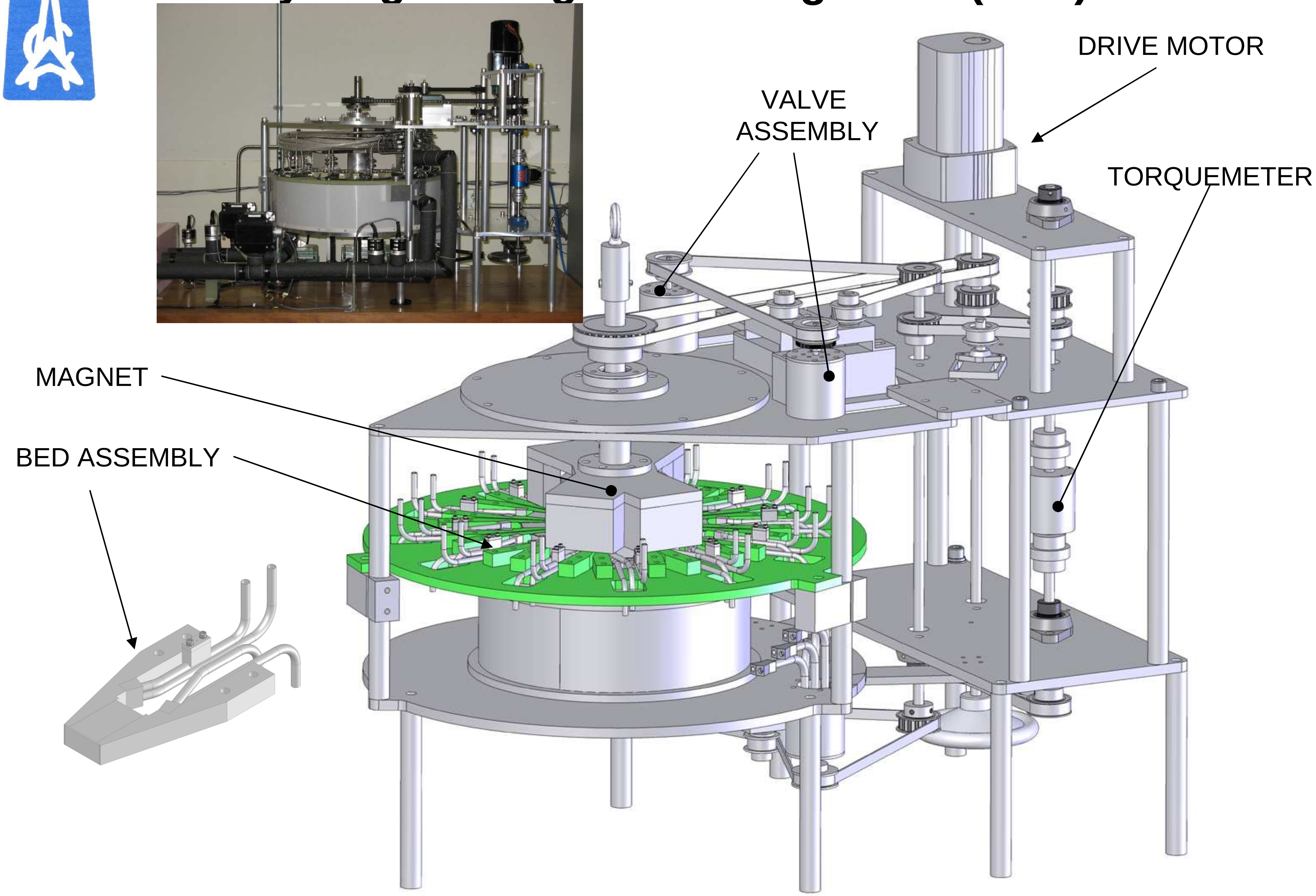
Modeling Magnetic Refrigeration

Dr. Steven Jacobs (primary author) s.jacobs@astronautics.com
Dr. Carl Zimm (presenter) c.zimm@astronautics.com
Astronautics Corporation of America
Madison and Milwaukee, WI USA

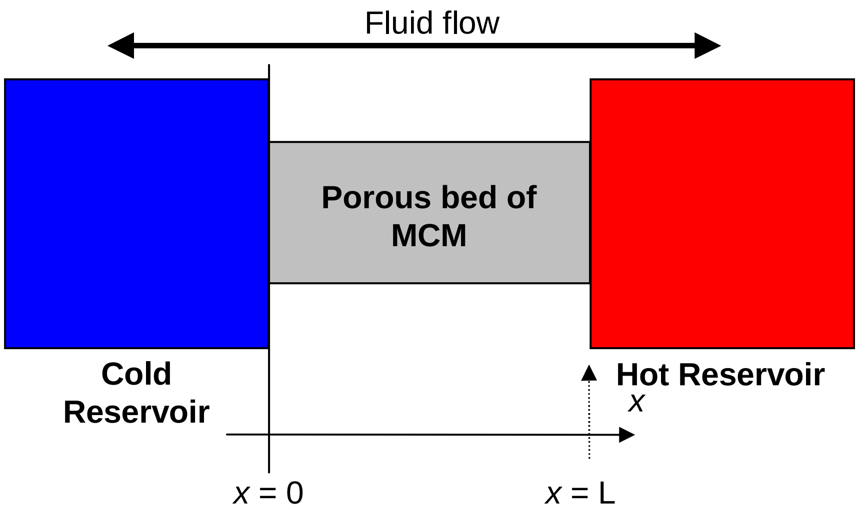
Delft Days on Magnetocalorics, Oct 30-31, 2008

machines, materials, *modeling*

Rotary Magnet Magnetic Refrigerator (BB2)



Modeling the Magnetic Refrigeration System[1]


Fluid flow

Porous bed of MCM

Cold Reservoir

Hot Reservoir

x

x = 0

x = L

Magnetic work done on MCM

$$(1-\varepsilon)\rho_b C_b \frac{\partial T_b}{\partial t} = \frac{\partial}{\partial x}\left(k_b \frac{\partial T_b}{\partial x}\right) + ha\left(T_f - T_b\right) - (1-\varepsilon)\rho_b T_b \frac{\partial S}{\partial B}\frac{\partial B}{\partial t}$$

