Operational Risk

Analysing and Scaling Operational Risk Loss Data

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Master Thesis

By

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1 Introduction

1.1 The importance of operational risk

In the last ten years, financial institutions have become more aware to their exposure to operational risk, next to their exposure to other types of risk. A widely accepted definition of operational risk is 'the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events' ([7], [8], [9], [10], [11] and [12]). Historical events have shown that financial losses occurred due to operational risk can be fatal. A 233 year old bank in the United Kingdom, Barings, had gone bankrupt due to $1.3 billion loss resulting from rogue trading activities of Nick Leeson [30]. Toshihide Iguchi trading scandal had brought Daiwa Bank to suffer $1.1 billion loss [24].

Figure 1 – operational risk: Nick Leeson and the bankruptcy of Barings

For this reason, financial institutions need to manage their exposure to operational risk to ensure its continuity. Nevertheless, unexpected losses can still happen even though the financial institutions completely manage their exposure to operational risk. One obvious example is the 9/11 attack to the World Trade Centre New York in 2001.

Figure 2 – operational risk: terrorist attack at September 9th, 2001 on World Trade Centre, New York

One approach to cover a financial institution's exposure to operational risk is by holding some amount of capital in order to cover the loss from such events. This approach was first applied to credit risk and explicitly given in Basel I Accord published in 1988. This Accord is issued by Basel Committee on Banking Supervision (a committee that represents the central banks of the G-10 countries). The Accord states that financial institutions must hold a separate capital (also known as a regulatory capital) for its exposure to credit risk. Later in 1995, the Basel Committee required financial institutions to reserve a regulatory capital charge for market risk, another type of risk considered important to be treated separately.

1.2 Quantifying operational risk

In June 1999 the Committee published a consultative document "A New Capital Adequacy Framework" [5], which proposed an explicit regulatory capital charge to operational risk following earlier capital treatments of credit risk and market risk. This framework is at latter time known as the Basel II Accord. Reflecting from major events due to operational risk in the last ten years, the Committee believes that the operational risk is too important and therefore should be treated separately within the capital framework. The framework is designed to be more risk sensitive in the way the regulatory capital requirements reflect underlying risks of banks. It is also aimed to recognise the improvement in risk measurement and control that have occurred.

As financial institutions start to put attention to operational risk management, while others have begun on earlier time, the Basel II Accord (1999) does not explicitly define the exact way to calculate the capital for operational risk (operational risk capital). Instead, the Committee let the financial institutions at that time to come up with ideas and intended to collect some of these ideas in the Committee's subsequent paper.

In January 2001, the Committee released a consultative document regarding Operational Risk [7], which followed by a working paper on the regulatory treatment of operational risk eight months later [8]. In this paper the Committee suggested several approaches to calculate the regulatory capital for the bank's exposure to operational risk. The approaches are given in a 'continuum' of increasing sophistication and risk sensitivity: (1) Basic Indicator Approach, (2) Standardised Approach and (3) Advanced Measurement Approaches (AMA). Full description about these approaches will be given in chapter 2. There are strict requirements that must be met before a bank is considered qualified to use AMA. One of Basel requirements to apply AMA in order to calculate operational risk capital is that a bank must collect its historical operational loss data. In general, AMA calls for greater cost than simpler approaches (collecting and maintaining loss data has already become an extra cost to the bank).

However, there are several benefits in using the advanced approaches. The Basel Committee has stated that the level of capital required under the AMA will be lower than under the simpler approaches to encourage banks to make the improvements in risk management and measurement needed to move toward the AMA [8]. The difference in the capital reserve could then be used for a bank's business activities. Additionally, the improvements in risk management and measurement are expected to lead the bank to a less number of losses or less severe losses in comparison to the number of losses or loss amounts in the past.

Second, qualifying for the advanced approaches is a positive value for the market participants, which in result is creating a good image for the bank. A final reason to use AMA is by following the movement in a bank’s peers. If other peer banks are using AMA, the bank will be more or less compulsory to apply also AMA in its operational risk measurement system. This is necessary to maintain the credibility of the bank in the eyes of the market and regulators (e.g. the central bank).

ABN AMRO, one of major banks in the Netherlands in particular and on international level, is also required to hold a separate capital for operational risk. Due to its position as an internationally active bank, ABN AMRO is committed to use AMA for its capital calculations. Since ABN AMRO bank is planning to apply AMA in the future to calculate its operational risk capital, operational losses occurred within the bank (internal data) are collected. At this moment ABN AMRO has not meet the minimum historical loss data needed to qualify for AMA (Basel requires collection of historical loss data of minimum 3 years).

In addition, the Committee also required banks to make use of data from other banks (external data) since loss experience of a bank alone might not be sufficient to represent the actual risk behaviour of the bank. The use of external data is compulsory, in particular when there is reason to believe that the bank is exposed to high severity-infrequent losses. ABN AMRO and several other banks have decided to join a consortium where they periodically send their internal loss data and receive loss data of other members. These loss data of other banks becomes the external data to ABN AMRO, which then can be used to its Operational Risk Modelling.

I got the opportunity to work as an intern for ABN AMRO in the Operational Risk Policy & Support (ORP&S) department. The two main tasks of this group are first, to construct policy, and second, to support, the operational risk management of the bank's Business Units[1] all over the world in the context of operational risk. Several examples are: training delivery, workshop, etc.

1.3 Goal of the thesis

During the internship I was given the task to analyse the historical operational loss data of the bank. Another task was to propose a technique to incorporate the external data into the internal data.

Therefore, the objectives of this thesis can be stated as follow:

1. Analyse the historical operational risk loss data of the bank
2. Propose a scaling mechanism to incorporate the external data into the internal data of the bank.

Directly including the external data into the internal loss data of the bank is not advised, because banks are very likely to be different in size, characteristic, control process, etc. The scaling mechanism is intended to remove banks' specific characteristics, so that the external data can be considered to have the same characteristics as the internal data. If this objective can be realised, we will be allowed to add the external data into the bank's internal data and to use both data altogether.

1.4 Methodology

In order to accomplish these goals, the first thing I did was a literature study on operational risk papers and Basel papers. This is necessary to: (1) give a description of operational risk as one of the important financial risks that are exposed to financial institutions, and (2) give a detailed explanation regarding the Basel Accord as the guideline to quantify operational risk.

[1] ABN AMRO Bank can be divided into smaller Business Units based on business activities (e.g. Business Unit Wholesale Clients) or region (e.g. Business Unit Brazil).

I have also spent my time to examine the characteristics and the differences between the internal and the external loss data. A full description of these loss data can be found in Chapter 3. A thorough study on the historical loss data of the bank has been performed in order to achieve the first goal. The internal loss data was investigated to test the following hypothesis:

Hypothesis: ‘There is a linear relationship between two attributes of an operational risk loss event, namely: (1) the loss amount and (2) the time interval between the moment an event is discovered and the moment the event is recognised as an operational risk loss event‘.

As a result, I could not find any linear relationship between attribute (1) and attribute (2) and therefore I had to reject the abovementioned hypothesis. I found out that attribute (2) is more related to the characteristic of a Business Unit and to the method to recognise the operational loss events in that Business Unit. The results of this study will be given in detail in Chapter 4.

To accomplish the second goal, we started by inspecting the relationship between the operational loss amount incurred in a financial institution within a certain time period and an indicator of size & exposure towards operational risk of that financial institution within a certain time period. We found that the power-law form can be used to explain this relationship.

Based on the existence of the power-law relationship, we were able to apply the scaling mechanism to remove financial institutions’ specific characteristics, so that the external data can be considered to have the same characteristics as the internal data. We would also show how to add the external data into the bank’s internal data and to use both data all together. Full explanation about this study will be given in Chapter 5.

1.5 Thesis structure

The thesis is organised as follows. In Chapter 2 we start by explaining different types of financial risks exposed by financial institutions. Afterwards, a definition of the operational risk and the way operational risk can be managed are presented. In Section 2.4, a description of the Basel II Accord is given, which leads to an explicit capital charge for operational risk. The description includes the approaches proposed by the Basel Committee to calculate the operational risk capital.

We continue by describing the characteristics of historical operational risk loss data, internal as well as external, and the differences between the two databases in Chapter 3. In Chapter 4, a comprehensive analysis of internal operational risk loss data will be presented. In this chapter, we describe the results of the linear relationship test between the loss amount and the IDR (Interval between Discovery and Recognition time) of operational loss events. The scaling mechanism to incorporate the external data into the internal data of the bank will be given in detail in Chapter 5. The conclusions are finally given in Chapter 6.

2 Operational Risk

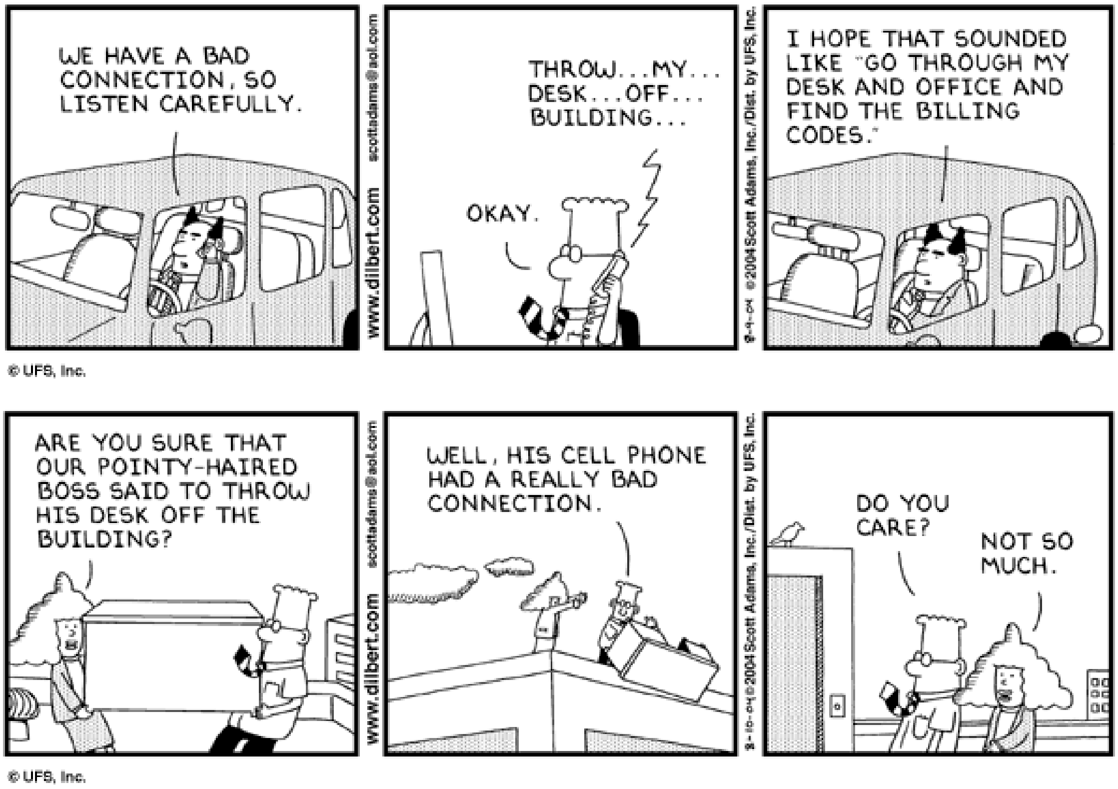


Figure 3 – An example of operational risk. Available at www.dilbert.com

In this chapter we will start by defining several types of financial risks exposed by financial institutions. We will continue with explaining operational risk as one of these financial risks, the need to manage operational risk by financial institutions. A description of the Basel Accord that serves as the guideline to manage and measure operational risk will be given in the subsequent section. In this section we will also describe the proposed approaches to quantify operational risk capital that banks must allocate to cover its exposure to operational risk. Finally, a review regarding these approaches will be given in the final section of this chapter.

2.1 Financial Risks

A financial institution is exposed to financial risks in its daily activities. Situations like system downturns, counterparty default, increase of oil price, terrorism attacks, etc. can bring significant losses to the institution. This can uppermost lead the institution to bankruptcy. It is therefore important to identify the exposure of financial institution to such risks. Afterward, necessary actions can be undertaken whereas its exposure to the risks is minimised (controlled).

In general, financial risks can be classified into the broad categories of credit risk, market risk, liquidity risk, legal risk and operational risk [24]. Financial institutions, in particular banks, are especially exposed to credit risk, which can be defined as the risk of loss due to counterparty default [3]. This condition is straightforward, since one of banks' main services is to provide loans to other parties. If a borrower defaults, the bank will suffer loss (which can range from minor to massive loss).

Market risk is also considered to be important to financial institutions. Market risk is defined as the risk of losses in on-balance- and off-balance-sheet positions arising from movements in market prices [4]. The risks subject to this definition are (a) the risks relevant to interest rate related instruments and equities in the trading book and (b) foreign exchange risk and commodities risk throughout the financial institution [4].

Liquidity risk takes two forms: market/ product liquidity and cash flow/funding. The first form arises when a transaction cannot be conducted at prevailing market prices due to insufficient market activity. The second form refers to the inability to meet cash flow obligations, which may force early liquidation of a financial institution [24].

Legal risk arises when counterparty does not have the legal or regulatory authority to engage in a transaction. This type of risk also includes compliance and regulatory risks, which relate to activities that might violate regulations of the government. Examples are market manipulation and insider trading [24].

The latest type of risk, operational risk, is actually the oldest risk facing banks and other financial institutions. Even before any financial institution decides on its first market trade or credit transaction, it will face operational risk long before then. Credit card fraud is a common example of loss resulting from operational risk. This kind of loss happens frequently within a bank, although the amount of loss might not be too significant. Nevertheless, operational risk can be one of the most devastating risk and the most difficult to foresee. The terrorism attack on the World Trade Centre on September 2001 and the Barings tragedy in 1995 are apparent examples of gigantic losses due to operational risk. High rate of technological change, globalisation, mergers and e-commerce have suggested that operational risk is increasing in financial institutions. The focus of this thesis is on operational risk.

2.2 Definition of operational risk

In the first place, operational risk was defined as other risk outside credit risk and market risk. Nevertheless, this definition was considered too broad and at present the definition of operational risk frequently used is the one proposed by the Basel Committee on Banking Supervision (BCBS)[2] in its 2001 paper [7]: ‘the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events’. Legal risk is included in this definition of operational risk. Legal risk itself includes, however is not limited to, exposure to fines, penalties, or disciplinary damages as result from supervisory actions, as well as private settlements [12]. On the other hand this definition excludes strategic and reputational risks. Strategic risk is where, for example, a loss results from a misguided business decision, while reputational risk is where decline in the firm’s value results from a damaged reputation.

[2] The Basel Committee on Banking Supervision is a committee of banking supervisory authorities, which was established by the central bank Governors of the Group of Ten countries (G-10) in 1975. It consists of senior representatives of bank supervisory authorities and central banks from Belgium, Canada, France, Germany, Italy, Japan, Luxembourg, The Netherlands, Spain, Sweden, Switzerland, United Kingdom and

2.3 The management of operational risk

In the past, financial institutions have focused mainly on credit and market risk management. The perception that operational risk has increased noticeably over recent years, while in the same time quantitative approaches to credit and market risk management do not take into account operational risk, have brought many banks to start paying more attention to operational risk. On the other hand, the fact that the risk of substantial loss from operational malfunctions was not adequately managed or measured has led regulators to issue guidelines to their members. The Basel Committee published The New Capital Adequacy Framework in 1999 to encourage improvement in operational risk management by introducing an explicit capital charge for operational risk and thereby generating incentives for banks to measure and monitor operational risk.

To begin with, in the next subchapter we shall describe thoroughly the Basel II Accord as the guideline for managing operational risk. As mentioned before, the Accord requires financial institutions to hold a capital amount to cover its exposure to operational risk. Proposed approaches to calculate the operational risk capital by the Basel Committee will be explained as well.

2.4 Basel Accord for Operational Risk

The Basel I Accord in 1988 is an agreement among the G-10 central banks that join the Basel Committee to apply capital standards to credit risk, the main risk incurred by banks, to banking industries. This Accord is followed by an Amendment in 1995 to adjust the regulatory capital charge to include market risk. The Accord was actually intended for internationally active banks in the G-10 countries. It has however been widely adopted and applied throughout the world; not only to internationally active banks, but also in many countries to domestic banks.

United States. It usually meets at the Bank for International Settlement in Basel, where its permanent secretariat is located.

In June 1999, Basel proposed a new capital adequacy framework in order to replace the 1988 Accord. The activities of banks (and thus their risk profiles) are becoming more diverse and complex due to globalisation of financial services, growing sophistication of financial technology. Developing banking practices at internationally active banks such as the growing use of e-commerce, outsourcing, highly automated technology and large-scale mergers may produce significant other type of risk, namely operational risk. In this new framework, the Committee proposed to develop capital charge for operational risk, another type of risk considered to be substantial next to credit and market risk. Together with the capital charges for credit risk and market risk, the sum of these three capital charges forms the economic capital (EC) of the bank.

In the January 2001 Consultative Document, operational risk was defined as 'the risk of direct or indirect loss resulting from inadequate or failed internal processes, people and systems or from external events'. As mentioned before, this definition includes legal risk, but strategic and reputational risks are not included for the purpose of a minimum regulatory operational risk capital charge [7].

However, there were concerns about the precise meaning of 'direct and indirect losses'. It is not the intention of the capital charge to cover all indirect losses or opportunity costs (as stated above that strategic and reputational risks are not included). The definition was then slightly revised to *'the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events'* [8]. This definition maintains the inclusion of legal risk and the exclusion of strategic and reputational risks.

The New Accord has put its attention again on internationally active banks, although the underlying principles should be suitable for application to banks of varying levels of complexity and sophistication. This new capital framework consists of three pillars, namely: *(1) minimum capital requirement, (2) supervisory review process, and (3) effective use of market discipline*. In reporting a revised capital framework to the extent of Basel I Accord, the importance of minimum regulatory capital continues to be recognised. An explicit capital charge to operational risk is added to the previous capital charge of credit risk and market risk, with a risk horizon of one year [12]. This is the first pillar of the framework.

The second pillar, *the supervisory review of capital adequacy*, is considered to be an integral and critical part of the operational risk capital framework. Several examples of supervisors are De Nederlandsche Bank in The Netherlands, Federal Reserve Bank in United States, Bank of England in England, etc. The goal of Pillar 2 is to ensure that the capital position of a bank is consistent with its overall risk profile and strategy. It enables also early intervention from supervisors if the capital charge does not provide a sufficient buffer against the risk. The Pillar 2 framework is based around four key and complementary principles [12]:

- A bank should have a process for assessing its overall capital adequacy in relation to its risk profile, as well as a strategy to maintain its capital levels.
- Supervisors should review and evaluate a bank's internal capital adequacy assessment and strategy, as well as its compliance with regulatory capital ratios. Supervisors should take supervisory action if they are not satisfied with the result of this process.
- Supervisors expect banks to operate above the minimum regulatory capital ratios, and should have the ability to require banks to hold capital in excess of the minimum regulatory capital.
- Supervisors should seek to intervene at an early stage to prevent capital from falling below the minimum levels required to support the risk characteristics of a particular bank and should require rapid remedial action if capital is not maintained or restored.

The third pillar, *market discipline*, will encourage high disclosure standards and enhance the role of market participants in encouraging banks to hold adequate capital. The purpose of Pillar 3 is to complement the minimum capital requirements (Pillar 1) and the supervisory review process (Pillar 2). The Basel Committee aims to encourage market discipline by developing a set of disclosure requirements which will allow market participants to assess key pieces of information on the scope of application, capital, risk exposures, risk assessment processes, and hence the capital adequacy of the financial institution. Such disclosures to the market have particular relevance under the Capital Framework, where the use of internal methodologies gives banks more discretion in assessing capital requirements. Generally, banks disclosures should be consistent with how senior management and the board of directors assess and manage the risks of the bank [12].

The disclosures should be made on a semi annual-basis, subject to some exceptions. Qualitative disclosures that provide a general summary of a bank's risk management objectives and policies, reporting system and definitions may be published on an annual basis. Large internationally active banks and other significant banks must disclose their total capital adequacy ratios on a quarterly basis. In all cases, banks should publish material information as soon as this becomes available.

The nature, components, features and composition of capital will provide market participants with important information about a bank's ability to absorb financial losses. A bank's risk exposure provides information about the stability of an institution's financial position and the sensitivity of its earnings potential to changes in underlying market conditions. Finally, market discipline carries out an essential role to ensure that the capital of banking institutions is hold at sufficient levels. Effective public disclosure enhances market discipline and allows market participants to assess a bank's capital adequacy and can serve as strong incentives to banks to carry out their business in a safe, sound and efficient manner [6].

Pillar three is thus proposed to enhance transparency and disclosure of information, which in follow will strengthen market discipline and effective banking supervision. The enclosure of the third Pillar in the new capital adequacy framework is intended to provide banking institutions specific guidance in the important area of capital. This will support and enhance both Pillar 1 and Pillar 2 of the framework by allowing the market to make an informed assessment of a bank's overall capital adequacy.

Quantifying Capital for Operational Risk

In the Consultative Document of Operational Risk, Basel proposes 3 following approaches to the capital assessment of operational risk in a continuum of increasing sophistication and risk sensitivity [7]:

1. The Basic Indicator Approach (BIA)
2. The Standardised Approach (STA)
3. Advance Measurement Approaches (AMA)

Banks are encouraged to move along the available approaches as they develop more sophisticated operational risk measurement systems and practices. Refinements to the Approaches are given in the '*Working Paper on the Regulatory Treatment of Operational Risk*' [8] as well as in the '*International Convergence of Capital Measurement and Capital Standards*' refined framework of Basel [12]. The measurement methodologies of each approach shall be explained broadly in the following sub-chapters.

2.4.1 The Basic Indicator Approach (BIA)

The Basic Indicator Approach is the simplest method, which estimates economic capital using a single exposure indicator (EI) that serves as a proxy for an institution's overall operational risk exposure. The Basel Committee proposed gross income as the exposure indicator. Gross Income is defined as net interest income plus net non-interest income. A detailed explanation of Gross Income is given in paragraph 650 of [12].

The Basic Indicator Approach is proposed to be applicable to any bank regardless of its complexity or sophistication. However, banks using this approach are encouraged to comply with the Committee's guidance on Sound Practices for the Management and

Supervision of Operational Risk, February 2003 [11]. Internationally active banks and banks with significant operational risk exposures are encouraged by the Committee to use a more sophisticated approach than the Basic Indicator Approach.

The capital for operational risk is equal to the average over the previous three years of a fixed percentage (denoted α) of positive annual gross income. Note that the three-year average is a modification proposed by Basel in [12] in comparison to previous papers of Basel ([7], [8]). For any year in which annual gross income is negative or zero, this figure should not be taken into account from both the numerator and denominator when calculating the average. The formula can be given as follows [12]:

$$K_{BIA} = [\Sigma\{GI_{1\ldots n} \times \alpha)]/n$$

Equation 1

K_{BIA} = the Basic Indicator Approach capital charge

GI = annual gross income, where positive, over the previous three years

n = number of the previous three years for which gross income is positive

α = 15%

The calculation of α is done in the following manner. The Committee has undertaken a data collection and analysis exercise – the Quantitative Impact Study (QIS) – to collect data to support the calibration of the capital charge. The QIS survey asked banks to provide information concerning their minimum regulatory capital, internal economic capital allocations, and gross income, both in the aggregate level and, in some cases, broken down by business lines. Minimum regulatory capital is calculated as 8 percent of a bank’s risk-weighted assets for the year in question [8]. The Committee examined then the relationship between economic capital allocated to operational risk and minimum regulatory capital. The result suggests that a reasonable level of the overall operational risk capital charge would be about 12 percent of minimum regulatory capital. It was first assumed that the capital charge under the BIA is based on an overall level of 12 percent of minimum regulatory capital. The data reported in the QIS concerning banks' minimum regulatory capital amounts and gross income was then used to calculate individual alphas for each bank for each year. The formula is given by [8]:

$$\alpha_{j,t} = \frac{12\% \times MRC_{j,t}}{GI_{j,t}}$$

Equation 2

In above equation, $MRC_{j,t}$ is minimum regulatory capital for bank j in year t and $GI_{j,t}$ is gross income for bank j in year t. Given these calculations, the distribution of alphas across banks in the sample was examined. Based on this distribution, the value of α is set by the Basel Committee to be 15% [12], so that the produced regulatory capital figures are approximately consistent with an overall capital standard of 12 percent of minimum regulatory capital. Variable α relates the industry wide level of minimum regulatory capital to the industry wide level of indicator (gross income).

2.4.2 The Standardised Approach (TSA)

The Standardised Approach can be seen as a further refinement along the range of approaches for operational risk capital. Banks' activities are divided into eight business lines: corporate finance, trading & sales, retail banking, commercial banking, payment & settlement, agency services, asset management, and retail brokerage (These business lines are given in Appendix I). Within each business line, an indicator is selected that reflects the size or volume of banks' activities in that area. The indicator can be seen as a proxy for the scale of business operations and thus the likely scale of operational exposure within each business line.

The Basel Committee proposes gross income to be used as the indicator in all business lines for the sake of simplicity, comparability, reduction of arbitrage possibilities. Next to these reasons, the most significant reason for using gross income is a lack of evidence of greater risk sensitivity of other indicators [8].

The capital charge within each business line is calculated by multiplying gross income by a “beta” factor. Beta serves as a proxy for the relationship between the industry’s operational risk loss experience for a given business line and the aggregate level of gross income for that business line. Please note that gross income is calculated for each business line, not for the whole institution, i.e. in asset management, the indicator is the gross income generated in the asset management business line. The three-year average of the summation of the regulatory capital charges across each of the business lines in each year will result as the total capital charge. The formula can be given as [12]:

$$K_{TSA} = \{\Sigma_{years 1-3} \max[\Sigma(GI_{1-8} \times \beta_{1-8}), 0]\} / 3$$

Equation 3

K_{TSA} = the Standardised Approach capital charge

$GI_{1\text{-}8}$ = annual gross income in a given year, as defined in the Basic Indicator Approach, for each of the eight business lines

$\beta_{1\text{-}8}$ = a fixed percentage, which is set by the Basel Committee, relating the level of required capital to the level of the gross income for each of the eight business lines

It is possible that in any given year, negative capital charges - resulting from negative gross income - in any business line may offset positive capital charges in other business lines without limit (national supervisors may implement a more conservative action to negative gross income) . Nevertheless, when the capital charge sum from all business lines within a given year is negative, the input to the numerator for that year will be zero (as given by the *max* function in the Equation 3 above). It is further noted in ([12], footnote 99] that if negative gross income distorts a bank’s Pillar 1 capital charge under the Basic Indicator and Standardised Approach, supervisors will consider appropriate supervisory action under Pillar 2.

The values of the betas are given in the following table [12]:

Business Lines	**Beta Factors**
Corporate finance (β_1)	18 %
Trading and sales (β_2)	18 %
Retail banking(β_3)	12 %
Commercial banking (β_4)	15 %
Payment and settlement (β_5)	18 %
Agency services (β_6)	15 %
Asset management(β_7)	12 %
Retail brokerage (β_8)	12 %

Table 1 – Beta values of each Line of Business

In order to calculate the beta factors, the baseline assumption was that the overall level of operational risk capital is 12 percent of minimum regulatory capital, the same with the BIA. The approach used was to estimate betas by business line for individual banks and then to examine the distribution of those betas across the sample banks. The QIS data regarding the distribution of operational risk economic capital and gross income across business lines was used for this purpose.

Information about the distribution of operational risk economic capital was used to distribute this regulatory capital amount across business lines. This business line regulatory capital figure was then divided by the business line gross income to arrive at a bank-specific beta for that business line. The following formula is used [8]:

$$\beta_{j,i} = \frac{12\% \times MRC_j \times OpRiskShare_{j,i}}{GI_{j,i}}$$

Equation 4

In above equation, $\beta_{j,i}$ is the beta for bank j and business line i, MRC_j is minimum regulatory capital for bank j, $OpRiskShare_{j,i}$ is the share of bank j's operational risk economic capital allocated to business line i. $GI_{j,i}$ is the gross income in business line i for bank j.

Under the standardised Approach, the minimum regulatory capital requirement for operational risk is calculated by dividing a bank's operations into eight separate business lines. The beta is thus relating the level of required capital to the level of the gross income for each of the 8 Lines of Business.

A bank that wants to use the Standardised Approach to calculate capital charge must meet the qualifying criteria given in paragraphs 660-662 of [12], while internationally active banks wish to use the Standardised Approach must meet the additional criteria in paragraph 663 of [12]. Basel mentions also in this paper another version of Standardised Approach, the Alternative Standardised Approach (ASA). National supervisor may allow a bank to use ASA only when this method provides an improvement, for example avoiding double counting of risk, compared to the original version. In ASA, the capital calculation of two business lines, retail banking and commercial banking, is different. For these business lines, average over the past three years of total outstanding loans and advances - multiplied by a fixed factor 'm' – is chosen as the exposure indicator (EI) instead of gross income. A broader description of ASA can be found in [12].

2.4.3 Advanced Measurement Approaches (AMA)

In comparison to the two approaches above, the Advanced Measurement Approaches are considered to be the most sophisticated approaches that currently are being developed for regulatory capital purposes. The operational risk capital requirement under the AMA would be based on an estimate derived from a bank's internal risk measurement system. The use of AMA will be subject to qualitative and quantitative standards set by the Committee in the revised Framework of June 2004 [12].

The qualitative standards would address the bank's operational risk management environment and the strategies to identify, assess, monitor, and control/mitigate operational risk. Internal operational risk measurement system must be closely integrated to the day-to day risk management processes, while regular reporting of operational risk exposures and loss experience to Business Unit management, senior management and board of directors is obligatory. Operational risk management system must be well documented, and auditor must perform regular reviews on both the operational risk management process and measurement systems.

The quantitative standards include a supervisory soundness standard that any internal risk measurement system must be consistent with the definition of operational risk. Basically, the quantitative standards have the following key features:

- The use of internal data
- The use of relevant external data
- Scenario analysis
- Business environment and internal control factors

The quantitative standards in short are given below:

- The operational risk capital requirement will be the unexpected loss in the total operational risk loss distribution corresponding to a confidence level of 99.9 percent and a risk horizon of one year. This approach is somewhat similar to the VAR (Value at Risk) method from the market risk world. VAR summarizes the expected maximum loss (or worst loss) over a target horizon within a given confidence interval [24]. A wide explanation regarding VAR can be found in [26]. Figure 4 illustrates the unexpected loss in the annual operational risk loss distribution [1]. The horizontal axis gives the possible annual operational risk loss amounts. The vertical axis gives the probability of the occurrence of particular loss amounts.

