# Pebble flow in a High Temperature Reactor

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# Preface

This thesis represents my graduation work for the Master in Applied Physics at Delft University of Technology. The thesis work was performed in the section Physics of Nuclear Reactors (PNR) of the faculty of Applied Sciences.

I would like to thank everybody at PNR for their support and suggestions. A special thanks to Brian Boer and Jan Leen Kloosterman for asking new questions each time and challenging me to get the most information out of my model. I would also like to thank August Winkelman and Vincent van Dijk, their questions about the experimental set-up kept me focused on the reality I was trying to simulate and Stavros Christoforou for being patient and help me decipher the PAPA code and try to get it to work.

Andrea Ooms

# Summary

For many applications it is relevant to know the pebble flow in a High Temperature Reactor. The pebble flow is needed to calculate the variations in physical properties, such as the power profile. The pebble flow is also of interest for fuel management. The aim of this thesis is to investigate whether a simple method would yield similar results to computational methods, which are physically accurate, to reduce CPU time.

First a static pebble bed is made with initial positions for the pebbles in the reactor, by placing pebbles at the lowest possible position in the core. The dynamics of a pebble bed are simulated by removing batches of pebbles at the bottom of the reactor and moving each remaining pebble to the lowest stable position; the pebble-flow model. For a reactor with a funnel, a funnel is made out of pebbles that are not moved.

The radial void fraction of the pebble bed, generated with the model for static packing, has a similar shape as found experimentally. For the tested reactor the average void fraction is 0.45, which is higher than the theoretical void fraction of 0.3954. This can be improved to 0.44 by applying the pebble-flow model without removing pebbles. Using a denser grid also improves the void fraction to 0.42. After the pebble-flow model has been applied, with removing pebbles, the radial void fraction still has the same shape and values as for the static packed pebble bed.

The movement of the pebbles is tested with a cylinder. The test shows that the expected result is generated when a large batch size is used. But when too many pebbles are removed, it is possible that cavities arise in the reactor. The cavities are most likely artificial and can be prevented by using a smaller batch size before applying the pebble-flow model; there is an optimum for the batch size.

The radial displacement of the pebbles is established with two different configurations: one with zones with an equal width and one with zones with an equal volume. The main difference between the two configurations is that in the case of equal volume the total amount of pebbles in each zone is the same, which makes it easier to interpret the results. After a complete pass of all pebbles in a reactor with 5000 pebbles, at least 26 % of all pebbles in a zone originates from that zone. This means that the pebbles are fairly mixed. It has been found that the mixing occurs quickly in the reactor.

The results from the pebble-flow model are not yet confirmed. Some results are compared with experimental results, but the model should still be compared to the Discrete Element Method.

# Contents

# Chapter 1

# Introduction

The Very High Temperature Reactor, or VHTR, is one of the generation IV reactors. This is a group of nuclear reactor designs being investigated for future development. The objective of the generation IV reactors is to be safer and to produce less waste than the reactors currently in use [1].

The VHTR is a graphite-moderated reactor and uses helium as a coolant. This makes it possible to generate a working fluid with higher temperature than with a reactor moderated and cooled by water. Because of the higher temperature the reactor is thermodynamically more efficient. The VHTR is filled with pebbles. These are billiard ball size graphite spheres with fuel, contained in TRISO particles, see figure 1.1.

In the past, several pebble bed reactors have been built and operated. These pebble bed reactors are all test facilities and are considered to be generation III$^+$ reactors. The temperatures in these reactors are lower than in the VHTR, therefore they are called High Temperature Reactors, HTR. At the moment there is just one HTR in operation, the HTR-10 in China [2]. The characteristics of the HTR-10 can be found in table 1.1. Because experimental data is available for the HTR, this thesis is about the HTR instead of the VHTR, although the pebble flow should not differ between the two types of reactor.

The VHTR is of interest as a nuclear reactor of the future because it is inherently safe, several mechanisms in the reactor will make a meltdown impossible. Most importantly, as all nuclear reactors currently in use, the reactor has a negative temperature feedback. This means that when the nuclear chain reaction, for whatever reason, generates more power (and thus heat) than nominal, this will cause the reactor to generate less power (and heat) without active intervention. For the same reason, the fission chain reaction will shut down when the coolant flow in the reactor is lost. Most of the nuclear reactors currently in operation have a negative temperature feedback, however they are not all inherently safe. Active intervention is needed when the reactor has been shut down and the coolant is lost. After a shut down, there will still be some heat generated in the reactor, caused by the decay of the fission products. Without cooling this heat causes the temperature in the reactor to rise, which may cause a melt down. Because of the graphite pebbles used, this is not the case for a VHTR. This has two reasons.

Table 1.1: Characteristics of the HTR-10 in China [2].

| | | |
|---|---|---|
| Reactor core diameter | 180 | cm |
| Number of fuel elements in core | 27 000 | |
| Average core height | 197 | cm |
| Pebble diameter | 6 | cm |
| Fuel management | Multi-pass | |
| Fuel charge tube diameter | 65 | mm |
| Fuel de-charge tube diameter | 500 | mm |
| Fuel (de-)charge tube position | Centre | |
| Fuel de-charge tube angle | 30 | ° |
| Thermal power | 10 | MW |
| Average helium temperature at reactor inlet | 250 | °C |
| Average helium temperature at reactor outlet | 700 | °C |
| Nuclear fuel | $UO_2$ | |
| Enrichment of fresh fuel element | 17 | % |

- The VHTR has a relative low power-density. This means that in a certain volume less heat is produced. Also after the reactor has shut down, less decay heat needs to be transported out of a certain volume.
- The high thermal capacity of graphite causes the reactor to heat up slowly after a shut down. The heat that is produced can be transported from the centre of the core to the outer reflector because the pebbles are in contact with each other, which enables conductance. Furthermore the heat produced can be transported from the reactor core by radiation. At the outer side of the reactor core, all the heat produced can radiate to the environment. Finally, heat can be transported through convection of the helium flow.

## 1.1 Pebble flow

For the operation of a pebble bed reactor it is helpful to know the pebble flow. This is needed for fuel management and to determine variation in physical properties in a certain location, because the density is, for example, lower in that location, rather than the average of such properties for the whole core. An example of a physical property for which the pebble flow is essential, is the core temperature distribution. The pebble flow affects the distribution of the fissile material in the reactor core. For example, if pebbles at the centre of the reactor move much faster than pebbles near the edge of the reactor, this can result in a lot of fissile material at the centre of the reactor, the pebbles here are renewed more often. Near the edge of the core, where the pebbles are renewed less often, a larger part of the fissile material is burned before the pebble exits the core. If this extreme situation would occur, the actual dimensions of the reactor differ from

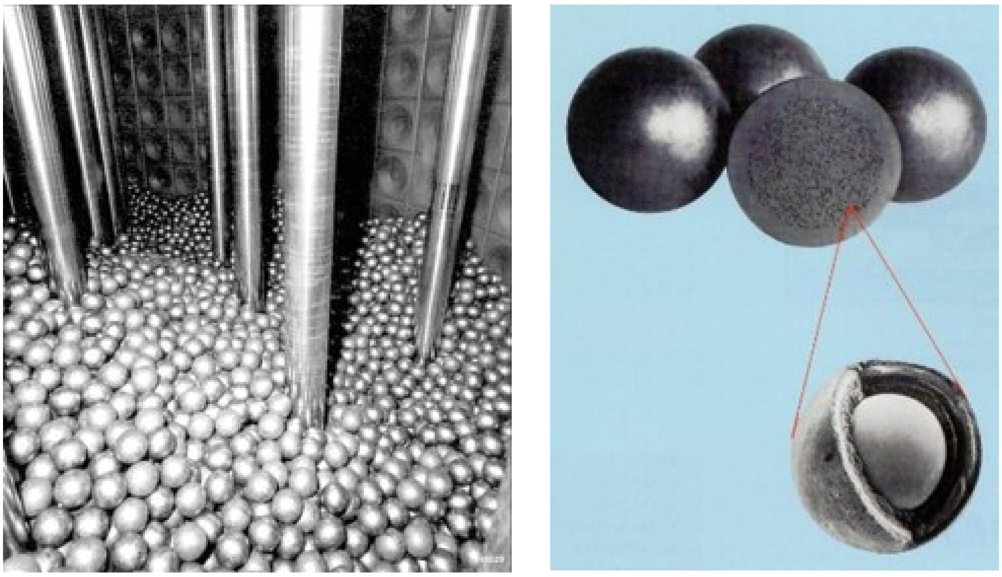

Figure 1.1: Pebbles. On the left; pebbles in a reactor with control rods. On the right; the cross section of a pebble, the highlighted part is a TRISO particle.