Rate of change of internal energy

Energy transport via conduction

Energy exchange between fluid and solid

Energy generation from viscous dissipation

$$\varepsilon\rho_f C_f \frac{\partial T_f}{\partial t} = \frac{\partial}{\partial x}\left(k_f \frac{\partial T_f}{\partial x}\right) + ha\left(T_b - T_f\right) - \frac{\Phi\rho_f}{A} C_f \frac{\partial T_f}{\partial x} + F$$

Energy transport via flow

[1] **Engelbrecht 2008 Ph.D. Thesis, UW ME Dept.**

Solving The MR Thermal Profile Equations

- ACA developed new and extremely efficient method for solving the thermal profile equations
- Easily handles layered beds with discontinuous properties
- Exploits smoothness of temperature profiles within layer by using high-order, rapidly converging techniques
- On the order of 100 times faster than other published methods
  - In some cases (low flow rate, small span) ~ 500 x faster
- Evaluation of refrigerator performance fast enough that:
  - Equation solver can be coupled to numerical optimization software to perform automated optimal design of refrigeration system
  - Large scale parameter studies can be performed

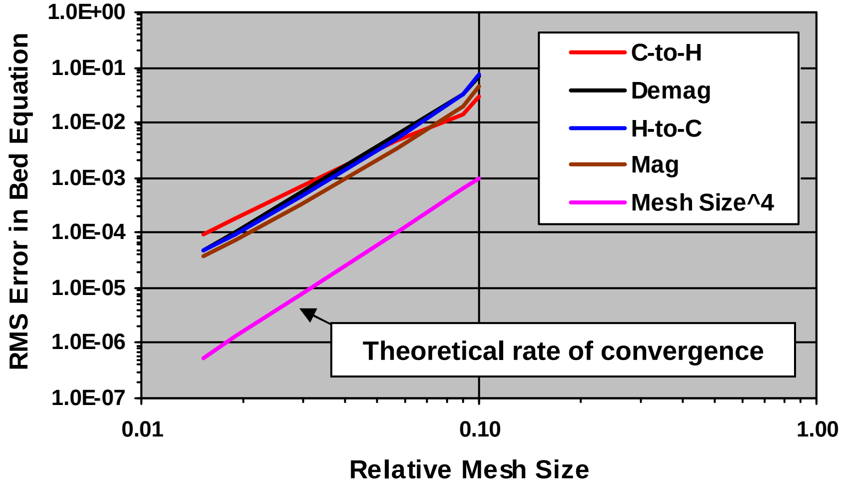
Software Validation

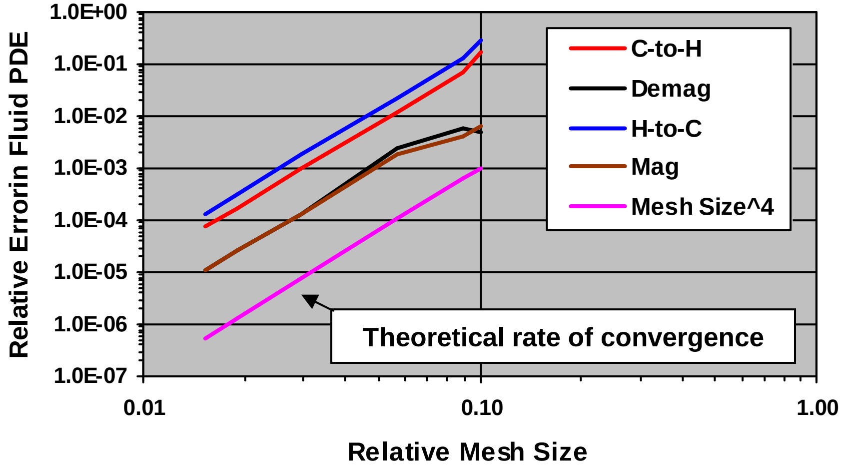
System with 5 layers of LaFeSiH (36% porosity), 120 RPM, 4 lit/min, 1.4 T field

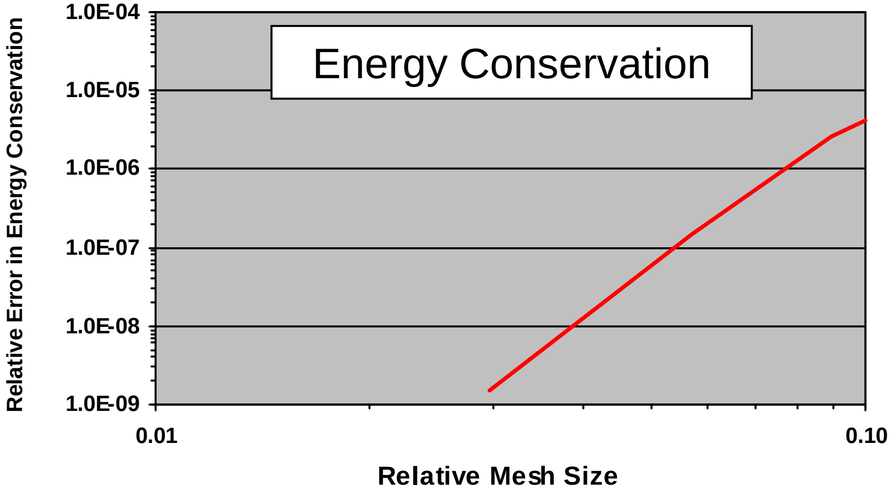
Convergence

Number of Terms	Heat Exhaust (W)	Cooling Load (W)
42	170.036471	153.624981
62	170.058153	153.646353
102	170.072145	153.660588
202	170.072665	153.661279
322	170.072670	153.661255
402	170.072670	153.661254

