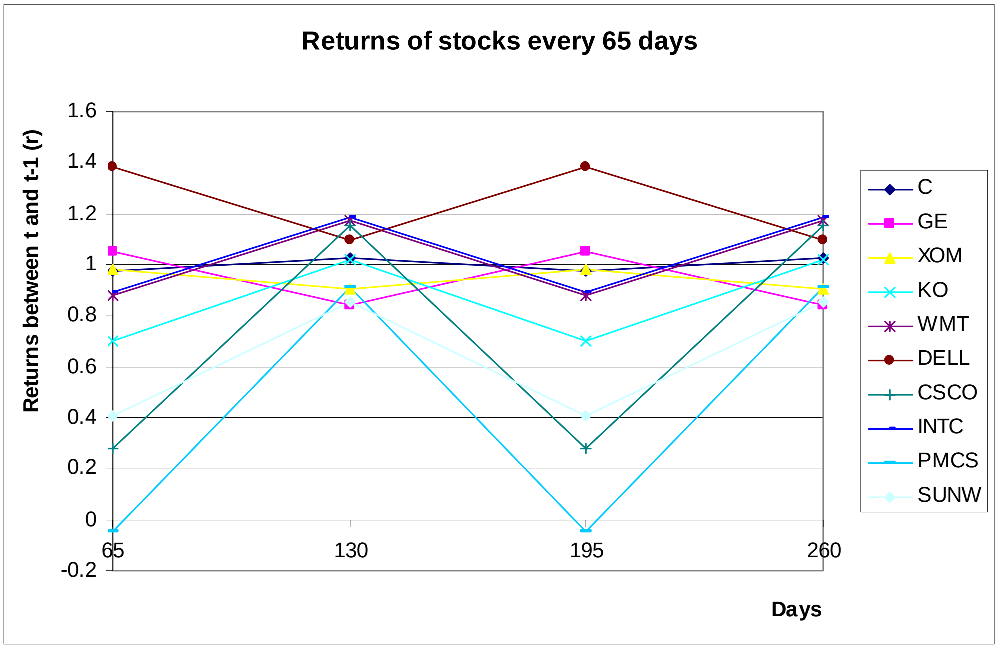


Period between 3rd January 2000 and 12th January 2001, 52 weeks history, transaction costs set at 5%.

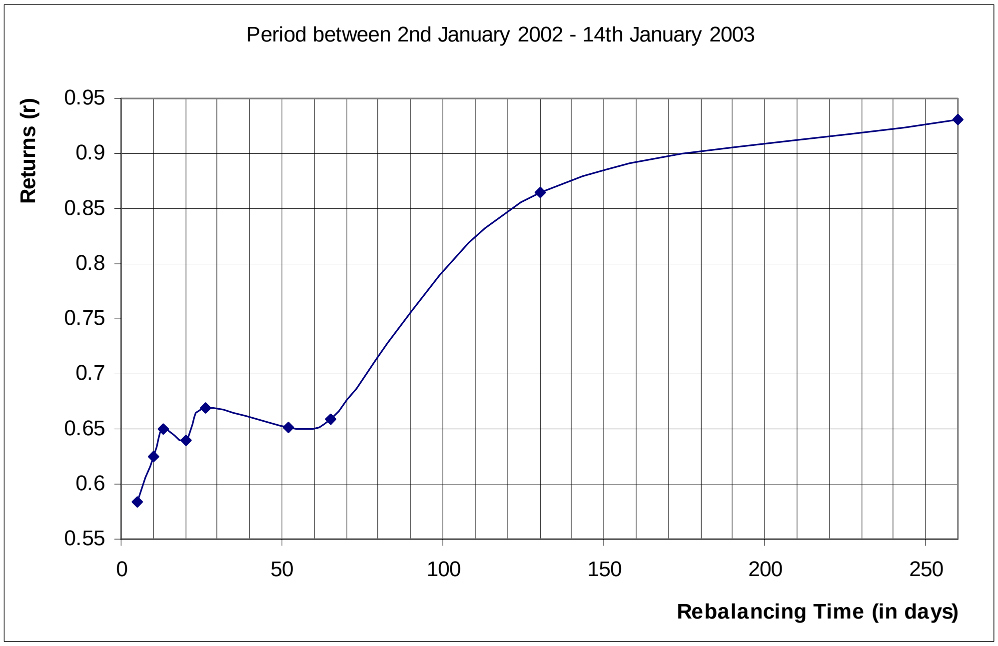


Period between 2nd January 2001 and 18th February 2002, 52 weeks history, transaction costs set at 5%.