the dimensions used to determine the safety aspects for the reactor. The pebbles with hardly any fissile material are now acting as a reflector and, effectively, the reactor core is smaller. This is of influence on the neutron flux distribution in the core and thus the temperature distribution. Hence, the determination of the pebble flow is important for safety. If there is not sufficient knowledge of the pebble flow, higher temperatures than assumed, may arise locally, for example at the centre of the core. This may lead to pebble failure and, possibly, to unexpected and dangerous situations because fissile material and fission products could move through the core [3].

Another reason why the pebble flow is of interest, is the core design. One of the designs that have been proposed, contains a central column of moderator pebbles. To use this design it is important to know what happens to the graphite pebbles: do they stay in the centre or will they be distributed homogeneously in the core? In other HTR designs the centre is made of graphite blocks, which has the disadvantage that it is more difficult to cool and that the reactor operation needs to be stopped to replace the graphite. If more details are known about the pebble flow, the option of having a central column made of graphite pebbles can be analysed.

Finally it is important to know the porosity of the pebble bed along the core. In regions with a higher porosity is more coolant, this makes the pebbles in these regions colder, which makes the power generated in these regions higher. It is known that in a static pebble bed the porosity is high along the reflector wall. Because of the reflector peak, the presence of more thermal neutrons directly next to the reflector, the power generated in this high porosity region becomes even higher. It is, however, not known how the porosity changes, in comparison to the static pebble bed, when the pebbles flow.

When the pebble flow is known this can also be used to determine the best loading scheme for the reactor. It might be that a more flat power profile can be achieved when fresh pebbles are loaded near the centre of the core. Or, as in the example given before, load new pebbles near the edge of the reactor and used pebbles in the centre. All these points are of importance if it is shown that the pebbles do not have a large radial displacement.

## 1.2 Outline of the thesis

The main analysis tool to investigate the pebble movement in the reactor core is computer simulation. The focus of this thesis is thus to simulate the pebble flow in the HTR and compare the results with currently existing simulation methods and results from experiments performed by others. Using simple methods, is it possible to get a similar result to computational methods which are physically more accurate?

In this thesis a computer model will be described to simulate the pebble flow in a pebble bed reactor. Before generating a new model a literature study has been performed to learn about the existing methods to simulate particle flow in (nuclear or chemical) reactors. The results of the literature study can be found in chapter 2. To be able to simulate the pebble flow, the simulation of a pebble bed packing in a stationary reactor was tackled first. There is a lot of experimental data for stationary reactors and this is a starting position for the pebble-flow model. After the correct starting position of a filled, but stationary, pebble bed is created, a computer model is applied that will move the pebbles downward, the pebble-flow model. The static packing is presented in chapter 3, the movement downwards is presented in chapter 4. From the results of the pebble-flow model, the radial displacement of the pebbles in the reactor is determined in chapter 5. The conclusions can be found in chapter 6.

# Chapter 2

# Theory of pebble flow

All the methods discussed in this thesis are models of the reality. They only estimate the real situation. There is however a lot of physics involved with the pebble flow in a pebble bed reactor, some of which is ignored in the different models.

The main principle, in reality, is conservation of energy. The reactor vessel is placed vertically, the pebbles thus move under the influence of gravity. The forces due to the other pebbles, the wall of the reactor vessel and the downward flow of the helium influence the movement of each individual pebble. Another important influence is friction. Pebbles at the outer side of the core are influenced differently by friction than pebbles at the centre of the core because the pebble has a different friction-factor to the reactor vessel than to another pebble. The contact surface will also be different as the pebbles are convex and the reactor vessel is concave. If there is a central column made of graphite blocks in the core, this will also be convex.

There are different forces working on each pebble and the reactor core as a whole. The different forces are (see figure 2.1);

**Gravity** The pebble experiences a force downward due to gravity. This is the mechanism for the pebble movement.

**Other pebbles and reactor vessel** Pebbles below and above the pebble cause force on it. The same holds for the reactor vessel, which keeps the whole core in place. This pressure is in the direction of the line connecting the centres of the pebbles and perpendicular to the reactor vessel. Each pebble is of some influence to each other pebble, directly or indirectly, through other pebbles in between.

**Friction** Because the pebbles move, friction is also of importance. The pebble can be in contact with two different kinds of materials, other graphite pebbles or the outer reflector. The friction is in the opposite direction of the movement of the pebble and will slow down the flow.

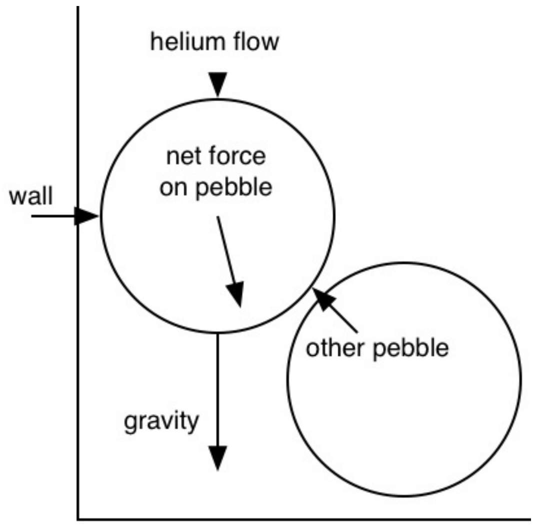


Figure 2.1: The forces acting on a pebble.

**Helium flow** A small contribution to the movement of the pebbles will be given by the helium flow in the reactor. The helium flow is in the same direction as the pebble flow and will thus increase the flow. This effect will be very small because helium is light. It will be neglected in the simulations described in this thesis.

## 2.1 Possible simulation methods of pebble flow

Not much research has been done into the composition of a pebble bed reactor, neither static nor dynamic, but the composition can be compared to that of chemical reactors. The static simulation of chemical reactors has been described in various studies, which may be applied to pebble bed reactors as well. A summary of the possible methods mentioned in this chapter can be found in appendix B.

**Discrete Element Method** The Discrete Element Method (DEM) describes the real physics of pebble flow as good as possible. This is done from the perspective of each pebble individually, all the forces on a pebble are summed and the resulting equations of motion are numerically integrated with a fixed time step. If the time step is sufficiently small, it may be assumed that only the direct neighbours of the pebble are of influence. This method can be used when the reactor is already filled with pebbles and their movements need to be determined [4–12].

It is also possible to use the DEM to randomly fill the reactor. To achieve this, the reactor will be filled with regularly located pebbles, possibly with a random initial velocity. The DEM is then used to move the pebbles in the reactor randomly until there is a stable situation [8].

**Spot model** An alternative method to model the flow, is the spot model. In this model the general idea is that there are voids in the pebble bed that move upwards. Since the voids move upwards, the pebbles move downwards in clusters. There is a strong correlation between the movements of the pebbles in a cluster. This model has been calibrated with the use of the DEM to determine the physical properties of the voids. This model is claimed to be a hundred times faster than DEM [12].

