Thermodynamics of Active Magnetic Regenerators

IESVic integrated energy systems

University of Victoria

The equations governing the physics of active magnetic regenerators are presented in simplified form. These are used as the basis for a numerical model which is compared to experimental results for validation.

Tom Burdyny, Andrew Rowe

ensys future energy today

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Overview

- AMR Thermodynamics
  - Governing behaviour, losses, magnetic cycle
  - **Goal: To find T_{Span} and Q_c as a function of T_H**
- Model based on simplified theory
- Comparison to experimental results
- Current issues
- Future work
- Summary

Motivation

- What is the purpose of a model based on simplified theory?
  - Quick solutions
    - Immediate sensitivity analysis
    - Optimization
  - Parameters linked to equipment sizing and costs
  - Beneficial for all researchers in a group

Governing Equations[1]

Fluid: $$m_f' c_p \frac{\partial T_f}{\partial \tilde{t}} + \dot{m} c_p \frac{\partial T_f}{\partial \tilde{x}} = \frac{\partial}{\partial \tilde{x}}\left(k_f A \frac{\partial T_f}{\partial \tilde{x}}\right) + hA'\left(T_s - T_f\right) \qquad (1)$$

Solid: $$M_s' c_B \frac{\partial T_s}{\partial \tilde{t}} + M_s' T_s \left(\frac{\partial m}{\partial T}\right)_H \frac{\partial \mu_0 H}{\partial \tilde{t}} = \frac{\partial}{\partial \tilde{x}}\left(k_s A \frac{\partial T_s}{\partial \tilde{x}}\right) + hA'\left(T_f - T_s\right) \qquad (2)$$

Assuming $T_f = T_s$ and $k_{eff} = k_f + k_s$:

$$\frac{\partial T}{\partial t} + \frac{\Phi}{R}\frac{\partial T}{\partial x} - \frac{1}{M_s' L c_B}\frac{\tau_B}{R}\frac{\partial}{\partial x}\left(\frac{k_{eff} A}{L}\frac{\partial T}{\partial x}\right) = \frac{1}{R}\frac{\partial T_{ad}}{\partial t} \qquad (3)$$

where R is the thermal mass ratio, Φ is utilization and T_{ad} is the adiabatic temperature change of the material.

[1]Rowe A. Thermodynamics of active magnetic regenerators: Part I. Cryogenics (2011), doi:10.1016/j.cryogenics.2011.09.005

Governing Equations cont.[1]

Against other terms conduction is then assumed to be small:

$$\frac{\partial T}{\partial t} + \frac{\Phi}{R}\frac{\partial T}{\partial x} - \frac{\partial T}{\partial t}\frac{1}{M_s L c_B}\frac{\partial}{\partial x}\left(k_{eff} A \frac{\partial T}{\partial x}\right) = \frac{1}{R}\frac{\partial T_{ad}}{\partial t} \tag{4}$$

Eq.(4) used to find the temperature at any point in the magnetic cycle relative to a single state point.

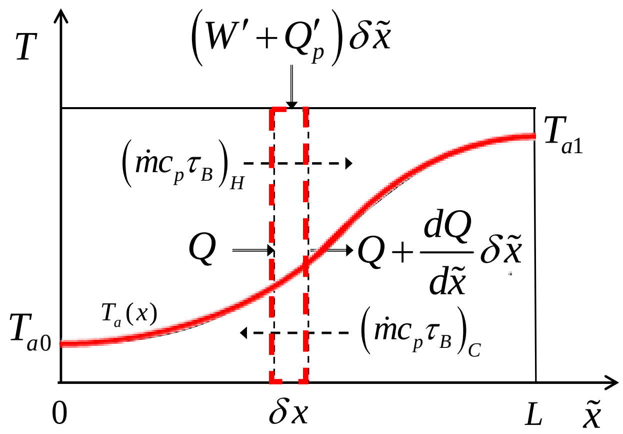
For convenience a secondary definition of utilization is:

$$U_H = \frac{\Phi_H}{R_H} \tag{5}$$

where the subscript H represents the utilization during a hot blow and C for a cold blow.

[1]Rowe A. Thermodynamics of active magnetic regenerators: Part I. Cryogenics (2011), doi:10.1016/j.cryogenics.2011.09.005

Energy Balance[2]



$$W' = \frac{M'_s c_s}{\tau_c}\frac{\Delta T}{T} c_{RH}\left[\left(\frac{R-1}{R}\right)\Delta T + \frac{\Phi}{R}\frac{dT}{dx}\right] \quad (6)$$

$$Q'_p = Q'_{eddy} + Q'_{amb} + Q'_{viscous} \quad (7)$$

$$Q = \frac{M'_s L c_s}{\tau_c}\left[\frac{\Phi}{R}\Delta T - K\frac{dT}{dx}\right] \quad (8)$$