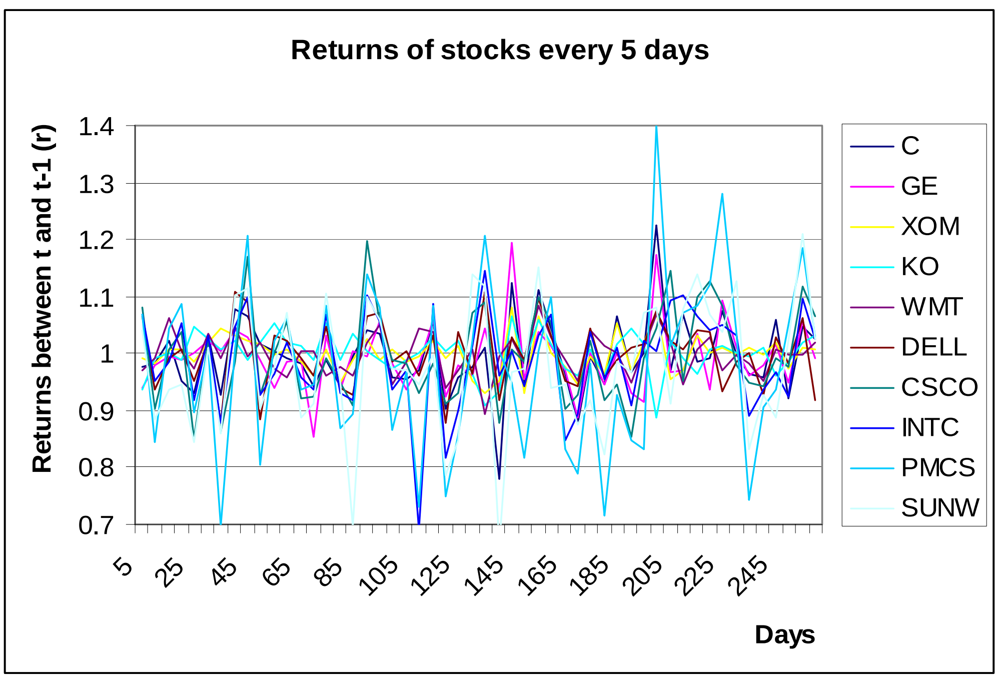
The point where the frequency is set to 65 days will further be examined.



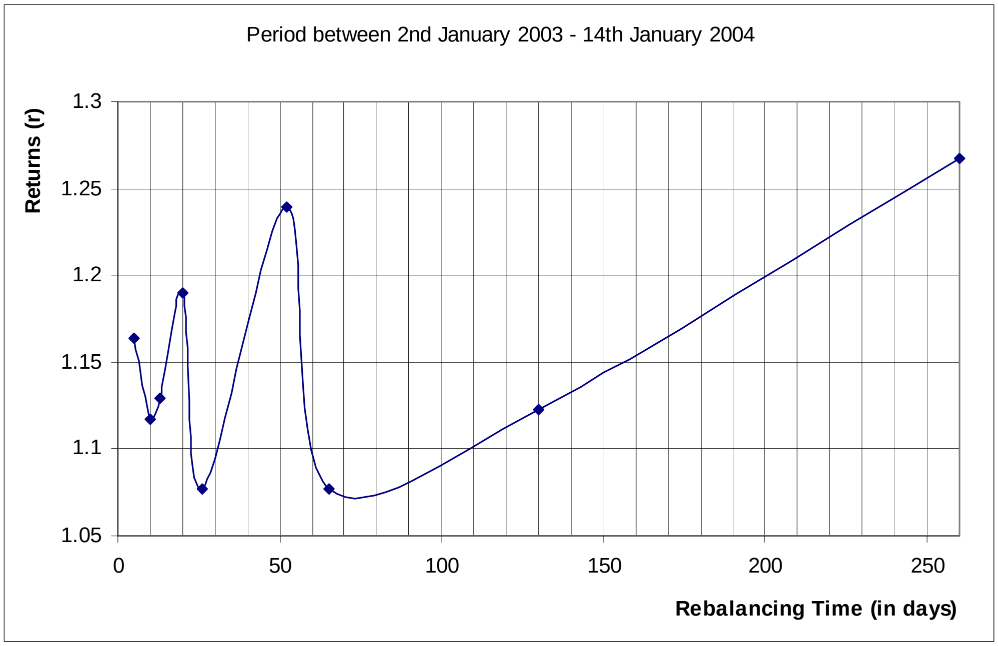
The returns of the stocks are very volatile that causes the performance of the portfolio to deviate every from the others. Big differences between the return of the stocks indicate that there is big correlation difference between the stocks. This will lead in the time of rebalancing the portfolio to large movement of the holdings of the stocks. The consequence is the increase in transaction costs and that will reduce the performance.