**Random point method** A possible method to generate a static filling, is to generate a very large number of random points in the reactor. When the whole reactor is filled, each point will be checked for the possibility to place a pebble without overlapping other pebbles or the wall of the reactor. In this manner the reactor is filled bottom up. The advantage of this method is that the void fraction generated, is the same as the void fraction found in experiments. This method does however present a disadvantage in the numerous random points that are needed to fill the whole reactor, the majority of which, will not be used. This takes up a lot of computer memory and for a large reactor it will be needed to fill the reactor with multiple batches of random points. Moreover, the radial porosity stabilises faster than the experimental results indicate [13].

**Rain model** The rain model is a physical method to fill the reactor core. The pebbles are dropped from a random position above the reactor and fall (or rain) down until they hit one of the pebbles already placed in the reactor. From here the pebble rolls over the pebble bed until it reaches a stable position. In this stable position, a pebble might be supported either by three pebbles or by two pebbles and the reactor wall. If applying this method fills the reactor, the porosity is higher than the experimental data because the only force taken into account is gravity [14].

**Rain model in combination with Monte Carlo** A different and less physical manner to implement the rain model, is to drop the pebble from a random position above the reactor and let it fall until it hits a pebble already placed. The falling pebble sticks to the pebble it hits and the process is repeated without the pebble sliding to a stable position. When the whole reactor is filled, it is shaken with the Monte Carlo method, i.e. a random pebble is moved into a random direction. If the pebble overlaps with another pebble in the new position, it will stay in its original position. If the pebble does not overlap with another pebble and the z-component is negative, it will stay at the new position. If the z-component is positive, there is a chance that it will stay in the new position, but it could also be placed back to its original position [15].

**Drop model** It is possible to simulate pebbles being dropped in the reactor from a certain given height. To save CPU time a group of M pebbles will be randomly generated each time step. The pebbles are generated at different heights and with a determined speed, as if they were generated at a random time in the time step. Creating M non-overlapping pebbles in a small cylinder above the reactor does this.

The initial velocity is dependent on the initial height of the pebble and equals the velocity that the pebble would have, if it had been dropped from a given height.

This method is that the height from which the pebbles are dropped can also be made time dependent. In this case, the distance that a pebble has to cross to reach the top of the pebble bed, is kept equal [16].

The difference between the rain model and the drop model is that in the rain model the pebbles are added separately and roll until the pebble is on a stable position before the next pebble is added. In the drop model multiple pebbles are added each time step.

**Lowest possible position** Another possible method, is to place the pebbles one at a time at the lowest possible position. A list is made for this purpose with all positions where pebbles do not overlap with other pebbles. After a pebble has been placed, the list is updated with new positions and original positions that overlap with the newly placed pebble are removed. This simulation does not yield the same radial porosity as in a real reactor [6, 14, 17].

A variation on this method that requires less CPU time, is to build the pebble bed layer by layer. This means that after a pebble is placed, no new possible positions are added; only the positions that overlap with the placed pebble are removed. When the list with possible positions is empty, a new list will be made and the process repeats. This speeds up the simulation, because updating the list of possible positions is the most time consuming step in the simulation.
There are some other possible variations on this method. A restriction can be made to the next pebble placed in the core. One example of this, is to alternately place a pebble in contact with the reactor wall and one in the middle of the reactor. Another possibility is to have a certain percentage touching the wall. The last method returned the best results compared to experimental data, but uses more CPU time than the other variations that used the lowest possible position [17].

**Eliminating overlap** It is also possible to place the total number of pebbles randomly in the reactor, disregarding overlap. Finding the biggest overlap and moving those two pebbles in opposite direction along the line connecting the centres will then eliminate the overlap. It is possible that this will create overlap between those pebbles and some other pebbles. This process is repeated until there is no more overlap found between the pebbles [18].

## 2.2 Current research on pebble flow

Some research has been carried out specifically on the pebble flow of the HTR. The following is a concise description of the state of affairs of research as it was known at the beginning of the present study.

**University of Potchefstroom** At the University of Potchefstroom in South Africa, Professor C. G. du Toit made simulations of a stationary pebble bed. He used the DEM to generate the bed and then calculated the radial porosity. Using this method, he showed that wall channelling could not be neglected [4]. By comparing the experimental data and simulations it is possible to test the simulations and to get an approximation of the radial porosity. That can be used for other simulations [9, 19]. Thus far only studies on stationary beds have been published.

**MIT** At the MIT in the United States, the department of nuclear energy carries out research on a modular pebble bed reactor. Part of this research is pebble flow. For this purpose, experiments have been done using an experimental set-up of half of a reactor. By means of different coloured marbles in various spots in the reactor, the flow of the pebbles is determined at different places in the reactor. By using a model of a whole reactor the data of the half model is validated [3]. This data has been used in chapter 4 and 5.

MIT also uses computer simulations to test new core designs. The simulation uses the DEM and yields the same results as experiments for similar situations. Much more information can be gathered from the data accumulated by simulation because all information about all pebbles is known [10].

At MIT a new simulation method for particle flow also investigated, the Spot model. The advantage of this model is that less CPU time is needed in comparison with the DEM and the results are similar. The Spot model has not been applied to the pebble bed reactor yet [12].

**Jülich Research Centre** At Jülich Research Centre in Germany the PAPA code, which is written by the University of Stuttgart, is used. PAPA is a $c^{++}$ code and it uses the DEM. This code is not written specifically for pebble flow in nuclear reactors.

Originally one of the objectives of this thesis was to compare the results in this thesis with the PAPA code. Unfortunately no comparison has been made because the PAPA code contained too much bugs to get useful results [11].

## 2.3 Conclusion

For this research it has been decided to use the method of the lowest possible position per layer to create a static packing of the reactor [17]. This method has been chosen, because the results are reasonably, according to the reference and it is possible to program it in a reasonable amount of time. The pebble flow will be modelled with a new method, which will be applied to this static packing. The general idea behind the method is that a pebble will move to the lowest stable position. More information about this method can be found in chapter 4.

# Chapter 3

# Static packing of pebbles in a HTR

In this chapter, the method used to model the static packing of the pebble bed reactor is discussed. As mentioned in chapter 2, the method of the lowest possible position per layer is used. A few adjustments have been made to the method to speed up the filling process. These changes result in a less dense packing, which can be corrected by applying the method for pebble flow on the static packing while the pebbles at the bottom of the reactor stay in place. In this manner, the pebbles move closer together and the packing will be denser. The flow chart of the program can be found in figure 3.7 and in appendix C.

## 3.1 Method used

The model uses a grid of points with a specified distance between them, on which the pebbles can be placed. Because the reactor is modelled with a funnel, the size of the grid changes the first layers of the model. A small example of the grid can be seen in figure 3.1.

For the first layer, the grid is made with a diameter equal to the diameter of the funnel exit. A point on this grid is randomly chosen and stored; a pebble is placed with this point as its centre. Next, a list is generated with possible positions, on the grid, for the next pebble, i.e. positions at the same height, that do not overlap with the pebble just placed. From this list, a position is randomly chosen and stored. The list with possible positions is up-dated; positions on which pebbles would overlap with the newly placed pebble, are removed. In this manner, the first layer is filled until the list with possible positions is empty.