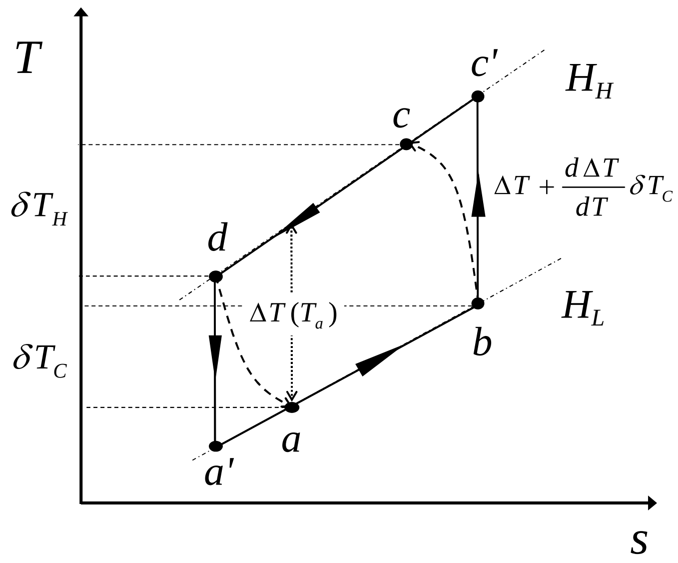
$$W' + Q'_p = \frac{dQ}{dx} \quad (9)$$

$$\frac{d^2T}{dx^2} = \left(\frac{1}{K}\right)\left[\frac{\Phi_{RH}}{R}\left(\frac{1}{c_{RH}}\frac{d\Delta T}{dT} - \frac{\Delta T}{T}\right)\frac{dT}{dx} - c_{RH}\left(\frac{R-1}{R}\right)\frac{\Delta T^2}{T} - \frac{\tau_c}{M'_s L c_s}Q_p\right] \quad (10)$$

Using the BC's, T_{a0} and T_{a1}, Eq.(10) can be solved for $T_a(x)$

[2]Rowe A. Thermodynamics of active magnetic regenerators: Part II. Cryogenics (2011), doi:10.1016/j.cryogenics.2011.09.007

Magnetic Cycle



- Using Eq.(4) and $T_a(x)$, temperatures T_b, T_c, T_d are found
- The fluid temperature, $T_f(x)$, is then: $T_f = \frac{1}{4}(T_a + T_b + T_c + T_d)$ (11)

$$T_{Span} = T_H - T_C = T_f(x = L) - T_f(x = 0)$$

Cooling Power

- Cooling Power is post-calculated

$$Q_c = Q_{amr} - Q_{losses} \tag{12}$$

$$\boxed{Q_c = Q_{amr} - Q_{NTU} - Q_{amb}} \tag{13}$$

$$Q_{amr} = \frac{M_s^{'} L c_s}{\tau_c}\left(\frac{\Phi}{R}\Delta T - K\frac{dT}{dx}\right) \tag{14}$$

$$Q_{NTU} = \frac{m_{disp}}{2}\omega c_p (T_H - T_C)\left(\frac{NTU/2}{(NTU/2)^2 + \left(m_f/m_{disp}\right)^2}\right) \tag{15}$$

$$Q_{amb} = \left(\frac{0.16W}{K}\right)(T_H - T_c) \tag{16}$$ [3]

[3]Tura, A. Cryogenic AMR Test Apparatus. CEC-ICMC, Keystone, Colorado, 2005

Model

Inputs

•T_{a0} and T_{a1} (Boundary Condition's)

•Operating parameters, fluid/solid properties

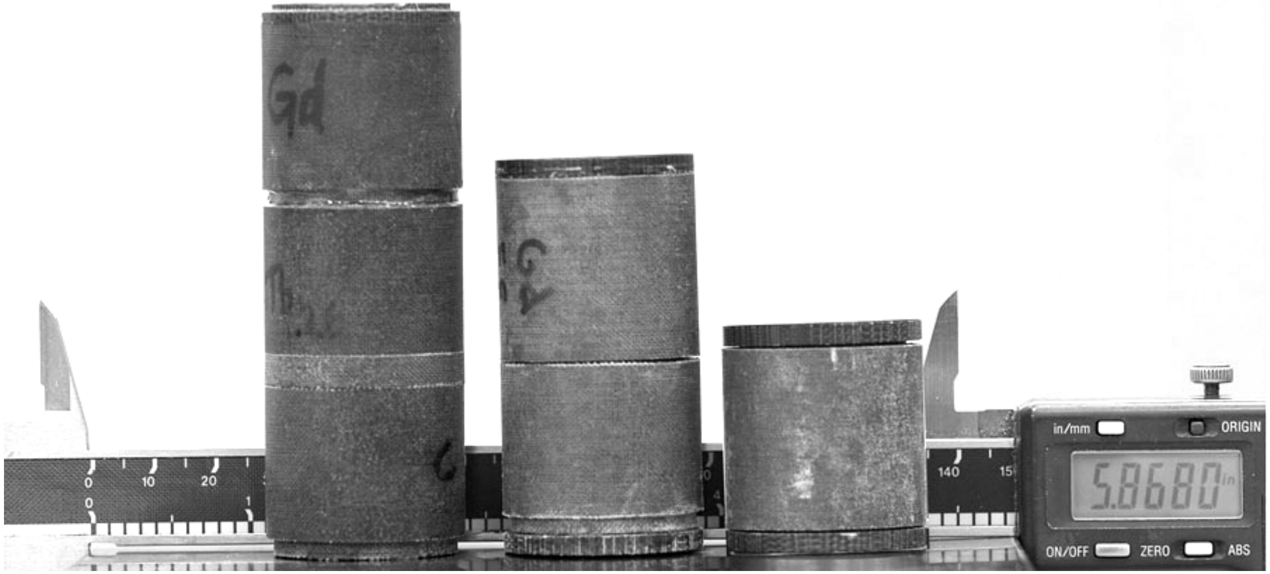
Outputs

•T_{Span}

•Cooling power, work, efficiency

Experimental Data[4]