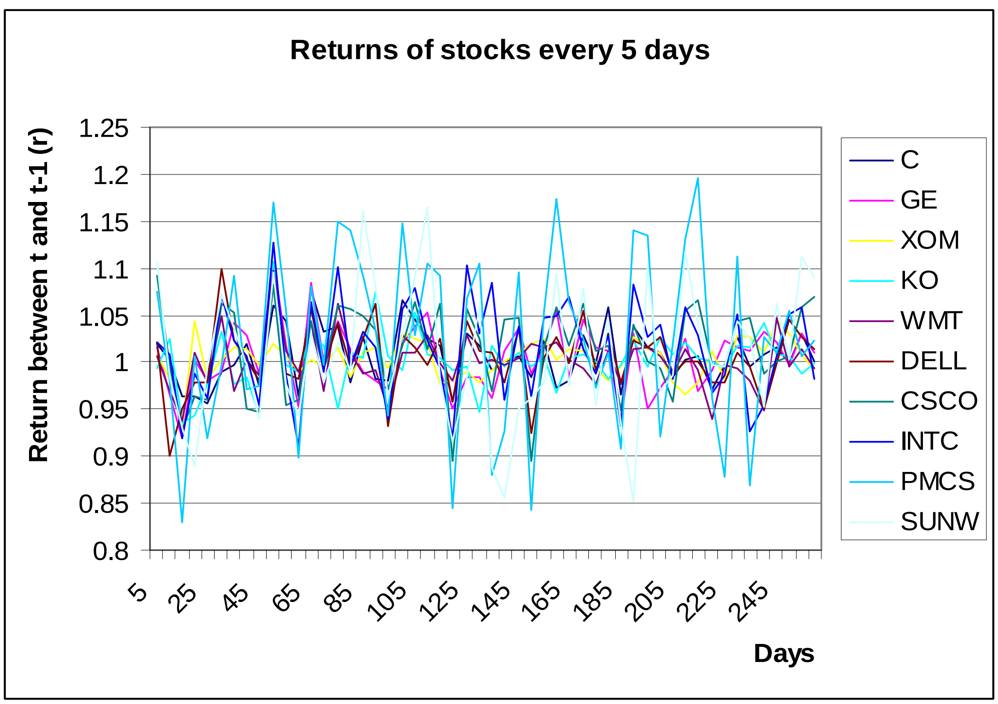
Period between 2nd January 2002 and 14th January 2003, 52 weeks history, transaction costs set at 5%.



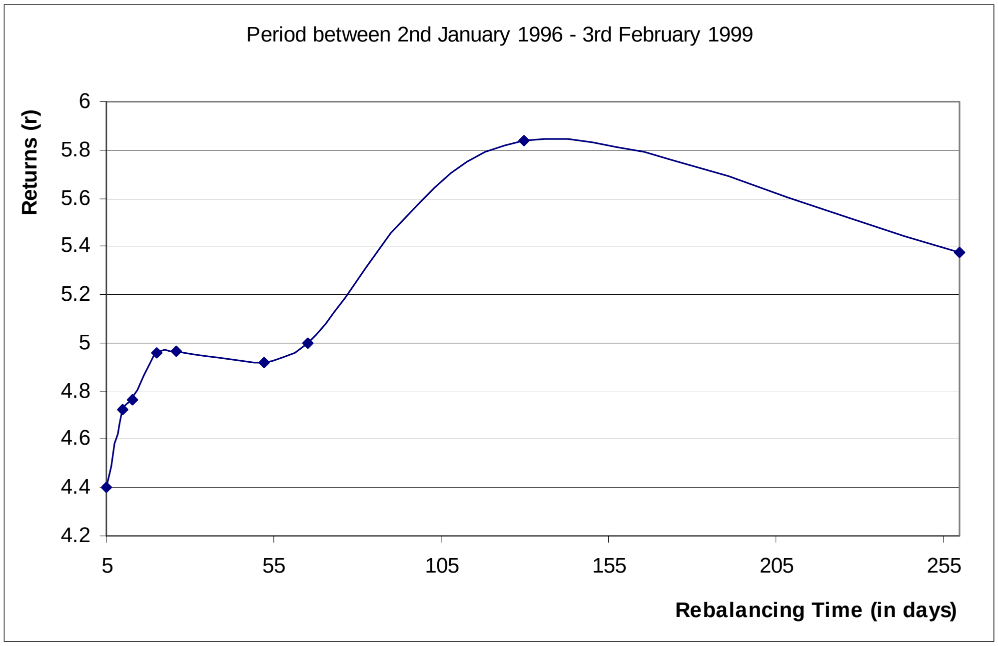
In this period the stocks don't move a lot in which the transaction costs will dominate the performance of the portfolio.