When there are no possible positions left on the grid, it moves up with a specified step. While in the funnel, the grid may be expanded to fit the diameter of the funnel at the given height. Because the step sizes in height and in radius are not necessarily the same, the grid does not have to be changed every time the grid is moved up. This is dependent on the angle of the funnel. If the diameter of the funnel at the given height

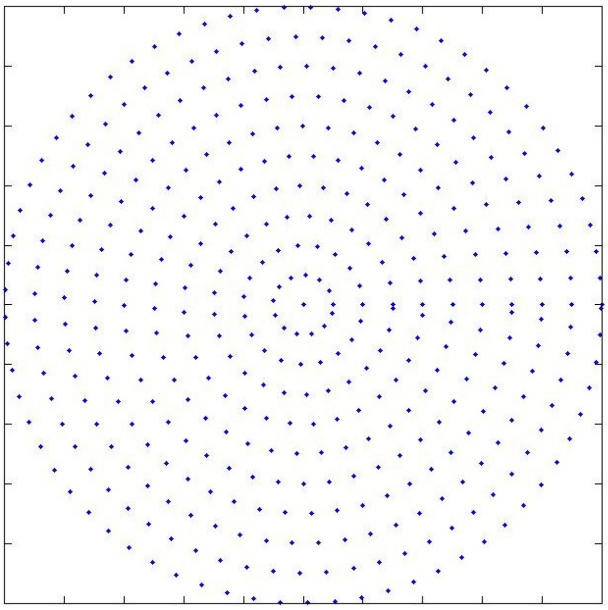

Figure 3.1: A small section of the grid used to generate the static packing.

is larger than the diameter of the grid plus two times the radial step size, the grid is expanded. On this height, a new list of possible positions is generated, comparing the grid points to the pebbles placed at lower positions. If there are no possible positions at the new height, the grid is moved up until there are positions available. After the first layer of the reactor is filled, the grid will have to be moved up a few times before a new pebble can be placed because the bottom of the reactor is filled. All those pebbles are at the same height and it is not possible for a pebble to be placed in-between. After a few layers of pebbles, at most new heights, it will be possible to place a new pebble, because the pebbles underneath are not all placed at the exact same height. This process is repeated until a given amount of pebbles is placed in the reactor.

## 3.2 Results

The result from filling the reactor with ten thousand pebbles, can be seen in figure 3.2. Here the reactor has a diameter of 100 cm, and the pebbles have a diameter of 6 cm. The funnel has an angle of $\frac{\pi}{6}$.

### 3.2.1 Cross-sections

The generated packing is verified by horizontal cross-sections at different heights of the reactor, see figure 3.3. With these cross-sections, it was confirmed that no pebbles overlap. The void fraction on a given height also follows from the cross-section. The void fraction, is the volume where no pebble is located, the void, divided by the total

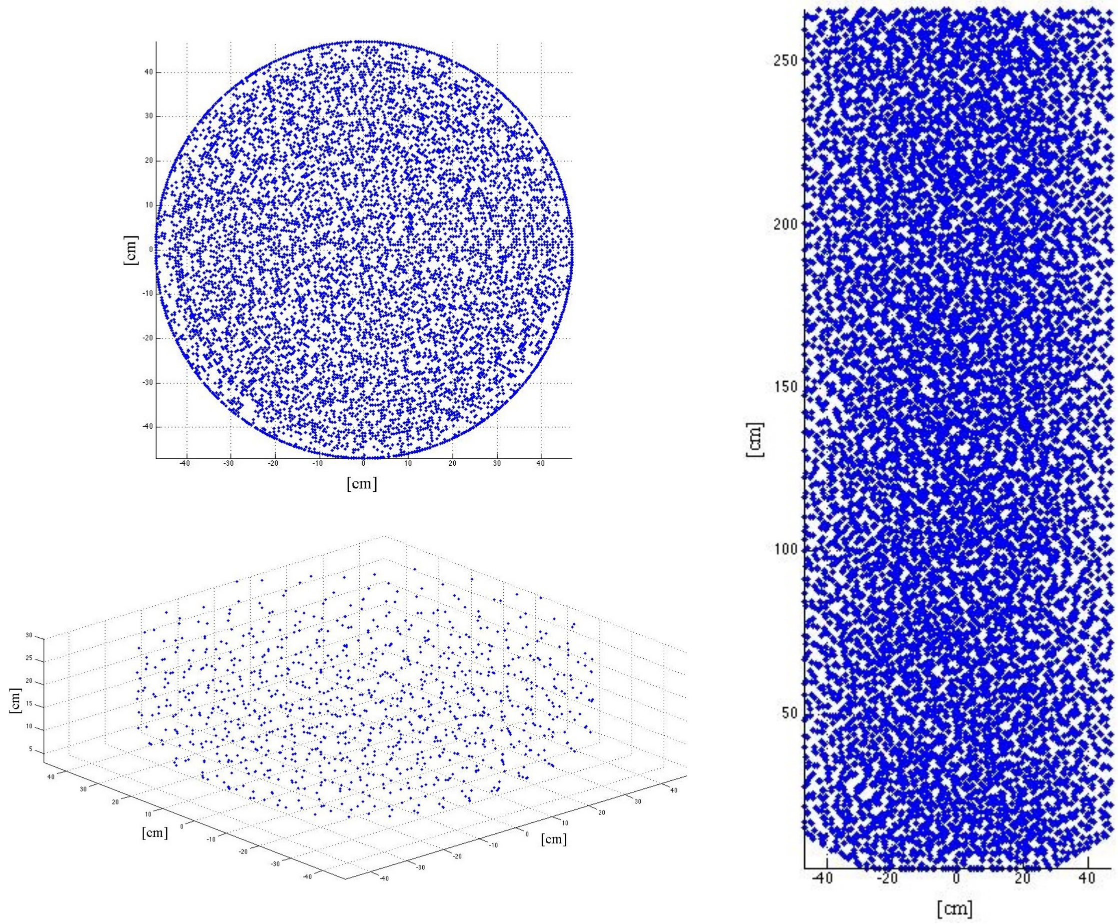


Figure 3.2: A generated static filling for a reactor. Top-left; the top view. Bottom-left; the funnel of the reactor. Right; side view. Each dot represents the centre of a pebble with a diameter of 6 cm.

Table 3.1: The influence of the density of the grid on the void fraction.

| distance between grid points | void fraction | relative calculation time |
|---|---|---|
| r | 0.76 | 0.0086 |
| $\frac{r}{5}$ | 0.48 | 0.14 |
| $\frac{r}{10}$ | 0.44 | 1 |
| $\frac{r}{20}$ | 0.42 | 118 |

volume of the reactor. In the case of the cross-section, it is the surface where no pebble is located, divided by the total surface of the reactor at the given height. The void fraction of the different cross-sections for the tested configuration, was found to be between 0.44 and 0.45, at the different heights of the reactor.

### 3.2.2 Overall void fraction

The theoretical average void fraction for randomly filled reactors is given by [20]

$$\epsilon = 0.375 + 0.34\frac{d_k}{D} \tag{3.1}$$

where $d_k$ is the pebble diameter and $D$ the diameter of the reactor. The range is not given for this formula. It can however be reasoned it is not valid for all ratio's of $\frac{d_k}{D}$. In the extreme case when $\frac{d_k}{D} = 1$, the void fraction is equal to 0.48, but formula 3.1 gives a void fraction of 0.71. When the void fraction of this extreme case, is taken as the maximal possible void fraction, it follows, the maximum ratio for which formula 3.1 can give a reasonable outcome is $\frac{d_k}{D} = 0.30$.

In the case that is shown in figure 3.2, $\frac{d_k}{D} = 0.06$ and the theoretical average void fraction is 0.3954, which is smaller than the void fraction found. The grid used causes this; the pebbles are not placed on the lowest possible position, but on the lowest available grid point. Pebbles are not necessarily in contact with each other. Using a less dense grid can show this. When a less dense grid is used, the void fraction should be higher and this should be visible in the cross-section. When a grid is used which contains half the grid points and a height step, twice the height step of the previous example, this effect is clearly visible. The void fraction at the different cross-sections is now between 0.48 and 0.49, which is also visible in figure 3.4. Note that the void fraction is now higher than the maximal possible void fraction. This is because the void fraction given in formula 3.1, gives the void fraction for a static packed pebble bed. In the configuration tested here, the pebbles are not in contact with each other, it is a physically impossible configuration. Therefore, the void fraction can be higher than the physical maximal. The influence of the grid and the distance between the pebbles can be seen in table 3.1.