- Data is from the SC-AMRTA at the University of Victoria
  - Working fluid is Helium (R~1)
  - Materials are Gd, GdTb and GdEr flakes
- Experiments use either single, double or triple pucks of material

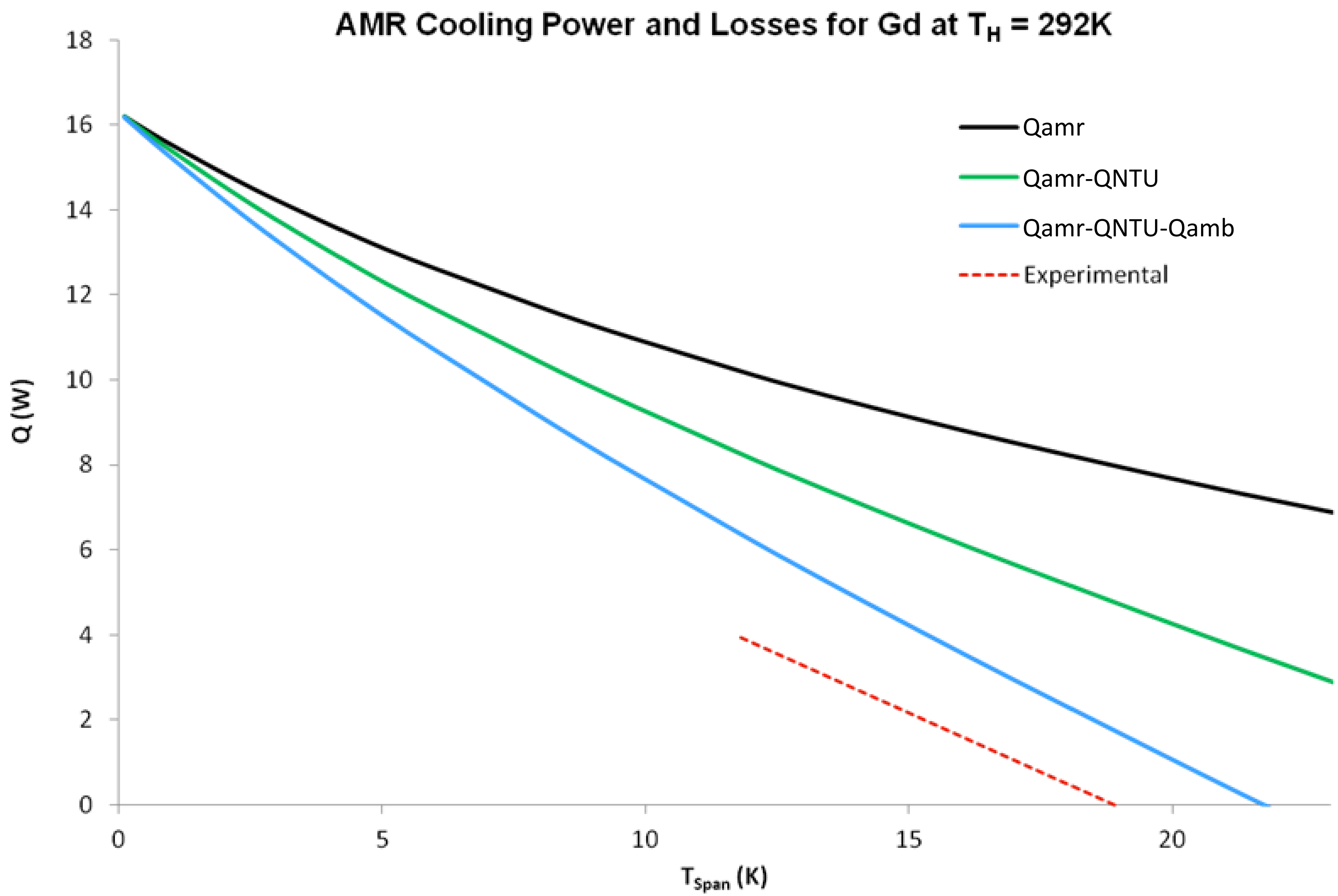


[4]Tura, A. Active Magnetic Regenerator Experimental Optimization. Master's Thesis, University of Victoria, 2005