Satisfaction of Equations

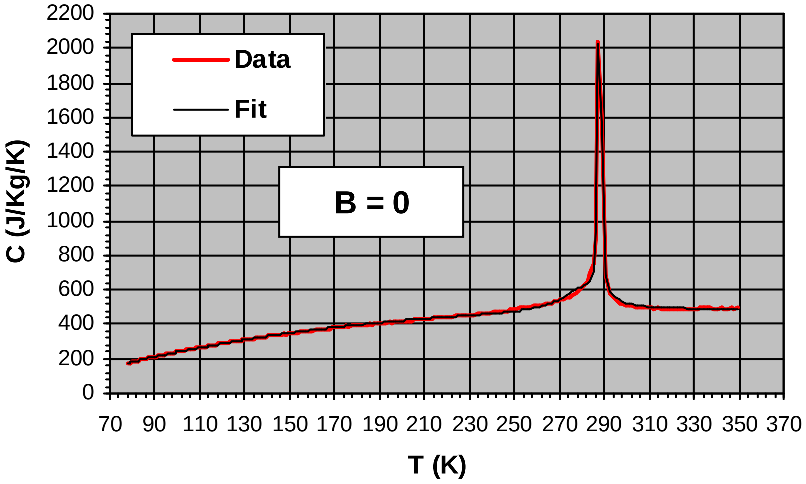


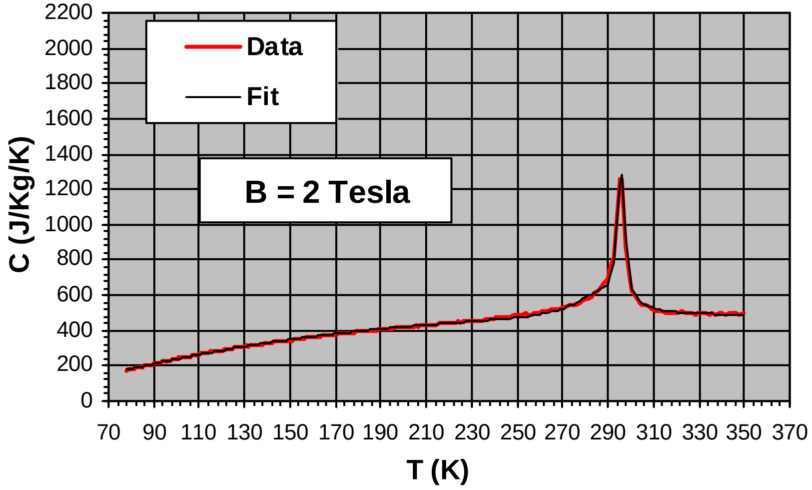




Modeling Magnetocaloric Properties

- Magnetocaloric properties
  - $C_b(B,T_b)$, $S(B,T_b)$
- High-order methods employed in solution of thermal profile equations requires smooth dependence on B, T_b
  - Precludes interpolation
- Should satisfy thermodynamic constraints
  - S = T integral of C/T
  - S decreasing function of B
- ACA developed new analytic representation for C such that S can be expressed in closed form and satisfies thermodynamic constraints
- Free parameters in representation chosen to fit to data and enforce thermodynamic constraints
- Excellent fit to C_b data for a variety of MCM (first-order, second-order)





Optimized System Design – Engineering Prototype (BB3)

- 5 layers of LaFeSiH with 37% porosity, particle diameter fixed at 205 microns
- Bed dimensions fixed by size of existing magnet
- Peak magnet field = 1.4 T
- Goal: maximize cooling power with pressure drop ≤ 15 PSI

System Parameter	Value
0-field Curie point 1st layer (K)	281.653
0-field Curie point 2nd layer (K)	284.670
0-field Curie point 3rd layer (K)	287.985
0-field Curie point 4th layer (K)	291.271
0-field Curie point 5th layer (K)	294.446
Flow rate (L/min)	3.07974
Frequency (Hz)	2.00000 (upper bound)
Hot reservoir temp (K)	298.15 (upper bound)

Performance Parameter	Value	Target
Total cooling load (W)	1476.4	≥ 750
COP (W/W)	8.671	≥ 5
Pressure drop (PSI)	15.053	≤ 15
Span (K)	14	≥ 10

Optimized System Design – Supplemental Electronics Cooler (SEC)