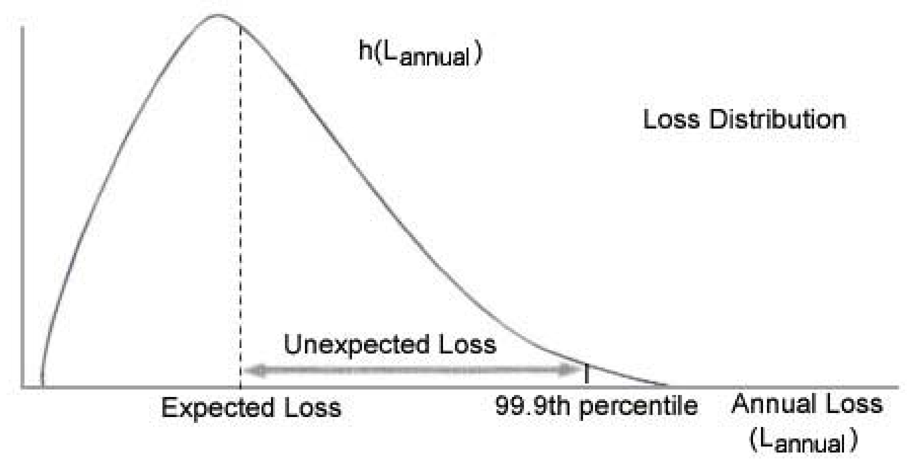


Figure 4 – Operational risk loss distribution

The unexpected loss is given by the difference between the 99.9th percentile and the expected loss in the annual operational loss distribution for the bank [1]. In this case, the 99.9th percentile is the VAR of annual operational risk loss amount within the confidence interval of 99.9%. The expected loss is nonetheless the mean of the operational loss distribution. Losses below the expected loss should be covered by provisions. Losses above the 99.9th percentile could bankrupt the firm, and therefore it is essential to control these losses. Capital charges are to cover losses in between these two limits. The term used by Basel for this is 'unexpected loss' [12]. If a bank is unable to show that its expected loss is covered by provisions, the capital calculation will then be the unexpected loss in the total loss distribution corresponding to a confidence level of 99.9% and a risk horizon of one year, plus the expected loss.

- A bank must be able to demonstrate that its approach captures potentially high severity loss events (the 'tail' of the operational risk loss distribution).
- A bank is required to calculate its regulatory capital requirement as the sum of expected loss and unexpected loss. It is allowed to base the minimum regulatory

capital on Unexpected Loss alone, subject to the condition that the bank can satisfy its national supervisor that its exposure to Expected Losses is sufficiently captured in its internal business practices.

- A bank may be allowed to use internally determined correlations' estimates in operational risk losses, provided systems for determining correlations are sound and implemented with integrity (conditional on satisfaction of the national supervisor).
- A bank must nevertheless be able to map its internal loss data into the supervisory categories defined in Appendix I and Appendix II for business lines and event type, respectively.
- Internal operational risk measures for regulatory capital purposes must be supported by a minimum of five-year observation period of historical internal loss data, although for the first transition period *a three-year historical data* is considered acceptable.
- A bank must have an appropriate gross loss threshold for internal data collection, e.g. € 10,000. Only losses equal or bigger than this amount are collected. This threshold may vary between banks and across business lines and/or event type within a bank. Nevertheless, thresholds chosen should be consistent with those thresholds used by peer banks.
- Next to the gross loss amounts, other information like the date of the event, recoveries of gross loss amounts, and descriptive information about the causes of the loss event should be collected. Information detail level will depend on the size of the gross loss amount.
- Relevant external data (public data and/or pooled industry data) must be used in a bank's operational risk measurement system. This is mainly necessary if the bank is exposed to high severity-low frequency losses. Determining which situations are suitable to use external data and methodologies to incorporate external data (e.g. *scaling*, qualitative adjustments, or to improve scenario analysis) should become a systematic process to a bank, subject to periodic review.
- Scenario analysis is used in combination with external data to assess a bank's exposure to events with high-severity losses. This approach draws on the knowledge

of experienced business managers and risk management experts to derive reasoned assessments of plausible severe losses. For example, these expert assessments could be used as parameters of an assumed statistical loss distribution. Scenario analysis should also be used to assess the impact of correlation assumptions rooted in the bank's operational risk measurement framework. In particular, the scenario analysis should also be used to evaluate potential losses arising from multiple simultaneous operational risk loss events. It must be noted that the assessments need to be validated and repeated over time; through comparison to actual loss experience to ensure their reasonableness.

- A bank's risk assessment methodology must denote *key business environment and internal control factors* that can change a bank's risk profile. By making use of these factors that directly reflects the quality of the bank's control and operating environments, more forward-looking risk assessments can be obtained.
- Finally, a bank is allowed to recognise the use of risk mitigation techniques (i.e. insurance) in calculating its regulatory capital charge.

Under AMA, a bank estimates the operational risk loss distribution for each business line/event type pair over some future horizon (e.g. one year). The operational risk capital requirement will then be the unexpected loss in the total loss distribution corresponding to a confidence level of 99.9% and a risk horizon of one year (or plus the expected loss, if a bank's expected loss is not covered by provisions). The overall operational loss distribution is usually based on assumptions about the frequency and severity of operational risk losses. The frequency of operational loss is the number of operational loss events occurred in a certain period (e.g. week, month, year, etc.). In other words, it is time-related. The severity of operational loss, on the contrary, is the financial loss amount for individual events. This means that the severity of operational loss is measured per individual event; thus it is not time-related.

A bank starts with estimating the shape of the frequency and severity distributions for each combination of business line and event type. This can be done by taking assumption of specific distributions for both the frequency and severity (e.g. Poisson distribution for the frequency and lognormal distribution for the severity [1]) or by empirically approximating the distributions with a technique such as Monte Carlo simulation. The obtained frequency and severity distributions are then compounded to produce the operational loss distribution. Monte Carlo simulation is the method used to generate an aggregate loss distribution from the frequency and severity distributions [1]. Because the operational loss distribution is aggregated from the frequency and severity distributions, the term 'Aggregate Loss distribution (ALD)' is sometimes used to denote the operational loss distribution.

The total capital charge can be calculated as the simple sum of the operational risk VAR (Value at Risk) for each business line/event type combination. Nevertheless, this calculation method implies an assumption of perfect correlation of losses across these pairs. Other methods that recognise non-perfect correlation might also be used. The correlation problem in operational risk is addressed in [21].

To conclude, the estimation of an operational loss distribution in AMA generally involves 3 steps [1]:

1. Estimating a frequency distribution

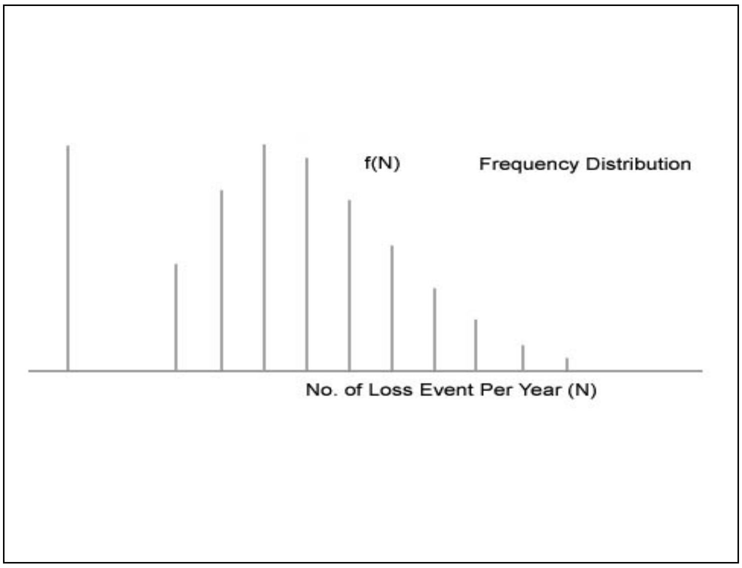


Figure 5 – Frequency distribution

The frequency distribution is given by the probability density function (hereby abbreviated p.d.f.) $f(N)$ above. The number of loss events per year is simply a single draw from the p.d.f. $f(N)$. For the years $1,\ldots,y$, the number of loss events for each year can be given by $n_1, n_2, \ldots, n_y$.

2. Estimating a severity distribution

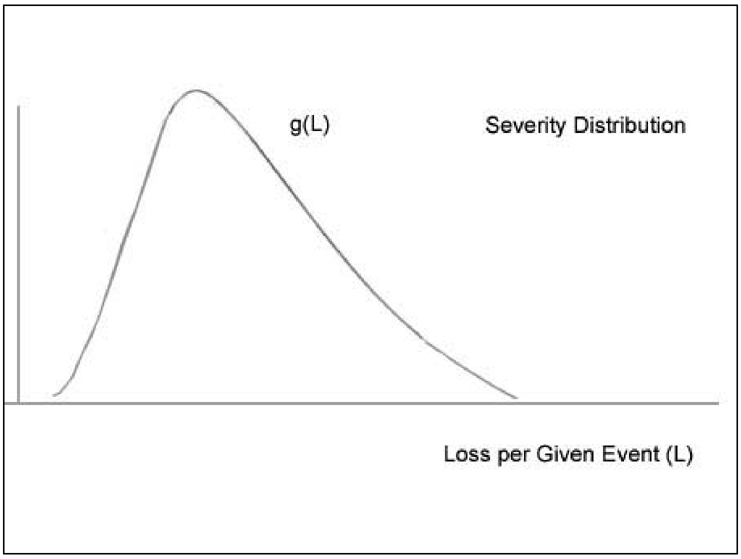


Figure 6 – Severity distribution

The severity distribution is given by the p.d.f. $g(L)$ above. Each loss amount L_i is then a single draw from p.d.f. $g(L)$.

3. Running a statistical simulation to produce a loss distribution

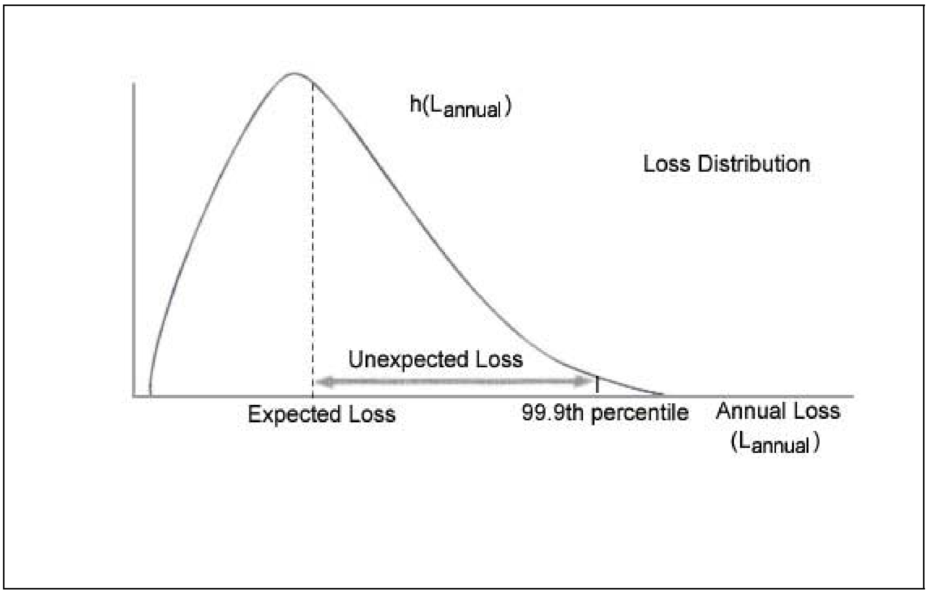


Figure 7 – Loss distribution

The total loss $L(y)$ for a particular year y can be calculated via:

$$L(y) = \sum_{i=1}^{n_y} L_i$$

Equation 5

If we run a statistical simulation so many times to get the total loss $L(y)$ value, the obtained total loss $L(y)$ values can then be used to produce a loss distribution such as the one in the Figure 7 above.

Basel was using the term Loss Distribution Approaches (LDA) to explain the method to estimate operational risk loss distribution by means of frequency and severity distributions [8]. The following two papers ([18] and [20]) addressed Loss Distribution Approaches in practice.

There are two main advantages in using AMA to quantify operational risk capital. The first advantage is the lower capital charge. The Basel Committee has stated that the level of capital required under the AMA will be *lower* than under the simpler approaches to encourage banks to make improvements in risk management and measurement needed to move toward the AMA [8]. The difference in the capital reserve could then be used for a bank's business activities. Additionally, the improvements in risk management and measurement are expected to lead the bank to a less number of losses or less severe losses in comparison to the number of losses or loss amounts in the past. The second advantage is that a bank will be allowed to recognise the risk mitigating impact of insurance in the measures of operational risk capital [12].

2.5 Review of the approaches to quantify operational risk capital

We have seen the approaches identified by the Basel Committee that can be used to calculate operational risk capital. The Basic Indicator Approach is simple and easy to use, but has as disadvantage that the resulting capital may not truly reflect the risk profile of the bank. We can also notice that the Basic Indicator Approach is a top-down approach, where the operational risk capital is calculated on the aggregate level and allocated afterward to the lines of business of the bank (lower level). Nevertheless, this might lead to an allocating problem, since the bank might need to determine what the best proportion is for each Line of Business.

In the Standardised Approach, the capital calculations are done at the level of lines of business of the bank instead of at the aggregate level. We can see that the Standardised

Approach is more a bottom-up approach, where calculations are done on a lower level and finally aggregated on the top level. A bank must however meet several qualitative standards before it is able to use the Standardised Approach. For internationally active banks that wish to use the Standardised Approach, the Basel Committee proposed several additional criteria. Therefore, this approach is considered to be more sensitive in comparison to the Basic Indicator Approach.

The Basel Accord expected internationally active banks and banks with significant operational risk exposures, to use an approach, that is more sophisticated than the Basic Indicator Approach and that is appropriate for the risk profile of the institution [12]. From all the approaches, AMA can be seen as the most sensitive approach since financial institutions must first meet the quantitative and qualitative standards before they are allowed to apply AMA to calculate the regulatory capital. The standards are more sophisticated in comparison to the standards to qualify for the Standardised Approach.

At this time, ABN AMRO is using the Standardised Approach to calculate its capital charge. However, ABN AMRO is also planning to apply AMA to calculate its operational risk capital in the future. One of the requirements to qualify for AMA is that a bank must have an internal risk measurement system and data collection of historical internal loss. Therefore, ABN AMRO has been collecting its operational loss data since January 2001. Basel requires a minimum three years of historical loss data during the first application, and afterwards a minimum data of five years. As mentioned before, financial institutions must also complement its historical internal data with relevant external data.

In the case of ABN AMRO, the external data is provided via a consortium that consists of 15 internationally active banks (inclusive ABN AMRO, the so-called ORX (**O**perational **R**isk data e**X**change). A member of this organisation will then have access to the operational risk loss information of all members, which is given in a standardised, anonymous and quality assured form. In the next chapter, we will describe the characteristics of operational risk loss data of the bank as well as of the ORX.

3 Operational Risk Loss Data

In comparison to the other two approaches (the Basic Indicator Approach and the Standardised Approach), the proposed Advanced Measurement Approaches (AMA) by Basel are intended to be the most risk sensitive approaches for quantifying operational risk capital and to relate the capital amount to the loss experience of each institution. As mentioned in the previous chapter, historical internal loss data is required for the capital calculations in the Advanced Measurement Approaches.

A bank is thus obliged to collect historical internal data in order to apply AMA. Nevertheless, historical internal loss data collected by banks may differ in several properties:

- Information detail level of operational loss events: the higher the detail level means more information can be obtained from the database, in other hand the cost (e.g. maintenance) might also become higher. A bank must find a way to find the best detail level to obtain data with adequate information and reasonable cost.
- Threshold level: loss events only with loss amounts greater than or equal to the threshold level are recorded in the database. The threshold level is set to balance two conflicting wishes: collecting loss events as many as possible while reducing costs by collecting only significant loss events.
- Method to identify operational loss event (e.g. event recognised directly after occurred, derived from general ledger)
- Incorporating external data into internal data (mapping and scaling problem)

In Section 3.2 we shall describe the characteristics of historical internal data of ABN AMRO. As for external operational loss data, it is needed to estimate the tail of the operational loss distribution of a bank implementing AMA, especially when there is a reason to believe that the bank is exposed to infrequent, but potentially severe losses. Another reason to use external loss data is that historical loss events in the bank might not be enough to obtain reliable results in terms of statistical analysis (e.g. parameters estimating process of a loss distribution will require the availability of enough loss data).

The Basel Committee mentions two types of external data:

1. Publicly released data: consist of losses that are considered too high not to be reported to public. Publicly released losses are usually of high amounts (for example: losses bigger than 1 million Euro) and can be found on the news
2. Peer-group data: collective data from a group of banks, where banks report their losses to a consortium and after some processing obtain their own internal losses plus loss data from other members.

Since ABN AMRO is a member of a consortium that provides historical operational loss data of the members, we will focus only on the second category of external data in this thesis. We will explain the characteristics of the peer-group data in Section 3.3. We begin in the next chapter with explaining the datasets we have attained in order to do our experiments.

3.1 The data sets

In order to carry out the analysis and scaling studies, we obtain the following datasets:

- Internal Data: historical loss data of ABN AMRO. This dataset contains the operational risk loss events of the 6 Business Units of the bank during the year 2002-2003. The Business Units are:
    a. Business Unit Asset Management & Trust (AM&T)
    b. Business Unit Brazil (BR)
    c. Business Unit Private Clients & New Growth Markets (PC&NGM)
    d. Business Unit Netherlands (NL)
    e. Business Unit Wholesale Clients (WCS)
    f. Business Unit North America (NA)

  We obtain operational risk loss events of the bank having Gross Loss amount of €20,000 or bigger (another way to mention it is a *threshold* value of €20,000). This is conformed to the €20,000 threshold value of the external data. Although the bank's loss data apply a threshold value of €5,000 – which means all loss events with Gross Loss amount of €5,000 or bigger are collected in the database –, we decide to match the threshold value with the threshold value of the external data. This is mainly done to make the threshold value consistent for the internal data as well the external data.

- External Data: historical loss data of peer-group banks in ORX consortium. This dataset consists of the operational risk loss events of the members of ORX during the year 2002-2003. ORX (**O**perational **R**isk data e**X**change) is a non-profit organisation initiated in January 2002, which is purposed to create a forum for the exchange of operational risk related loss information between its members in a standardised, anonymous and quality assured form. ORX intends to deliver high quality data of operational risk loss information, which closely aligns with regulatory requirements for business lines and event categories. There are strict requirements that must be met before a bank can join the ORX. An independent custodian is given the task to make the data anonymous, and afterward the data transmission is done in high security levels (the data is also only available to its members). The current members of the ORX consortium are [36] :

  1. ABN AMRO, Amsterdam
  2. Banca Intesa, Milan
  3. Banco Bilbao Vizcaya Argentaria, Madrid
  4. Bank of America
  5. The Bank of Nova Scotia, Toronto
  6. BNP Paribas, Paris
  7. Commerzbank AG, Frankfurt
  8. Danske Bank A/S, Copenhagen
  9. Deutsche Bank AG, Frankfurt
  10. Euroclear Bank, Brussels
  11. Fortis, Brussels
  12. HBOS plc, Edinburgh

13. ING, Amsterdam
14. JPMorganChase, New York
15. SanPaoloIMI SpA, Turin

In the following two sections we will explain the features of the internal loss data of ABN AMRO, followed by the features of the ORX data.

3.2 Internal Operational Risk Loss Data

Historic operational loss events of ABN AMRO are collected in the Corporate Loss Database (CLD), which started to be used since January 2001. The second version, CLD II, is rolled out in 2003 and provides improvements to increase data quality issue. In the CLD, not only the amount of loss event is recorded, but also a wide range of information in connection with the event is entered in the database, e.g.:

- Description of the background and circumstances of an operational loss event
- Loss amount in Euro and original currency. Losses not in Euro are converted to Euro by using the currency rate on the date of Occurrence of the incident.
- Name and location of Business Unit
- Event type
- Date of occurrence (the date when the event happened),
- Date of detection / discovery (the date when an ABN AMRO employee discovers that the event occurred)
- CLD entry date (the date when the operational risk loss event is recorded in the CLD)
- Direct recovery and indirect recovery (Insurance)
- Effect types, etc

For the purpose of operational risk loss quantification and the pooling of loss data across banks, distinctions are made between the causes, events, and effects of operational risk loss events. An *event* is something that happens at a certain moment in time. If we use the definition of operational risk as a basis, an event is defined as ‘a malfunction of internal processes, human behaviour, systems or an external fact’.

An event has one or more causes. A *cause* is ‘a prevailing circumstance that increases the probability of an event’. Causes are the circumstances that can lead to an event. If more than one cause is entered the first one will be displayed as ‘Primary Cause’, and any additional causes will be shown under ‘Other Causes’.

An event has a certain effect, ‘the impact of an event on the organisation’. Effects are normally described in term of (monetary) losses, but could also be described in more qualitative terms, for instance reputation damage. The effect types used in CLDII are:

- P&L Effect (in case of operational risk loss events).
- Regulatory Action (fines / licence revocations)
- Reputation Damage (in case the incident was mentioned in the newspaper)
- Client Complaint (in case a complain from a client was received)
- Other Effects (downtime, rework, opportunity costs, etc.)
- Not applicable (only in specific cases of ‘Near Misses’).

Operational Risk Near Misses is defined in ABN AMRO as operational risk events that did not materialise in a loss, i.e. for which no financial loss has been incurred. In other words, an operational risk near miss is a malfunction of internal processes, human behaviour, system or an external fact not leading to a Profit and Loss effect.

Finally, an operational risk *loss event* is defined as ‘a malfunction of internal processes, human behaviour, systems or an external fact leading to a financial loss (i.e. a negative impact on the earnings or equity value of the firm)’. ABN AMRO is using the event type categories of Basel in its internal loss data (Basel event type categories are given in Appendix II). The Level 2 of the Event Type categories are extended to meet the specific needs of data quality of the bank.

There are two other dates that are not recorded in the CLD, but are also noteworthy. These dates are:

1. Accounting date (the date when the loss amount of an operational risk loss event is booked in the General Ledger)
2. Recognition date (the date when an ABN AMRO employee acknowledges that the event qualifies for being registered in the CLD). Recognition of an operational risk loss event must be done as close as possible to the discovery of the event. Subsequently, the entry of a loss event in the CLD must happen as close as possible to the recognition date. Dates of occurrence, detection, and recognition are ideally not too far from each other. In ABN AMRO, the CLD entry date of an operational loss event is regarded the same as the recognition date.

The minimum requirements to report operational risk loss event in the CLD are:

1. Report loss events with a gross loss above € 5,000 (mandatory threshold). Operational risk losses below €5,000 and operational risk near misses may be recorded in the CLD but are not compulsory by the ORP&S.
2. Comply to the “Coverage, Completeness, Correctness” constraint:
    a. Coverage: All Business Units in all countries are able to report loss data
    b. Completeness: All events and all loss amounts are reported
    c. Correctness: All information per event is accurate and complete

Assuming full Coverage, operational risk loss recognition can be done by:

- Effect based: based on (analysis) of financial effects, events can be identified
- Event based: based on the identification of events, related effects can be collected.

There are three methods of recognition used in the bank:

1. Method 1 - Event Recognition: an employee discovers an event that leads to a financial loss.

2. Method 2 - Derived from General Ledger input: Accounting and / or Finance staffs assess accounting entries and recognise operational risk loss events.
3. Method 3 - Derived from General Ledger output: the operational risk management can refine operational risk loss data from General Ledger postings by using some kind of pre-defined logic.

Each method has advantages and disadvantages in regards to achieve full coverage, completeness, and correctness of the CLD data. The level of Coverage will be high for all methods when the CLD is run appropriately and the Finance and Accounting procedures are well written and distributed. The General Ledger Output method will most likely result in a high completeness level assuming that the pre-defined logic works adequately. The Completeness level on the other two methods will rely on the awareness and discipline of business staff for the 'Event Recognition' method, and of accounting staff for the 'General Ledger input' method.

In 'Event Recognition' method, the events are identified at the source, so that the employees from the Business Units will be aware of the details of the incidents. This in return will imply a high level of correctness. CLD entry takes place before General Ledger entry. Immediate recognition after discovery of an event is also possible in this method, which will lead to a small time interval between date of detection and date of recognition. It is possible that discovery and recognition of an operational risk loss event happen on the same day. However, reconciliation of the loss event to General Ledger data can become a cumbersome task.

Using General Ledger input, event recognition process is via accounting procedure and time of recognition is on a later period compared to the first method. Entry to the CLD is done by administrative unit and takes place after the General Ledger posting. The disadvantage is that loss data might not always be complete. It is easy to reconcile to General Ledger data, since they come from the same source. Correctness of the data depends on received information.

The third method is done by monthly upload of General Ledger data. Data enrichment can be done by CLD co-ordinator. CLD entry is the latest in comparison to the previous two methods. To ensure completeness, this method will depend very much on the accounting structure. Multiple entries in General Ledger per event will lead to multiple events. In the term of correctness, inconsistent identification of events might happen. Others disadvantages of this method are the difficulties to do event/cause categorisation and poor loss descriptions.

To conclude, no single method will meet full completeness and/or correctness requirements of the CLD. The best approach is to combine them by:

- Event recognition is the basis method in collecting operational risk loss data
- General Ledger inputs can be used to maximise completeness
- Finally, General Ledger outputs can be used for comparison/cross checking at aggregate level.

The following table gives the method used in each Business Unit to recognise an operational risk loss event:

Business Unit	Method of operational risk loss event recognition
BU AM&T	Method 1
BU BR	Method 2
BU PC&NGM	Method 1
BU NL	Method 1 (Starting from January 1st 2004 method 2 is applied)
BU WCS	Method 1
BU NA	Method 3

Table 2 – Method of recognition in each Business Unit of the bank

The internal database of the bank (CLD) has achieved the completeness level since 1 January 2004. The bank is currently striving to accomplish the correctness level. In the next subchapter we will describe the characteristics and guidelines of reporting of the external data.

3.3 External Operational Risk Loss Data

As mentioned before, the ORX peer-group data will serve as the external data to the bank. In the ORX consortium, the members are allowed to use different definitions and methodologies for internal loss recording, nevertheless required to use ORX standards in order to submit data to ORX. Submission of operational loss data is done on a quarterly basis, starting from joining the ORX (January 2002 for founding members). Basically, every member may choose which part of their internal data they want to submit. In this case, ABN AMRO decides to submit only the retail banking and asset management lines of business. After validation process and anonymity procedures are carried out, the total data are returned back to all members so those banks can use these data to their internal modelling.

In ORX Reporting Standards, an operational risk event is defined as ‘an incident leading to the actual outcome(s) of a business process to differ from the expected outcome(s), due to inadequate or failed processes, people and systems, or due to external facts or circumstances’ [28]. Losses arising from flawed strategic or discretionary processes are not recordable in the ORX database. Such losses are considered the result of business-strategic risk. Legal risk (the risk of loss resulting from failure to comply with laws as well as ethical standards and contractual obligations) is also included in the definition of operational risk. This is in line with the specification of the Basel Committee.

Any write-down due to loss of recourse may be considered credit loss. Nevertheless, events must be recorded in ORX where an operational risk event is a principal driver of the size of the loss. Losses in this “overlap” category type must be marked as “C” in the appropriate ORX submission field, as far as the amount is already in the credit risk database. Market risk events are not reportable to ORX, but reportable losses can occur when operational risk events cause losses in the marketplace (e.g. a security is sold when a buy was intended, etc…)[3]. Reputational risk (damage to the firm's reputation in the marketplace, the shareholder community, etc.) is not recordable as an operational risk loss. This is true where:

1. the entire impact of an event is reputational
2. reputational damage is only one impact of an event that has recordable losses as other impact (in this case only the recordable losses are submitted to ORX)

It is possible for an event to have multiple losses (or effects). In this case it is necessary to find the root event. Root event is the original incident without which none of the losses would have occurred. These effects can be divided into the following effect types (reporting this field is optional):

1. Legal liability
2. Regulatory action
3. Loss or damage to assets
4. Restitution
5. Loss of recourse
6. Write-down

An operational risk loss is 'a negative impact on the earnings or equity value of the firm' from an operational risk event. In general, an operational risk event is not subject to ORX reporting unless it has a quantifiable impact. Such impacts may be reflected anywhere in the books of the firm, and multiple impacts must be aggregated for submission.

Events excluded from ORX reporting are:

- Near misses (these are events that did not lead to operational risk losses)
- Opportunity costs / lost future business
- Events causing only reputational damage
- Events causing only gains or timing losses/timing impacts

[3] At present, ORX will not require these amounts to be marked as "market risk-related".

Timing impacts arising from operational risk events are not subject to reporting in ORX submissions. A timing impact is a temporary distortion to the aggregate P&L of a firm in a particular reporting period that can be fully corrected when later discovered. It results in P&L being shifted from one period to another. For example: a bank employee executes a payment transaction to a client twice. The client discovers receiving the payment twice and returns the money to the bank and thus the mistake is corrected[4].

The *threshold* amount, that determines whether an event should be reported to the ORX or not, is set to be € 20,000 by the ORX Committee. This threshold is applied to the Gross Loss amount, which means that Gross Loss amount greater than or equal to € 20,000 will be reported to the ORX. When reporting to ORX, member using other currencies other than EURO must convert their loss amounts to Euro base on the exchange rate as of the internal booking date of the event. In case of multiple events, or events with recoveries, the historic rate will be used, which means the rate applied to the first loss booked. Again, the accounting date must be the main driver for reporting purposes.

In some instances, operational risk losses can be reduced by recoveries. A recovery is 'an independent occurrence, separate from the original event, in which funds are recovered or contributed, usually from or by a third party'. The reporting threshold for ORX submissions applies to the gross amount of a loss. Please note that the recovery from an event can exceed the amount initially written-down; in such case a gain would occur in the event reported.