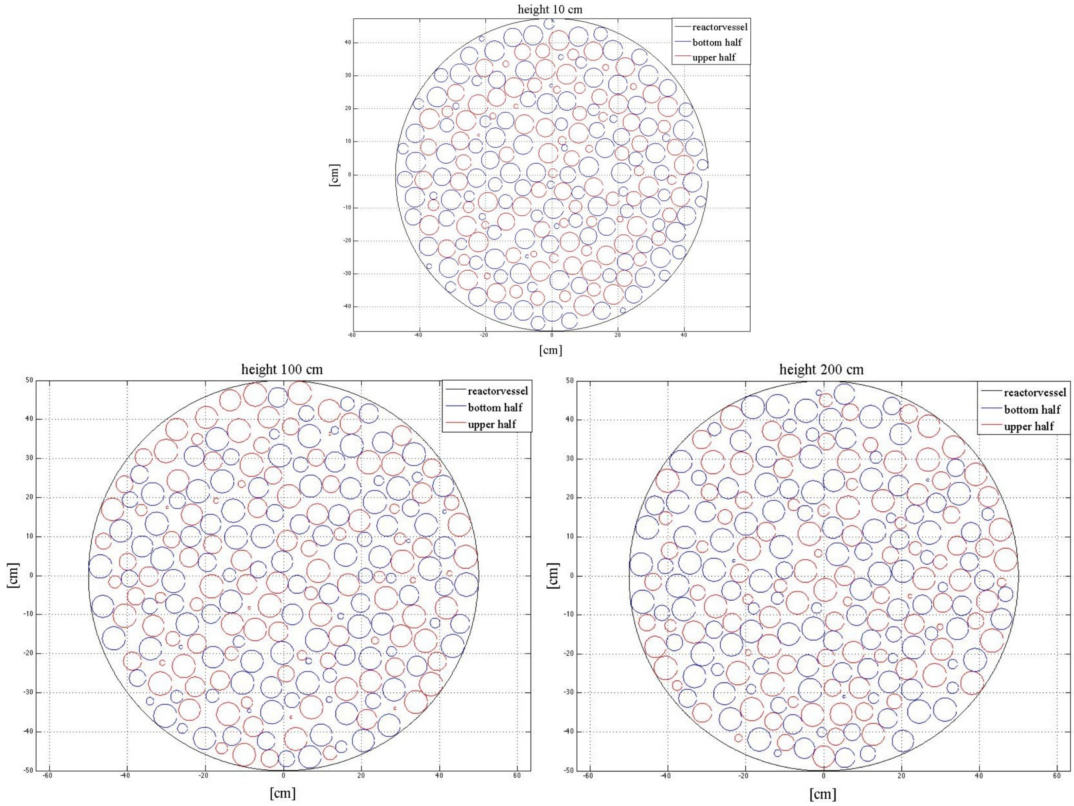


Figure 3.3: Cross-sections at different heights of the filling as shown in figure 3.2. The black circle shows the location of the reactor wall, the cross-sections of the pebbles are blue if it goes through the bottom half of the pebble and red if it goes through the upper half.

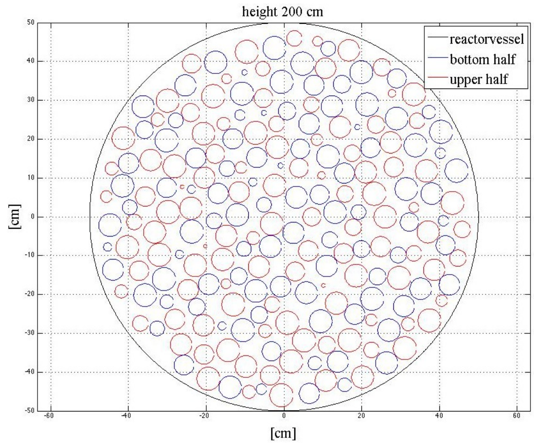


Figure 3.4: Cross-section with a less dense grid.

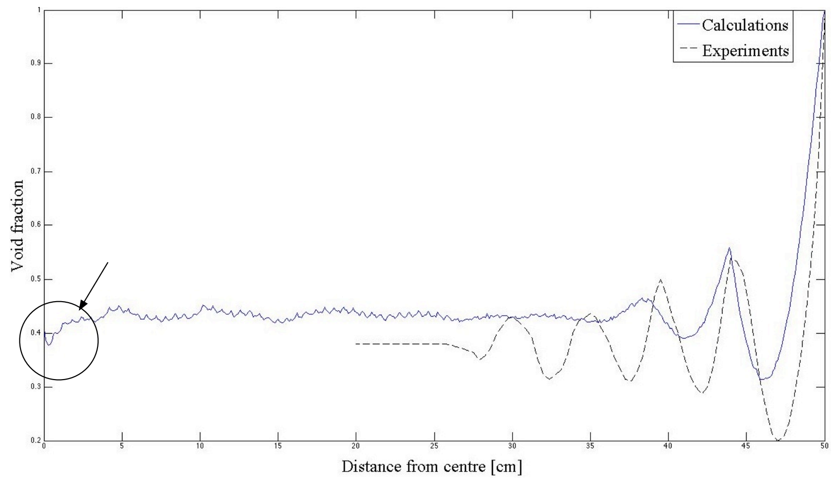


Figure 3.5: The radial void fraction of a static reactor, including experimental results with $\frac{d_k}{D} = \infty$ (extracted from [21]). The arrow highlights the dip found in the radial void fraction in the centre of the reactor.

### 3.2.3 Radial void fraction

The previously mentioned void fraction, is the average void fraction at a given height. The radial void fraction is also of interest. For the packing shown in figure 3.2, the radial void fraction is calculated by averaging the void fraction at points with the same radius at different angles. The pebbles in the funnel, are not taken into account because that would interfere with the effect of the reactor wall. The results are shown in figure 3.5 with results from experiments as a comparison [21]. Both graphs have a similar shape, they show the effect from wall channelling. The overall void fraction of the reactor is 0.45, higher than the theoretical overall void fraction. This is also visible in figure 3.5 where the experimental results are below the results of the calculations.

Besides the effects of wall channelling at the side of the reactor, another minimum is observed and highlighted in figure 3.5, at the centre of the reactor core. When the same graph is made for other random realizations of the reactor core, this point shows a high or a low value. The packing shows a different kind of behaviour there; which is caused by the way figure 3.5 is generated. Each point is calculated by taking the average of the void fraction, at different angles, at the same radial distance from the centre. At the centre of the reactor, it is not possible to average the value and thus you get the void fraction at that point, which might be higher or lower than the average void fraction. This gives a large statistical error. In figure 3.6 it can be seen, the centre of the core is not extraordinary. The figure shows the void fraction as a function of the position in the reactor. The value at the centre of the reactor does not differ from other points in the reactor. In figure 3.6 the effect of wall channelling is also visible, it are the darker rings close to the outer reflector. It is more visible than in figure 3.5, in figure 3.6 around five

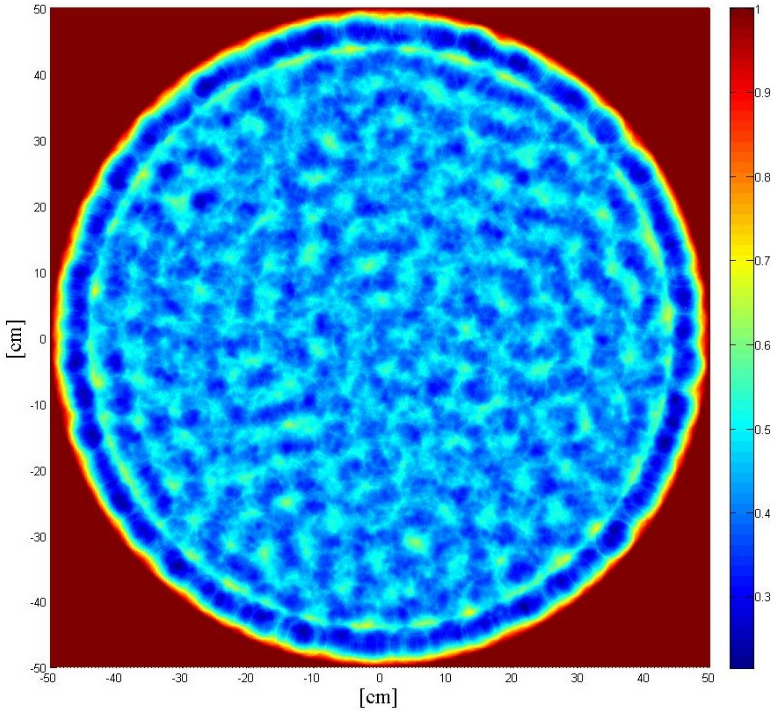


Figure 3.6: The void fraction of a static reactor.

rings are visible, while figure 3.5 only shows three fluctuations. Experiments also show that the effect of wall-channelling is visible for five-pebble diameters [21].