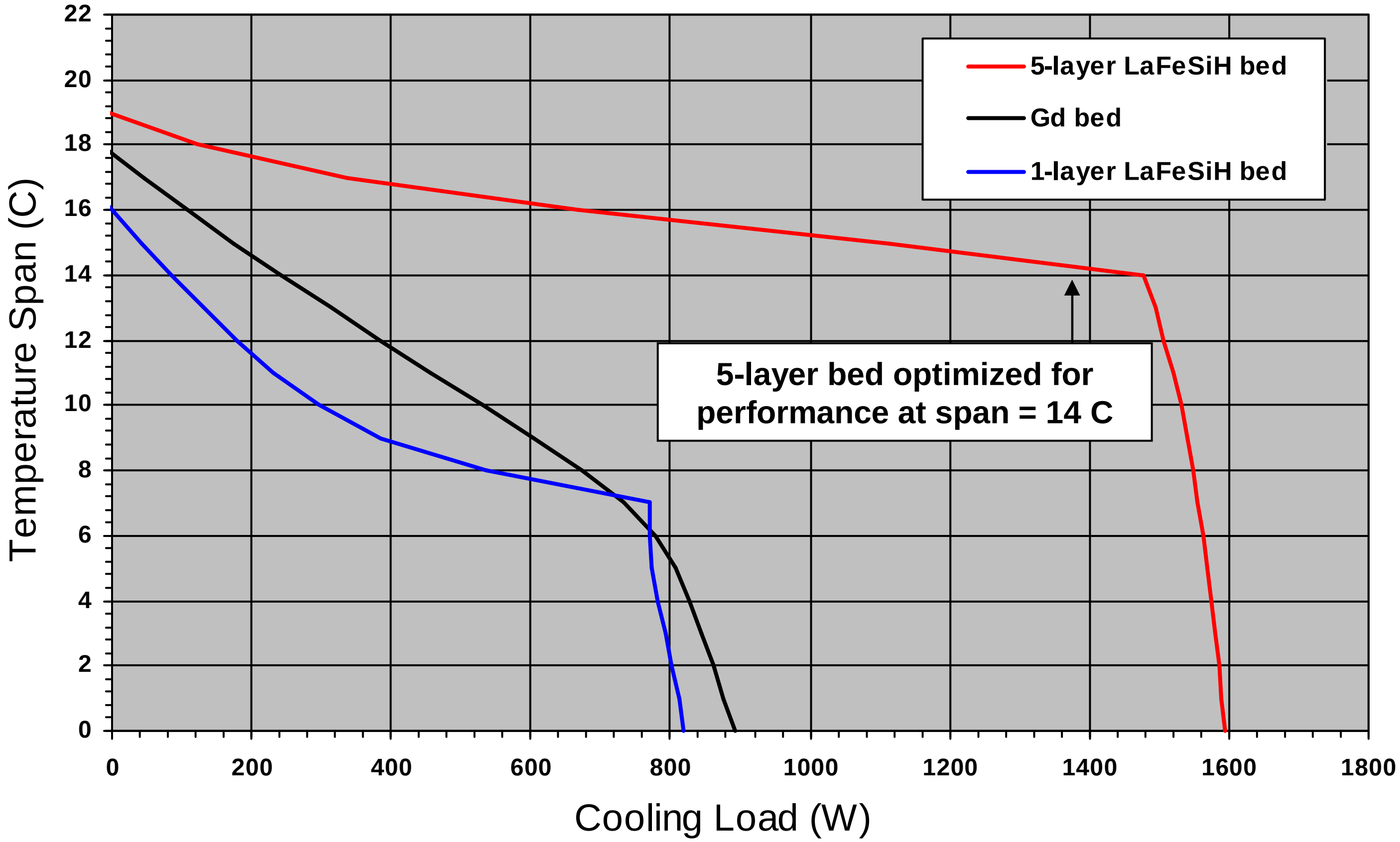
- Layered bed of LaFeSiH
- Bed dimensions can be chosen by optimization
- Peak magnet field = 1.5 T
- Hot reservoir temperature fixed at 44 C
- Goal: minimize cold outlet temperature with pressure drop ≤ 20 PSI, COP ≥ 10, and cooling power ≥ 3 kW

System parameter	Value	Notes
Porosity	0.37	upper bound
Magnet rotation frequency	2.0 Hz	upper bound
Particle diameter	250 microns	upper bound
Flow rate	4.791 lit/min	
Cold reservoir temperature	304.911 K	
Bed length	40.03 mm	
Bed cross-sectional area	6.5812 sq. cm	
Number of layers	6	
0-field Curie point 1st layer	280.803 K	
0-field Curie point 2nd layer	283.066 K	
0-field Curie point 3rd layer	285.671 K	
0-field Curie point 4th layer	288.292 K	
0-field Curie point 5th layer	290.870 K	
0-field Curie point 6th layer	293.258 K	

Performance Parameter	Value	Design Target
Total cooling power	2.999 kW	≥ 3 kW
COP	10.37 W/W	≥10 W/W
Pressure drop	20.15 PSI	≤ 20 PSI
Cold outlet temperature	29.5 C	≤ 33 C

The Advantages of a Layered Bed

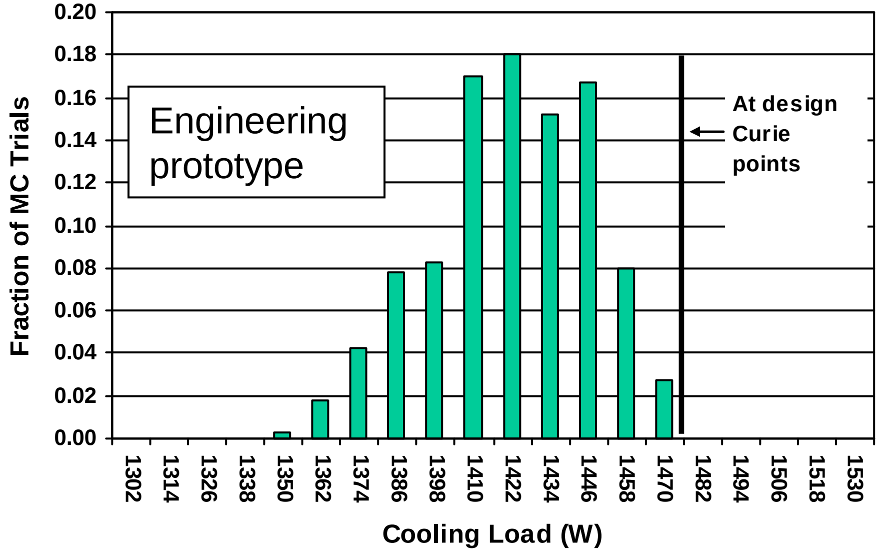
T_{Hot} = 25 C 120 RPM   3.1 lit/min   Bed Volume = 15.6 cm^3 1.4 T Field