There are two types of recovery, direct or indirect. An indirect recovery is one that has been paid for in advance, while a direct recovery is one that is obtained without such payment. For example: a misdirected wire transfer not detected for several months, and once discovered the payment is not immediately returned on a voluntary basis. The firm

[4] At present, there is no time limitation for this kind of events.

books a loss. After negotiation, however, the firm is able to regain the funds; this is booked as a direct recovery.

Three dates must be submitted with each event record:

1. Date of occurrence: when the event happened or first began
2. Discovery date: when the event was first identified
3. Accounting date: when a loss was first posted to the General Ledger. If the loss is not posted to the General Ledger, use the recognition date of the loss (which could be different from the discovery date of the event) instead. The accounting data is the driver field for reporting purposes. If multiple losses are posted at different times in the General Ledger, the firs accounting data is always used.

Event Classification

ORX losses reflect underlying events. ORX event classification is consistent with Basel categories, although certain details differ (For more details, see Appendix IV). The principal requirement for ORX event classification is consistency, according to agreed rules and definitions. One key issue is to determine the proper event category for complex, multi-effect operational risk events. If ambiguity in classifying events exists, we should ask the question “Is this a Basel event/loss type? If Basel category fits the answer, that’s fine. If not, a second question should be asked: What has lead to that event/loss? The answer to that second question would then fit within a Basel category.

The key point here is to avoid the question “Why”. The reason is because why-type questions sometimes get you to the wrong cause of the event/loss. An example might explain this. Let us consider a programming bug that has lead to an OR event. This case fits clearly to Business Disruptions & Systems Failure. Nonetheless, if we ask why and the answer is because of human error, this would send us to Execution, Delivery and Process Mgt (other Task Misperformance) or Clients, Product and Business Practices (Product Flaws).

In its reporting standards [28], the workgroup of ORX mentioned that it has observed certain inconsistencies in the Basel categories. The order of categories in the ORX standards has therefore been modified slightly to group similar event categories together more closely. These adjustments are implemented since the first quarter of 2004.

The definition of event types of level 1 in the ORX standards is given below, and words given in italic means the modifications in ORX categories in comparison to the Basel categories. Full description of event types of ORX is given in [28].

1. Internal Fraud: losses due to acts of a type intended to defraud, misappropriate property or circumvent regulations, the law or company policy, excluding diversity/discrimination events, which involves at least one internal party, excluding malicious damage. Examples: bribes, insider trading (not on firm's account), etc.
2. External Fraud: losses due to acts of a type intended to defraud, misappropriate property or circumvent the law, by a third party without the assistance of an internal party, excluding malicious damage. Examples: robbery, misappropriation of assets, etc.
3. *Malicious Damage*: losses due to acts of malice, spite, terrorism or the like, with no profit intention.
4. Employment Practices and Workplace Safety: losses arising from acts inconsistent with employment, health or safety laws or agreements, from payment of personal injury claims, or from diversity discrimination events.
5. Clients, Products & Business Practices: losses arising from an unintentional or negligent failure to meet a professional obligation to specific clients (including fiduciary and suitability requirements), or from the nature or design of a product.
6. Disaster *and Public Safety*: losses arising from loss or damage to physical assets from natural disaster or other events.
7. *Technology and Infrastructure Failures*: losses arising from disruption of business or system failures.

8. Execution, Delivery & Process Management: losses from failed transactions processing or process management.

During the year of 2002 and 2003, the operational loss data in the ORX has the following attributes:

1. Line of Business
2. Event Category
3. Gross Loss
4. Direct Recovery
5. Net after Direct Recovery (= Gross Loss – Direct Recovery)
6. Indirect Recovery
7. Net after All Recovery (= Gross Loss – Direct Recovery – Indirect Recovery)
8. 6 cells of Effect Types (The net loss after all recovery from number 7 can be categorised into one or more effect types of loss)
9. Date of Occurrence
10. Date of Recognition
11. Credit or Market Risk related (whether this loss event is related to credit risk and/or market risk)

Starting 2004, date of discovery of operational loss must also be submitted to ORX database. Risk events should be given in Level 2 of Event Types categories and assigned to Level 2 Lines of Business (BL) categories. ORX business lines categories are based on the Basel Accord categories, with some variations (See Appendix III for overview and BL definitions). Banks must also report their exposure indicators (Gross Income and Full Time Employee) during each submission.

3.4 Comparison of Internal Data and External Data

The previous two sub-chapters give overviews over the Bank's historical internal data (CLD) and respectively the external data of peer-group Banks (ORX). We can see that the underlying definitions are quite similar to each other. Nevertheless, several obvious differences between the two data are:

1. The detail level of internal data is higher than of external data.
2. In external data we do not know which loss belongs to which banks, since this information is not available.
3. CLD works with Business Unit, while ORX works with Line of Business from the Basel specification. In addition, starting from 2004 the categorisation of Event Type and Line of Business in ORX is slightly modified from the Basel categories.

In the following section we are going to take a look at the proportion of the frequency and the severity of operational loss, for each combination of the internal Business Units and event types, to the total amount of the internal data. The same thing is done for each combination of the external Lines of Business and event types. We will start with the internal Business Units.

3.4.1 Internal Business Units (BU)

3.4.1.1 The Frequency of Operational Loss (number of loss events)

The following table gives the proportion of the number of loss events, for each combination of the internal Business Units and event types (Level 1), to the total number of loss events of ABN AMRO during the year 2002-2003:

INSERT TABLE A HERE[5]

From the table above we can see that Business Unit North America has suffered the highest number of operational risk loss events, followed by Business Unit Brazil. In

[5] The information in this table is considered to be confidential. It is available subject to the permission in writing from the author or from the Group Operational Risk Policy & Support (ORP&S) of ABN AMRO Bank, The Netherlands.

contrary, Business Unit Asset Management & Trust (AM&T) experienced the lowest number of operational risk loss events. It is noteworthy that the loss events occurred in Business Units Brazil and North America are almost 10 times higher than Business Unit AM&T. We can also see that events from the event type ‘Execution, Delivery and Process Management’ dominate the loss events from the bank. On the other hand, event type ‘Damage to Physical Assets’ has the least number of events, followed by the event type ‘Business Disruption and System Failures’.

3.4.1.2 The Severity of Operational Loss (loss amount of loss events)

The following table gives the proportion of the loss amount of operational risk loss events, for each combination of the internal Business Units and event types (Level 1), to the total loss amount of operational risk loss events of ABN AMRO during the year 2002-2003:

INSERT TABLE B HERE

From the table above we can see that Business Unit North America has suffered the biggest loss amount of operational risk loss events, followed by Business Unit Wholesale Clients (WCS). Even though the number of loss events (frequency) of Business Unit Brazil is the second-highest, the severity of the operational losses in Business Unit Brazil is the second lowest from all Business Units. In contrary, Business Unit Asset Management & Trust experienced the lowest number of operational risk loss events. It is noteworthy that the loss events occurred in Business Units Brazil and North America are almost 10 times higher than Business Unit AM&T.

We can also see that events from the event type ‘Execution, Delivery and Process Management’ dominate the loss events from the bank. On the other hand, event type ‘Damage to Physical Assets’ has the least number of events, followed by the event type ‘Business Disruption and System Failures’.

3.4.2 External Lines of Business (BL)

3.4.2.1 The Frequency of Operational Loss (number of loss events)

The following table gives the proportion of the number of loss events, for each combination of the external Lines of Business and event types (Level 1), to the total number of loss events of ORX members during the year 2002-2003:

INSERT TABLE C HERE

From this table we can see that 50.76% (the half) of the total number of loss events occurred within Line of Business Retail Banking and 23.87% (almost one-fourth) of the losses took place in Line of Business Trading and Sales. Corporate Finance and Retail Brokerage suffer the least number of loss events, with 0.9% and 1.6% respectively.

It is obvious also that event type ‘Execution, Delivery and Process Management’ dominates the loss events (53.13%) of the bank, followed by event type ‘External Fraud’. We have seen a similar characteristic in the internal data. Nevertheless, the proportion of event type ‘Employment Practices and Workplace Safety’ (5.33%) is not as big as the proportion in the internal data (18.61%).

3.4.2.2 The Severity of Operational Loss (loss amount of loss events)

The following table gives the proportion of the loss amount of operational risk loss events, for each combination of external Lines of Business and event types (Level 1), to the total loss amount of operational risk loss events of ORX members during the year 2002-2003:

INSERT TABLE D HERE

We can observe from this table that 30.19% of the total loss amount took place in Trading and Sales Line of Business and 29.64% occurred in Retail Banking. Commercial Banking contributes 22.32% of the total severity of operational loss. Agency Services and Payment and Settlement suffer the least amount of loss severity; respectively 2.11% and 2.21% of the total losses.

From the event type point of view, event type ‘Execution, Delivery and Process Management’ has the biggest amount of loss severity (42.71%), the same as in the internal data. This is followed by event type ‘Clients, Products and Business Practices’ with 24.73%. The second position of is different to the internal data, where ‘Internal Fraud’ and ‘External Fraud’ contribute 16.08% and 11.45% respectively to the total severity amount of operational loss. In the internal data the event type ‘External Fraud’ is on the second position, noticeably with 36.75%. This value is higher than three times of the proportion of External Fraud in the ORX data. ABN AMRO scores better in the severity of operational loss of event types ‘Internal Fraud’ (2.60%) and ‘Clients, Products and Business Practices’ (11.14%), in comparison to the proportions in the ORX data (16.08% and 24.73% respectively).

3.5 Review

The loss information of external data is given in combination of Line of Business and event type categorisation. On the other hand, the loss internal data is given in combination of Business Unit and event type categorisation. While the level 1 event type categorisation is the same to the one used by the bank, it is necessary to map the internal Business Units to the lines of business categorisation in advance before comparing both data.

At this moment the mapping mechanism to map the Business Units of the bank to the lines of business of ORX can be given by the following table:

	Line of Business								
Business Unit	Corporate Finance	Trading and Sales	Retail Banking	Commercial Banking	Payment and Settlement	Agency Services	Asset Management	Retail Brokerage	Grand Total
BU Asset Management & Trust						9%	91%		100%
BU Brazil		7%	87%	6%					100%
BU PC&NGM			100%						100%
BU NL			52%	48%					100%
BU WCS	7%	59%		20%	13%	1%			100%
BU North America		5%	47%	47%				1%	100%

Table 3 – Mapping matrix of ABN AMRO Business Unit to Basel Line of Business

By means of the above table, we can translate the internal Business Units into the provided lines of business types of Basel. Nevertheless, it is not advised to directly compare both data or directly incorporate the external data into the internal data. This is mainly due to the specific internal risk profile and characteristic of each bank submitting its loss information to ORX, which is very likely to be different to each other. In Chapter 5 we will propose a scaling mechanism to solve this problem. Beforehand, we will analyse the internal loss data in the next chapter, whether there exists a linear relationship between the loss severity and the time interval between the occurrence and recognition time of operational risk loss events.

4 Analysis of Operational Risk Loss Data

In the preceding chapter we have explained the characteristics of both the internal data of the bank and the peer-group data (ORX) that serves as the external data. These two types of historical loss data are required by the bank in order to move towards AMA to calculate its operational risk capital. In this chapter, we will perform an analysis on the historical loss data of the bank. We do a linear relationship test in the next section between the loss amount of an operational risk loss event and the time interval between time of discovery and time of recognition of the loss event. This test is done on the aggregate level of the bank (the bank as a whole). Since the loss amount cannot explain the behaviour of the time interval between discovery and recognition of loss events, we decided to look at other elements that might explain the behaviour of the time interval between discovery and recognition time. For this reason, we look at two aspects of each Business Unit of the bank, namely the method of recognition and the Business Unit characteristic. The results of this test will be given in the subsequent section.

4.1 Linear Relationship Test

In this section, we will perform a linear relationship test between two attributes of an operational risk event. We are particularly interested in testing the following hypothesis:

Hypothesis: 'There is a linear relationship between two attributes of an operational risk loss event, namely: (1) the loss amount and (2) the time interval between the moment an event is discovered and the moment the event is recognised as an operational risk loss event'.

We have seen that every operational loss event will possess the following characteristics: time of occurrence, time of discovery, and time of recognition. When an operational loss event is discovered, the time the event occurred must first be obtained. The event is afterwards recognised as an operational risk loss event. This loss event is also entered in the internal database and booked in the General Ledger. Ideally, the time interval between the time (or in general the date) the event occurred and the time the event is discovered and recognised as an operational loss event should be minimum.

An investigation usually takes place after an operational risk event is discovered (especially to operational risk events with large loss amount). The investigation is done in order to find out the cause of the event. After investigation, necessary actions can be taken so that the probability of occurrence of similar events in the future can be minimised. An investigation will consume some time and the time needed to investigate might vary a lot. It can take only several hours, which means that the loss event is recognised in the same day as it was discovered. In other occasions, the investigation might need months (or even years) before it can be finalised.

When an operational loss with small amount occurs, it is expected that an investigation is less likely to happen. This is mainly in consideration of the cost, since an investigation might cost more than the loss occurred. As results, the time needed to recognise this kind of event should be shorter than the time needed to recognise the loss events with high severity. Therefore we think that small losses will tend to have a small interval between time of discovery and recognition. At the same time we also expect that large losses will tend to have a high time interval between discovery and recognition, and this higher interval is mainly caused by the time needed to investigate.

We denote the time interval between discovery and recognition time as IDR (Interval Discovery Recognition time). The loss amount is the Gross Loss amount of the operational risk loss event. We are going to study whether our hypothesis conforms to the reality. This study is done only to the internal data of the bank; since in the external data samples the date of discovery of operational loss events is not included (we obtain the external data only for the period between 2002 and 2003). Starting from the year 2004 the date of discovery of operational loss events is also included in the ORX data. This means that in the future this study can also be done on the basis of the ORX data. We will start by analysing the loss events of the bank as a whole, and then looking into the loss events of each Business Unit separately.

ABN AMRO Bank

The following tables give the descriptive statistics of the Gross Loss amount of ABN AMRO during the year 2002 and 2003.

GROSS LOSS (in Euro)	
Mean	€ 188,060.92
Median	€ 37,391.74
Mode	€ 25,000
Standard Deviation	€ 2,380,815.60
Sum	€ 382,892,038.64
Number of events	2,036

Table 4 – The descriptive statistics of gross loss amount of all Business Units of the bank in year 2002 and 2003

We can see from the above table that there are 2,036 operational risk loss events occurred during the year 2002-2003. The total loss amount is € 382,892,038.64 with an average amount of € 188,060.92. The most prevailing events have the loss amount of €25,000. The stated minimum loss amount is €20,000 because we set the threshold value to be €20,000. The standard deviation of the loss amount is €2,380 million.

The following table gives the descriptive statistics of the variable IDR (interval between time of decision and recognition of ABN AMRO during the year 2002 and 2003. The IDR is given in workdays[6].

[6] There are 5 workdays in a week (Monday to Friday), respectively [4 weeks * 5 =] 20 workdays in a month and more or less 250 workdays in a year (the workdays in a year are less than [52 weeks* 5 =] 260 due to holidays).

IDR (Interval Decision and Recognition time) in workdays	
Mean	88.70
Median	29
Mode	1
Standard Deviation	162.07
Sample Variance	26,267.11
Range	2,459
Minimum	1
Maximum	2,460
Sum	180,599
Count	2,036

Table 5 –The descriptive statistics of IDR (Interval Discovery and Recognition time) of all Business Units of the bank in year 2002 and 2003

The average time interval between the discovery and recognition time is 89 workdays (88.70), or two and a half months. The maximum value of IDR is 2,460 workdays, which is almost equal to 10 years. Please note that a time interval of 1 workday means that a loss event is discovered and recognised on the same workday. The minimum possible value for IDR is thus 1 workday.

The following figure plots the severity of losses in the y-axis and the time interval between discovery and recognition (IDR) in the x-axis.

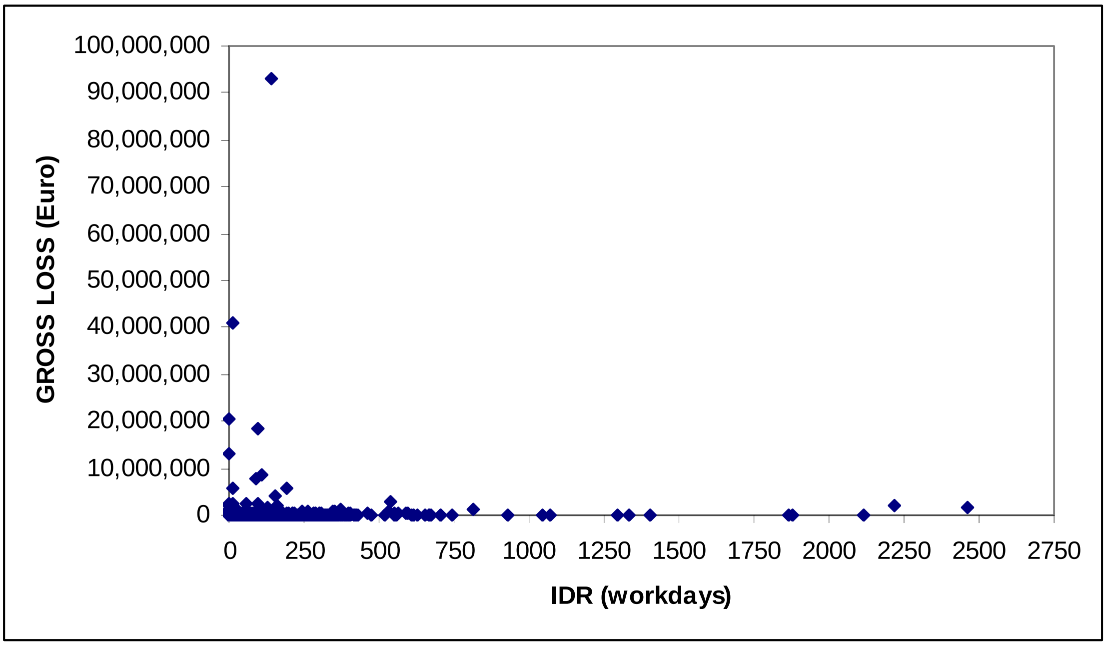


Figure 8 – Severity and IDR (Interval Discovery and Recognition time)

From this figure we can only see that the maximum time interval is less than 10 years (2500 days), which is not the property of the loss event with the maximum loss amount but instead of a loss event with loss amount of less than €100,000. The loss event with maximum loss severity has an IDR value of less than 250 workdays, which means it was recognised less than one year after discovered. We can observe that most of the loss events have IDR value of between 0 and 750 workdays (3 years). It is also noteworthy that all loss events bigger than 10 million euro are also recognised less than one year after discovered. This figure clearly indicates that there is no linear relationship between high loss severity and high interval between discovery and recognition time (high IDR).

The IDR value of small amount loss events are distributed between minimum to maximum value. From the figure above we can see that the density (concentration of loss events) of loss events with loss severity less than €10,000,000 and IDR values less than 500 workdays is higher than the density of other combinations of loss severity and IDR value. Because the figure does not give an overview of the losses less than €10,000,000, we need to explore into the region of smaller losses and find out whether we can find the linear relationship between small losses and small values of IDR.

For this reason, we divide the loss events into different categories of loss severity and IDR value. The severity of operational loss is divided into 8 categories:

1. Losses greater than or equal to €20,000[7] and less than €100,000
2. Losses greater than or equal to €100,000 and less than €500,000
3. Losses greater than or equal to €500,000 and less than €1,000,000
4. Losses greater than or equal to €1,000,000 and less than €5,000,000
5. Losses greater than or equal to €5,000,000 and less than €10,000,000
6. Losses greater than or equal to €10,000,000 and less than €50,000,000
7. Losses greater than or equal to €50,000,000 and less than €100,000,000
8. Losses greater than or equal to €100,000,000

The IDR value is also divided into 8 categories:

a. IDR greater than or equal to 1 workday and less than 6 workdays / losses recognised within 1 week time after discovered
b. IDR greater than or equal to 6 workdays and less than 21 workdays / losses recognised between 1 week and 1 month time after discovered
c. IDR greater than or equal to 21 workdays and less than 41 workdays / losses recognised between 1 month and 2 months time after discovered
d. IDR greater than or equal to 41 workdays and less than 61 workdays / losses recognised between 2 months and 3 months time after discovered
e. IDR greater than or equal to 61 workdays and less than 121 workdays / losses recognised between 3 months and 6 months time after discovered
f. IDR greater than or equal to 121 workdays and less than 251 workdays / losses recognised between 6 months and 1 year time after discovered
g. IDR greater than or equal to 251 workdays and less than 751 workdays / losses recognised between 1 year and 2 years time after discovered

[7] This value depends on the threshold value chosen. We choose a threshold value of €20,000 in this study.

h. IDR greater than or equal to 751 workdays / losses recognised more than 3 years time after discovered

It is obviously possible to use other categorisation for the loss severity and the IDR value, since the categories above are chosen intuitively. The number of loss events for various combinations of loss severity and IDR value will then be filled in the following matrix:

	IDR (Interval Decision and Recognition time) in workdays								
Loss Value	**>= 1 & < 6**	**>= 6 & < 21**	**>= 21 & < 41**	**>= 41 & < 61**	**>= 61 & < 121**	**>= 121 & < 251**	**>= 251 & < 751**	**>= 751**	**Grand Total**
>= €100,000,000	Region 57	Region 58	Region 59	Region 60	Region 61	Region 62	Region 63	Region 64	
>= €50,000,000 & < €100,000,000	Region 49	Region 50	Region 51	Region 52	Region 53	Region 54	Region 55	Region 56	
>= €10,000,000 & < €50,000,000	Region 41	Region 42	Region 43	Region 44	Region 45	Region 46	Region 47	Region 48	
>= €5,000,000 & < €10,000,000	Region 33	Region 34	Region 35	Region 36	Region 37	Region 38	Region 39	Region 40	
>= €1,000,000 & < €5,000,000	Region 25	Region 26	Region 27	Region 28	Region 29	Region 30	Region 31	Region 32	
>= €500,000 & < €1,000,000	Region 17	Region 18	Region 19	Region 20	Region 21	Region 22	Region 23	Region 24	
>= €100,000 & < €500,000	Region 9	Region 10	Region 11	Region 12	Region 13	Region 14	Region 15	Region 16	
>= €20,000 & < €100,000	Region 1	Region 2	Region 3	Region 4	Region 5	Region 6	Region 7	Region 8	
Grand Total									

Table 6 – Matrix of number of loss events for various combinations of loss severity and IDR value

There are in total 64 regions in this matrix. We number the region starting from 1 in the lower-left corner to the 64 in the upper-right corner. If the density of region 1 is much higher than the densities of region 2-8, it will indicate that the losses ‘between €20,000 and less than €100,000’ are most likely to be recognised less than 6 workdays after discovery. In other words, small losses are most likely to have small IDR values.

On the other hand, if the density of region 64 is much higher in comparison to the densities of region 57-63, it will indicate that the losses with severity of ‘€100,000,000 or higher’ are most likely to be recognised ‘within 3 years or more (>=751 workdays)’ after discovery. In other words, losses with high severity are most likely to have high values of IDR values. Accordingly, our hypothesis of linear relationship would expect the data points to fall around the diagonal regions of the matrix (Region 1-10-19-28-37-46-55-64) above.

The following tables give the number of loss events within each combination of the loss severity and time interval IDR for ABN AMRO data for the year 2002 and 2003.

INSERT TABLE E HERE

There is no losses with severity of '€100,000,000 and higher'. The highest severity amount of the loss events occurred in the bank is given in the category "between €50 million and less than €100 million". We can see from the above table that this loss event falls under the category of loss severity 'between €50,000,000 and less than €100,000,000' and the IDR value is 'between 121 and 250 workdays'. We can thus confirm from this table that there is no linear relationship between high amount of loss severity and high value of IDR.

The next table gives the density of loss events for various combinations of loss severity and IDR:

INSERT TABLE F HERE

We can see from this table that for losses 'between €20,000 and less than €100,000', the IDR value of 'less than 6 workdays' is not dominant; the density of this region is only 10.5%. The densities of the regions with IDR value 'between 6 and 20 workdays' and IDR value 'between 21 and 40 workdays' are bigger. These proportions are 17.8% and 24.6%, respectively.

For the losses ‘between €100,000 and less than €500,000’, the density of the region with IDR value ‘less than 6 workdays’ is the highest (=2.8%) in comparison to other regions with higher IDR values. The density is however not significantly higher than the densities of the regions with IDR value ‘between 21 and 40 workdays’ (=2.6%) and IDR value ‘between 121 and 250 workdays’ (=2.3%).

Based on these results it can be seen that we cannot find any linear relationship between small losses and small IDR values. We therefore have to reject the hypothesis suggested before. We can however notice two properties from the earlier table:

1. From all operational risk loss events occurred in the bank, more than a quarter of the losses are recognised between 1 and two months after discovered (between 21 and 40 workdays). This characteristic holds also for loss events with loss amount between €20,000 and less than €100,000.
2. A proportion of 60.9% (13.7+19.7+27.5) of the loss events are recognised within 2 months after discovered, 28.7% (6.4+8.6+13.7) are recognised between 2 months and 1 year, and the rest 10.5% (9.9+0.6) have a time interval of more than 1 year between discovery and recognition time.

In order to explain the behaviour of the IDR values of operational risk loss events, the following two attributes are considered:

1. Method of Recognition. We have seen in Table 2 from Section 3.2 that not all Business Units apply the same approach to recognise operational risk loss event. Recall that there are 3 methods of recognition:
   a. Method 1 - Event Recognition. Immediate recognition after discovery of the event. This should lead to a small value of IDR.
   b. Method 2 - Derived from General Ledger Input: CLD entry (Recognition) is done after General Ledger posting. The IDR value should be higher than the first method.
   c. Method 3 - Derived from General Ledger Output: Monthly upload of General Ledger data. This will lead to a late CLD entry (Recognition). The

IDR value should be the highest in comparison to the two previous methods.

We provide again the Table 2 that is given in Chapter 3.2:

Business Unit	**Method of operational risk loss events recognition**
BU AM&T	Method 1
BU BR	Method 2
BU PC&NGM	Method 1
BU NL	Method 1 (Starting from January 1st 2004 method 2 is applied)
BU WCS	Method 1
BU NA	Method 3

2. Characteristic of a Business Unit. The duration of an investigation process might be dependent on the characteristic of a Business Unit. Two possibilities might arise:
   a. Type 1 - Business Units where the clients are: (1) high net-worth individuals and institutional investors, i.e. BU Private Clients (PC) and BU Asset Management & Trust (AM&T), and (2) major international corporations and financial institutions, i.e. BU Wholesale Clients (WCS). In this type of Business Unit the senior management of the Business Unit may require the operational risk management unit to investigate most of the loss events thoroughly. This is necessary when a client requests an explanation regarding the cause behind operational risk loss events. A detailed and satisfactory explanation may be necessary to calm down the client. The loss events in these Business Units are likely to have high values of IDR.
   b. Type 2 - Business Units where the clients are individual and small-to-medium-sized businesses and requiring day-to-day banking: i.e. BU Brazil, BU Netherlands, BU New Growth Markets (NGM) and BU North America. In this type of Business Unit the result of an investigation process is mainly intended for intern use of the Business Unit or the bank at the aggregate level. The senior management in this respect may require the operational risk management unit to provide the investigation end result within a certain time

period. Consequently, operational risk loss events might not be fully investigated or the investigation process is terminated in order to meet the reporting deadline from the senior management. This condition will result in recognising loss events, even though the investigation process might not be finished yet. The loss events in these Business Units are likely to have low values of IDR.

More information regarding the Business Units of the bank can be found in [34]. We will look at these two attributes to find out whether they can explain the behaviour of IDR values in each Business Unit of the bank. This study is done in the following section.

4.2 Relationship Test within each Business Unit

We will start by looking at BU Asset Management & Trust. The same matrix from Table 6 will be used to give the number of loss events with various combinations of loss severity amount and IDR value. Nevertheless, the number of loss events within each cell will be given in percentage value instead of real number.

Business Unit Asset Management & Trust (AM&T)

The following table gives the density of loss events for each combination of loss severity amount and IDR value for BU Asset Management & Trust.

INSERT TABLE G HERE

In this Business Unit the highest density (20.3%) is given by two IDR categories; IDR value ‘between 121 and 250 workdays’ and IDR value ‘between 251 and 750 workdays’. These high density values are expected due to the characteristic of this Business Unit, namely of Type 1. The loss events in this type of Business Unit are investigated in detail.

The densities of the categories with IDR value ‘between 6 and 20 workdays’ and IDR value between 21 and 40 workdays’ are also high, 15.9% and 18.8% respectively. The high densities values of these two IDR value categories are very likely because BU AM&T applies Method 1 to recognise its operational risk loss events.

Business Unit Brazil (BU BR)

The following table gives the density of loss events for each combination of loss severity amount and IDR value for BU Brazil.

INSERT TABLE H HERE

In this Business Unit the highest density (58%) is given by the category of IDR value ‘between 21 and 40 workdays’. The second highest density (41%) is given by the category of IDR value ‘between 6 and 21 workdays’. The density of these two IDR categories dominates the densities of the other IDR categories. Again, this result is in line to the Type 2 of BU Brazil characteristic. The losses of this Business Unit are recognised quickly. But there are no loss events that are recognised within 1 week after discovery. This is very likely due to the use of Method 2 of loss events recognition, where the recognition moment is later than the recognition time in Method 1.

Business Unit Private Clients & New Growth Markets (BU PC&NGM)

The following table gives the density of loss events for each combination of loss severity amount and IDR value for BU PC&NGM.

INSERT TABLE I HERE

In this Business Unit the highest density (26.6%) is given by the IDR values category of ‘between 251 and 750 workdays’. The second highest density (22.3%) is given by the category of IDR value ‘between 121 and 250 workdays’. The IDR value category ‘between 61 and 121 workdays’ also has a rather high value, 13.8%. These high density values are more in line to the Business Unit characteristic of BU Private Clients, namely of Type 1.

The high density values of the categories of IDR value ‘between 6 and 20 workdays’ (8%) and IDR value between 21 and 40 workdays’ (16.5%), may come from 2 sources: the characteristic of Business Unit NGM (Type 2) and the application of Method 1 of loss events recognition in Business Unit PC&NGM.

Business Unit Netherlands (BU NL)

The following table gives the density of loss events for each combination of loss severity amount and IDR value for BU NL.