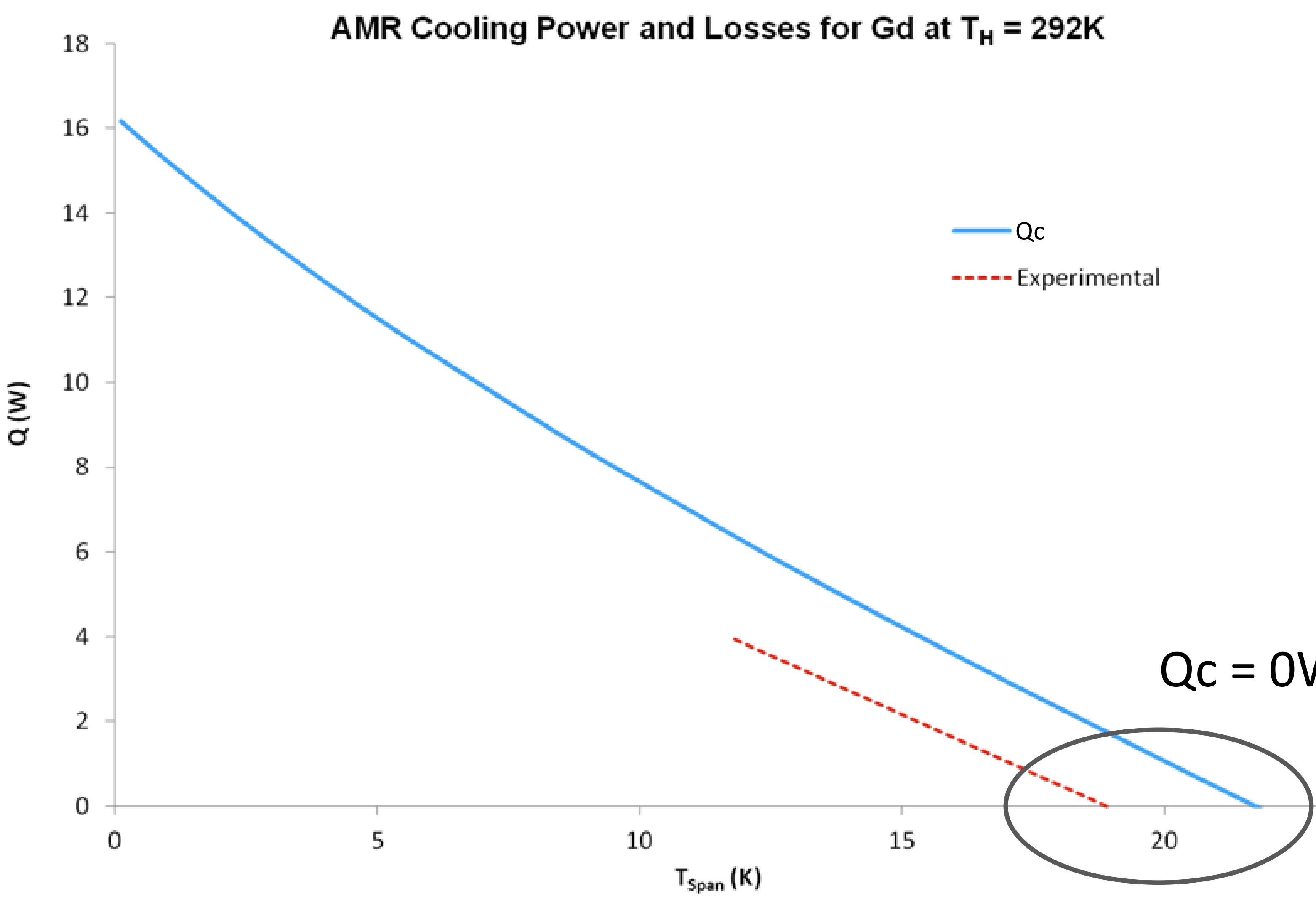
Model Results Summary

- Comparison to experimental results
  - Q_c vs T_{Span}
  - T_{Span} vs T_H
  - Regenerator temperatures profile
- Effect of utilization (U) on calculated results