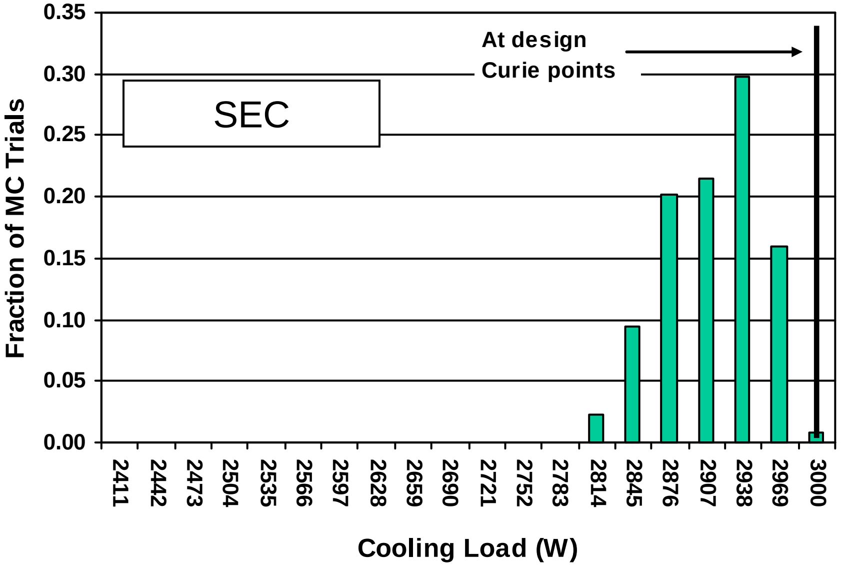


*Note how rapidly the 5-layer bed performance deteriorates as the span goes above its design span

Sensitivity of Optimized Designs to Random Variation in Curie Temperatures

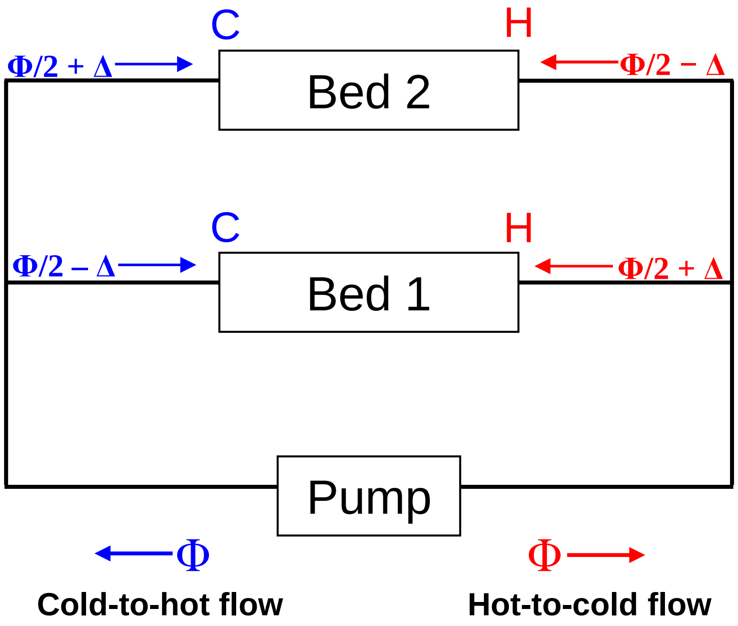
- Lack of control during fabrication will cause variation in Curie temperatures
  - Strong sensitivity to Curie temperatures → low fabrication yield
- Curie temperatures may change over time while in service
  - Strong sensitivity to Curie temperatures → short service life
- Use Monte-Carlo analysis to evaluate sensitivity in performance to random Curie point variation of up to ± 0.5 K
- Cooling load dropped < 8% over 400 Monte-Carlo trials
- No trial exceeded the cooling load of the design – establishes validity of optimization process





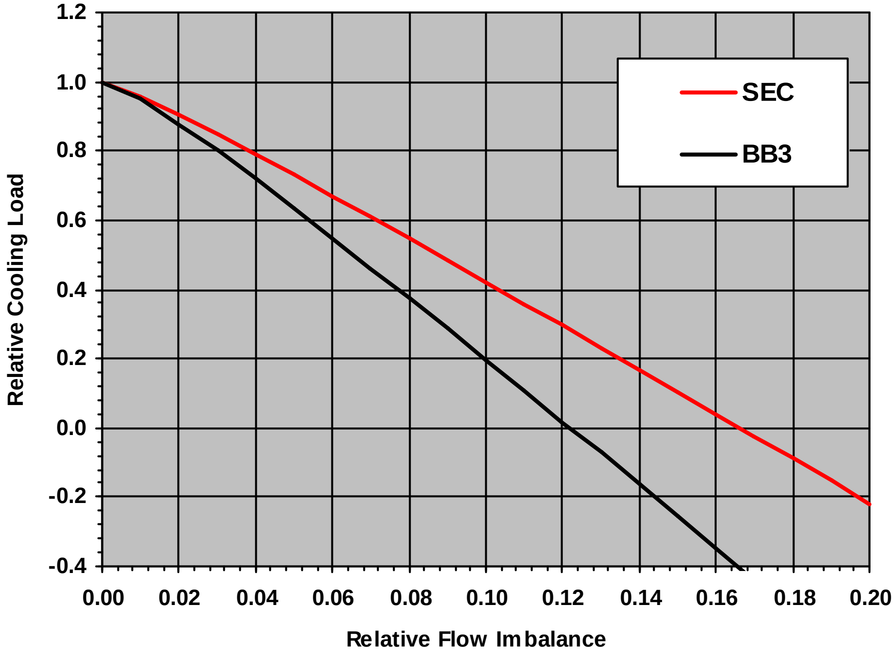
Sensitivity of Optimized designs to Flow Imbalance