INSERT TABLE J HERE

In Business Unit Netherlands the highest density (83.7%) is given by the category of IDR value ‘less than 6 workdays’. The significantly high density value of this IDR category is coming from the Type 2 characteristic of Business Unit Netherlands and the application of Method 1 to recognise operational risk loss events. The loss events within the category of IDR value ‘between 251 and 751 workdays’ might be considered as outliers, because the density is only 7.4% and much smaller in comparison to the highest density value.

Business Unit Wholesale Clients (BU WCS)

The following table gives the density of loss events for each combination of loss severity amount and IDR value for BU WCS.

INSERT TABLE K HERE

In this Business Unit the highest density (26.9%) is given by the IDR values category of ‘between 121 and 250 workdays’. The third highest density (17.2%) is given by the category of IDR value ‘between 61 and 120 workdays’. The IDR value category ‘between 251 and 750 workdays’ also has a rather high value, 14.7%. These high density values are in line to the Type 1 Business Unit characteristic of Business Unit Wholesale Clients.

The high density values of the categories of IDR value ‘between 6 and 20 workdays’ (17.6%) and IDR value between 21 and 40 workdays’ (10.1%), are very likely due to the application of Method 1 of loss events recognition in this Business Unit.

Business Unit North America (BU NA)

The following table gives the density of loss events for each combination of loss severity amount and IDR value for BU NA.

INSERT TABLE L HERE

In Business Unit North America there are six IDR value categories with density value higher than 12%. The highest density (22.4%) is given by the IDR value category of ‘between 121 and 250 workdays’. This is most likely due to the application of Method 3 of loss events recognition, which results in a late recognition time. The high density values of the categories of IDR value ‘between 61 and 120 workdays’ (14.9%) and IDR value ‘between 251 and 750 workdays’ (12.1%) are also likely based on the same reason.

The second highest density value is given by the IDR value category of ‘between 21 and 40 workdays’ (20.9%). This is most likely due to the Type 2 characteristic of Business Unit North America. The high density values of the categories of IDR value ‘between 6 and 20 workdays' (12.7%) and IDR value 'between 41 and 60 workdays' (15.1%) are also likely to come from the same motivation.

4.3 Discussion

We have seen from the previous section that the behaviour of IDR values of operational risk loss events can be explained by two attributes:

1. Business Unit characteristic
2. Method of recognition

We notice also that the IDR value category with highest density value(s) can be explained by the Business Unit characteristic in most cases. In the case of Business Unit PC&NGM, the highest density value is explained by the characteristic of Business Unit PC (Type 1), not the characteristic of Business Unit NGM (Type 2). These two Business Units might need to be split to clarify the results.

However, the IDR value category with highest density value in Business Unit North America is explained by the method of recognition and not by the Business Unit characteristic. Since Business Unit North America is the only Business Unit that applies Method 3 to recognise loss events, this exception might be due to this method of recognition.

In this chapter we have analysed the historical loss data of the bank. We will study a scaling mechanism in the next chapter, which can be used to incorporate the external peer-group data into the internal loss data.

5 Scaling Operational Risk Loss Data

In this chapter we will start by explaining why a scaling mechanism is needed in order to be able to incorporate the external operational risk loss data into the internal data of the bank. The scaling mechanism, which will be proposed in this chapter, has been applied subject to the assumption that there is a universal power-law relationship between the operational risk loss amount within a certain time period and an indicator of size & exposure towards operational risk within a certain time period of different financial institutions. The statistical tests we have performed support the idea that such universal power-law relationship is present. Afterwards, we will present how the scaling mechanism can be used to incorporate the external data into the internal data of the bank.

Remember that an operational loss amount is coming from two elements, namely the frequency and the severity. Therefore, a similar experiment concerning the power-law relationship is carried out to the frequency and the severity elements. We will study whether the power-law relationship is coming from the frequency element or the severity element.

5.1 Background

In general terms, operational risk modelling may be considered as modelling loss distributions. The loss distribution portrays operational risk loss events. The end goal is to use the fitted distributions in order to make inferences about future behaviour of losses, and to determine the risk profile of the bank (e.g. operational risk capital calculation). A future risk profile is usually estimated using internally experienced loss information of a group of events (Event Categories). This information is gathered by the various Lines of Business of the bank.

Generally speaking, it is very likely that bank’s historical loss data may not fully represent the bank’s exposure to operational risk losses. We can think of two possible reasons. First, there are not enough historical loss event numbers for estimating reliable parameters. Second, there are only a few or even no high severity losses that have happened in the past, which are required to estimate the tail part of the loss distribution. A good estimation of the tail of the loss distribution is essential, especially if there is reason to believe that the bank is exposed to high severity-low frequency losses (this has been mentioned before in Section 2.4.3).

Therefore, the Basel Accord requires the use of relevant external data in a bank's operational risk measurement system [12]. Direct utilisation of external data is not advised, since each individual external loss data comes from a bank that has its own risk profile and characteristics, such as the size of the bank providing the external data, the target-market this bank is concentrating on, the level of control system when events happened. Furthermore, each Line of Business of a bank is very likely to vary to each other in terms of risk profile and internal characteristics. Different banks in size may present a greater or lesser number of loss events or operational loss amount in a certain period of time.

Another reason why external data should not be directly incorporated into internal loss database of the bank is due to the different threshold values used within banks. This problem is addressed in [14]. We do not experience this problem in our experiment since we are using equal threshold values for the external and internal loss data (namely €20,000).

Results of previous works concerning the problem of how to incorporate the external data into the internal data are given in [13] and [19]. One major assumption used is that the external data are drawn from the same (probabilistic) distribution as the internal data. The authors admit, though, that it is not obvious whether probability distributions for internal and external data are identical.

In this chapter, we are going to explain a scaling mechanism that can be used to add the external data into the internal data of the bank. The scaling mechanism is intended to remove banks' specific characteristics, so that the external data can be considered to have the same characteristics as the internal data. If this objective can be realised, we will be allowed to add the external data into the bank's internal data and to use both data altogether.

However, the scaling mechanism can only be applied subject to the assumption of the existence of a universal power-law relationship between the operational loss amount within a certain time period and the size & exposure towards operational risk within a certain time period of different financial institutions. There are two underlying reasons to use the power-law form in order to explain the relationship between the operational loss amount within a certain time period and the size & exposure towards operational risk within a certain time period of different financial institutions:

- Previous study over the relationship between the severity of operational loss and the size of a firm can be found in [31]. The results of this study suggest that: (1) there is a relation between the size of a firm and the severity of operational loss in that firm and (2) the power-law relationship can be used to explain the relation between the firm size and the severity of operational loss in that firm.
- Power-law form also occurs in some statistical distributions of quantities. Several examples can be found in the world of Physics ([33], [34]), World Wide Web [15], Economics ([2], [17], [22], and [29]) and Finance [16].

While the study in [31] examines only the severity of operational loss, we will study whether a universal power-law relationship, between the operational loss amount within a certain time period and the size & exposure towards operational risk within a certain time period of different financial institutions, is present.

Additionally, the study in [31] has investigated at the financial institution (view each financial institution or each bank as a single entity) level. In our study, we will consider each Line of Business of a bank as a single entity. In other words, the study is done at the Line of Business level.

The choice of examining at the Lines of Business level is particularly based on the information available from the external data (ORX). We can only tell from which Line of Business, but not from which bank, an operational loss amount comes from. This information is not given away in the ORX operational loss data. Remember that in our data set, the internal loss data of the bank is given per Business Unit. For the reason of simplicity, we use directly the Business Units of the bank instead of mapping them into the Basel Lines of Business categorisation. Gross Income is chosen as the indicator for the size and exposure to operational risk of a Line of Business. Other possible indicators are transaction volumes and number of employees.

Remember that an operational loss amount is coming from two elements, namely the frequency and the severity. A similar experiment concerning the power-law relationship is carried out to the frequency and the severity elements. Therefore, we shall also study whether the power-law relationship is coming from the frequency or the severity element.

It is expected that the power-law relationship is coming from the frequency element of the operational loss. For example, it is very likely that the bigger the size of a Line of Business, the higher the number of transactions in that Line of Business will be. Therefore, the number of operational losses occurred (frequency) in that Line of Business is very likely to be greater in comparison to the frequency of operational loss in another Line of Business, which size is smaller or carries out fewer transactions.

From the severity element, there is no general evidence that big losses always occur at bigger institutions. We have seen that in the case of operational risk, small financial institutions (i.e. small incomes and number of employees) can still experience a catastrophic loss. Barings disaster [30] is again an obvious example. It is not expected that the probability of a big single loss in a big bank would be higher than a small single loss in a small bank. There is no fundamental relation between severity and size that would support that small banks are more likely to undergo a single small loss than big banks.

The result of a previous study [31] indicates that there is a power-law relationship between the size of a firm and the severity of operational loss in that firm. Nevertheless, the study was done using a database of publicly reported operational losses. This database contains 4700 operational losses with severity in excess of $1 million. It is still not clear however, whether a similar relationship also exists for the operational losses with severity less than $1 million. In our study, we will look at the operational losses with severity greater than or equal to €20,000. We will start by explaining the power-law relationship and the scaling mechanism in the next section.

5.2 Components of Operational Risk

We know from previous chapters that a bank can be divided into various Lines of Business or Business Units. In the world of operational risk, it is possible to view each Line of Business as a single financial institution. Therefore, we will examine the operational risk loss at the Line of Business level than at the financial institution level in our study.

The operational risk loss of a Line of Business of a bank can be thought of as coming from two components: the common component and the idiosyncratic component [32]. The common component refers to the risk of change in factors such as similar macro economic, geopolitical and culture environment, the general human nature or tendency to err, etc. The probability distribution characteristics of this component will be common to all Lines of Business. In other words, we assume that:

The common component of operational risk losses is equivalent for all Lines of Business.

Assumption 1

On the other hand, the idiosyncratic component refers to the risk arising from the Line of Business specific factors, such as size & exposure towards operational risk. Therefore, for each Line of Business, operational risk loss amount L_T within a certain period of time T (weekly, monthly, yearly, etc.) is a random variable that can be expressed as a function of the two components aforementioned [32]:

$$L_T = l((R_{idio})_T\ ,(R_{com})_T)$$

Equation 6

In this equation, the random variable L_T gives the loss amount suffered from the operational risk events, $(R_{com})_T$ is a variable measuring the common component and $(R_{idio})_T$ is a variable measuring the Line of Business idiosyncratic component; all within the time period T.

From Equation 6, a particular random draw of operational loss amount L_T from among all Lines of Business depends not only on the value of the common risk $(R_{com})_T$, but also on the value of the idiosyncratic risk $(R_{idio})_T$ (i.e. which Line of Business that loss event comes from). This means that when a Line of Business wants to draw an operational loss amount L_T from other Lines of Business, the idiosyncratic risk $(R_{idio})_T$ of those Lines of Business will become an issue.

If we know how to remove the idiosyncratic component from the distribution of operational loss amount L_T of a Line of Business at its Line of Business specific level, the variability of operational loss data will only be contributed from the changing values of $(R_{com})_T$. Because the common component is equivalent for all Lines of Business, we can then incorporate the operational loss data from other Lines of Business into the internal use of a Line of Business and vice versa.

However, in order to solve the issue of idiosyncratic component, explicit knowledge of the relation $L_T = l((R_{idio})_T, (R_{com})_T)$ is required. Let us assume that the operational loss function $l((R_{idio})_T, (R_{com})_T)$ is a product between a function of $(R_{idio})_T$ and a function of $(R_{com})_T$:

$$l((R_{idio})_T, (R_{com})_T) = g((R_{idio})_T) \times h((R_{com})_T)$$

Equation 7

In the equation above, $g((R_{idio})_T)$ is the function of $(R_{idio})_T$ and $h((R_{com})_T)$ is the function of $(R_{com})_T$. Thus, we assume that:

The function $l((R_{idio})_T, (R_{com})_T)$ is a product between a function of $(R_{idio})_T$ and a function of $(R_{com})_T$, which can be stated as $l((R_{idio})_T, (R_{com})_T) = g((R_{idio})_T) \times h((R_{com})_T)$.

Assumption 2

5.2.1 Power-law form

Now, let us assume that the function $g((R_{idio})_T)$ can be given by the following power-law form $(R_{idio})_T^{\lambda}$. Our third assumption is thus:

The function $g((R_{idio})_T)$ can be given by the power-law form $(R_{idio})_T^{\lambda}$, where parameter $(R_{idio})_T$ is considered to be constant.

Assumption 3

Equation 7 now becomes:

$$l((R_{idio})_T\ ,(R_{com})_T) = (R_{idio})_T^{\lambda} \times h((R_{com})_T)$$

Equation 8

We can see from Equation 8 that the relationship between the operational loss function $l((R_{idio})_T\ ,(R_{com})_T)$ and the idiosyncratic component -Line of Business specific factors- $(R_{idio})_T$ is of a power-law form. The common component function $h((R_{com})_T)$ can be obtained by dividing the operational loss function $l((R_{idio})_T\ ,(R_{com})_T)$ with $(R_{idio})_T^{\lambda}$. Please notice that if the value of variable $\lambda = 1$, a linear form of $(R_{idio})_T$ is obtained. This will suggest a linear relationship between the operational loss function and the idiosyncratic component of a Line of Business.

On the other hand, if the value of variable λ is 0, the value of $(R_{idio})_T^{\lambda}$ will be equal to 1. In other words, this will suggest that no power-law relationship exists between the operational loss function and the idiosyncratic component of a Line of Business. For every Line of Business S, we can then rewrite Equation 8 as:

$$l((R_{idio})_T\ ,(R_{com})_T)_S = (R_{idio})_{T,S}^{\lambda} \times h((R_{com})_T) \quad ; S = BL01, BL02, \ldots$$

Equation 9

5.2.2 Scaling Mechanism

Please note that the function $h((R_{com})_T)$ is equivalent for all Lines of Business, due to Assumption 1. We can then relate any Line of Business with each other by:

$$\frac{l((R_{idio})_T\ ,(R_{com})_T)_{BL01}}{(R_{idio})_{T,BL01}^{\lambda}} = \frac{l((R_{idio})_T\ ,(R_{com})_T)_{BL02}}{(R_{idio})_{T,BL02}^{\lambda}} = \ldots = h((R_{com})_T)$$

Equation 10

Now recall Equation 6, where $L_T = l((R_{idio})_T, (R_{com})_T)$ for each Line of Business. We can then rewrite Equation 10 as:

$$\frac{L_{T,BL01}}{(R_{idio})^{\lambda}_{T,BL01}} = \frac{L_{T,BL02}}{(R_{idio})^{\lambda}_{T,BL02}} = \ldots = L_{T,STANDARD}$$

Equation 11

In the equation above we have $L_{T,STANDARD} = h((R_{com})_T)$, i.e. the operational loss of the Line of Business with $(R_{idio})_T = 1$!

We can thus scale each operational loss amount $L_{T,S}$ of Line of Business S by dividing it with $(R_{idio})^{\lambda}_{T,S}$ of that Line of Business to obtain the operational loss standard, which is given by the random variable $L_{T,STANDARD}$. The operational loss standard is only influenced by the common component, which can be seen from $L_{T,STANDARD} = h((R_{com})_T)$. Transforming each actual loss amount from every Line of Business by means of Equation 11, we will obtain its operational loss standard form that is no longer affected by the idiosyncratic component. In other words, the operational loss amounts of different Lines of Business can be utilised together in their operational loss standard form. Equation 11 can thus be seen as our scaling mechanism.

5.2.3 Scaling Probability Density Function, Mean, and Standard Deviation

Remember that our scaling mechanism (Equation 11) can be applied subject to the existence of a universal power-law relationship between the operational loss amount within time period T and the idiosyncratic component within time period T of all Lines of Business. Therefore, statistical testing can be used in order to proof the existence of a universal power-law relationship of all Lines of Business.

First of all, we can rewrite Equation 9 as:

$$L_{T,S} = (R_{idio})^{\lambda}_{T,S} \times L_{T,STANDARD} \quad ; S = BL01, BL02, \ldots$$

Equation 12

This equation (together with Equation 11) actually implies the scaling of the probability density function of operational loss $L_{T,S}$ of different Lines of Business S into a standard probability density function of $L_{T,STANDARD}$. The two important characteristics of a probability density function, the mean and the standard deviation, are expected to be scaled as well. Remember that $L_{T,S}$ and $L_{T,STANDARD}$ are random variables and $(R_{idio})^{\lambda}_{T,S}$ is constant. From Equation 12, we derive [23]:

$$\mu(L_{T,S}) = (R_{idio})^{\lambda_\mu}_{T,S} \times \mu(L_{T,STANDARD}) \quad ; S = BL01, BL02, \ldots$$
$$\sigma(L_{T,S}) = (R_{idio})^{\lambda_\sigma}_{T,S} \times \sigma(L_{T,STANDARD}) \quad ; S = BL01, BL02, \ldots$$

Equation 13

The mean value of $L_{T,S}$ can be scaled into the mean value of $L_{T,STANDARD}$ by means of the constant parameter $(R_{idio})^{\lambda_\mu}_{T,S}$ for each Line of Business. Furthermore, the standard deviation value of $L_{T,S}$ can be scaled into the standard deviation value of $L_{T,STANDARD}$ as well by means of the constant parameter $(R_{idio})^{\lambda_\sigma}_{T,S}$ for each Line of Business.

Now, let assume that the mean and the standard deviation of the probability density function scale in the same way as the probability density function of operational loss. We can rewrite this as:

The mean and the standard deviation of the probability density function scale in the same way as the probability density function of operational loss. This means that λ, λ_μ and λ_σ will have the same value.

Assumption 4

By this assumption, we can estimate the value of variable λ via the value of variables λ_μ and λ_σ.

5.2.4 Estimate the value of λ via λ_μ and λ_σ

Let us take a linear model of Equation 12, which can be obtained by taking the logarithmic value on both sides:

$$\log\left[L_{T,S}\right] = \lambda \times \log\left[\left(R_{idio}\right)_{T,S}\right] + \log\left[L_{T,STANDARD}\right] \quad ; S = BL01, BL02, \ldots$$

Equation 14

This can be done as well for Equation 13:

$$\log\left[\mu\left(L_{T,S}\right)\right] = \lambda_\mu \times \log\left[\left(R_{idio}\right)_{T,S}\right] + \log\left[\mu\left(L_{T,STANDARD}\right)\right] \quad ; S = BL01, BL02, \ldots$$
$$\log\left[\sigma\left(L_{T,S}\right)\right] = \lambda_\sigma \times \log\left[\left(R_{idio}\right)_{T,S}\right] + \log\left[\sigma\left(L_{T,STANDARD}\right)\right] \quad ; S = BL01, BL02, \ldots$$

Equation 15

We can rewrite Equation 15 as:

$$l_S = C \times r_S + p \quad ; S = BL01, BL02, \ldots$$

Equation 16

In case of the mean value, we have:

$$l = \log\left[\mu\left(L_{T,S}\right)\right]$$

$$C = \lambda_{\mu}$$

$$r = \log\left[\left(R_{idio}\right)_{T,S}\right]$$

$$p = \log\left[\mu\left(L_{T,STANDARD}\right)\right]$$

In case of the standard deviation value, we have:

$$l = \log\left[\sigma\left(L_{T,S}\right)\right]$$

$$C = \lambda_{\sigma}$$

$$r = \log\left[\left(R_{idio}\right)_{T,S}\right]$$

$$p = \log\left[\sigma\left(L_{T,STANDARD}\right)\right]$$

Based on Assumption 4, we will estimate the value of variable λ via the value of variables λ_{μ} and λ_{σ}. Therefore, we apply Equation 16 to estimate the value of variable λ_{μ} using the mean value of the operational loss function $L_{T,S}$ of each Line of Business S. A similar experiment is done to estimate the value of variable λ_{σ}, using the standard deviation value of the operational loss function.

If the estimate of both λ_{μ} and λ_{σ} is zero, by means of Assumption 4, it will indicate the value of variable λ in the operational loss function to be zero as well. This condition will suggest that no power-law relationship exists between the operational loss function and the idiosyncratic component of a Line of Business. Therefore, it is necessary to study whether the estimates of λ_{μ} and λ_{σ} are significantly different from zero. It can be done by applying a *t*-test [25] and using a null hypothesis that the value of variable C in Equation 16 (λ_μ and λ_σ) is equal to zero. If the value of variables λ_μ and λ_σ is significantly different from zero, this will suggest the existence of a power-law relationship and the null hypothesis must be rejected.

However, if the estimates of λ_μ and λ_σ are different to each other; such condition will suggest that the mean and the standard deviation values scale differently. As a result, Assumption 4 must be rejected. The value of variable λ in the operational loss probability density function can still be estimated by means of Equation 12 below:

$$L_{T,S} = \left(R_{idio}\right)_{T,S}^{\lambda} \times L_{T,STANDARD} \quad ; S = BL01, BL02, \ldots$$

Nonetheless, a future study examining the different way of scaling between the mean and the standard deviation values is necessary. The result of this study might be essential to decide to which extent the estimate of the value of variable λ via Equation 12 is valid. We will not consider this issue in our study.

The value of variable λ_μ and λ_σ can be estimated by a standard regression of Equation 16. It is also possible to make the log×log plot in order to calculate the angular coefficient of the regression line. The angular coefficient of the regression line will be equal to the variable C (λ_μ and λ_σ) in Equation 16. The results of our estimate on variables λ_μ and λ_σ will be given in the following sections.

5.3 Data set

In this section, we will use the internal and external loss data from the year 2003. Remember that in our data sets, the internal loss data of the bank is given per Business Unit. The external loss data on the other hand is given for every Line of Business. As mentioned before, for the reason of simplicity, we directly use the Business Units of the bank instead of mapping them into the Basel Lines of Business categorisation.

The Gross Income of each Line of Business and Business Unit in year 2003 is chosen as the indicator of size & exposure towards operational risk of each Line of Business and Business Unit.

The following codes will be applied to denote the bank's Business Units and the Lines of Business of the external data:

Code	Business Unit
BU 01	BU Asset Management & Trust
BU 02	BU Brazil
BU 03	BU PC & NGM
BU 04	BU NL
BU 05	BU WCS
BU 06	BU North America
Code	**Line of Business**
BL01	Corporate Finance
BL02	Trading and Sales
BL03	Retail Banking
BL04	Commercial Banking
BL05	Payment and Settlement
BL06	Agency Services
BL07	Asset Management
BL08	Retail Brokerage

Table 7 – Code of Business Units and Lines of Business

5.4 Estimating variable λ_μ and λ_σ

In order to estimate the value of variable λ_μ and λ_σ, using the internal Business Units and the external Lines of Business, recall Equation 16:

$$l_S = C \times r_S + p \quad ; S = BU\ 01, \ldots, BU\ 06, BL01, \ldots, BL08$$

For the mean value of the different Lines of Business, we have:

$$l = \log[\mu(L_{T,S})]$$

$$C = \lambda_\mu$$

$$r = \log[(R_{idio})_{T,S}]$$

$$p = \log[\mu(L_{T,STANDARD})]$$

For the standard deviation value of different Lines of Business, we have:

$$l = \log[\sigma(L_{T,S})]$$

$$C = \lambda_\sigma$$

$$r = \log[(R_{idio})_{T,S}]$$

$$p = \log[\sigma(L_{T,STANDARD})]$$

Let us consider the operational loss amount per week in year 2003 for each Line of Business and Business Unit S, which can be given by variable $L_{WEEK,S}$. We use '$weeknum(\ )$' function in Microsoft Excel to determine to which week in year 2003 each operational loss amount belongs. The '$weeknum(\ )$' function returns a number that indicates where the week falls numerically within a year. However, the '$weeknum(\ )$' function considers the week containing January 1 to be the first week of the year. This will result that there are 53 weeks in a year.

Let us recall Assumption 3:

The function $g\left(\left(R_{idio}\right)_T\right)$ *can be given by the power-law form* $\left(R_{idio}\right)_T^{\lambda}$*, where parameter* $\left(R_{idio}\right)_T$ *is considered to be constant.*

The Gross Income in year 2003 is obtained for each Line of Business and Business Unit. We divide the Gross Income in year 2003 with the number of weeks in year 2003 to obtain the Gross Income per week in year 2003. Since the use of '$weeknum(\;)$' function leads to 53 weeks in a year, let us assume that there are 53 weeks in one year. Thus, the Gross Income per week is obtained by dividing the Gross Income in 2003 of each Line of Business and Business Unit with 53. The Gross Income per week in year 2003 is used to represent $\left(R_{idio}\right)_{WEEK,S}$, the size & exposure towards operational risk per week for each Line of Business and Business Unit S.

We obtain the log values of the mean and standard deviation of operational loss amount per week and the log values of the Gross Income per week in year 2003 for different Lines of Business and Business Units. The information is provided in the following table.

INSERT TABLE M HERE

5.4.1 Estimating variable λ_μ

We will first apply the regression between the log values of the mean of operational loss amount per week and the log values of the Gross Income per week, only using the external Lines of Business. For this reason, we rewrite Equation 16 as:

$$l_{EXTERNAL} = C \times r_{EXTERNAL} + p \quad ; EXTERNAL = BL01, \ldots, BL08$$

$$l = \log\left[\mu\left(L_{WEEK,\,EXTERNAL}\right)\right]$$

$$C = \lambda_\mu$$

$$r = \log\left[\left(R_{idio}\right)_{WEEK,EXTERNAL}\right]$$
$$p = \log\left[\mu\left(L_{WEEK,STANDARD}\right)\right]$$

Afterwards, the regression is run only using the internal Business Units. The regression of Equation 16 becomes:

$$l_{INTERNAL} = C \times r_{INTERNAL} + p \quad ; INTERNAL = BU\ 01, \ldots, BU\ 06$$
$$l = \log\left[\mu\left(L_{WEEK,INTERNAL}\right)\right]$$
$$C = \lambda_{\mu}$$
$$r = \log\left[\left(R_{idio}\right)_{WEEK,INTERNAL}\right]$$
$$p = \log\left[\mu\left(L_{WEEK,STANDARD}\right)\right]$$

The lxr plot, the regression line that best describes the internal data, and the regression line that best describes the external data are given in the following figure:

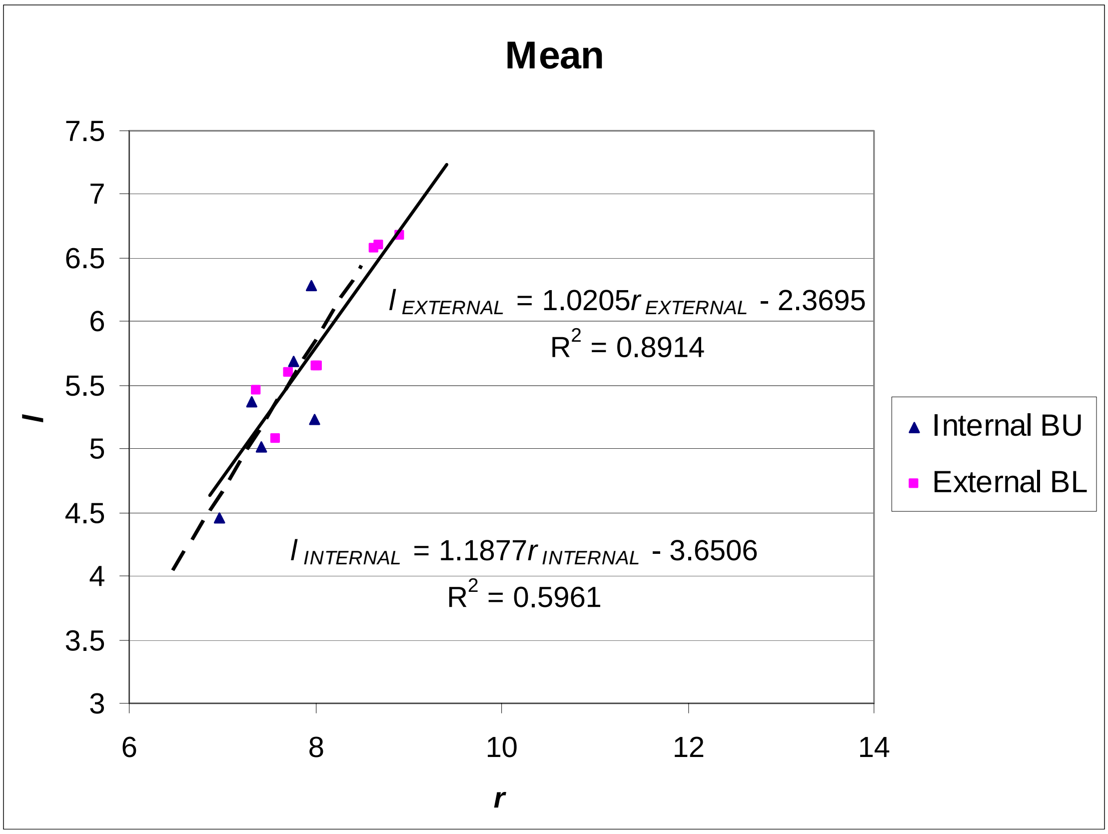


Figure 9 – *l* x *r* plot. The horizontal axis gives the variable *r* value, while the vertical axis gives the variable *l* value. Internal Business Units are given in triangles, and the regression line that best fits the internal data is given by the dash line. External Lines of Business are given in squares, with solid line as the regression line that best fits these Lines of Business.

The first impression from the figure above is that the regression lines of the external Lines of Business and the internal Business Units are almost in line to each other. This condition may support the choice of using the internal Business Units directly, instead of mapping them into the Basel Lines of Business categorisation.

The regression results are described in detail in the subsequent sections. We start with the regression result of the external Lines of Business (external data).

5.4.1.1 External data

The results of the regression on the Lines of Business of the external data are shown in the following table:

R-Square	***Regression results***	***Coefficients***	***Standard Error***	***t Stat***	***P-value***
0.8914	Intercept (*p*)	-2.3695	1.1828	-2.0033	0.0920
	C	1.0205	0.1455	7.0163	0.0004

Table 8

The first column gives the R^2 value of the regression line. An R^2 value of 0.8914 suggests that around 89.14% of variable l is attributable to variable r for the external Lines of Business. The third column gives the regression coefficients value for the variables C (1.0205) and p (-2.3695). The standard errors of these coefficients are given in the fourth column. The standard error of a variable is a measure of how far it is likely to be from its expected value [37].