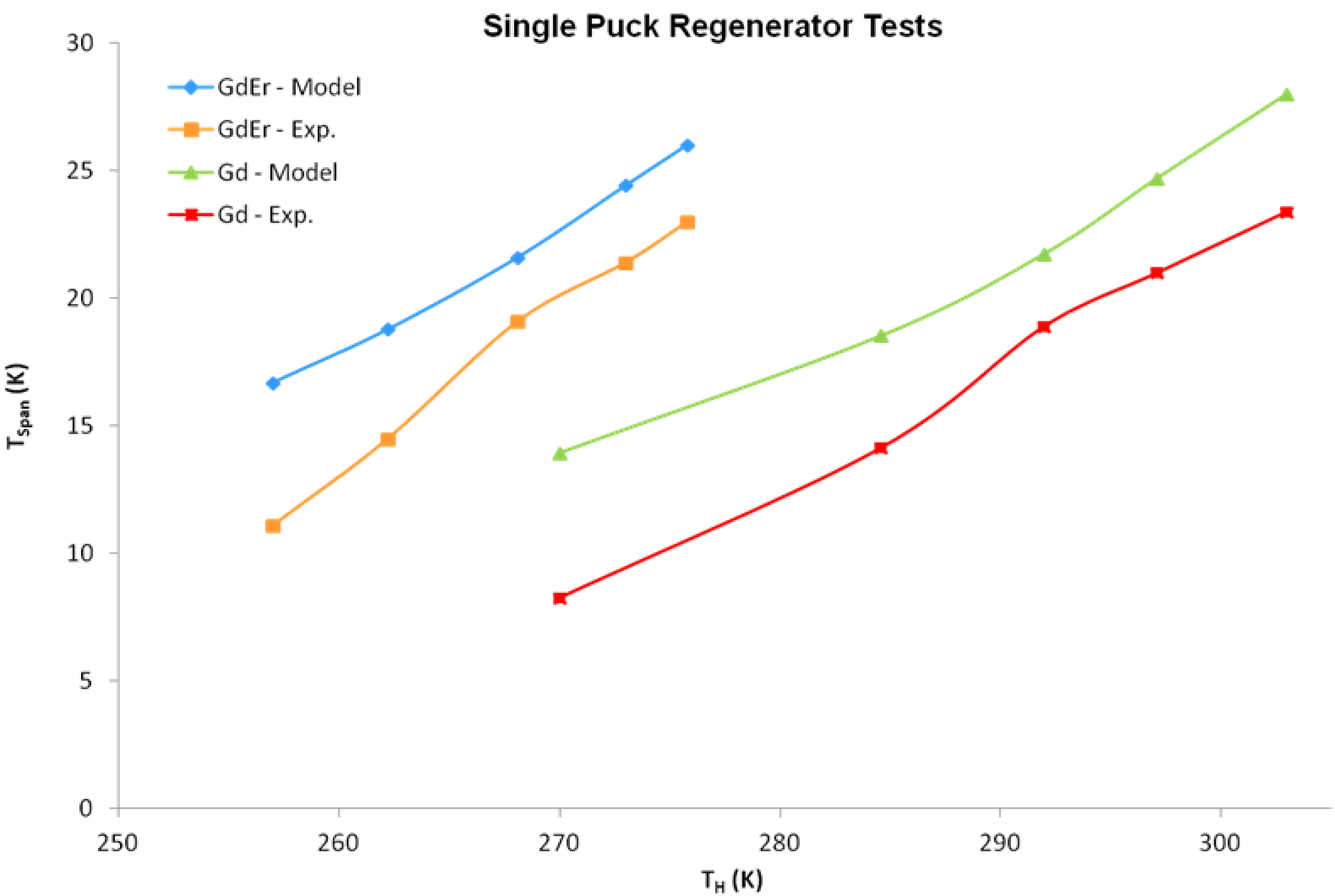
Results: Q_c vs T_{Span} (T_H=292K)



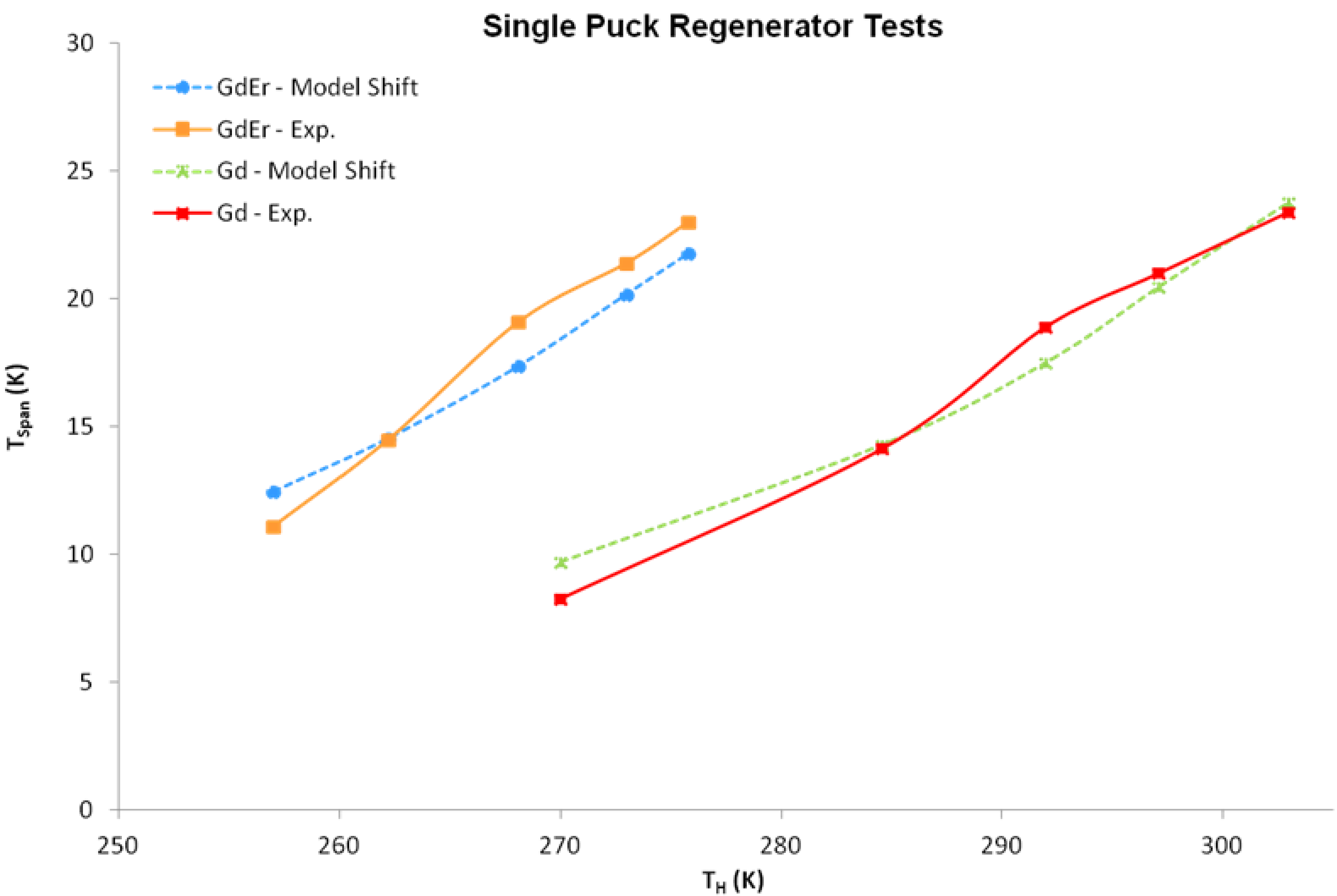
Results: Q_c vs T_{Span} (T_H=292K)