### 3.2.4 Improvement void fraction

The overall void fraction in the reactor core can be improved by applying the pebble-flow model without removing pebbles. The pebble-flow model is described in chapter 4. When this model is applied, the pebbles will be placed at stable positions touching at least two other pebbles or the bottom reflector of the reactor. This decreases the void fraction because the pebbles are now in contact with each other. When this is done on the previously mentioned configuration, the overall void fraction in the cylinder above the funnel, reduces to 0.44, which is still higher than the theoretical void fraction. It is however not beneficial to apply the pebble-flow model multiple times. After the first time, the pebbles are already touching each other and because of that the model will not improve the void fraction anymore.

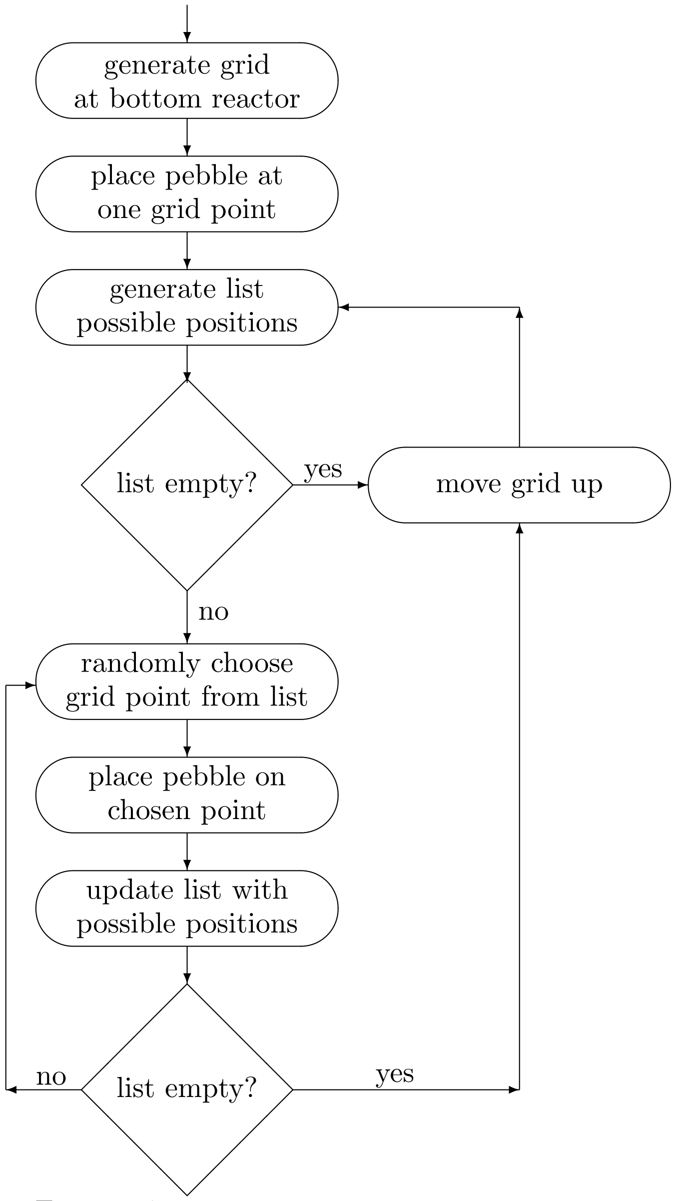

Figure 3.7: The flowchart for the static packing.

# Chapter 4

# Dynamic packing of pebbles in a HTR

For the dynamics of the pebble bed, a new method has been chosen. Although the method does not involve the real physics of the dynamics of the pebble bed, it costs less CPU time and should give an outcome comparable to the DEM.

The general idea behind the method is, that a pebble will move to the lowest stable position available in its neighbourhood. This can be at the bottom of the reactor, on top of three other pebbles or on top of two pebbles and against the reactor wall. A pebble moves to this position instantly and this position will be used to determine the stable position for the next pebble, if necessary. This way, all pebbles are moved to a stable position. Then the lowest pebbles in the reactor will be removed and the pebbles are moved to their new lowest stable position. Because the pebbles are moved instantly, they do not have a real velocity. If the velocity of the pebbles is desired, the distance the pebble has been moved, should be divided by the time needed in reality, to remove the amount of pebbles that have been removed before the pebble-flow model was applied.

## 4.1 Method used

The matrix with pebble positions is ordered according to the height of the centre of the pebbles, from bottom to top and randomly in the other directions. The pebbles are selected to be moved according to the order in which they are in the matrix. That means that the pebbles at the bottom of the reactor, are considered to be moved first. For clarity, the selected pebble is called pebble M from now on. If pebble M is not already at the bottom of the reactor (which means that the height of the centre of pebble M is higher than the radius of the pebble), a stable position, pebbles in the neighbourhood of pebble M are sought. This means all pebbles that have their centre in a cylinder around the selected pebble are highlighted. Only pebbles that have already been moved to a stable position themselves, are included in the search, because than the pebble will also remain in a stable position after the other calculation steps. This is allowed because the pebbles are ordered from bottom to top, this way all pebbles at

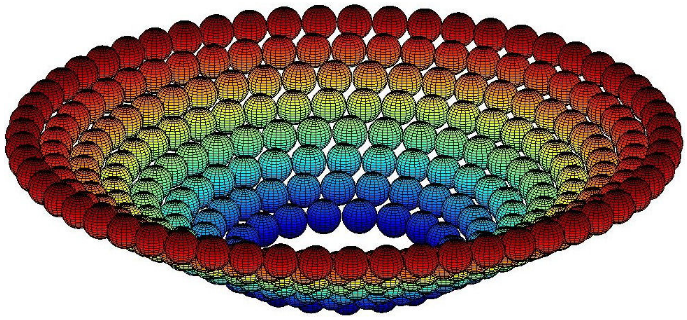

Figure 4.1: A funnel made by pebbles. The colour displays the height of the pebble in the reactor.

lower positions are included. Pebble M is then moved to a stable or lower position in the reactor. The possibilities from the new position, depend on how much pebbles are found in the neighbourhood of pebble M. The possible actions in the different cases will be discussed next. The flow chart for the model can be found in figure 4.13 and the formula's used to program the situations described next are placed in appendix D.

### 4.1.1 Funnel

When the reactor contains a funnel, there is interaction between the pebbles and the funnel. Because it proved to be difficult to solve this interaction analytically, an approximation has been used. The approximation utilises a funnel made of pebbles. An earlier approximation, used the height of pebble M and the pebbles found, to estimate the diameter of the funnel at the final position of pebble M. Because this resulted in strange funnel shapes and pebbles moving upward, instead of downward, the new method has been adopted.

Before the pebbles are moved, the funnel is generated. An example of such a funnel can be found in figure 4.1. The pebbles that build up the funnel are never moved, but are always taken into account when pebble M is nearby, also if they are above pebble M. This funnel approximates the effect of a real funnel, this approximation is allowed because friction is not taken into account in the pebble-flow model. If friction was taken into account, a funnel made of pebbles would give a different result than a real funnel because the contact area and material is different.

The overall shape of the pebbles in the funnel, is the same as in a real funnel. An individual pebble may however, be a little bit outside the real funnel. There might also be pebbles in positions that are not really stable, for example when in contact with two pebbles from the funnel and one of the real pebbles. In reality this means, the pebble is in contact with one pebble and the funnel, which is not a stable position.