Flow Imbalance in a 2-Bed System


C
H
Φ/2 + Δ
Bed 2
Φ/2 − Δ
C
H
Φ/2 − Δ
Bed 1
Φ/2 + Δ
Pump
Φ
Φ

Cold-to-hot flow

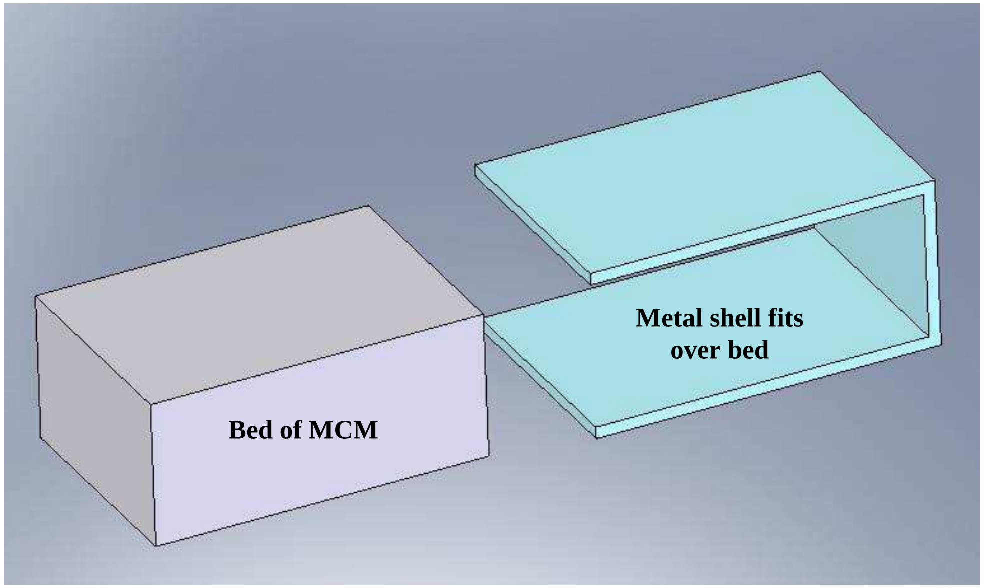
Hot-to-cold flow

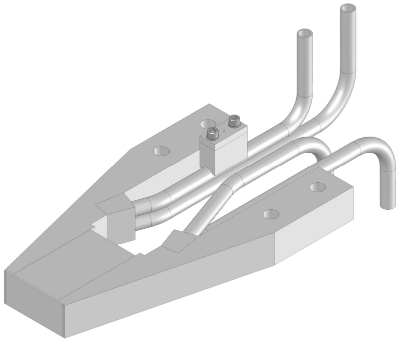


Systems are sensitive to flow imbalance

- 5% imbalance reduces cooling load by 27% for SEC, 37% for BB3
- Must design flow control system to minimize any possible flow imbalance

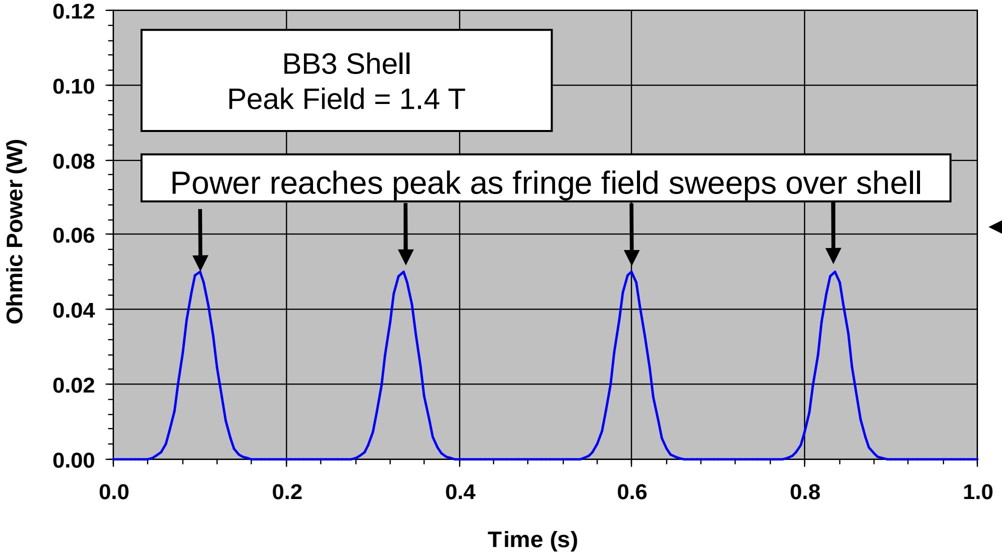
The Effect of Eddy Currents





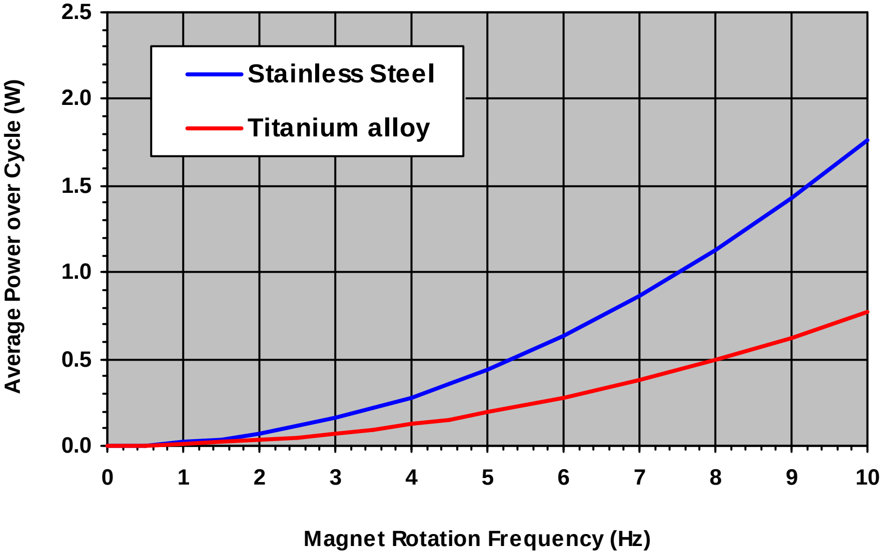
- Bed and metal shell used to seal bed and attach it to plenum are subjected to strong, time-varying magnetic field which generates eddy currents
- Eddy currents cause Joule heating as they flow in resistive shell and bed
- Does this effect need to be included in thermal profile equations?
- Can metal with higher conductivity be used for bed shell, other bed parts?