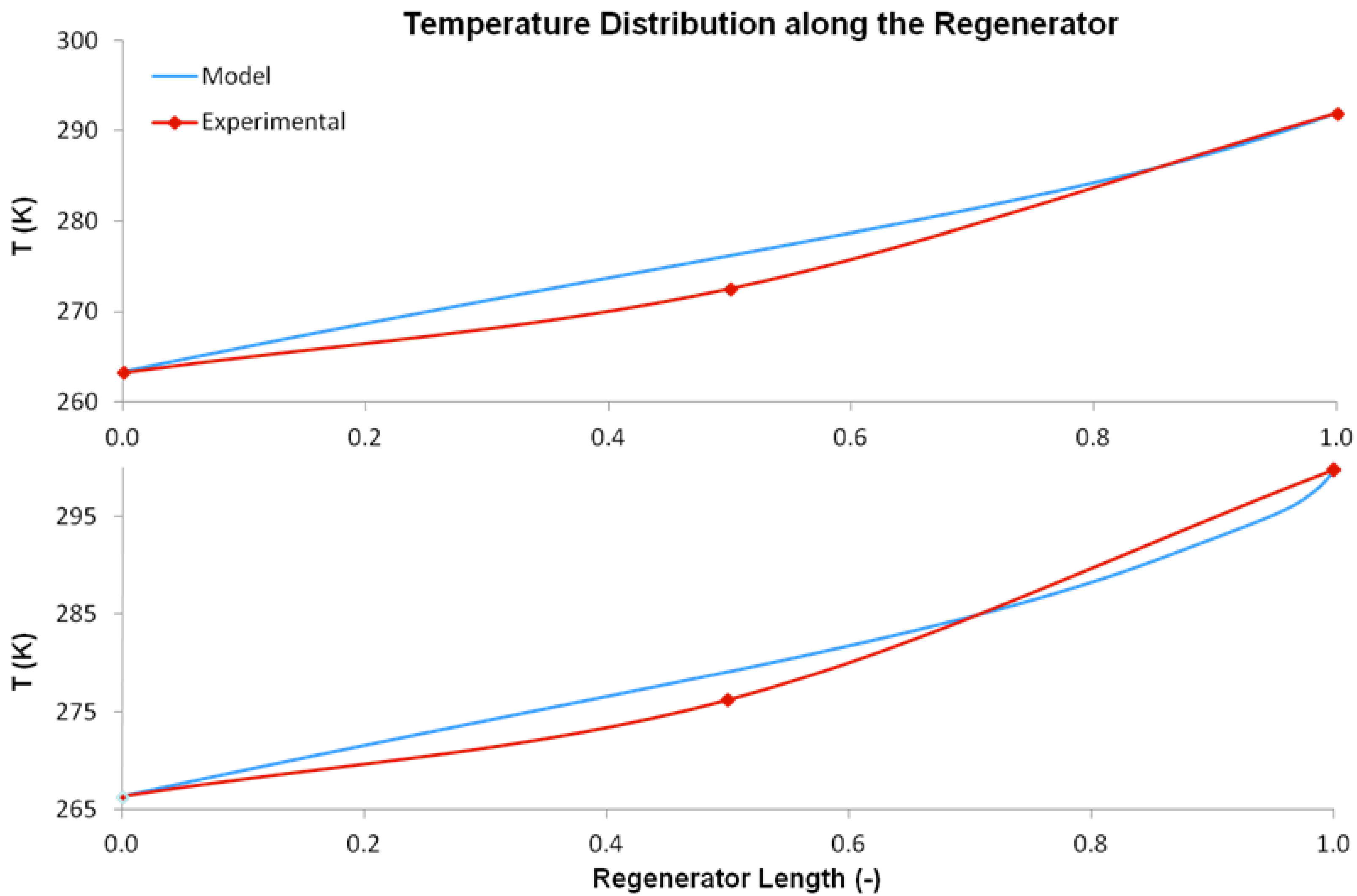
Results: T_H vs T_{Span} (Q_c = 0W)