We run the *t*-test to examine whether the coefficient value for the variable C is significantly different from zero, using a null hypothesis that the coefficient value for the variable C is equal to zero. The program automatically performs the same test on the coefficient value of p. The fifth column gives the *t-Statistics* values, and the significance of the *t-Statistics* value (denoted as the *P-value*) is given in the sixth column.

Within the 95% confidence interval, the coefficient value is considered statistically significant different from zero if the *P-value* is less than 0.05 (also known as the *alpha* level [27], pp. 29-30). The 90%, 99.95% and 99.99% confidence intervals correspond to the *alpha* levels of 0.10, 0.0005 and 0.0001, respectively.

The *P-value* of coefficient C in the sixth column is definitely less than 0.05, which means that the coefficient C is significantly different from zero within the 95% confidence interval. The coefficient C is also significant within the confidence interval of 99.95%, because the *P-value* is slightly less than 0.0005. This result suggests that there is a power-law relationship between the operational loss amount per week and the size & exposure towards operational risk per week of the external Lines of Business. The idiosyncratic component can explain a big proportion of the variability in the operational loss amount per week, as we can see from the high value of R^2.

5.4.1.2 Internal data

The results of the regression on the Business Units of the internal data are shown in the following table:

R-Square	*Regression results*	*Coefficients*	*Standard Error*	*t Stat*	*P-value*
0.5961	Intercept (*p*)	-3.6506	3.7060	-0.9851	0.3804
	C	1.1877	0.4889	2.4294	0.0720

Table 9

The R^2 value of 0.5961 suggests that around 59.61% of variable l is attributable to variable r for the internal Business Units. The coefficient values are respectively 1.1877 for the variable C, and -3.6506 for the variable p.

The *P-value* of coefficient C is 0.0720 (slightly higher than 0.05), so we cannot reject the null hypothesis that the coefficient C is equal to zero within the 95% confidence interval. Nevertheless, we can reject the null hypothesis that the coefficient C is equal to zero within the 90% confidence interval (the *P-value* of coefficient C is less than the alpha level of 0.10). The idiosyncratic component can explain a big proportion (59.61%) of the variability in the operational loss amount per week, which can be seen from the rather high value of R^2.

In other words, this result suggests that the power-law relationship between the operational loss amount per week and the size & exposure towards operational risk per week of the internal Business Units is present, conditional on a lower confidence interval than the confidence interval of the external data. The relationship existence subject to a lower confidence interval than the confidence interval when using the external data might be due to the choice of categorising losses per Business Units instead of per Lines of Business of the bank. Recall from Section 1.2 that separation of the bank into several Business Units is based on business activities or region. A Business Unit may thus be constructed from several Lines of Business.

5.4.1.3 Combination of external data and internal data

Finally, we perform the regression on the combination of the external Lines of Business and the internal Business Units. Equation 16 will be used for this purpose and will have the following form:

$$l_S = C \times r_S + p \quad ; S = BU\ 01, \ldots, BU\ 06, BL01, \ldots, BL08$$

$$l = \log\left[\mu\left(L_{WEEK,\,S}\right)\right]$$

$$C = \lambda_\mu$$

$$r = \log\left[\left(R_{idio}\right)_{WEEK,\,S}\right]$$

$$p = \log\left[\mu\left(L_{WEEK,STANDARD}\right)\right]$$

The lxr plot and the regression line that best describes the combination of the internal data and the external data are given in the following figure.

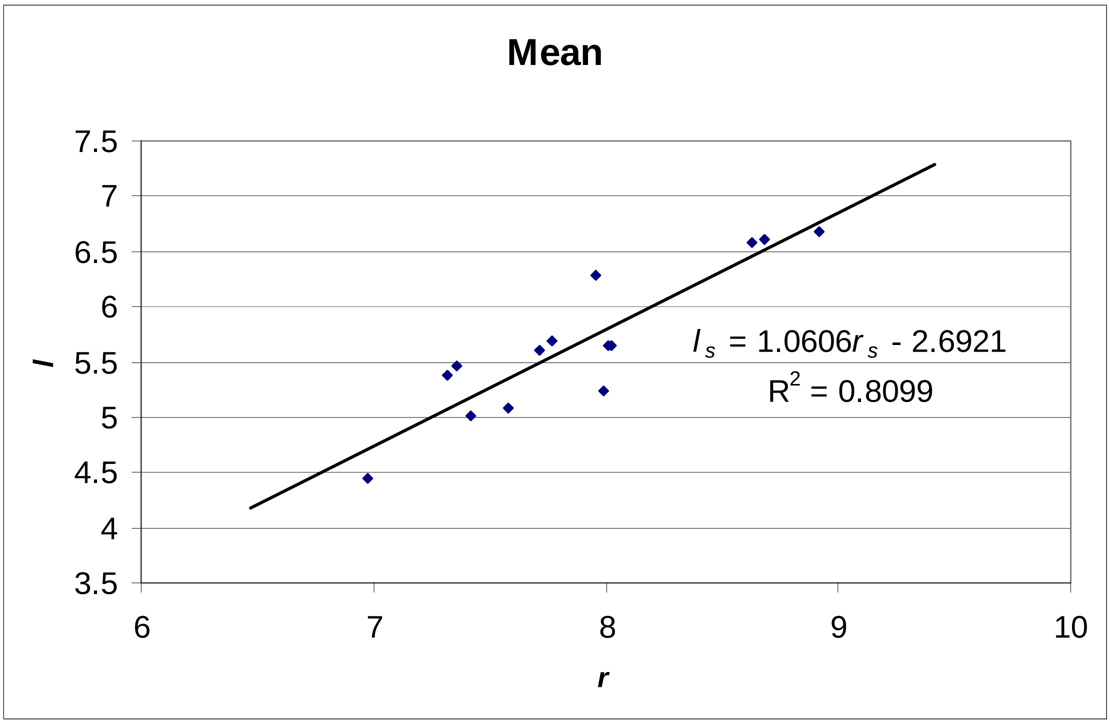


Figure 10 – *l* x *r* plot. The horizontal axis gives the variable *r* value, while the vertical axis gives the variable *l* value. The regression line that best fits the combination of the two data is given by the solid line.

The results of the regression on the combination of the external Lines of Business and the internal Business Units are shown in the next table:

R-Square	*Regression results*	*Coefficients*	*Standard Error*	*t Stat*	*P-value*
0.8099	Intercept (p)	-2.6921	1.1718	-2.2975	0.0404
	C	1.0606	0.1483	7.1511	0.0000

Table 10

The coefficient values are respectively 1.0606 for the variable C, and -2.6921 for the variable p. The R^2 value is 0.8099. This suggests that around 80.99% of variable l is attributable to variable r. The value of R^2 lies in the middle between the R^2 value of the experiment using only the external data and the R^2 value of the experiment using only the internal data.

The *P-value* of C in the sixth column is smaller than 0.05 and indicates that the coefficient C is significantly different from zero within the 95% confidence interval. Not only that, the coefficient C is also significant within the 99.99% confidence interval, because the *P-value* is slightly less than 0.0001.

The confidence interval is increasing when we perform the regression on the combination of internal Business Units and external Lines of Business. This result suggests that there exists a universal power-law relationship between the operational loss amount per week and the size & exposure towards operational risk per week of the combination of external Lines of Business & internal Business Units.

5.4.2 Estimating variable λ_σ

In this section we will first apply the regression between the log values of the standard deviation of operational loss amount per week and the log values of the Gross Income per week, only using the external Lines of Business. For this reason, we can rewrite Equation 16 as:

$$l_{EXTERNAL} = C \times r_{EXTERNAL} + p \quad ; EXTERNAL = BL01, \ldots, BL08$$
$$l = \log\left[\sigma\left(L_{WEEK,\, EXTERNAL}\right)\right]$$
$$C = \lambda_\sigma$$
$$r = \log\left[\left(R_{idio}\right)_{WEEK, EXTERNAL}\right]$$
$$p = \log\left[\sigma\left(L_{WEEK, STANDARD}\right)\right]$$

Afterwards, the regression is run only using the internal Business Units. The regression of Equation 16 becomes:

$$l_{INTERNAL} = C \times r_{INTERNAL} + p \quad ; INTERNAL = BU\ 01, \ldots, BU\ 06$$

$l = \log[\sigma(L_{WEEK,INTERNAL})]$

$C = \lambda_{\sigma}$

$r = \log[(R_{idio})_{WEEK,INTERNAL}]$

$p = \log[\sigma(L_{WEEK,STANDARD})]$

The lxr plot, the regression line that best describes the internal data, and the regression line that best describes the external data are given in the following figure:

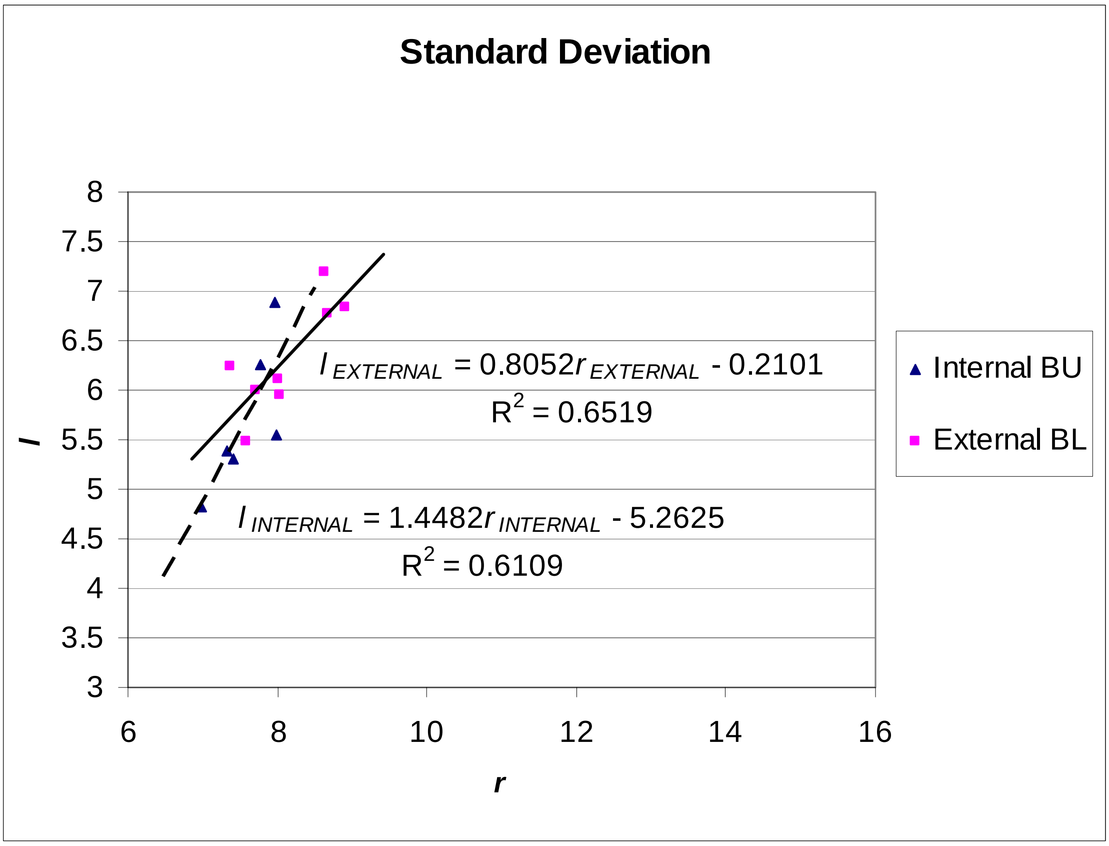


Figure 11 – *l* x *r* plot. The horizontal axis gives the variable *r* value, while the vertical axis gives the variable *l* value. Internal Business Units are given in triangles, and the regression line that best fits the internal data is given by the dash line. External Lines of Business are given in squares, with solid line as the regression line that best fits these Lines of Business.

The regression results are described in detail in the subsequent sections. We start with the regression result of the external Lines of Business (external data).

5.4.2.1 External data

The results of the regression on the Lines of Business of the external data are shown in the following table:

R-Square	Regression results	Coefficients	Standard Error	t Stat	P-value
0.6519	Intercept (*p*)	-0.2101	1.9534	-0.1075	0.9179
	C	0.8052	0.2402	3.3521	0.0154

Table 11

An R^2 value of 0.6519 suggests that around 89.14% of variable l is attributable to variable r for the external Lines of Business. The *P-value* of coefficient C in the sixth column is definitely less than 0.05, which means that the coefficient C is significantly different from zero within the 95% confidence interval. This result suggests that there is a power-law relationship between the operational loss amount per week and the size & exposure towards operational risk per week of the external Lines of Business. The idiosyncratic component can explain a big proportion of the variability in the operational loss amount per week, as we can see from the rather high value of R^2.

5.4.2.2 Internal data

The results of the regression on the Business Units of the internal data are shown in the following table:

R-Square	Regression results	Coefficients	Standard Error	t Stat	P-value
0.6109	Intercept (*p*)	-5.2625	4.3808	-1.2013	0.2959
	C	1.4482	0.5779	2.5060	0.0663

Table 12

The R^2 value of 0.6109 suggests that around 61.09% of variable l is attributable to variable r for the internal Business Units. The *P-value* of coefficient C is 0.0663 (slightly higher than 0.05), so we cannot reject the null hypothesis that the coefficient C is equal to zero within the 95% confidence interval. Nevertheless, we can reject the null hypothesis that the coefficient C is equal to zero within the 90% confidence interval (the *P-value* of coefficient C is less than the alpha level of 0.10). The idiosyncratic component can explain a big proportion (61.09%) of the variability in the operational loss amount per week, which can be seen from the rather high value of R^2.

Once more, this result suggests that the power-law relationship between the operational loss amount per week and the size & exposure towards operational risk per week of the internal Business Units is present, conditional on a lower confidence interval than the confidence interval of the external data. The relationship existence subject to a lower confidence interval than the confidence interval when using the external data might be yet again due to the choice of categorising losses per Business Units instead of per Lines of Business of the bank.

5.4.2.3 Combination of external data and internal data

Finally, we perform the regression on the combination of the external Lines of Business and the internal Business Units. Equation 16 will be used for this purpose and will have the following form:

$$l_S = C \times r_S + p \quad ; S = BU\ 01, \ldots, BU\ 06, BL01, \ldots, BL08$$

$$l = \log\left[\sigma\left(L_{WEEK,\,S}\right)\right]$$

$$C = \lambda_\sigma$$

$$r = \log\left[\left(R_{idio}\right)_{WEEK,\,S}\right]$$

$$p = \log\left[\sigma\left(L_{WEEK,STANDARD}\right)\right]$$

The lxr plot and the regression line that best describes the combination of the internal data and the external data are given in the following figure.

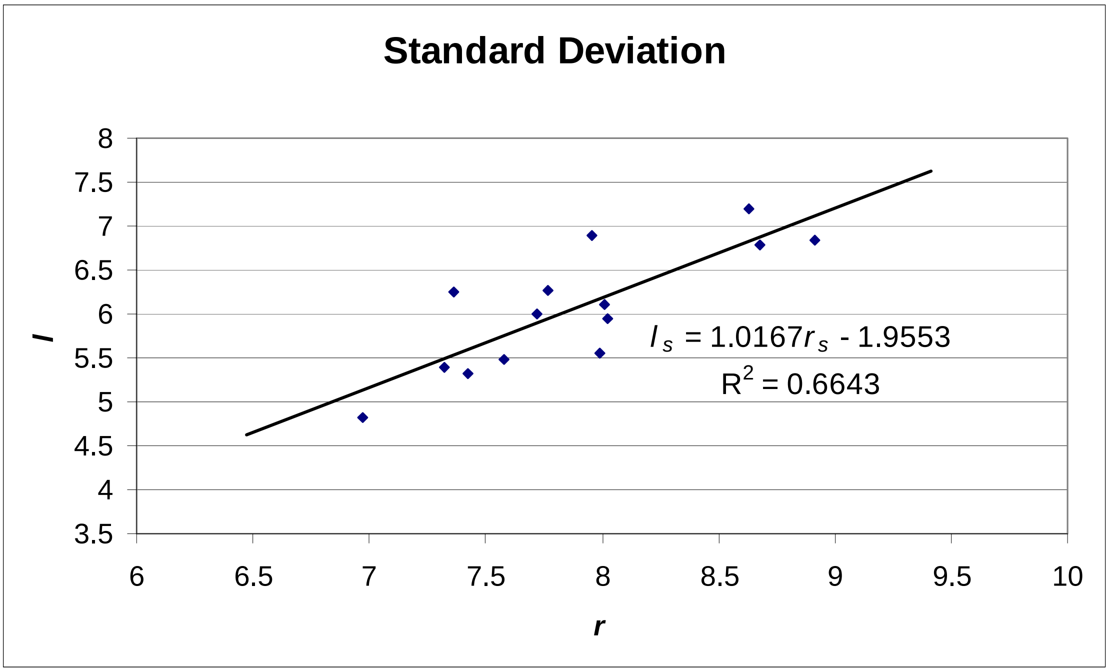


Figure 12 – *l* x *r* plot. The horizontal axis gives the variable *r* value, while the vertical axis gives the variable *l* value. The regression line that best fits the combination of the two data is given by the solid line.

The results of the regression on the combination of the external Lines of Business and the internal Business Units are shown in the next table:

R-Square	*Regression results*	*Coefficients*	*Standard Error*	*t Stat*	*P-value*
0.6643	Intercept (*p*)	-1.9553	1.6482	-1.1864	0.2584
	C	1.0167	0.2086	4.8732	0.0004

Table 13

The R^2 value of 0.6643 suggests that around 66.43% of variable l is attributable to variable r. The value of R^2 is higher than the R^2 value of the experiment using only the external data and the R^2 value of the experiment using only the internal data. The *P-value* of C in the sixth column is smaller than 0.05 and indicates that the coefficient C is significantly different from zero within the 95% confidence interval. Not only that, the coefficient C is also significant within the 99.95% confidence interval, because the *P-value* is slightly less than 0.0005.

The confidence interval is increasing when we perform the regression on the combination of internal Business Units and external Lines of Business. This result suggests that there exists a universal power-law relationship between the operational loss amount per week and the size & exposure towards operational risk per week of the combination of external Lines of Business & internal Business Units.

5.4.3 Summary

We have run the regressions in the previous sections. The following results are obtained:

1. There *exists* a power-law relationship between the operational loss amount per week and the size & exposure towards operational risk per week of the external Lines of Business.
2. There also *exists* a power-law relationship between the operational loss amount per week and the size & exposure towards operational risk per week of the internal Business Units is also present, although on a lower confidence interval than the confidence interval of the external data. The existence of the power-law relationship, subject to a lower confidence interval than the confidence interval when using the external data, might be due to the choice of categorising losses per Business Units instead of per Lines of Business of the bank.
3. There is a *universal* power-law relationship between the operational loss amount per week and the size & exposure towards operational risk per week of the combination of external Lines of Business & internal Business Units.
4. We have estimated the value of variables λ_μ and λ_σ via the mean and standard deviation values, respectively, of operational loss amount per week of the combination of external Lines of Business & internal Business Units. Inserting the estimates of variables λ_μ and λ_σ into Equation 13, we have:

$$\mu(L_{WEEK,S}) = (R_{idio})_{WEEK,S}^{1.0606} \times \mu(L_{WEEK,STANDARD}) \quad ; S = BU\ 01,\ldots,BU\ 06, BL01,\ldots,BL08$$

$$\sigma(L_{WEEK,S}) = (R_{idio})_{WEEK,S}^{1.0167} \times \sigma(L_{WEEK,STANDARD}) \quad ; S = BU\ 01,\ldots,BU\ 06, BL01,\ldots,BL08$$

The estimates of variable λ_μ and variable λ_σ can be seen as equivalent, even though they are slightly dissimilar in the second decimal. Please note that we have estimated the mean and standard deviation of operational loss amount per week of different Lines of Business and Business Units by using only the loss data of year 2003. The slightly dissimilarity between the estimates of λ_μ and λ_σ might be caused by the slightly difference between the estimated mean and standard deviation and the true mean and standard deviation of operational loss amount per week of different Lines of Business and Business Units. This problem can be circumvented by using more loss data (e.g. several years) to have good estimates of the true mean and standard deviation of operational loss amount per week of different Lines of Business and Business Units.

Since the estimates of λ_μ and λ_σ are more or less the same, this result suggests that the mean value scale in the same way as the standard deviation value. Based on Assumption 4, the estimates of λ_μ and λ_σ can be used to represent the value of variable λ in the operational loss function. Because the two estimates are slightly dissimilar, we use the average of the two estimates (1.0387) as the value of variable λ. We can then enter the value of variable λ into Equation 12 as follow:

$$L_{WEEK,S} = (R_{idio})_{WEEK,S}^{1.0387} \times L_{WEEK,STANDARD} \quad ; S = BU\ 01,\ldots,BU\ 06, BL01,\ldots,BL08$$

Equation 17

It is important to remark that the estimates of λ_μ and λ_σ are very close to 1. This suggests that the universal power-law relationship can be very likely regarded as a linear form. In other words, the operational loss amount per week relates almost linearly to the size & exposure towards operational risk per week of the combination of external Lines of Business & internal Business Units.

Please note that even though the mean and standard deviation values scale more or less in the same way, this does not guarantee that the probability density function of operational loss scale in the same way as the mean and the standard deviation values. In the future, it is necessary to estimate the value of variable λ by employing Equation 12:

$$L_{WEEK,S} = \left(R_{idio}\right)^{\lambda}_{WEEK,S} \times L_{WEEK,STANDARD} \quad ; S = BU\ 01, \dots, BU\ 06, BL01, \dots, BL08,$$

Using this equation, the regression will be done on each operational loss amount per week data, instead of only on the mean and standard deviation values, against the idiosyncratic component of every Line of Business and Business Unit. Thus, Equation 14 will be used:

$$\log\left[L_{WEEK,S}\right] = \lambda \times \log\left[\left(R_{idio}\right)_{WEEK,S}\right] + \log\left[L_{WEEK,STANDARD}\right] \quad ; S = BU\ 01, \dots, BU\ 06, BL01, \dots, BL08$$

If the estimate of λ is equivalent to the estimates of λ_{μ} and λ_{σ}, we can then say that the probability density function of operational loss scale in the same way as the mean and the standard deviation values. In other words, we can then conclude that Assumption 4 is valid.

5.5 What to do with $L_{WEEK,STANDARD}$

Before we show an example of utilising the operational loss standard per week ($L_{WEEK,STANDARD}$), let us first observe the probability density function of the operational loss amount per week. The number of operational loss data of different Business Units and Lines of Business in year 2003 is provided in the following table.

INSERT TABLE N HERE

We can see from this table that there is only a small number of operational loss data in certain Lines of Business or Business Units. The discrete probability density function (p.d.f.) of the operational loss amount per week L_{WEEK} for each Business Unit and Line of Business is plotted in the following figure:

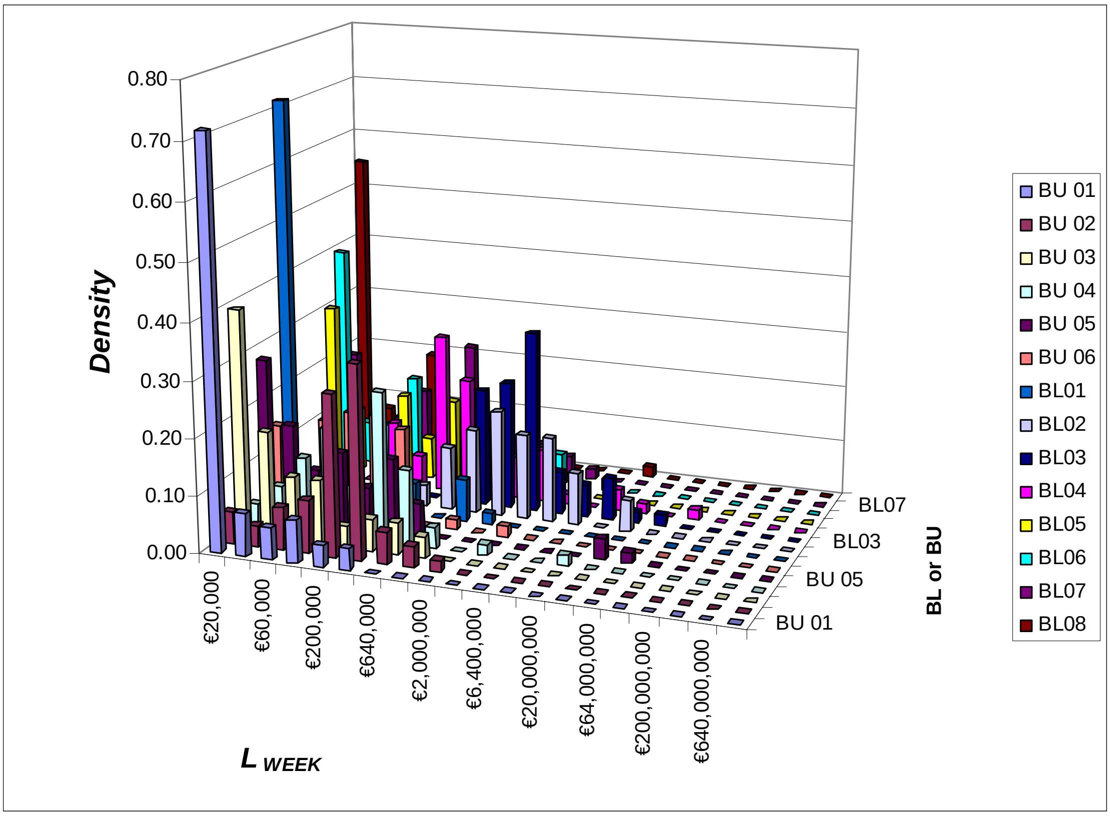
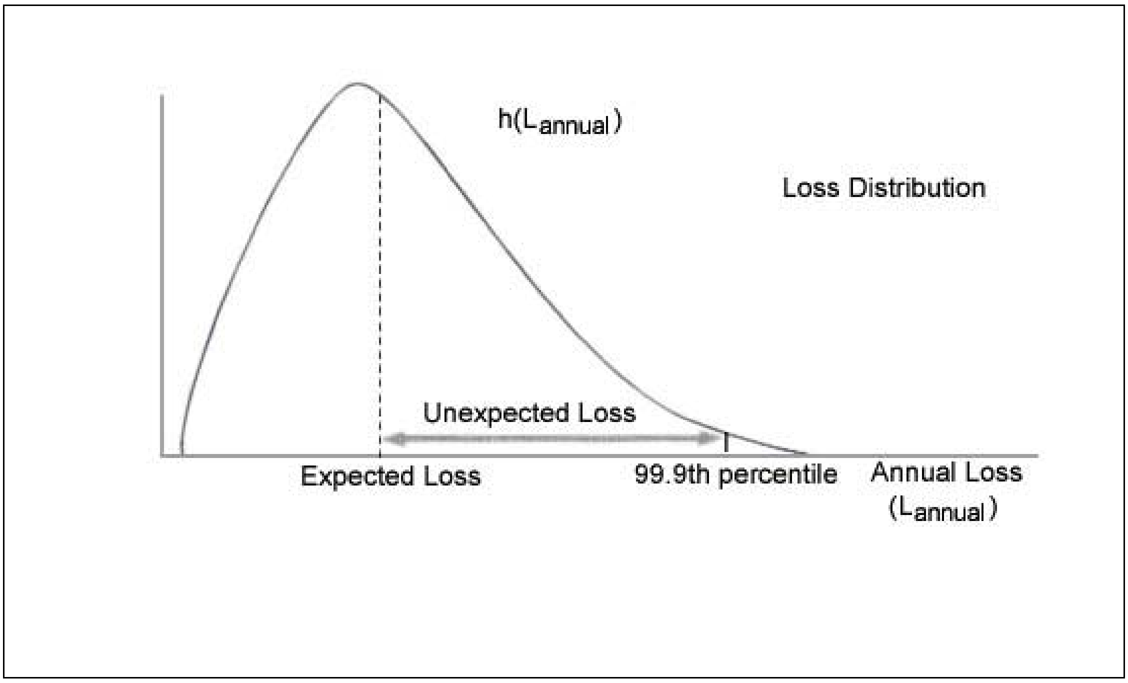


Figure 13 – Discrete probability density function of operational loss amount per week (L_{WEEK}) for each Business Unit and Line of Business. The x-axis gives the L_{WEEK} , the y-axis gives the density of each particular range of L_{WEEK} .

We can see that the probability density functions of operational loss amount per week for the external Lines of Business and the internal Business Units are quite different to each other. Since we only use the operational loss data of year 2003, the operational loss data in a Business Unit or Line of Business is very likely not enough to obtain a good estimate of the true discrete probability density function of operational loss amount per week for that Business Unit or Line of Business.