### 4.1.2 Movements

If there are pebbles in the neighbourhood of pebble M, but all at the bottom of the reactor, it is possible there is a stable position independent of the pebbles found. This would mean that pebble M is placed at the bottom of the reactor, not overlapping any pebble already at the bottom. To achieve this, a similar grid as used to generate the static filling of the reactor, is used. The grid has the same diameter as the cylinder in which the pebbles found are located and is centred around pebble M. A grid point, which does not overlap with one of the pebbles placed, is sought. If there is such a grid point, there is a stable position at the bottom of the reactor and pebble M is placed there.

If there is no free space at the bottom of the reactor, it is possible that the new position of pebble M, depending on the pebbles found, overlaps with pebbles already placed at the bottom of the reactor. This is caused by the fact that a pebble at the bottom of the reactor is always on a stable position. When pebbles are placed in the core on a position that is not considered to be stable, for example next to one pebble, they might overlap with other pebbles in this temporary position. Because pebbles are supposed to be moved away from this unstable position before any other pebble is moved it does not matter if there is a small overlap. If the temporary position is at the bottom of the reactor, however, this is considered to be a stable position and the pebble will not be moved again. Preventing pebbles to be placed on temporary positions if all the pebbles found are at the bottom of the reactor eliminates this situation. The pebble will always be placed on top of the pebbles found and not next to pebbles found.

The movements of the pebbles in other situations are discussed next. The movement chosen, depends on the amount of pebbles found near pebble M. The description of the movements is ordered by this amount.

**Zero pebbles** Pebble M is moved a distance, equal to its radius downward. After pebble M has been moved, it is not on a stable position yet (unless it was moved onto the bottom of the reactor). The process of finding pebbles nearby, will be repeated for pebble M, until pebble M ends up in a stable position.

**One pebble** Pebble M is placed next to the pebble found. Pebble M is moved along the line connecting the centres of the two pebbles in the horizontal plane. Pebble M moves away from the centre of the pebble found, until the horizontal distance between the two pebbles is equal to the diameter of the pebbles, see figure 4.2. Pebble M is then moved down, to the same height as that of the pebble found. This way, it is simulated that the pebble rolls over the side of the pebble found.

When the pebble found is close to the side of the reactor, placing pebble M next to it, might mean pebble M will overlap with the wall. To prevent this from happening, pebble M is placed at a distance equal to the radius of the pebbles from the wall of the reactor. Pebble M will not pass the highest point of the other pebble, this decides the direction pebble M will be moved in, see figure 4.3.

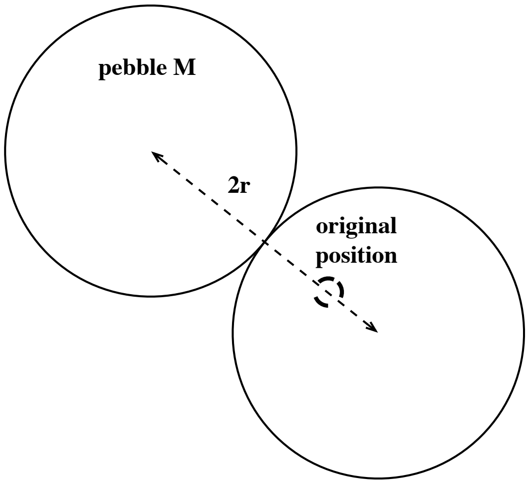


Figure 4.2: Top view of the final placement of pebble M when one pebble is near.

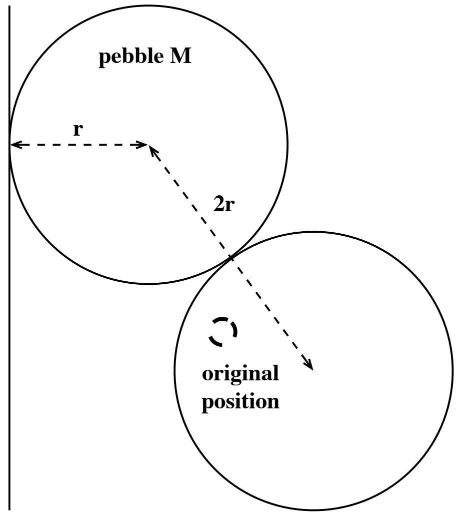


Figure 4.3: Top view of the final placement of pebble M near the wall of the reactor when one pebble is near.

When just one pebble is found, pebble M will not reach a stable position, unless it is placed at the bottom of the reactor. The process of finding pebbles nearby, will be repeated for pebble M until the pebble ends up in a stable position.

**Two pebbles** In this case it is possible that pebble M will get positioned on a stable position. This is the case when the pebbles are close to the reflector wall and pebble M is between the pebbles found and the wall. Pebble M will rest on the two pebbles and against the wall.

If the pebbles found and pebble M are not close to the wall, pebble M is moved to a location as low as possible. This is done by placing pebble M next to the highest pebble in the horizontal plane, like in the case when one pebble is found, but at the height of the lowest pebble. It is possible that the new position of pebble M, will overlap with the lowest pebble. In that case, pebble M has to be moved again, to eliminate the overlap. Pebble M is moved in the same direction it has already been moved, see figure 4.4 for the top view. This simulates the conservation of momentum, pebble M slides over both pebbles without changing direction. At its final position pebble M is touching the lower of the two pebbles found. Because this does not produce a stable position for pebble M, the process of finding pebbles nearby will be repeated for pebble M until a stable position is found.

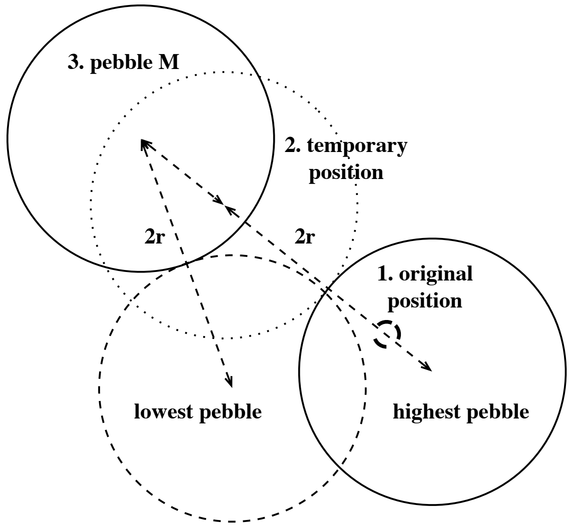


Figure 4.4: Top view of the final placement of pebble M when two pebbles are near and pebble M overlaps with the lowest pebble after the initial movement of the pebble.

**Three pebbles** In this case, pebble M will always end up at a stable position. Depending on the position of pebble M and the pebbles found, pebble M will be placed on top of the three pebbles or on the two highest pebbles and against the wall of the reactor, as described in the case when two pebbles are found.

**More than three pebbles** The same will happen as when three pebbles are found. To select the two or three support pebbles, the final position of pebble M is calculated, for all possible combinations. The lowest position for pebble M calculated, that does not overlap with any of the other pebbles, is selected as the lowest stable position for pebble M.

## 4.2 Generating flow lines

When the pebble-flow model is applied to a static packing, generated as discussed in chapter 3, it is possible to see the vertical displacement of the pebbles. The vertical displacement can be made visible in flow lines. To generate flow lines, the reactor has been split in small rings with equal height and width. For each pebble it is determined in which ring the original position is. The difference in height, between the original and the current position, is then calculated. If the pebble is no longer in the reactor, the current height is taken to be one pebble diameter below the bottom of the reactor. For each ring, the average vertical displacement is calculated. The overall flow lines show this average displacement projected from the left top of each ring.

To get a better estimate of the vertical displacement of the pebbles, an average can be taken over many displacements. Each time the pebble-flow model is applied, the vertical displacement in the different sections of the reactor, is recorded. The pebbles are labelled, with their current position, before the pebble-flow model is applied again. These labels are used as the reference to determine the vertical displacement. This vertical displacement is averaged over the total number of recordings to generate the average flow lines.