Results: T_H vs T_{Span} (Q_c = 0W)

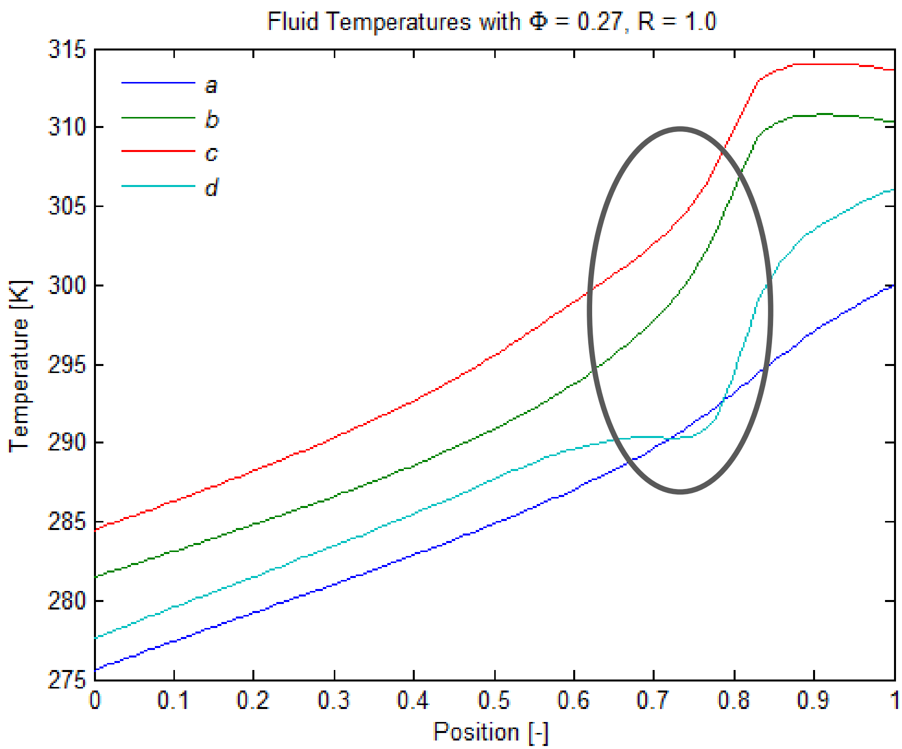


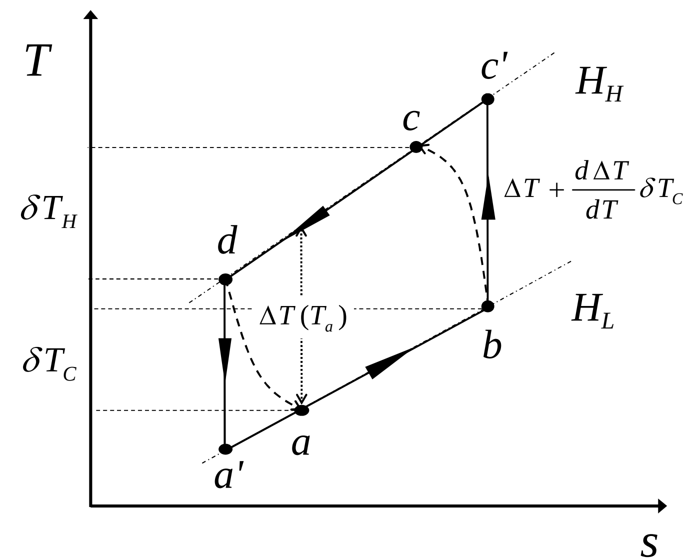
Results: Interface Temperature



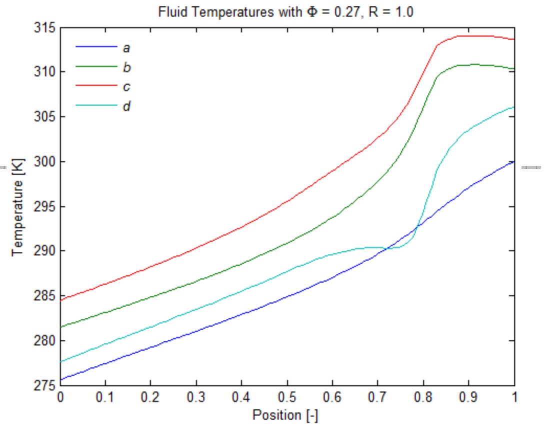
Results: Effect of Utilization, U

- Near the Curie point curves destabilize for high U
- Prevents T_{Span} from being calculated at T_H = 310.7K