The probability density function of operational loss amount per week is expected to be the same with the probability density function of operational loss amount per year, because the underlying components are the same. The true probability density function of operational loss amount per year is expected to be similar to the one given in Figure 4, which is also given below:



Remember that the probability density function in this figure is a continuous function. The operational loss amounts per week of each Line of Business and Business Unit can be scaled into variable $L_{WEEK,STANDARD}$ by means of Equation 11 as follow:

$$\frac{L_{WEEK,BU\,01}}{\left(R_{idio}\right)^{1.0387}_{WEEK,BU\,01}} = \ldots = \frac{L_{WEEK,BU\,06}}{\left(R_{idio}\right)^{1.0387}_{WEEK,BU\,06}} = \frac{L_{WEEK,BL01}}{\left(R_{idio}\right)^{1.0387}_{WEEK,BL01}} = \ldots = \frac{L_{WEEK,BL08}}{\left(R_{idio}\right)^{1.0387}_{WEEK,BL08}} = L_{WEEK,STANDARD}$$

The discrete standard probability density function for each Line of Business and Business Unit after transformation can be given in the following figure:

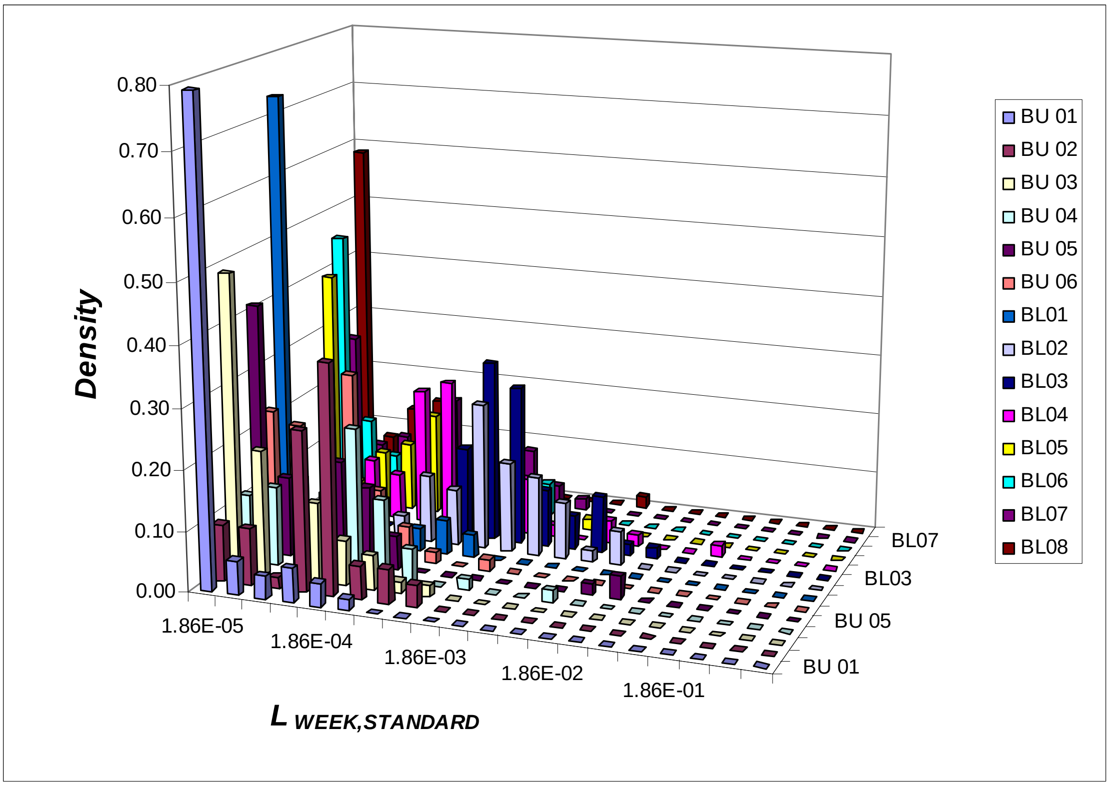


Figure 14 – Discrete standard probability density function of operational loss amount per week ($L_{WEEK,STANDARD}$) for each Business Unit and Line of Business. The x-axis gives the $L_{WEEK,STANDARD}$, the y-axis gives the density of each particular range of $L_{WEEK,STANDARD}$.

We can see that the standard probability density functions after scaling are quite similar to the probability density functions before scaling. The mean and standard deviation values of the standard probability density function of different Business Units and Lines of Business are given in the following table:

INSERT TABLE O HERE

Please note that after scaling, the operational loss standard is only influenced by the common component and is no longer affected by the idiosyncratic component. In other words, the operational loss amounts of different Lines of Business can be utilised together in their operational loss standard form. Therefore, if we put together the operational loss standard from all Lines of Business and Business Units, the combined discrete probability density function of $L_{WEEK,STANDARD}$ will be given by the following figure:

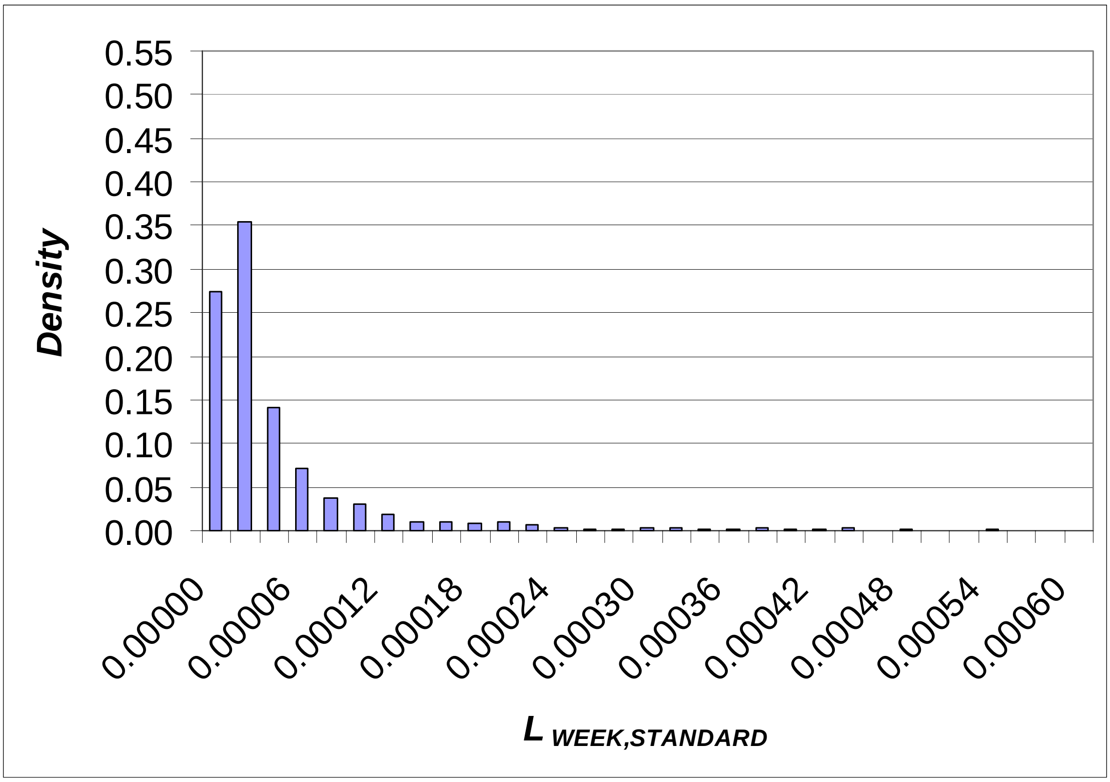


Figure 15 – Discrete probability density function of $L_{WEEK,STANDARD}$. The x-axis gives the $L_{WEEK,STANDARD}$, the y-axis gives the density of each particular range of $L_{WEEK,STANDARD}$.

The shape of this combined discrete probability density function is quite similar and comparable to the shape of the expected true continuous probability density function of operational loss per year, which is given in the previous two pages. This result suggests that after putting together the operational loss standard from all Lines of Business and

Business Units, we have a probability density function that is very similar to the expected true probability density function.

Therefore, the operational risk capital amount can then be calculated on basis of this probability density function. In the next section, we will show an application of $L_{WEEK,STANDARD}$ in calculating the operational risk capital amount.

5.5.1 Utilising $L_{WEEK,STANDARD}$ to calculate operational risk capital

In this section, we will show you how to utilise the $L_{WEEK,STANDARD}$ in order to calculate the operational risk capital of each Line of Business and Business Unit. The Value at Risk (VAR) of loss standard $L_{T,STANDARD}$ within time period T can be given by the following formula:

$$VAR_{T,CI,STANDARD} = L_{T,STANDARD,J} \quad ; J = CI \times N$$

Equation 18

$VAR_{T,CI,STANDARD} =$ Value at Risk in the $L_{T,STANDARD}$ world; the expected maximum loss or worst loss over a target horizon T within a confidence interval CI

$L_{T,STANDARD,J} =$ The J^{th} sorted loss standard within time period T, where $J = CI \times N$

$N =$ The number of data points in the $L_{T,STANDARD}$

The sorted loss standard per week $L_{WEEK,STANDARD}$ data is given in the following figure:

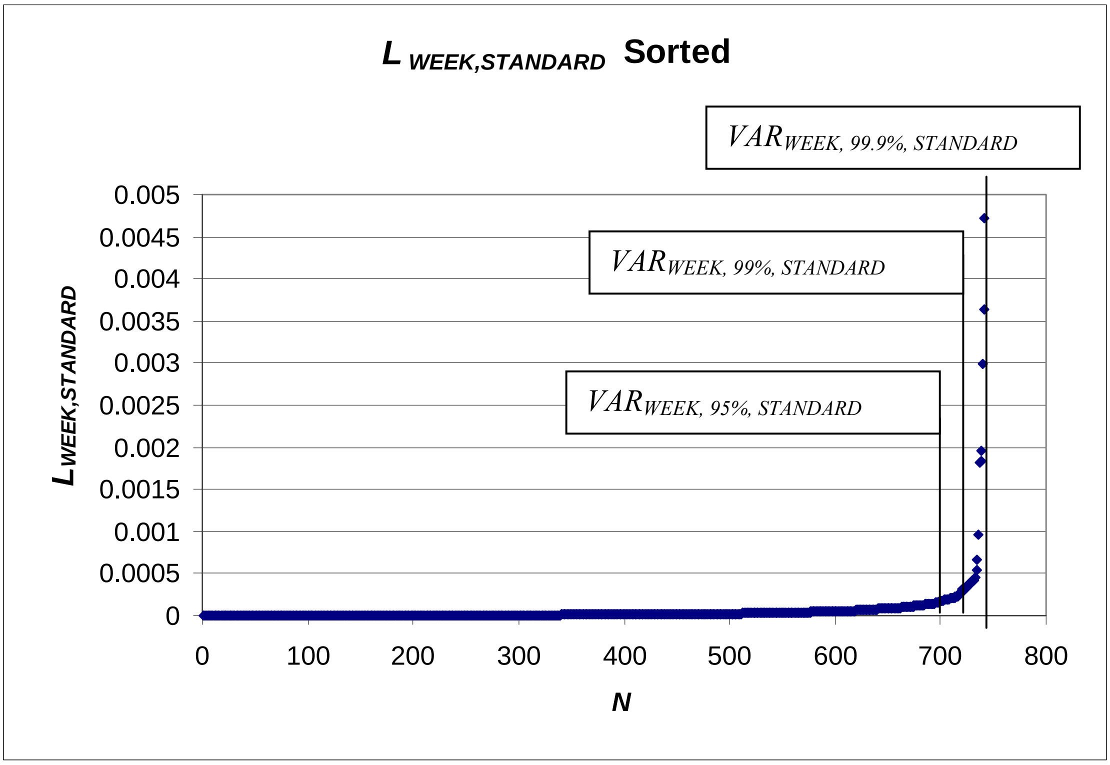


Figure 16 – Sorted values of $L_{WEEK,STANDARD}$

The VAR of loss standard per week for three different confidence interval values is given in the following table:

Confidence Interval	***J***	***VAR***
95%	705	0.00019
99%	735	0.00067
99.9%	741	0.00363

Table 14

Recall from Section 2.4.3 that the operational risk capital requirement will be given by the difference (unexpected loss) between the VAR within the confidence interval of 99.9% and the mean of the operational loss distribution (expected loss) for a time horizon of one year.

In our case, we use a time horizon of one week. In case of the $VAR_{WEEK, 99.9\%, STANDARD}$, the 741st data of sorted loss standard $L_{WEEK,STANDARD}$ is 0.00363. The mean of the sorted loss standard per week $L_{WEEK,STANDARD}$ is 0.00006. The difference (unexpected loss) will then be 0.00357. Thus, this figure is the operational risk capital standard with a time horizon of one week ($ORC_{WEEK,STANDARD}$). The value of $ORC_{WEEK,STANDARD}$ for the other two confidence interval values is also given in the following table:

Confidence Interval	**ORC$_{WEEK,STANDARD}$**
95%	0.00013
99%	0.00061
99.9%	0.00357

Table 15

Let us recall Equation 17:

$$L_{WEEK,S} = \left(R_{idio}\right)_{WEEK,S}^{1.0387} \times L_{WEEK,STANDARD} \quad ; S = BU\ 01, \ldots, BU\ 06, BL01, \ldots, BL08$$

To transform the operational risk capital standard with a time horizon of one week into the operational risk capital amount with a time horizon of one week for each Line of Business and Business Unit, we can rewrite Equation 17 as:

$$ORC_{WEEK,S} = \left(R_{idio}\right)_{WEEK,S}^{1.0387} \times ORC_{WEEK,STANDARD} \quad ; S = BU\ 01, \ldots, BU\ 06, BL01, \ldots, BL08$$

Equation 19

$ORC_{WEEK,S} =$ The operational risk capital amount with a time horizon of one week for each Line of Business or Business Unit S

$ORC_{WEEK,STANDARD} =$ The operational risk capital standard with a time horizon of one week

$\left(R_{idio}\right)_{WEEK,S}^{1.0387} =$ The idiosyncratic component (size & exposure towards operational risk) per week of each Line of Business and Business Unit S, to the power of λ (1.0387)

If the idiosyncratic component per week of Line of Business BL01 is, for example, €50,000,000 the operational risk capital amount with a time horizon of one week and confidence interval 99.9% for that Line of Business can be simply calculated by:

$$ORC_{WEEK,BL01} = (€50{,}000{,}000)^{1.0387} \times 0.00357$$

The result is €354,569.16. For the other two confidence interval values, we have:

Confidence Interval	$ORC_{WEEK,STANDARD}$	ORC_{WEEK}
95%	0.00013	€12,474.62
99%	0.00061	€60,357.84
99.9%	0.00357	€354,569.16

Table 16

The operational risk capital amount with a time horizon of one week for the rest of the Lines of Business and Business Units can be calculated in the same way. If we can assume that the operational risk capital amount with a time horizon of one year is simply the operational risk capital amount with a time horizon of one week multiplied by the number of weeks in a year, we will have:

$$ORC_{YEAR} = ORC_{WEEK} \times w$$

Equation 20

$ORC_{YEAR} =$ The operational risk capital amount with a time horizon of one year

$ORC_{WEEK} =$ The operational risk capital amount with a time horizon of one week

$w =$ The number of weeks in one year

Let us assume that there are 53 weeks in one year, which is based on the '$weeknum(\ )$' function we used before. The operational risk capital amount with a time horizon of one year for Line of Business BL01 will be given by the following figure: €18,787,293.90.

The operational risk capital amount with a time horizon of one year for the other two confidence interval values is given in the following table:

Confidence Interval	**ORC_{YEAR}**
95%	€661,154.79
99%	€3,198,965.28
99.9%	€18,792,165.23

Once more, the operational risk capital amount with a time horizon of one year for the rest of the Lines of Business and Business Units can be calculated in the same way.

5.5.2 Summary

In the previous section we have shown that the operational loss amount per week of all Lines of Business and Business Units can be combined together, after being scaled into the operational loss *standard* per week. This is possible, since the idiosyncratic components of different Lines of Business and Business Units no longer becomes an issue in the operational loss *standard* per week data.

We also showed an example of how to utilise the operational loss standard per week, $L_{WEEK,STANDARD}$, in order to calculate the operational risk capital amount with a time horizon of one week of each Line of Business and Business Unit. The operational risk capital calculation can also be done for different confidence intervals.

The operational loss capital amount with a time horizon of one week of each Line of Business and Business Unit will simply be the operational loss capital standard with a time horizon of one week multiplied by the idiosyncratic component (size & exposure towards operational risk) per week of each Line of Business and Business Unit, to the power of λ. In our case, the value of variable λ is (1.0387).

If we can assume that the operational risk capital amount with a time horizon of one year is simply the operational risk capital amount with a time horizon of one week multiplied by the number of weeks in a year, the operational loss capital amount with a time horizon of one year of each Line of Business and Business Unit will be obtained immediately.

5.6 Frequency and Severity of Operational Loss

Let us recall from Section 2.4.3 that an operational risk loss distribution is usually estimated by a compound of the frequency and severity distributions. It is interesting to observe whether the power-law relationship - between the operational loss amount per week and the size & exposure towards operational risk per week of the external Lines of Business, the internal Business Units, and the combination of external Lines of Business & internal Business Units - comes from the frequency element, the severity element, or even from both elements. For this reason, a similar experiment is conducted to the frequency and the severity distributions. We will start with the frequency element in the next section.

5.6.1 Frequency of Operational Loss per week

In this section we will study the relationship between the frequency (number of loss events) of operational loss per week and the size & exposure towards operational risk per week of different Lines of Business and Business Units. In the following equations, the operational loss amount per week will be replaced by the frequency of operational loss per week. Equation 12, Equation 13, Equation 14, and Equation 15 can thus be modified as:

$$Freq_{T,S} = \left(R_{idio}\right)_{T,S}^{\lambda} \times Freq_{T,STANDARD} \quad ; S = BL01, BL02, \ldots$$

Equation 21

$Freq_{T,S}$ = The frequency of operational loss per week within time period T for each Line of Business and Business Unit S

$Freq_{T,STANDARD}$ = The frequency of operational loss per week standard within time period T

$(R_{idio})^{\lambda}_{T,S}$ = The idiosyncratic component within time period T for each Line of Business and Business Unit S

$$\mu(Freq_{T,S}) = (R_{idio})^{\lambda_\mu}_{T,S} \times \mu(Freq_{T,STANDARD}) \quad ; S = BL01, BL02, \ldots$$
$$\sigma(Freq_{T,S}) = (R_{idio})^{\lambda_\sigma}_{T,S} \times \sigma(Freq_{T,STANDARD}) \quad ; S = BL01, BL02, \ldots$$

Equation 22

$$\log[Freq_{T,S}] = \lambda \times \log[(R_{idio})_{T,S}] + \log[Freq_{T,STANDARD}] \quad ; S = BL01, BL02, \ldots$$

Equation 23

$$\log[\mu(Freq_{T,S})] = \lambda_\mu \times \log[(R_{idio})_{T,S}] + \log[\mu(Freq_{T,STANDARD})] \quad ; S = BL01, BL02, \ldots$$
$$\log[\sigma(Freq_{T,S})] = \lambda_\sigma \times \log[(R_{idio})_{T,S}] + \log[\sigma(Freq_{T,STANDARD})] \quad ; S = BL01, BL02, \ldots$$

Equation 24

Finally, Equation 16 can be as well modified as:

$$l_S = C \times r_S + p \quad ; S = BL01, BL02, \ldots$$

Equation 25

In case of the mean value, we have:

$l = \log[\mu(Freq_{T,S})]$

$C = \lambda_\mu$

$r = \log\left[\left(R_{idio}\right)_{T,S}\right]$

$p = \log\left[\mu\left(Freq_{T,STANDARD}\right)\right]$

In case of the standard deviation value, we have:

$l = \log\left[\sigma\left(Freq_{T,S}\right)\right]$

$C = \lambda_{\sigma}$

$r = \log\left[\left(R_{idio}\right)_{T,S}\right]$

$p = \log\left[\sigma\left(Freq_{T,STANDARD}\right)\right]$

We obtain the log values of the mean and standard deviation of the frequency of operational loss per week and the log values of the Gross Income per week in year 2003 for different Lines of Business and Business Units. The information is provided in the following table.

INSERT TABLE P HERE

5.6.1.1 Estimating λ_{μ}

We will first apply the regression between the log values of the mean of the frequency of operational loss per week and the log values of the Gross Income per week, only using the external Lines of Business. For this reason, we can rewrite Equation 25 as:

$l_{EXTERNAL} = C \times r_{EXTERNAL} + p \quad ; EXTERNAL = BL01, \ldots, BL08$

$l = \log\left[\mu\left(Freq_{WEEK,\,EXTERNAL}\right)\right]$

$C = \lambda_{\mu}$

$r = \log\left[\left(R_{idio}\right)_{WEEK,EXTERNAL}\right]$

$p = \log\left[\mu\left(Freq_{WEEK,STANDARD}\right)\right]$

Afterwards, the regression is run only using the internal Business Units. The regression of Equation 25 becomes:

$$l_{INTERNAL} = \lambda \times r_{INTERNAL} + p \quad ; INTERNAL = BU\ 01, \ldots, BU\ 06$$
$$l = \log\left[\mu\left(Freq_{WEEK,INTERNAL}\right)\right]$$
$$C = \lambda_{\mu}$$
$$r = \log\left[\left(R_{idio}\right)_{WEEK,INTERNAL}\right]$$
$$p = \log\left[\mu\left(Freq_{WEEK,STANDARD}\right)\right]$$

The lxr plot, the regression line that best describes the internal data, and the regression line that best describes the external data are given in the following figure.

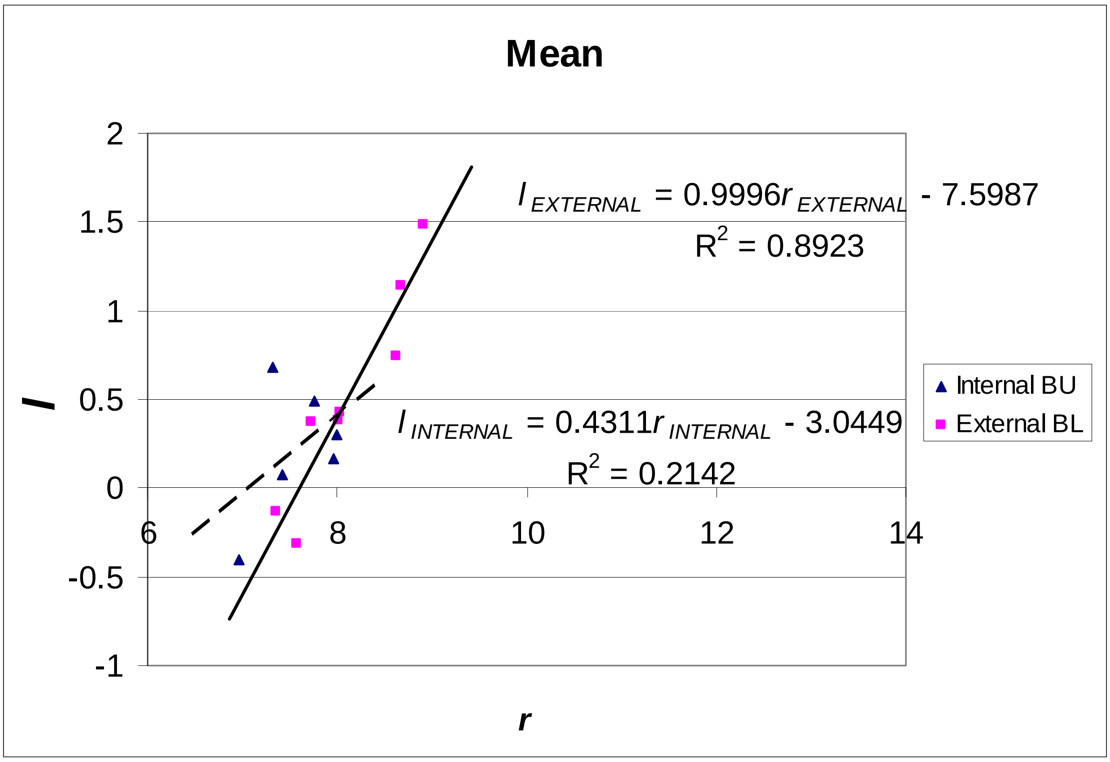


Figure 17 – *l* x *r* plot. The horizontal axis gives the variable *r* value, while the vertical axis gives the variable *l* value. Internal Business Units are given in triangles, and the regression line that best fits

the internal data is given by the dash line. External Lines of Business are given in squares, with solid line as the regression line that best fits these Lines of Business.

5.6.1.1.1 External data

The results of the regression on the Lines of Business of the external data are shown in the following table:

R-Square	***Regression results***	***Coefficients***	***Standard Error***	***t Stat***	***P-value***
0.8923	Intercept (*p*)	-7.5987	1.1529	-6.5907	0.0006
	C	0.9996	0.1418	7.0507	0.0004

Table 17

The R^2 value is 0.8923, which that around 89.23% of variable l is attributable to variable r for the external Lines of Business. The *P-value* of coefficient C in the sixth column is definitely less than 0.05, which means that the coefficient C is significantly different from zero within the 95% confidence interval. The coefficient C is also significant within the confidence interval of 99.95%, because the *P-value* is slightly less than 0.0005. This result suggests that there is a power-law relationship between the frequency of operational loss per week and the size & exposure towards operational risk per week of the external Lines of Business. The idiosyncratic component can explain a big proportion of the variability in the frequency of operational loss per week, as we can see from the high value of R^2. The same regression is done to the Business Units of the internal data.

5.6.1.1.2 Internal data

The results of the regression on the Business Units of the internal data are shown in the following table:

R-Square	*Regression results*	*Coefficients*	*Standard Error*	*t Stat*	*P-value*
0.2142	Intercept (p)	-3.0449	3.1297	-0.9729	0.3857
	C	0.4311	0.4129	1.0442	0.3553

Table 18

The R^2 value of 0.2142 suggests that only around 21.42% of variable l is attributable to variable r for the internal Business Units. In other words, the idiosyncratic component can explain merely a small proportion of the variability in the frequency of operational loss per week, as we can see from the low value of R^2 (0.2142). The *P-value* of the coefficient C is 0.3553 (absolutely higher than 0.05), so we cannot reject the null hypothesis that the coefficient C is equal to zero within the 95% confidence interval. This result suggests that a power-law relationship between the frequency of operational loss per week and the size & exposure towards operational risk per week of the internal Business Units is not present. The same regression is performed again, this time on the combination of the external Lines of Business and the internal Business Units.

5.6.1.1.3 Combination of external data and internal data

Finally, we perform the regression on the combination of the external Lines of Business and the internal Business Units. Equation 25 will be used for this purpose and will have the following form:

$$l_S = C \times r_S + p \quad ; S = BU\ 01, \ldots, BU\ 06, BL01, \ldots, BL08$$

$$l = \log\left[\mu\left(Freq_{WEEK,\,S}\right)\right]$$

$$C = \lambda_\mu$$

$$r = \log\left[\left(R_{idio}\right)_{WEEK,\,S}\right]$$

$$p = \log\left[\mu\left(Freq_{WEEK,STANDARD}\right)\right]$$

The $l \times r$ graph and the regression line that best describe the combination of the internal and the external data are given in the following figure:

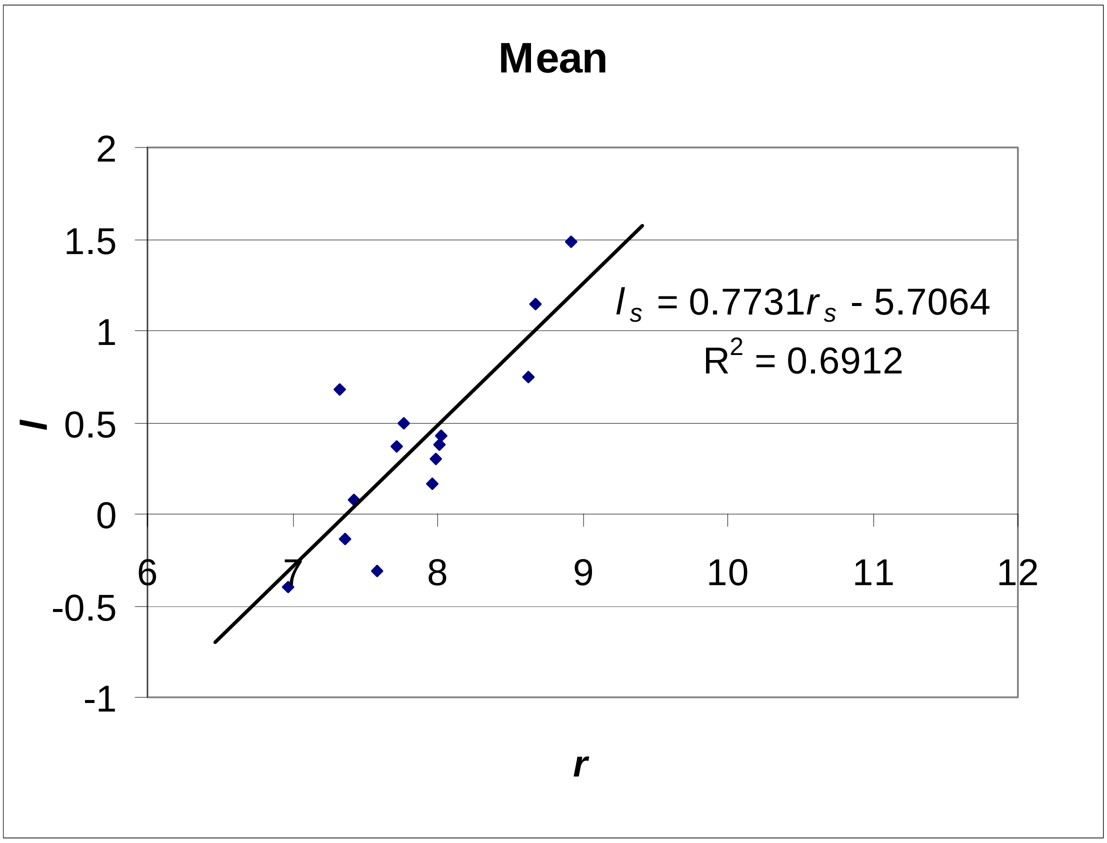


Figure 18 – *l* x *r* plot. The horizontal axis gives the variable *r* value, while the vertical axis gives the variable *l* value. The regression line that best fits the combination of the two data is given by the solid line.

The results of the regression on the combination of the external Lines of Business and the internal Business Units are shown in the next table:

R-Square	*Regression results*	*Coefficients*	*Standard Error*	*t Stat*	*P-value*
0.6912	Intercept (*p*)	-5.7064	1.1784	-4.8425	0.0004
	C	0.7731	0.1492	5.1831	0.0002

Table 19

The R^2 value is 0.6912. This suggests that around 69.12% of variable l is attributable to variable r. The value of R^2 lies in the middle between the R^2 value of the experiment using only the external data and the R^2 value of the experiment using only the internal data. The *P-value* of C in the sixth column is smaller than 0.05 and indicates that the coefficient C is significantly different from zero within the 95% confidence interval. Not only that, the coefficient C is also significant within the 99.95% confidence interval, because the *P-value* is less than 0.0005.

We find a power-law relationship between the frequency of operational loss per week and the size & exposure towards operational risk per week when performing the regression on the combination of the external Lines of Business and the internal Business Units. The confidence interval in this regression is also higher than the confidence interval when we perform the regression only on the external Lines of Business. This result suggests that there exists a universal power-law relationship between the frequency of operational loss per week and the size & exposure towards operational risk per week of the combination of external Lines of Business & internal Business Units.

The reason why a power-law relationship between the frequency of operational loss per week and the size & exposure towards operational risk per week of the internal Business Units is not present is potentially due to the choice of categorising losses per Business Units instead of per internal Lines of Business.