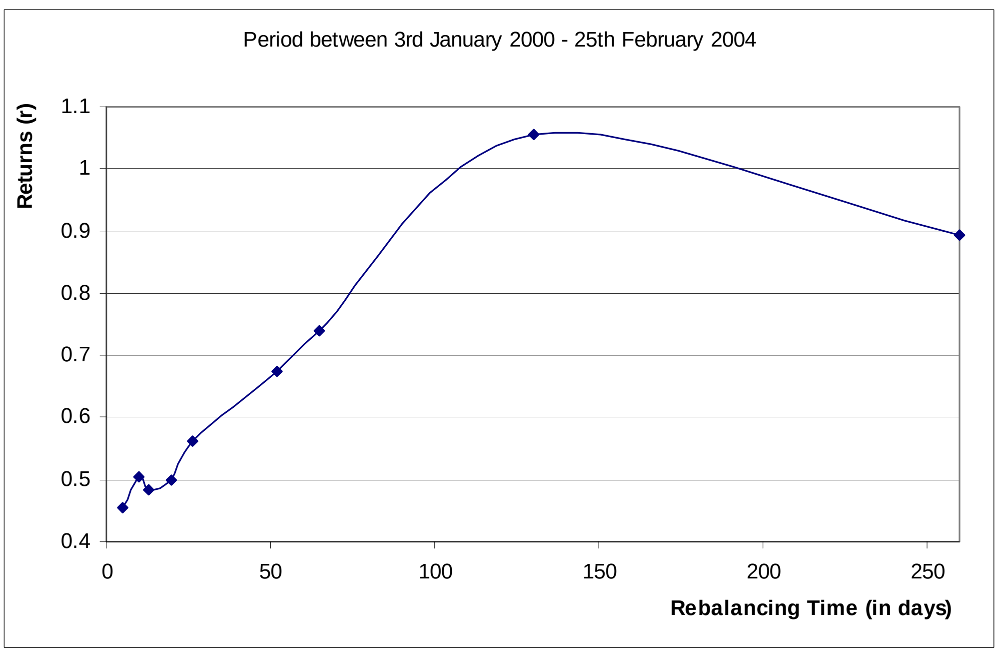
Period between 2nd January 2003 and 14th January 2004, 52 weeks history, transaction costs set at 5%.



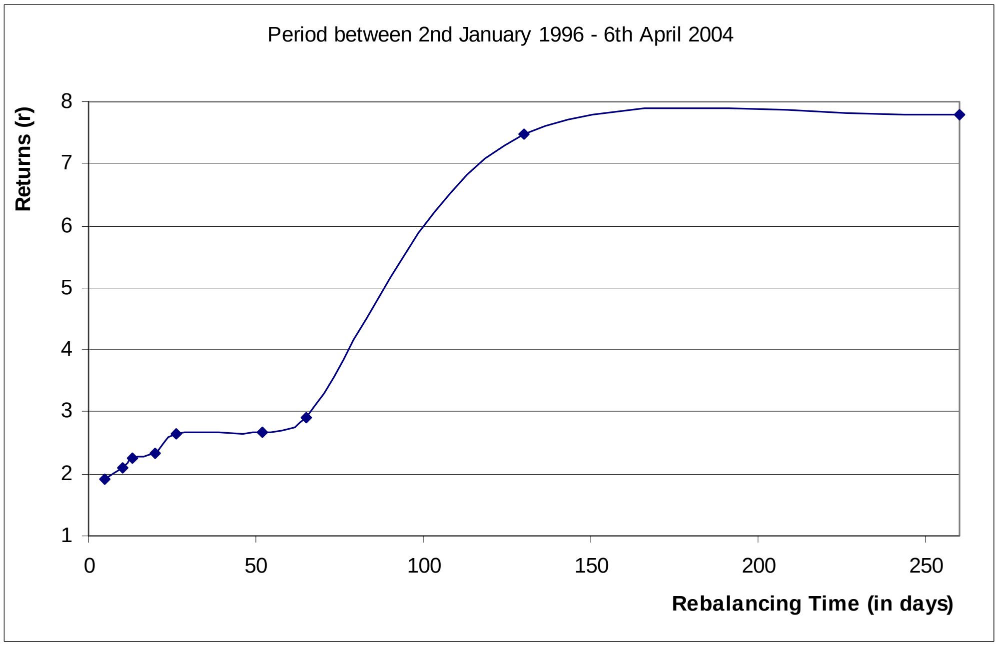
Similar cause as the previous year.