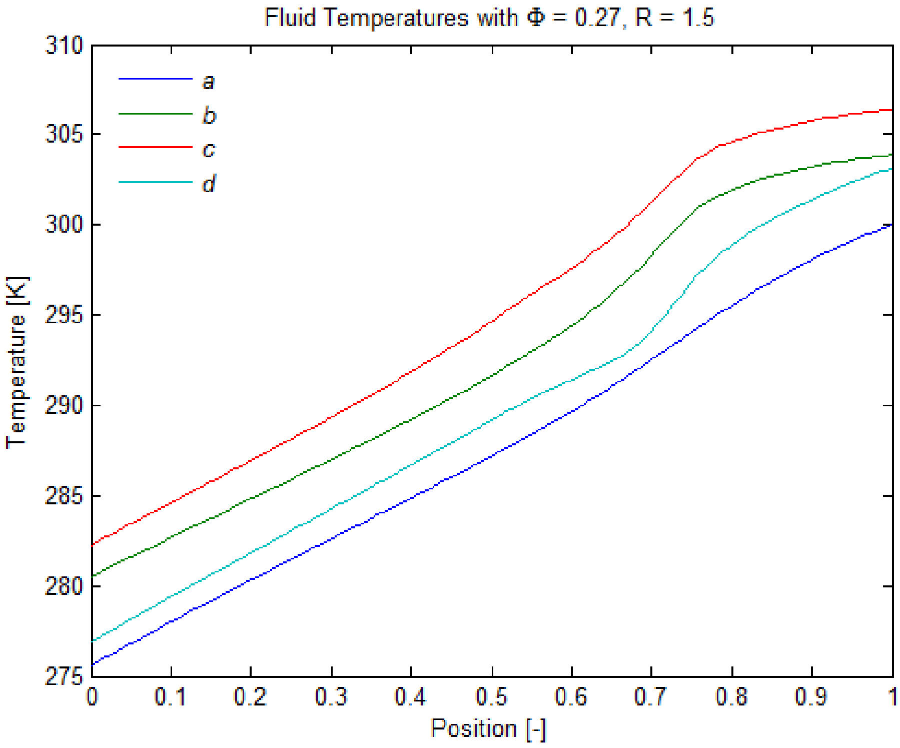


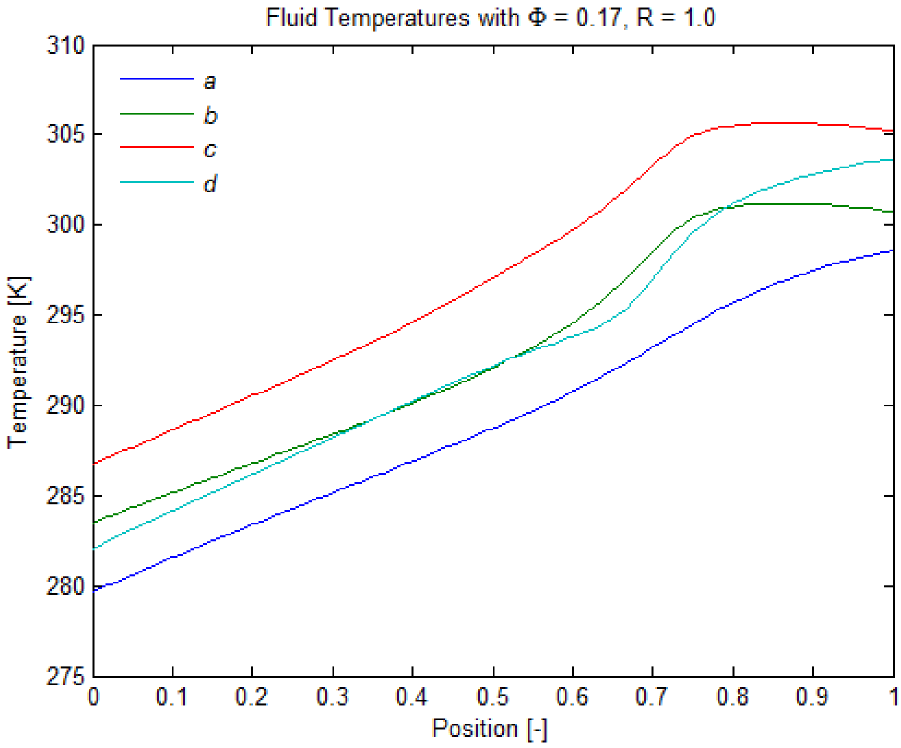


Results: Effect of Utilization



- R = 1 for He, R = 1.5 for water
- Φ = 0.27 at 9.5atm, Φ = 0.17 at 6atm





Comments

- Other losses
  - Demagnetization
  - Other heat leaks
- Model over predicts by a consistent amount
- Simulations take ~3s for each set of BC's
- For a desired T_H value multiple simulations (i.e. various BC's) need to be performed and the data interpolated
  - One specific data point (T_H, T_C) takes ~15s

Future Work

- Allow for the user to choose a specific field strength
- Account for further losses
- Layering
- Use model in conjunction with:
    - Costing
    - Optimization of parameters

Summary

- Simplified AMR thermodynamics were presented
- A model was created using this theory
- Model compared to experimental results
    - Shows similarities to experiments
    - Requires further data comparisons (GdTb)