5.6.1.2 Estimating λ_σ

In this section we will first apply the regression between the log values of the standard deviation of the frequency of operational loss per week and the log values of the Gross Income per week, only using the external Lines of Business. For this reason, we can rewrite Equation 25 as:

$$l_{EXTERNAL} = C \times r_{EXTERNAL} + p \quad ; EXTERNAL = BL01, \ldots, BL08$$
$$l = \log\left[\sigma\left(Freq_{WEEK,\,EXTERNAL}\right)\right]$$

$$C = \lambda_{\sigma}$$
$$r = \log\left[(R_{idio})_{WEEK,EXTERNAL}\right]$$
$$p = \log\left[\sigma\left(Freq_{WEEK,STANDARD}\right)\right]$$

Afterwards, the regression is run only using the internal Business Units. The regression of Equation 25 becomes:

$$l_{INTERNAL} = C \times r_{INTERNAL} + p \quad ; INTERNAL = BU\ 01, \ldots, BU\ 06$$
$$l = \log\left[\sigma\left(Freq_{WEEK,INTERNAL}\right)\right]$$
$$C = \lambda_{\sigma}$$
$$r = \log\left[(R_{idio})_{WEEK,INTERNAL}\right]$$
$$p = \log\left[\sigma\left(Freq_{WEEK,STANDARD}\right)\right]$$

The lxr plot, the regression line that best describes the internal data, and the regression line that best describes the external data are given in the following figure:

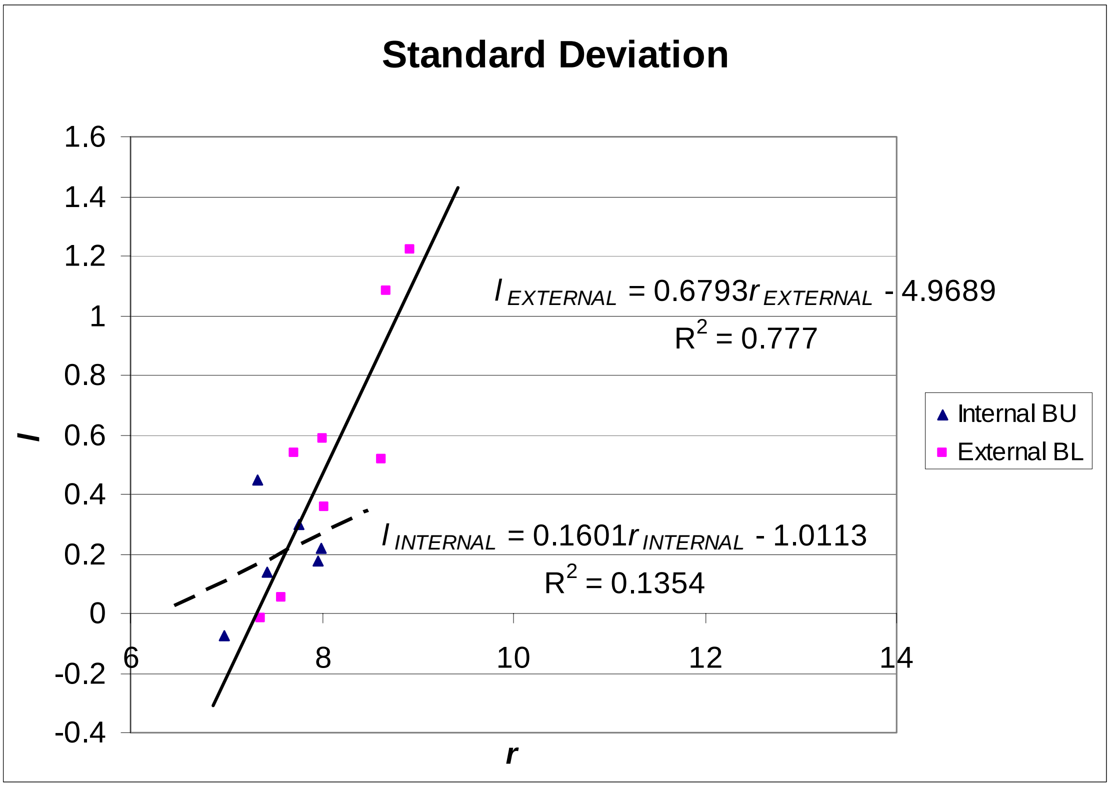


Figure 19 – *l* x *r* plot. The horizontal axis gives the variable *r* value, while the vertical axis gives the variable *l* value. Internal Business Units are given in triangles, and the regression line that best fits the internal data is given by the dash line. External Lines of Business are given in squares, with solid line as the regression line that best fits these Lines of Business.

The regression results are described in detail in the subsequent sections. We start with the regression result of the external Lines of Business (external data).

5.6.1.2.1 External data

The results of the regression on the Lines of Business of the external data are shown in the following table:

R-Square	***Regression results***	***Coefficients***	***Standard Error***	***t Stat***	***P-value***
0.7770	Intercept (p)	-4.9689	1.2082	-4.1127	0.0063
	C	0.6793	0.1486	4.5722	0.0038

Table 20

An R^2 value of 0.7770 suggests that around 77.70% of variable l is attributable to variable r for the external Lines of Business. The *P-value* of coefficient C in the sixth column is definitely less than 0.05, which means that the coefficient C is significantly different from zero within the 95% confidence interval. This result suggests that there is a power-law relationship between the frequency of operational loss per week and the size & exposure towards operational risk per week of the external Lines of Business. The idiosyncratic component can explain a big proportion of the variability in the frequency of operational loss per week, as we can see from the rather high value of R^2.

5.6.1.2.2 Internal data

The results of the regression on the Business Units of the internal data are shown in the following table:

R-Square	***Regression results***	***Coefficients***	***Standard Error***	***t Stat***	***P-value***
0.1354	Intercept (*p*)	-1.0113	1.5330	-0.6597	0.5455
	C	0.1601	0.2022	0.7916	0.4729

Table 21

The R^2 value of 0.1354 suggests that around 13.54% of variable l is attributable to variable r for the internal Business Units. In other words, the idiosyncratic component can explain merely a small proportion of the variability in the frequency of operational loss per week, as we can see from the low value of R^2 (0.2142). The *P-value* of the coefficient C is 0.4729 (absolutely higher than 0.05), so we cannot reject the null hypothesis that the coefficient C is equal to zero within the 95% confidence interval. This result suggests that a power-law relationship between the frequency of operational loss per week and the size & exposure towards operational risk per week of the internal Business Units is not present. The same regression is performed again, this time on the combination of the external Lines of Business and the internal Business Units.

5.6.1.2.3 Combination of external data and internal data

Finally, we perform the regression on the combination of the external Lines of Business and the internal Business Units. Equation 25 will be used for this purpose and will have the following form:

$$l_S = C \times r_S + p \quad ; S = BU\ 01, \ldots, BU\ 06, BL01, \ldots, BL08$$

$$l = \log\left[\sigma\left(Freq_{WEEK,\,S}\right)\right]$$

$$C = \lambda_\sigma$$

$$r = \log\left[\left(R_{idio}\right)_{WEEK,\,S}\right]$$

$$p = \log\left[\sigma\left(Freq_{WEEK,STANDARD}\right)\right]$$

The lxr plot and the regression line that best describes the combination of the internal data and the external data are given in the following figure.

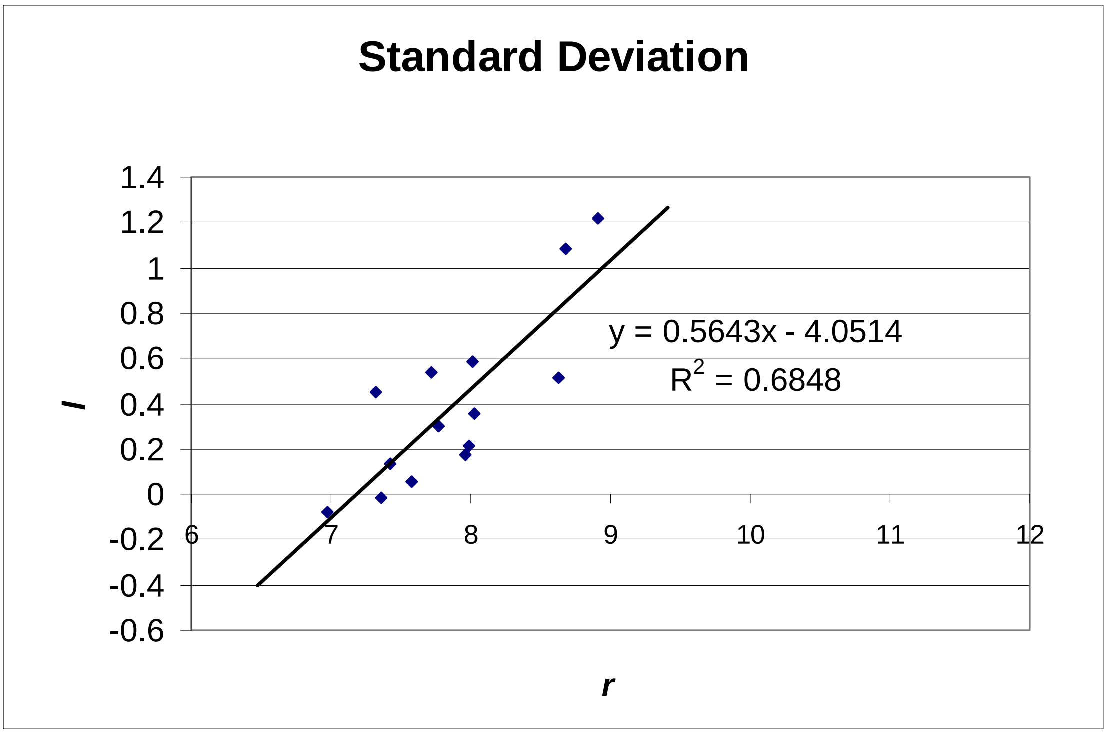


Figure 20 – *l* x *r* plot. The horizontal axis gives the variable *r* value, while the vertical axis gives the variable *l* value. The regression line that best fits the combination of the two data is given by the solid line.

The results of the regression on the combination of the external Lines of Business and the internal Business Units are shown in the next table:

R-Square	*Regression results*	*Coefficients*	*Standard Error*	*t Stat*	*P-value*
0.6848	Intercept (p)	-4.0514	0.8732	-4.6397	0.0006
	C	0.5643	0.1105	5.1058	0.0003

Table 22

The R^2 value of 0.6848 suggests that around 68.48% of variable l is attributable to variable r. The value of R^2 lies in the middle between the R^2 value of the experiment using only the external data and the R^2 value of the experiment using only the internal data. The *P-value* of C in the sixth column is smaller than 0.05 and indicates that the coefficient C is significantly different from zero within the 95% confidence interval. Not only that, the coefficient C is also significant within the 99.95% confidence interval, because the *P-value* is less than 0.0005.

The confidence interval is increasing when we perform the regression on the combination of internal Business Units and external Lines of Business. This result suggests that there exists a universal power-law relationship between the frequency of operational loss per week and the size & exposure towards operational risk per week of the combination of external Lines of Business & internal Business Units.

The reason why a power-law relationship between the frequency of operational loss per week and the size & exposure towards operational risk per week of the internal Business Units is not present is potentially due to the choice of categorising losses per Business Units instead of per internal Lines of Business.

5.6.1.3 Summary

In this section we applied the same regressions, which we have done on the operational loss amount per week, on the frequency of operational loss per week. The results suggest that:

1. There *exists* a power-law relationship between the frequency of operational loss per week and the size & exposure towards operational risk per week of the external Lines of Business.
2. A power-law relationship between the frequency of operational loss per week and the size & exposure towards operational risk per week of the internal Business Units is *not present*.
3. There is a *universal* power-law relationship between the frequency of operational loss per week and the size & exposure towards operational risk per week of the combination of external Lines of Business & internal Business Units.
4. The reason why the power-law relationship between the frequency of operational loss per week and the size & exposure towards operational risk per week of the

internal Business Units is not present is potentially due to the choice of categorising losses per Business Units instead of per internal Lines of Business.

5. We have estimated the value of variables λ_μ and λ_σ via the mean and standard deviation values, respectively, of the frequency of operational loss per week of the combination of external Lines of Business & internal Business Units. Inserting these estimates into Equation 22, we have:

$$\mu\left(Freq_{WEEK,S}\right)=\left(R_{idio}\right)_{WEEK,S}^{0.7731}\times\mu\left(Freq_{WEEK,STANDARD}\right)\quad ;S=BU\ 01,\ldots,BU\ 06,BL01,\ldots,BL08$$
$$\sigma\left(Freq_{WEEK,S}\right)=\left(R_{idio}\right)_{WEEK,S}^{0.5643}\times\sigma\left(Freq_{WEEK,STANDARD}\right)\quad ;S=BU\ 01,\ldots,BU\ 06,BL01,\ldots,BL08$$

The estimate of λ_μ and λ_σ is different to each other. This condition suggests that the mean and the standard deviation values scale differently. As a result, Assumption 4 must be rejected and is not valid for the frequency element of operational loss per week. The value of variable λ in the probability density function of the frequency of operational loss per week can still be estimated by means of Equation 21 below:

$$Freq_{T,S}=\left(R_{idio}\right)_{T,S}^{\lambda}\times Freq_{T,STANDARD}\quad ;S=BU\ 01,\ldots,BU\ 06,BL01,\ldots,BL08$$

However, as we have mentioned before, a future study examining the different way of scaling between the mean and the standard deviation values is necessary. The result of this study might be essential to decide to which extent the estimate of value of variable λ via Equation 21 is valid. In the next section, we conduct a similar experiment to the severity element of operational loss.

5.6.2 Severity of Operational Loss

In the first place, we must be aware of the fact that severity of operational loss is the financial loss amount of individual events, which means that severity is measured per individual event and thus not time-related. Therefore, in this section we will study the relationship between the severity of operational loss and the size & exposure towards operational risk of different Lines of Business and Business Units, without any time period T. In the following equations, the operational loss amount per week will be replaced by the severity of operational loss. Equation 12, Equation 13, Equation 14, and Equation 15 can thus be modified as:

$$Sev_S = \left(R_{idio}\right)_S^{\lambda} \times Sev_{STANDARD} \quad ; S = BL01, BL02, \ldots$$

Equation 26

$Sev_S =$ The severity of operational loss for each Line of Business and Business Unit S

$Sev_{STANDARD} =$ The severity of operational loss standard

$\left(R_{idio}\right)_S^{\lambda} =$ The idiosyncratic component of each Line of Business and Business Unit S

The Gross Income in year 2003 will be used to represent the idiosyncratic component of each Line of Business and Business Unit S.

$$\mu\left(Sev_S\right) = \left(R_{idio}\right)_S^{\lambda_\mu} \times \mu\left(Freq_{STANDARD}\right) \quad ; S = BL01, BL02, \ldots$$
$$\sigma\left(Sev_S\right) = \left(R_{idio}\right)_S^{\lambda_\sigma} \times \sigma\left(Sev_{STANDARD}\right) \quad ; S = BL01, BL02, \ldots$$

Equation 27

$$\log\left[Sev_S\right] = \lambda \times \log\left[\left(R_{idio}\right)_S\right] + \log\left[Sev_{STANDARD}\right] \quad ; S = BL01, BL02, \ldots$$

Equation 28

$$\log\left[\mu\left(Sev_S\right)\right] = \lambda_\mu \times \log\left[\left(R_{idio}\right)_S\right] + \log\left[\mu\left(Sev_{STANDARD}\right)\right] \quad ; S = BL01, BL02, \ldots$$
$$\log\left[\sigma\left(Sev_S\right)\right] = \lambda_\sigma \times \log\left[\left(R_{idio}\right)_S\right] + \log\left[\sigma\left(Sev_{STANDARD}\right)\right] \quad ; S = BL01, BL02, \ldots$$

Equation 29

Finally, Equation 16 can be as well modified as:

$$l_S = C \times r_S + p \quad ; S = BL01, BL02, \ldots$$

Equation 30

In case of the mean value, we have:

$$l = \log[\mu(Sev_S)]$$

$$C = \lambda_\mu$$

$$r = \log[(R_{idio})_S]$$

$$p = \log[\mu(Sev_{STANDARD})]$$

In case of the standard deviation value, we have:

$$l = \log[\sigma(Sev_S)]$$

$$C = \lambda_\sigma$$

$$r = \log[(R_{idio})_S]$$

$$p = \log[\sigma(Sev_{STANDARD})]$$

We obtain the log values of the mean and standard deviation of the severity of operational loss and the log values of the Gross Income in year 2003 for different Lines of Business and Business Units. The information is provided in the following table.

INSERT TABLE Q HERE

5.6.2.1 Estimating λ_μ

We will first apply the regression between the log values of the mean of the severity of operational loss and the log values of the Gross Income, only using the external Lines of Business. For this reason, we can rewrite Equation 30 as:

$$l_{EXTERNAL} = C \times r_{EXTERNAL} + p \quad ; EXTERNAL = BL01, \ldots, BL08$$

$$l = \log[\mu(Sev_{EXTERNAL})]$$
$$C = \lambda_{\mu}$$
$$r = \log[(R_{idio})_{EXTERNAL}]$$
$$p = \log[\mu(Sev_{STANDARD})]$$

Afterwards, the regression is run only using the internal Business Units. The regression of Equation 30 becomes:

$$l_{INTERNAL} = C \times r_{INTERNAL} + p \quad ; INTERNAL = BU\ 01, \ldots, BU\ 06$$
$$l = \log[\mu(Sev_{INTERNAL})]$$
$$C = \lambda_{\mu}$$
$$r = \log[(R_{idio})_{INTERNAL}]$$
$$p = \log[\mu(Sev_{STANDARD})]$$

The lxr plot, the regression line that best describes the internal data, and the regression line that best describes the external data are given in the following figure.

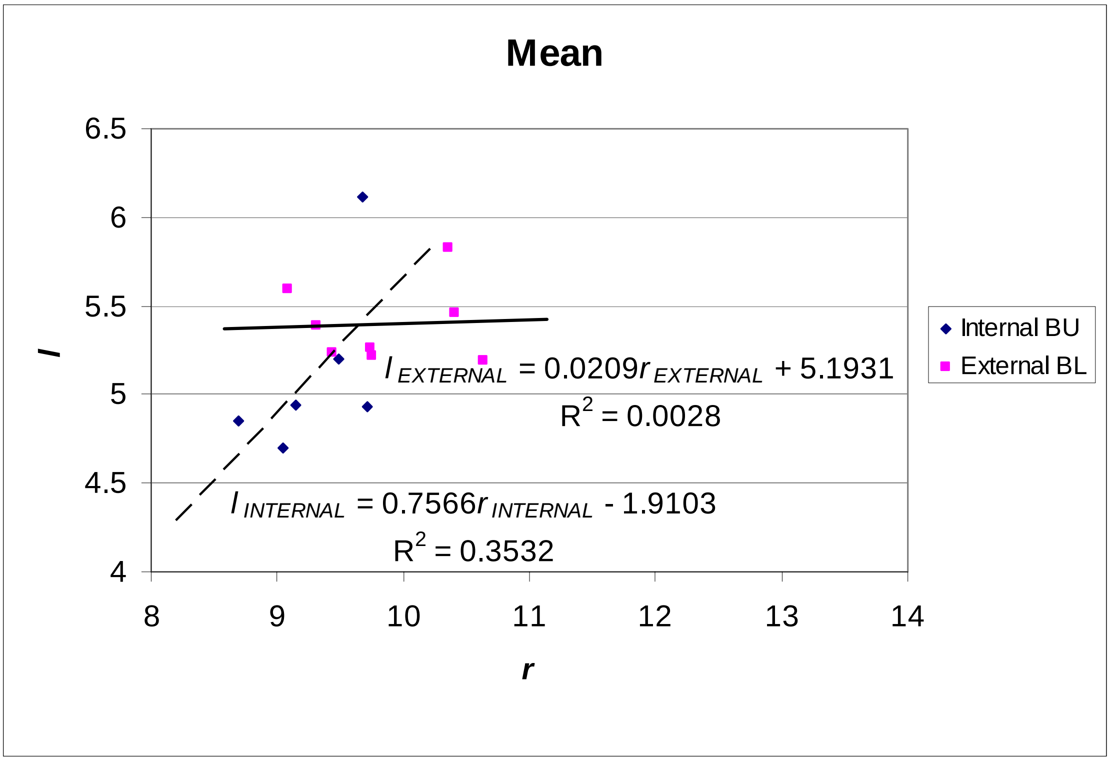


Figure 21 – *l* x *r* plot. The horizontal axis gives the variable *r* value, while the vertical axis gives the variable *l* value. Internal Business Units are given in triangles, and the regression line that best fits the internal data is given by the dash line. External Lines of Business are given in squares, with solid line as the regression line that best fits these Lines of Business.

5.6.2.1.1 External data

The results of the regression on the Lines of Business of the external data are shown in the following table:

R-Square	*Regression results*	*Coefficients*	*Standard Error*	*t Stat*	*P-value*
0.0028	Intercept (*p*)	5.1931	1.5766	3.2938	0.0165
	C	0.0209	0.1600	0.1306	0.9004

Table 23

The R^2 value is 0.0028, which suggests that only 0.28% of variable l is attributable to variable r for the external Lines of Business. The *P-value* of coefficient C in the sixth column is absolutely higher than 0.05, so we cannot reject the null hypothesis that the coefficient C is equal to zero within the 95% confidence interval. This result indicates that a power-law relationship between the severity of operational loss and the size & exposure towards operational risk of the external Lines of Business is not present. Additionally, the idiosyncratic component can hardly be used to explain the variability in the severity of operational loss due to the very small value of R^2. We continue with performing the same regression on the Business Units of the internal data.

5.6.2.1.2 Internal data

The results of the regression on the Business Units of the internal data are shown in the following table:

R-Square	***Regression results***	***Coefficients***	***Standard Error***	***t Stat***	***P-value***
0.3532	Intercept (*p*)	-1.9103	4.7626	-0.4011	0.7088
	C	0.7566	0.5119	1.4779	0.2135

Table 24

The R^2 value of 0.3532 suggests that only around 35.32% of variable l is attributable to variable r for the internal Business Units. In other words, the idiosyncratic component can explain a rather small proportion of the variability in the severity of operational loss, as we can see from the low value of R^2. The *P-value* of the coefficient C is 0.2135 (absolutely higher than 0.05), so we cannot reject the null hypothesis that the coefficient C is equal to zero within the 95% confidence interval. This result suggests that a power-law relationship between the severity of operational loss and the size & exposure towards operational risk of the internal Business Units is not present. The same regression is performed again, this time on the combination of the external Lines of Business and the internal Business Units.

5.6.2.1.3 Combination of external data and internal data

Finally, we perform the regression on the combination of the external Lines of Business and the internal Business Units. Equation 30 will be used for this purpose and will have the following form:

$$l_S = C \times r_S + p \quad ; S = BU\ 01, \ldots, BU\ 06, BL01, \ldots, BL08$$

$$l = \log\left[\mu\left(Sev_S\right)\right]$$

$$C = \lambda_\mu$$

$$r = \log\left[\left(R_{idio}\right)_S\right]$$

$$p = \log\left[\mu\left(Sev_{STANDARD}\right)\right]$$

The $l \times r$ graph and the regression line that best describe the combination of the internal and the external data are given in the following figure:

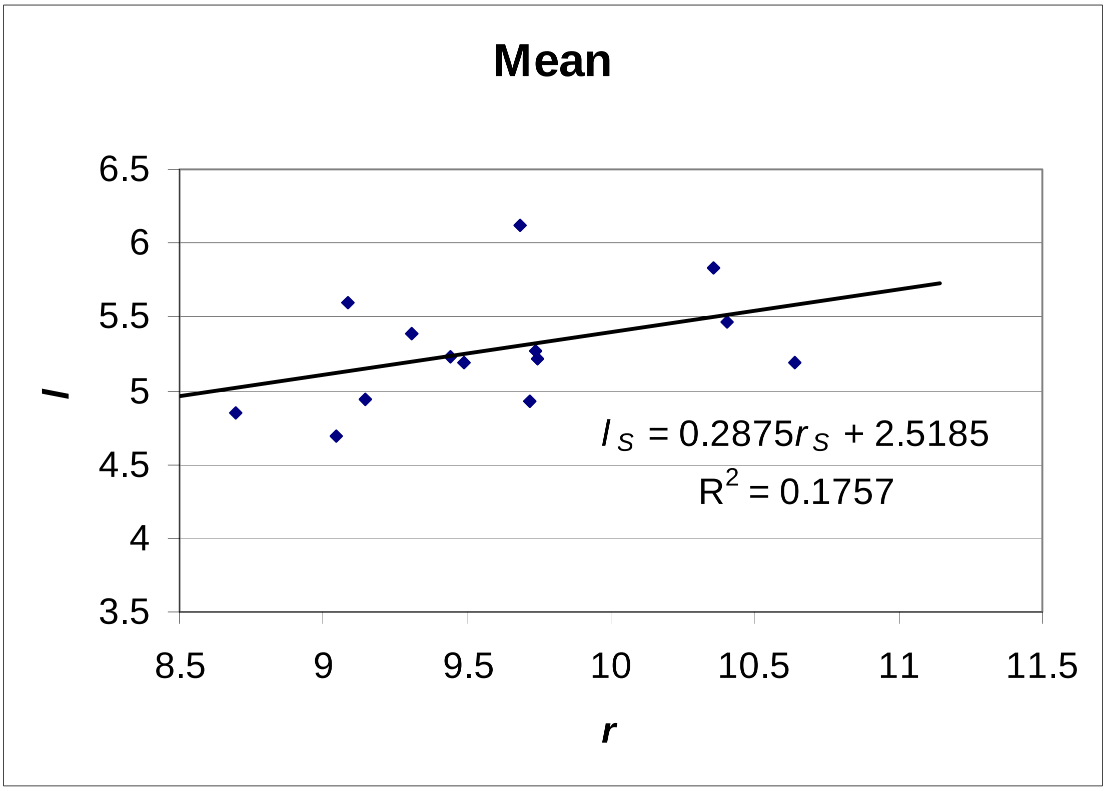


Figure 22 – *l* x *r* plot. The horizontal axis gives the variable *r* value, while the vertical axis gives the variable *l* value. The regression line that best fits the combination of the two data is given by the solid line.

The results of the regression on the combination of the external Lines of Business and the internal Business Units are shown in the next table:

R-Square	*Regression results*	*Coefficients*	*Standard Error*	*t Stat*	*P-value*
0.1757	Intercept (*p*)	2.5185	1.7300	1.4558	0.1711
	C	0.2875	0.1798	1.5991	0.1358

Table 25

The R^2 value is 0.1757. This suggests that around 17.57% of variable l is attributable to variable r. The value of R^2 lies in the middle between the R^2 value of the experiment using only the external data and the R^2 value of the experiment using only the internal data. The *P-value* of C in the sixth column is definitely higher than 0.05, so we cannot reject the null hypothesis that the coefficient C is equal to zero within the 95% confidence interval.

We also cannot reject the null hypothesis that the coefficient C is equal to zero within the 90% confidence interval, because the *P-value* is higher than 0.10. This result indicates that there is no power-law relationship between the severity of operational loss and the size & exposure towards operational risk of the combination of external Lines of Business & internal Business Units.

5.6.2.2 Estimating λ_σ

In this section we will first apply the regression between the log values of the standard deviation of the severity of operational loss and the log values of the Gross Income, only using the external Lines of Business. For this reason, we can rewrite Equation 30 as:

$$l_{EXTERNAL} = C \times r_{EXTERNAL} + p \quad ; EXTERNAL = BL01, \ldots, BL08$$
$$l = \log[\sigma(Sev_{EXTERNAL})]$$
$$C = \lambda_\sigma$$
$$r = \log[(R_{idio})_{EXTERNAL}]$$
$$p = \log[\sigma(Sev_{STANDARD})]$$

Afterwards, the regression is run only using the internal Business Units. The regression of Equation 30 becomes:

$$l_{INTERNAL} = C \times r_{INTERNAL} + p \quad ; INTERNAL = BU\,01, \ldots, BU\,06$$
$$l = \log[\sigma(Sev_{INTERNAL})]$$
$$C = \lambda_\sigma$$
$$r = \log[(R_{idio})_{INTERNAL}]$$
$$p = \log[\sigma(Sev_{STANDARD})]$$

The lxr plot, the regression line that best describes the internal data, and the regression line that best describes the external data are given in the following figure:

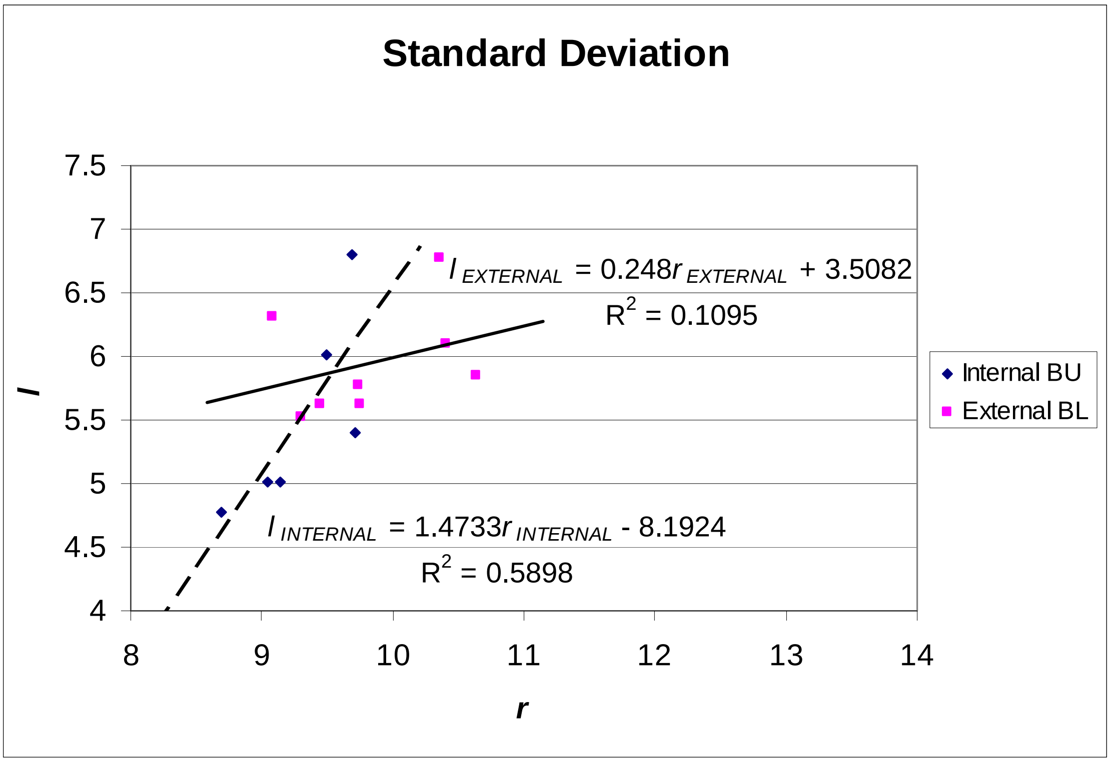


Figure 23 – *l* x *r* plot. The horizontal axis gives the variable *r* value, while the vertical axis gives the variable *l* value. Internal Business Units are given in triangles, and the regression line that best fits the internal data is given by the dash line. External Lines of Business are given in squares, with solid line as the regression line that best fits these Lines of Business.