Period between 2nd January 1996 and 3rd February 1999, 52 weeks history, transaction costs set at 5%.



Period between 3rd January 2000 and 25th February 2004, 52 weeks history, transaction costs set at 5%.



Period between 3rd January 1996 and 6th April 2004, 52 weeks history, transaction costs set at 5%.
This chart represents the performance of the portfolio over 2 kind of market. The bull market before year 2000 and the bear market after 2000.

6. Conclusion and suggestions

6.1 Conclusion

The expected curve of assumption 1 can be seen in some of the results. It depends on the stock movement and the covariance matrix at that time. In the results too frequent updating the portfolio shows a negative impact in the performance. This can be explained by the insufficient stock returns to cover the transaction costs even if we have perfect information. Another point is that large covariance matrix changes can have a negative impact on the portfolio. Strong changes in the covariance matrix will lead to big changes in the portfolio at one time and the other time. This will increase the transaction costs due to bigger order value. If the stock returns are lower than the transaction costs (in this case set at 5%) the most logical step is not the update the portfolio. Update it later when the stocks perspectives are better than the transaction costs.
Here beneath a summary of all results are put in one table. The majority of the results conclude that updating the portfolio 1 or 2 times a year is the sufficient enough.

Period	Optimal interval
1996-1997	130 days
1997-1998	13 days
1998-1999	52 days
1999-2000	260 days
2000-2001	130 days
2001-2002	26 days
2002-2003	260 days
2003-2004	260 days
1996-1999	130 days
2000-2004	130 days
1996-2004	260 days

6.2 Suggestions for further research

The experiment ran here can be used as a foundation for further research. Some suggestions are:

- Make the experiment more realistic by
    1. Only rebalance the portfolio if the perspective of the new portfolio is better than the current portfolio.
    2. No perfect information. Calculating the expected returns of the stocks. This can be done using the theories previous discussed in chapter 2. The CAPM, APT and the Wiener process are examples to calculate it.
- Using different kind of methods to calculate the covariance matrix like:
    - Exponential declining weight (Riskmetrics)
    - Structural risk models[11].
- Using variable historical data to calculate the covariance matrix and the expected

returns. Like for every frequency of rebalancing the portfolio at least 5 data are required. For example rebalancing every year requires 5 years of historical data and for every month 5 month of data.

- Research the relationship between the covariance matrix and the return of the portfolio along with the impact of transaction costs.

References

[1] H.M. Markowitz, “Portfolio Selection”, Journal of Finance 7 (March 1952), pp. 77-91.

[2] R. Brealey & S. Myers, “Principles of Corporate Finance”, 7th edition, 2003, McGraw-Hill, pp. 168.

[3] Index Funds Advisors, URL:<http://www.ifa.com/12steps/Step2/step2page2.asp>, used in 9-feb-2005.

[4] R. Brealey & S. Myers, “Principles of Corporate Finance”, 7th edition, 2003, McGraw-Hill, pp. 169-170.
R.E. Oberuc, “Dynamic Portfolio Theory and Management”, 2004, McGraw-Hill, pp.3-7.

[5] C.R.S. Dougherty, “Introduction to Econometrics”, 2nd edition, 2002, Oxford University Press, pp. 36.

[6] R.C. Grinold & R.N. Kahn, “Active Portfolio Management, 2nd edition, 1999, McGraw-Hill, pp. 32-33.
MoneyChimp, URL<http://www.moneychimp.com/articles/risk/sharpe_ratio.htm>, used in 9-feb-2005.

[7] MoneyChimp, URL<http://www.moneychimp.com/articles/risk/sharpe_ratio.htm>, used in 9-feb-2005.

[8] R. Brealey & S. Myers, “Principles of Corporate Finance”, 7th edition, 2003, McGraw-Hill, pp. 195-196.

[9] R. Brealey & S. Myers, “Principles of Corporate Finance”, 7th edition, 2003, McGraw-Hill, pp. 204-205.
R.C. Grinold & R.N. Kahn, “Active Portfolio Management”, 2nd edition, 1999, McGraw-Hill, pp. 173-181.

[10] R. Brealey & S. Myers, “Principles of Corporate Finance”, 7th edition, 2003, McGraw-Hill, pp. 218-222.

[11] R.C. Grinold & R.N. Kahn, “Active Portfolio Management”, 2nd edition, 1999, McGraw-Hill, chapter 3.