Eddy Currents in the Bed Shell



Instantaneous power developed in stainless steel bed shell at 2 Hz

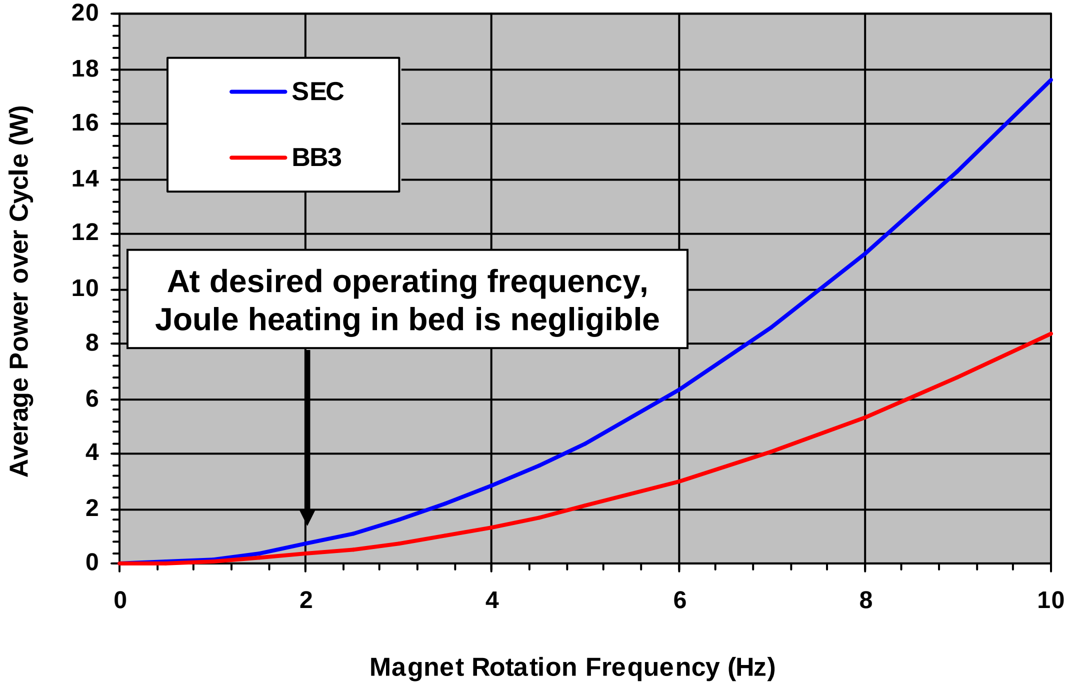
Average power developed in bed shell over one cycle



At desired operating frequency (2 Hz) Joule heating is negligible with either metal

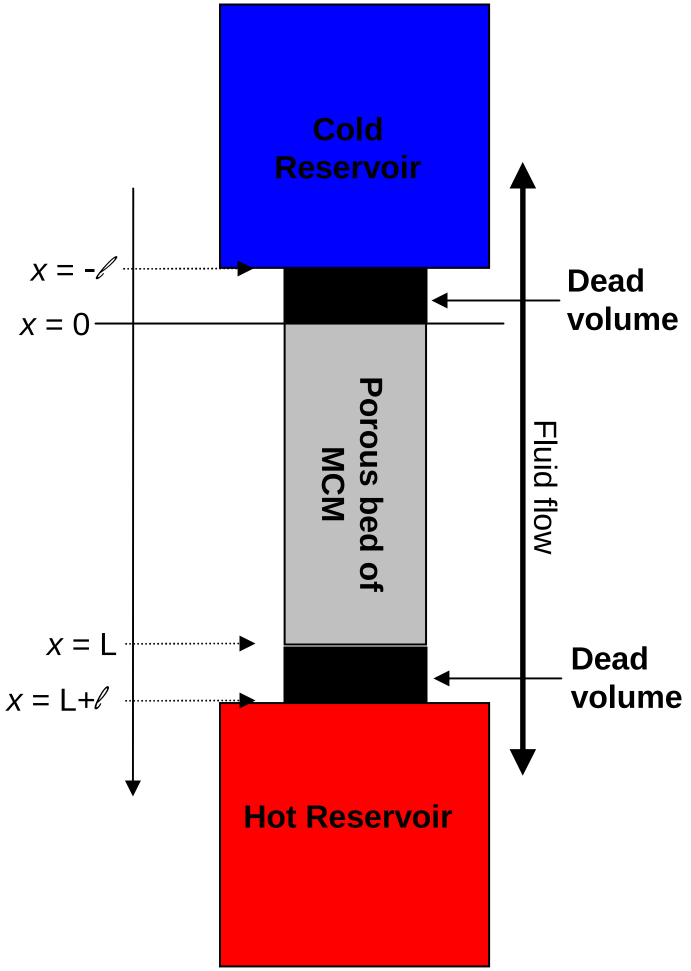
Eddy Currents in the Bed

- Difficult to model eddy current generation in bed exactly because electrical conductivity of porous bed is complicated function of position
- Get an upper bound on the eddy current generation by treating bed as solid block of material with conductivity $(1 - \varepsilon) \times \sigma$ where ε = bed porosity, σ = conductivity of spherical particles composing the bed
- Assume $\sigma = 1.4 \times 10^{-6}$ mhos/m (same as stainless steel)