The regression results are described in detail in the subsequent sections. We start with the regression result of the external Lines of Business (external data).

5.6.2.2.1 External data

The results of the regression on the Lines of Business of the external data are shown in the following table:

R-Square	*Regression results*	*Coefficients*	*Standard Error*	*t Stat*	*P-value*
0.1095	Intercept (*p*)	3.5082	2.8459	1.2327	0.2638
	C	0.2480	0.2888	0.8587	0.4235

Table 26

An R^2 value of 0.1095 suggests that only around 10.95% of variable l is attributable to variable r for the external Lines of Business. The *P-value* of coefficient C in the sixth column is absolutely higher than 0.05, so we cannot reject the null hypothesis that the coefficient C is equal to zero within the 95% confidence interval. This result indicates that a power-law relationship between the severity of operational loss and the size & exposure towards operational risk of the external Lines of Business is not present. Additionally, the idiosyncratic component can hardly be used to explain the variability in the severity of operational loss due to the very small value of R^2. We continue with performing the same regression on the Business Units of the internal data.

5.6.2.2.2 Internal data

The results of the regression on the Business Units of the internal data are shown in the following table:

R-Square	*Regression results*	*Coefficients*	*Standard Error*	*t Stat*	*P-value*
0.5898	Intercept (*p*)	-8.1924	5.7151	-1.4335	0.2250
	C	1.4733	0.6143	2.3983	0.0745

Table 27

The R^2 value of 0.5898 suggests that around 58.98% of variable l is attributable to variable r for the internal Business Units. The *P-value* of coefficient C is 0.0745 (slightly higher than 0.05), so we cannot reject the null hypothesis that the coefficient λ is equal to zero within the 95% confidence interval. Nevertheless, we can reject the null hypothesis that the coefficient C is equal to zero within the 90% confidence interval (the *P-value* of coefficient λ is less than the alpha level of 0.10). The idiosyncratic component can explain a big proportion (58.98%) of the variability in the severity of operational loss, which can be seen from the rather high value of R^2.

This result suggests that a power-law relationship between the severity of operational loss and the size & exposure towards operational risk of the internal Business Units is present, conditional on a 90% confidence interval. However, when we estimated the value of variable λ_μ, we cannot find a power-law relationship between the severity of operational loss and the size & exposure towards operational risk of the internal Business Units.

5.6.2.2.3 Combination of external data and internal data

Finally, we perform the regression on the combination of the external Lines of Business and the internal Business Units. Equation 30 will be used for this purpose and will have the following form:

$$l_S = C \times r_S + p \quad ; S = BU\ 01, \ldots, BU\ 06, BL01, \ldots, BL08$$
$$l = \log\left[\sigma\left(Sev_S\right)\right]$$
$$C = \lambda_\sigma$$
$$r = \log\left[\left(R_{idio}\right)_S\right]$$
$$p = \log\left[\sigma\left(Sev_{STANDARD}\right)\right]$$

The lxr plot and the regression line that best describes the combination of the internal data and the external data are given in the following figure.

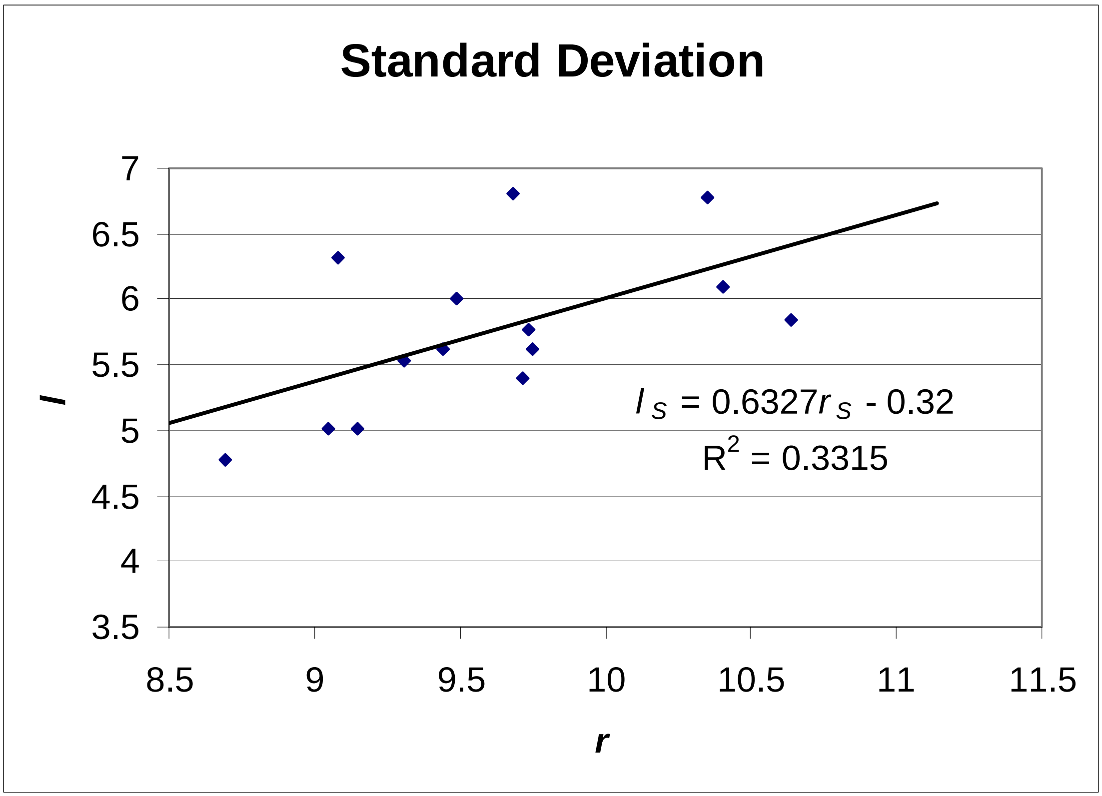


Figure 24 – *l* x *r* plot. The horizontal axis gives the variable *r* value, while the vertical axis gives the variable *l* value. The regression line that best fits the combination of the two data is given by the solid line.

The results of the regression on the combination of the external Lines of Business and the internal Business Units are shown in the next table:

R-Square	*Regression results*	*Coefficients*	*Standard Error*	*t Stat*	*P-value*
0.3315	Intercept (*p*)	-0.3200	2.4954	-0.1282	0.9001
	C	0.6327	0.2594	2.4395	0.0312

Table 28

The R^2 value of 0.3315 suggests that around 33.15% of variable l is attributable to variable r. The value of R^2 lies in the middle between the R^2 value of the experiment using only the external data and the R^2 value of the experiment using only the internal data. The *P-value* of C in the sixth column is smaller than 0.05 and indicates that the coefficient C is significantly different from zero within the 95% confidence interval.

This result suggests that there exists a universal power-law relationship between the severity of operational loss and the size & exposure towards operational risk of the combination of external Lines of Business & internal Business Units. The confidence interval (95%) is bigger than the confidence interval when we perform the regression only on the internal Business Units (90%). Nonetheless, when we estimated the value of variable λ_{μ}, we cannot find a power-law relationship between the severity of operational loss and the size & exposure towards operational risk of the combination of external Lines of Business & internal Business Units.

5.6.2.3 Summary

In this section we applied the same regressions, which we have done on the operational loss amount per week and on the frequency of operational loss per week, on the severity of operational loss. When we apply the regression to estimate the value of λ_{μ}, the results suggest that:

1. A power-law relationship between the severity of operational loss and the size & exposure towards operational risk of the external Lines of Business is *not present*.
2. A power-law relationship between the severity of operational loss and the size & exposure towards operational risk of the internal Business Units is *not present* as well.
3. There is *no power-law relationship* between the severity of operational loss and the size & exposure towards operational risk of the combination of external Lines of Business & internal Business Units.

When we apply the regression to estimate the value of λ_{σ}, the results suggest:

1. A power-law relationship between the severity of operational loss and the size & exposure towards operational risk of the external Lines of Business is *not present*.

2. A power-law relationship between the severity of operational loss and the size & exposure towards operational risk of the internal Business Units is *present, conditional on a 90% confidence interval*.
3. There is *a universal* power-law relationship between the severity of operational loss and the size & exposure towards operational risk of the combination of external Lines of Business & internal Business Units (within the 95% confidence interval).

Thus, it is noticeable that there is *no* power-law relationship between the severity of operational loss and the size & exposure towards operational risk of the external Lines of Business.

For the internal Business Units or the combination of external Lines of Business & internal Business Units, the results of the regression when estimating the value of λ_σ suggest the presence of a universal power-law relationship. On the other hand, the results of the regression when estimating the value of λ_μ suggest no presence of a universal power-law relationship. As a result, Assumption 4 must be rejected and is not valid for the severity element of operational loss of the internal Business Units or of the combination of external Lines of Business & internal Business Units.

At this moment, we cannot reveal whether a power-law relationship is present between the severity of operational loss and the idiosyncratic component of the internal Business Units. We also cannot reveal whether a power-law relationship is present between the severity of operational loss and the idiosyncratic component of the combination of external Lines of Business & internal Business Units. In future study, it is necessary to apply the regression on each severity data of operational loss. Thus, Equation 26 should be used:

$$Sev_S = \left(R_{idio}\right)_S^{\lambda} \times Sev_{STANDARD} \quad ; S = BU\ 01, \ldots, BU\ 06, BL01, \ldots, BL08$$

The result of this regression will suggest whether a power-law relationship is present between the probability density function of the severity of operational loss and the idiosyncratic component of the internal Business Units and of the combination of external Lines of Business & internal Business Units.

5.7 Conclusion and Remarks

5.7.1 Power-law relationship and scaling mechanism

In previous sections we have examined the relationship between the operational loss amount per week and the size & exposure towards operational risk of the external Lines of Business, of the internal Business Units, and of the combination of external Lines of Business & internal Business Units. The results suggest that:

1. There *exists* a power-law relationship between the operational loss amount per week and the size & exposure towards operational risk per week of the external Lines of Business.
2. There also *exists* a power-law relationship between the operational loss amount per week and the size & exposure towards operational risk per week of the internal Business Units is also present, although on a lower confidence interval than the confidence interval of the external data. The existence of the power-law relationship, subject to a lower confidence interval than the confidence interval when using the external data, might be due to the choice of categorising losses per Business Units instead of per Lines of Business of the bank.
3. There is a *universal* power-law relationship between the operational loss amount per week and the size & exposure towards operational risk per week of the combination of external Lines of Business & internal Business Units.

The estimates of variable λ_{μ} and variable λ_{σ} can be seen as equivalent, even though they are slightly dissimilar in the second decimal. This result suggests that the mean value scale in the same way as the standard deviation value. Based on Assumption 4, the estimates of λ_μ and λ_σ can be used to represent the value of variable λ in the operational loss function.

It is important to remark that the estimates of λ_μ and λ_σ are very close to 1. This suggests that the universal power-law relationship can be very likely regarded as a linear form. In other words, the operational loss amount per week relates almost linearly to the size & exposure towards operational risk per week of the combination of external Lines of Business & internal Business Units.

Please note that even though the mean and standard deviation values scale more or less in the same way, this does not guarantee that the probability density function of operational loss scale in the same way as the mean and the standard deviation values. In the future, it is necessary to estimate the value of variable λ for the probability density function.

It can be done by applying the regression on each operational loss amount per week data, instead of only on the mean and standard deviation values, against the idiosyncratic component of every Line of Business and Business Unit. If the estimate of λ is equivalent to the estimates of λ_μ and λ_σ, we can then say that the probability density function of operational loss scale in the same way as the mean and the standard deviation values. In other words, we can then conclude that Assumption 4 is valid.

We have shown how the scaling mechanism transforms the operational loss amount per week of each Line of Business into the operational loss standard per week. Afterwards, the operational loss standard per week of all Lines of Business and Business Units can be combined together, since the idiosyncratic components of different Lines of Business and Business Units no longer becomes an issue in the operational loss *standard* per week data. By means of this scaling mechanism, the operational loss data of other Lines of Business can thus be incorporated into the operational loss data of a Line of Business.

We also showed an example of how to utilise the operational loss standard per week, in order to calculate the operational risk capital amount with a time horizon of one week of each Line of Business and Business Unit. The operational risk capital calculation can also be done for different confidence intervals. If we can assume that the operational risk capital amount with a time horizon of one year is simply the operational risk capital amount with a time horizon of one week multiplied by the number of weeks in a year, the operational loss capital amount with a time horizon of one year of each Line of Business and Business Unit will be obtained immediately.

5.7.2 Frequency of Operational Loss

We have as well examined the relationship between the frequency of operational loss per week and the size & exposure towards operational risk per week of the external Lines of Business, the internal Business Units, and the combination of external Lines of Business & internal Business Units. The results suggest that:

1. There *exists* a power-law relationship between the frequency of operational loss per week and the size & exposure towards operational risk per week of the external Lines of Business.
2. A power-law relationship between the frequency of operational loss per week and the size & exposure towards operational risk per week of the internal Business Units is *not present*.
3. There is a *universal* power-law relationship between the frequency of operational loss per week and the size & exposure towards operational risk per week of the combination of external Lines of Business & internal Business Units.
4. The reason why the power-law relationship between the frequency of operational loss per week and the size & exposure towards operational risk per week of the internal Business Units is not present is potentially due to the choice of categorising losses per Business Units instead of per internal Lines of Business.
5. The estimate of λ_μ and λ_σ is different to each other. This condition suggests that the mean and the standard deviation values scale differently. As a result,

Assumption 4 must be rejected and is not valid for the frequency element of operational loss per week.

5.7.3 Severity of Operational Loss

Finally, we examined the relationship between the severity of operational loss and the size & exposures towards operational risk of the external Lines of Business, the internal Business Units, and the combination of external Lines of Business & internal Business Units. When we apply the regression to estimate the value of λ_{μ}, the results suggest that:

1. A power-law relationship between the severity of operational loss and the size & exposure towards operational risk of the external Lines of Business is *not present*.
2. A power-law relationship between the severity of operational loss and the size & exposure towards operational risk of the internal Business Units is *not present* as well.
3. There is *no power-law relationship* between the severity of operational loss and the size & exposure towards operational risk of the combination of external Lines of Business & internal Business Units.

When we apply the regression to estimate the value of λ_{σ}, the results suggest:

4. A power-law relationship between the severity of operational loss and the size & exposure towards operational risk of the external Lines of Business is *not present*.
5. A power-law relationship between the severity of operational loss and the size & exposure towards operational risk of the internal Business Units is *present, conditional on a 90% confidence interval*.
6. There is *a universal* power-law relationship between the severity of operational loss and the size & exposure towards operational risk of the combination of external Lines of Business & internal Business Units (within the 95% confidence interval).

Thus, it is noticeable that there is *no* power-law relationship between the severity of operational loss and the size & exposure towards operational risk of the external Lines of Business.

For the internal Business Units or the combination of external Lines of Business & internal Business Units, the results of the regression when estimating the value of λ_{σ} suggest the presence of a universal power-law relationship. On the other hand, the results of the regression when estimating the value of λ_{μ} suggest no presence of a universal power-law relationship. As a result, Assumption 4 must be rejected and is not valid for the severity element of operational loss of the internal Business Units or of the combination of external Lines of Business & internal Business Units.

Based on these results, we cannot reveal whether a power-law relationship is present between the severity of operational loss and the idiosyncratic component of the internal Business Units. We also cannot reveal whether a power-law relationship is present between the severity of operational loss and the idiosyncratic component of the combination of external Lines of Business & internal Business Units.

In future study, it is necessary to apply the regression on each severity data of operational loss. The result of this regression will suggest whether a power-law relationship is present between the probability density function of the severity of operational loss and the idiosyncratic component of the internal Business Units and of the combination of external Lines of Business & internal Business Units.

5.7.4 Power-law relationship in the Frequency and Severity of Operational Loss

Bringing our results together, we can conclude that:

1. The power-law relationship between the operational loss amount per week and the size & exposure towards operational risk per week of the external Lines of Business is *coming* from the frequency element, not the severity element.

2. The power-law relationship between the operational loss amount per week and the size & exposure towards operational risk per week of the internal Business Units is *not coming* from the frequency element. However, the contribution from the severity element is *not clear yet*.
3. The universal power-law relationship between the operational loss amount per week and the size & exposure towards operational risk per week of the combination of external Lines of Business & internal Business Units is *coming* from the frequency element; while the contribution from the severity element is *not yet evident*.

6 Conclusions and Future Research

6.1 Summary

Financial institutions face financial risks in their business activities. One type of financial risk considered important to be managed by financial institutions is the operational risk. Operational risk is usually defined as 'the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events'.

Financial institutions, banks in particular, are required to allocate a separate capital to cover their exposure to operational risk. This capital is acknowledged as operational risk capital and becomes a part of the total regulatory capital a financial institution must hold. The other capital reserves are necessary to cover its exposure to credit risk and market risk, respectively. The capital framework published by the Basel Committee in June 2004 serves as the guidelines for financial institutions relating to the regulatory capital calculations.

For the purpose of operational risk capital calculations, the Basel Committee identified three possible approaches that range from a simple method to more sophisticated ones. These approaches are:

1. Basic Indicator Approach
2. Standardised Approach
3. Advanced Measurement Approaches (AMA)

AMA is considered to be the most sophisticated approach to calculate the operational risk capital of a bank. In order to be qualified to use AMA, financial institutions must meet quantitative and qualitative standards set by the Basel Committee in the Capital Framework of June 2004 (the Basel II Accord). One of the quantitative standards is that a financial institution must collect its historical operational risk loss data. The capital calculation will be done on the basis of this internal loss data.

Nevertheless, the Basel Committee also required banks to make use of data from other banks (external data) since loss experience of a bank alone might not be sufficient to represent the actual risk behaviour of the bank. The use of external data is compulsory, in particular when there is reason to believe that the bank is exposed to high severe-infrequent losses. We obtain the external data via a consortium of which historical loss data of 15 banks, inclusive ABN AMRO, are collected.

The objectives of this thesis were defined as:

1. Analyse the historical operational risk loss data of the bank as well as the external data.
2. Propose a scaling mechanism to incorporate the external data into the internal data of the bank.

6.2 Conclusions

In order to accomplish the first goal, we first analysed the characteristics and the differences between the internal and external loss data. Afterwards, a thorough study on the historical loss data of the bank has been done and given in Chapter 4. The internal loss data has been investigated to test the following hypothesis:

Hypothesis: ‘There is a linear relationship between two variables of an operational risk loss event, namely: (1) the loss amount and (2) the time interval between the moment an event is discovered and the moment the event is recognised as an operational risk loss event‘.

As a result, we cannot find a linear relationship between variable (1) and (2) and finally have to reject the abovementioned hypothesis. Furthermore, we find out that the variable (2) is more related to the characteristic of a Business Unit and the method to recognise the operational loss events in that Business Unit.

To accomplish the second goal, we started by inspecting the relationship between the operational loss amount incurred in a financial institution within a certain time period and an indicator of size & exposure towards operational risk of that financial institution within a certain time period. We found that the power-law form can be used to explain this relationship.

Based on the existence of the power-law relationship, we were able to apply the scaling mechanism to remove financial institutions' specific characteristics, so that the external data can be considered to have the same characteristics as the internal data. Instead of investigating at the aggregate level (view each bank as a single entity), we have chosen to investigate at the Line of Business level. The choice of examining on the Lines of Business level is particularly based on the information available from the external data. We can only tell from which Line of Business, but not from which bank, an operational loss comes from. This information is not given away in the external data. In our data set, the internal loss data of the bank is given per Business Unit. For the reason of simplicity, we use directly the Business Units of the bank instead of mapping them into the Basel Lines of Business categorisation. We use Gross Income as the indicator for the size and exposure to operational risk of a Line of Business.

We have shown how to apply the scaling mechanism, by transforming the operational loss data into the operational loss standard data, in order to incorporate the external Lines of Business operational loss data into the internal Business Units operational loss data. Additionally, the choice of investigating at the Line of Business Level gives also the possibility to:

1. Incorporate the operational loss data of other external Lines of Business into a single external Line of Business, when using only the external data.
2. Incorporate the operational loss data of other internal Business Units into a single internal Business Unit of the bank, when using only the internal data. Thus, an internal Business Unit (eventually internal Line of Business) of a bank should not directly use the operational loss data of other internal Business Units, even

though these Business Units are originated from the same bank. The reason is because each internal Business Unit also has its specific characteristics.

We have also given an example of how to utilise both data altogether in order to calculate the operational risk capital amount with a time horizon of one year of each Line of Business and Business Unit. First, the operational loss amount per week of all Lines of Business and Business Units, by transforming them into the operational loss standard per week, can be used to calculate the operational risk capital standard with a time horizon of one week. We can do this, since the idiosyncratic components of different Lines of Business and Business Units no longer becomes an issue in the operational loss standard per week data. If we can assume that the operational risk capital amount with a time horizon of one year is simply the operational risk capital amount with a time horizon of one week multiplied by the number of weeks in a year, the operational loss capital amount with a time horizon of one year of each Line of Business and Business Unit will be obtained immediately.

We have tried to observe whether the power-law relationship - between the operational loss amount per week and the size & exposure towards operational risk per week of the external Lines of Business, of the internal Business Units, and of the combination of external Lines of Business & internal Business Units - comes from the frequency element, the severity element, or even from both elements. The obtained results suggest that:

1. The power-law relationship between the operational loss amount per week and the size & exposure towards operational risk per week of the external Lines of Business is *coming* from the frequency element, *not* the severity element.
2. The power-law relationship between the operational loss amount per week and the size & exposure towards operational risk per week of the internal Business Units is *not* coming from the frequency element. However, the contribution from the severity element is *not obvious yet*.

3. The universal power-law relationship between the operational loss amount per week and the size & exposure towards operational risk per week of the combination of external Lines of Business & internal Business Units is *coming* from the frequency element. However, the contribution from the severity element is *not obvious yet*.

6.3 Future Research

Extensions to the current study are widely open. In Chapter 4, we have only tested our hypothesis on the historical loss data of the bank. It is of course interesting to see whether we can also reject our hypothesis if we use the external loss data. If the hypothesis is rejected when using the external data, it will be also very fascinating to study whether the time interval between discovery and recognition time is related to the Line of Business characteristics and the method of recognition, as if in the case of the internal loss data.

We can also extend our study concerning the scaling mechanism, for example by:

1. Mapping the Business Units of the bank first into the Basel Lines of Business and use these Lines of Business as the internal data of the bank.
2. Testing the existence of universal power-law relationship by using other loss data than the external data and the bank's internal data. For example: publicly released data.
3. Estimating the value of the variable λ by running the regressions on each operational loss amount per week data, each frequency of operational loss per week data, and each severity of operational loss data against the idiosyncratic component of the external Lines of Business, of the internal Business Units and of the combination of external Lines of Business & internal Business Units.
4. Studying the development of the value of variable λ across time.

The Basel Committee proposes gross income to be used as the indicator to represent a financial institution's exposure towards operational risk for the sake of simplicity, comparability, reduction of arbitrage possibilities. Next to these reasons, the most significant reason for using gross income is a lack of evidence of greater risk sensitivity of other indicators. If reliable information of other indicators can be obtained, intelligent systems may be applied to study which indicators are important and attributable to the operational loss behaviour of a financial institution.

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Appendices

Appendix I

Basel Mapping of Business Lines [12]

<table>
<tr><th>Code</th><th>Level 1</th><th>Level 2</th><th>Activity groups</th></tr>
<tr><td rowspan="4">BL01</td><td rowspan="4">Corporate Finance</td><td>Corporate Finance</td><td rowspan="4">Mergers and Acquisitions, Underwriting, Privatisations, Securitisation, Research, Debt (Government, High Yield) Equity, Syndications, IPO, Secondary Private Placements</td></tr>
<tr><td>Municipal/Government Finance</td></tr>
<tr><td>Merchant Banking</td></tr>
<tr><td>Advisory Services</td></tr>
<tr><td rowspan="4">BL02</td><td rowspan="4">Trading and Sales</td><td>Sales</td><td rowspan="4">Fixed Income, equity, foreign exchanges, commodities, credit, funding, own position securities, lending and repos, brokerage, debt, prime brokerage</td></tr>
<tr><td>Market Making</td></tr>
<tr><td>Proprietary Positions</td></tr>
<tr><td>Treasury</td></tr>
<tr><td rowspan="3">BL03</td><td rowspan="3">Retail Banking</td><td>Retail Banking</td><td>Retail lending and deposits, banking services, trust and estates</td></tr>
<tr><td>Private Banking</td><td>Private lending and deposits, banking services, trust and estates, investment advice</td></tr>
<tr><td>Card Services</td><td>Merchant/Commercial/Corporate cards, private labels and retail</td></tr>
<tr><td>BL04</td><td>Commercial Banking</td><td>Commercial banking</td><td>Project finance, real estate, export finance, trade finance, factoring, leasing, lends, guarantees, bills of exchange</td></tr>
<tr><td>BL05</td><td>Payment and Settlement</td><td>External Clients</td><td>Payments and collections, funds transfer, clearing and settlement</td></tr>
<tr><td rowspan="3">BL06</td><td rowspan="3">Agency Services</td><td>Custody</td><td>Escrow, Depository Receipts, Securities lending (Customers) Corporate actions</td></tr>
<tr><td>Corporate Agency</td><td>Issuer and paying agents</td></tr>
<tr><td>Corporate Trust</td><td></td></tr>
<tr><td rowspan="2">BL07</td><td rowspan="2">Asset Management</td><td>Discretionary Fund Management</td><td>Pooled, segregated, retail, institutional, closed, open, private equity</td></tr>
<tr><td>Non-Discretionary Fund Management</td><td>Pooled, segregated, retail, institutional, closed, open</td></tr>
<tr><td>BL08</td><td>Retail Brokerage</td><td>Retail Brokerage</td><td>Execution and full service</td></tr>
</table>

Appendix II

Basel Loss Event Type Classification [12]

Code	Event-Type Category (Level 1)	Definition	Categories (Level 2)
EL01	Internal Fraud	Losses due to acts of a type intended to defraud, misappropriate property or circumvent regulations, the law or company policy, excluding diversity / discrimination events, which involves at least one internal party	Unauthorized Activity
			Theft and Fraud
EL02	External Fraud	Losses due to acts of a type intended to defraud, misappropriate property or circumvent the law, by a third party	Theft and Fraud
			Systems Security
EL03	Employment Practices and Workplace Safety	Losses arising from acts inconsistent with employment, health or safety laws or agreements, from payment of personal injury claims, or from diversity / discrimination events	Employee Relations
			Safe Environment
			Diversity and Discrimination
EL04	Clients, Products and Business Practices	Losses arising from an unintentional or negligent failure to meet a professional obligation to specific clients (including fiduciary and suitability requirements), or from the nature or design of a product	Suitability, Disclosure, and Fiduciary
			Improper Business or Market Practices
			Product Flaws
			Selection, Sponsorship, and Exposure
			Advisory Activity

Code	Event-Type Category (Level 1)	Definition	Categories (Level 2)
EL05	Damage to Physical Assets	Losses arising from loss or damages to physical assets from natural disaster or other events	Disasters and Other Events
EL06	Business Disruption and System Failures	Losses arising from disruption of business or system failures	Systems
EL07	Execution, Delivery, and Process Management	Losses from failed transaction processing or process management, from relations with trade counterparties and vendors	Transaction Capture, Execution, and Maintenance
			Monitoring and Reporting
			Customer Intake and Documentation
			Customer/Client Account Management
			Trade Counterparties
			Vendors and Suppliers

Appendix III

Adjusted ORX Lines of Business Grids

Code	Level 1	Level 2
BL01	Corporate Finance	Corporate Finance
		Municipal/Government Finance
		Advisory Services
BL02	Trading and Sales	Equities
		Global Markets
		Corporate Investments
		Treasury
BL03	Retail Banking	Retail Banking
		Card Services
BL09	Private Banking	Private Banking
BL04	Commercial Banking	Commercial banking
BL05	Clearing	Cash clearing
		Securities clearing
BL06	Agency Services	Custody
		Corporate Trust & Agency
		Custom Services
BL07	Asset Management	Fund Management
BL08	Retail Brokerage	Retail Brokerage
BL10	Corporate Items	Corporate Items
BL11	Multiple Lines of Business	Multiple Lines of Business

Appendix IV

Adjusted ORX Event Type Grids

Code	Level 1	Level 2
EL01	Internal Fraud	Unauthorized Activity
		Internal Theft and Fraud
		Internal Systems Security (for profit)
EL02	External Fraud	External Theft and Fraud
		External Systems Security (for profit)
EL03	Employment Practices and Workplace Safety	Employee Relations
		Safe Workplace Environment
		Employment Diversity and Discrimination
EL04	Clients, Products and Business Practices	Suitability, Disclosure, and Fiduciary
		Improper Business or Market Practices
		Product Flaws
		Selection, Sponsorship, and Exposure
		Advisory Activity
EL05	Disasters and Public Safety	Disasters and Other Events
		Accidents and Public Safety
EL06	Technology and Infrastructure Failures	Technology and Infrastructure Failures
EL07	Execution, Delivery, and Process Management	Transaction Capture, Execution, and Maintenance
		Monitoring and Reporting
		Customer Intake and Documentation
		Customer/Client Account Management
EL08	Malicious Damage	Wilful Damage and Terrorism
		Systems Security - Wilful Damage