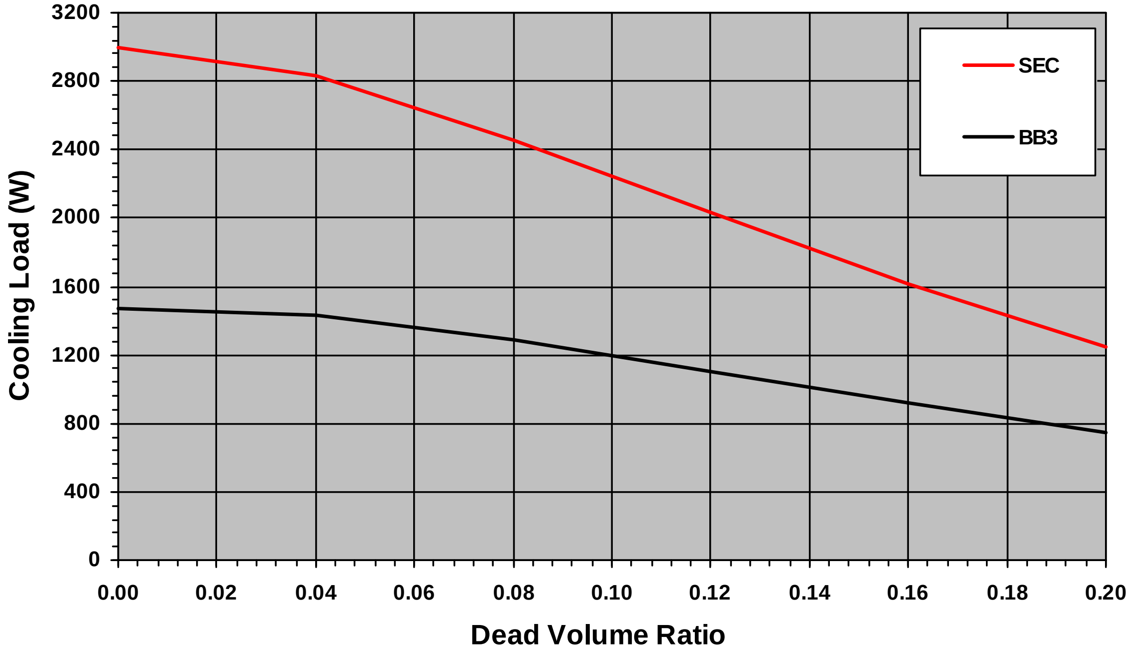


Simulating the Effect of Dead Volume

- Dead volume is the volume at each end of the bed where substantial mixing can occur between inlet and outlet fluids, which are generally at different temperatures
- Model dead volume by adding a layer at each end of the bed composed of packed, inert particles
  - Particles are thermally conductive but have no magnetocaloric effect
  - Dead volume layers have same porosity as active portion of bed
- Particles serve to “mix” the inlet and outlet fluids, simulating the enhanced thermal interaction between them due to their complex flow pattern
- The “Dead Volume Ratio” is the ratio of the volume of one dead volume layer to the fluid volume of the active bed layers = ℓ / L



The Effect of Dead Volume on Optimized System Performance



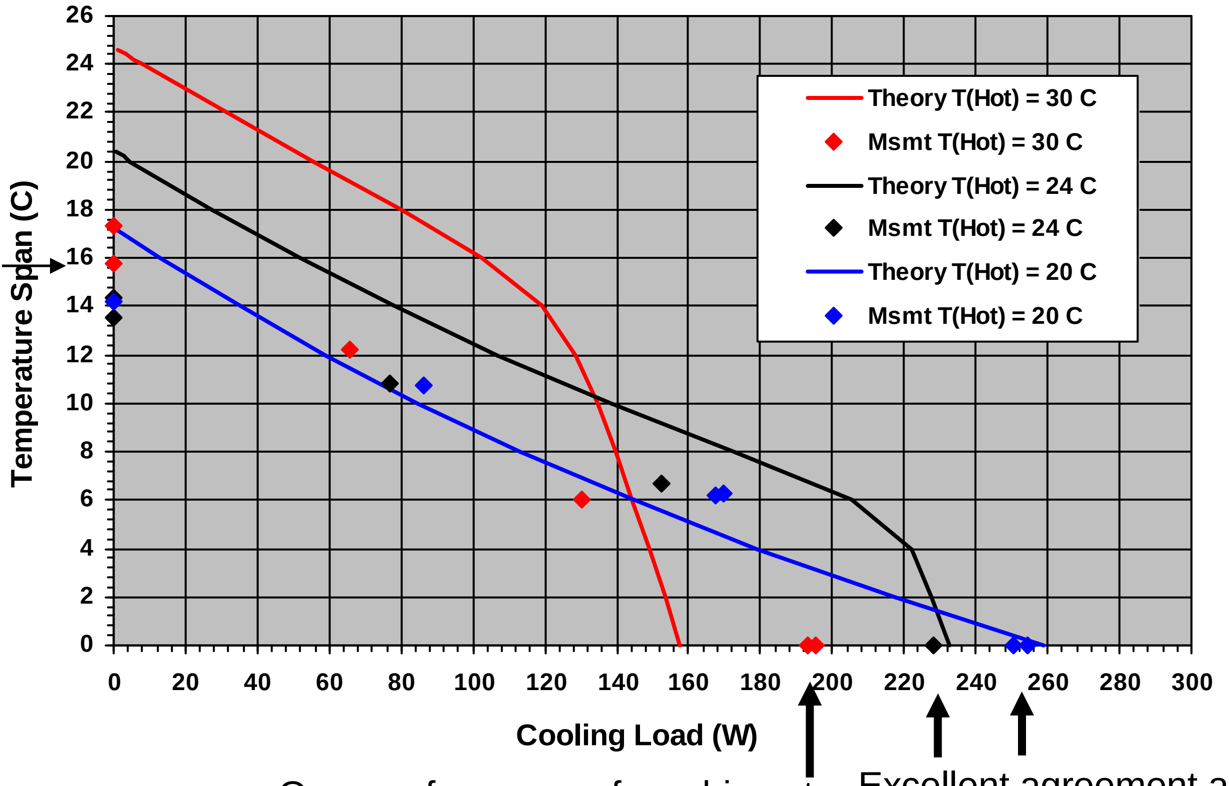
The presence of dead volume can significantly degrade machine performance

Dead volume has a substantial detrimental effect at small temperature spans where the cooling load is largest and therefore the temperature difference between inlet and outlet fluids is largest

As the span increases and cooling load decreases, the inlet and outlet temperature difference decreases so the effect of dead volume decreases

Theoretical and Experimental Performance of BB2 with Gd Beds

60 RPM 0.75 lit/min 1.4 T Peak Field



Under-performance of machine at large span is consistent with 4% flow imbalance (see next slide)

Over-performance of machine at zero span probably due to convective cooling to ambient at this relatively high temperature

Excellent agreement at zero span precludes existence of dead volume

Theoretical and Experimental Performance of BB2 with Gd Beds

4% Flow Imbalance