## 4.3 Removal of pebbles

To initiate the movement in the core, pebbles are removed from the bottom. The amount of pebbles in each batch that is removed before the code is applied, can influence the flow lines. With a large batch size, the calculation will obviously use less CPU time. A cylinder has been generated with a diameter of 18.03 pebble diameters. The cylinder does not have a funnel and is filled with 5000 pebbles. These characteristics have been chosen, so the results can be compared to experimental results that are currently being generated at TU Delft. Because the pebble-flow model does not include friction, the flow in the cylinder should be constant over the diameter of the core. Three ways are used to find the best way to remove the pebbles, every time 500 (10 %) pebbles are removed. The vertical movement of the pebbles has been determined without adding new pebbles, because the pebbles are moved separately and only pebbles that are below pebble M are taken into account. First the batch size is one pebble, then the batch size is ten pebbles. Finally, each time, all the pebbles on the bottom of the core are in a batch. By removing all the pebbles from the bottom it is unlikely that exactly 500 pebbles will be removed. The pebbles removed in each batch depend on the amount of pebbles on the bottom of the reactor. In this case at least 500 pebbles are removed.

The overall flow lines of the tests presented, can be seen in figure 4.5. The graphs show that, the batch size removed, has a large influence on the flow lines. The left figure shows a lot of variation and some flow lines rise above their original positions. This means that pebbles have been moved upward instead of downward. This is because the pebbles are moved one at a time. When one pebble is removed, one of the other pebbles will take its place. This does not have to be at the exact same place the original pebble was positioned. If the pebble is placed on a different position, it might overlap with a pebble that is not moved jet. This pebble will then have to be moved up a little bit. When more pebbles are removed at a time, there is more space for the pebbles to move down and it is less likely pebbles will have to be moved up to eliminate overlap. It is as if the core is tilted a little bit.

As explained before, the expected result is a constant vertical displacement over the diameter of the core. The best result can be seen in the figure where the pebbles are removed by layer. Here the lines are, apart from some minor fluctuations, horizontal. When the batch size is ten pebbles, the flow lines also look acceptable, they are reasonably constant.

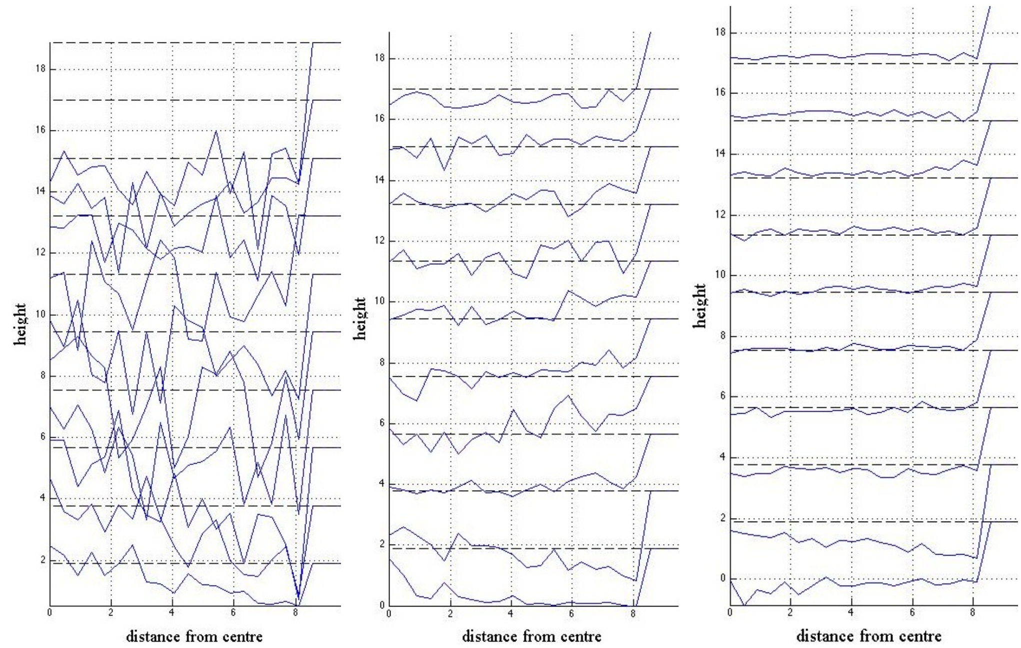


Figure 4.5: The vertical displacements of pebbles in a cylinder after 500 pebbles have been removed. On the left; pebbles are removed one at a time. In the middle; pebbles are removed ten at a time. On the right; pebbles are removed by layer, in this case 648 pebbles were removed.

The figure clearly shows that, removing one pebble at a time, does not give the best results. Removing the pebbles by layer gives much better results. This is more like the situation in a real cylinder, here pebbles that are at the same height will also leave the reactor at the same time.

In further calculations the pebbles will be removed by layer. However, it has to be noted that this test was executed on a reactor without funnel. This means that each layer has the diameter of the cylinder and a batch contains around 150 pebbles. When there is a funnel in the reactor, the diameter of the bottom plane of the reactor, where the layers are removed, is a lot smaller. This means the CPU time needed depends on the exit diameter of the funnel.

## 4.4 Results

### 4.4.1 Vertical displacement

#### Cylinder

To calculate the average flow lines, the cylinder from the previous example has had one complete pass through, all 5000 pebbles have been removed and new pebbles have been added to keep the total amount of pebbles in the reactor at 5000. Each new pebble is placed above the highest pebble in the reactor, at the horizontal position of the pebble that has just been removed. By adding the pebbles on these positions, they do not overlap, which would be likely with randomly chosen positions because the number of pebbles added is enough to fill the entire cross section of the reactor at the give height.

The result of this complete pass through can be seen in figure 4.6. The flow lines in this figure are smoother than in the right graph of figure 4.5. This is because in figure 4.6 averages are shown and the figure shows the result of a complete pass through.

Figure 4.6 shows the flow lines relative to the distance from the centre. This means, only half of the reactor is shown in the graph. The lowest line however, does look symmetric. The small peak, high lighted in graph 4.6, is probably caused by the same effect seen in, the highlighted part of, figure 3.5. The flow lines are averaged over rings with the same width. There is a large statistical error at the centre, because there are less pebbles in the centre disc than in the other rings. Some times, there was no pebble in this region, which decreases the average.

### Reactor with funnel

Of interest to a real reactor design, is also the vertical displacement in a cylinder with a funnel. As explained, this case will take more calculation steps, because there are less pebbles at the bottom of the reactor. Less pebbles are replaced before the pebble-flow model is applied. For this reason, only 500 pebbles have been removed from the reactor with funnel. The funnel has an angle of $\frac{\pi}{6}$ and the exit diameter of the funnel is 5 pebble diameters. In that case, there are around 8 pebbles at the bottom layer.

The result can be seen in figure 4.7. Because less pebbles have been removed, the average vertical displacement of the pebbles is smaller. To improve the graph the values of the average vertical displacements have been multiplied by ten. This makes the vertical displacements comparable with figure 4.6.

The most remarkable points figure 4.7 are at the side, by the pebbles touching the wall of the reactor. Close to the funnel edge, the flow lines near the wall of the reactor are steep. The pebbles move into the funnel. At the top of the reactor, there are positions where, on average, the pebbles move upward. It is expected that pebbles at the centre of the core move down faster than the pebbles near the outer reflector. Close to the funnel and on the side of the funnel, pebbles near the side of the reactor need to move in the radial direction to the funnel exit before they can move down. This radial movement might cause the upward movement, although that movement is at the bottom of the reactor. The best indication that this upward flow near the edge of the reactor is caused by the funnel, is the fact that no upward flow is visible in figure 4.6, where no funnel is present.

Another interesting observation from figure 4.7, is the top flow line. This flow line shows the same characteristics as the other flow lines, but it seems to be less wide, there is a dip, highlighted in figure 4.7. The new pebbles added to the reactor can explain this. To prevent overlap in the initial positions of the pebbles, they are placed at the

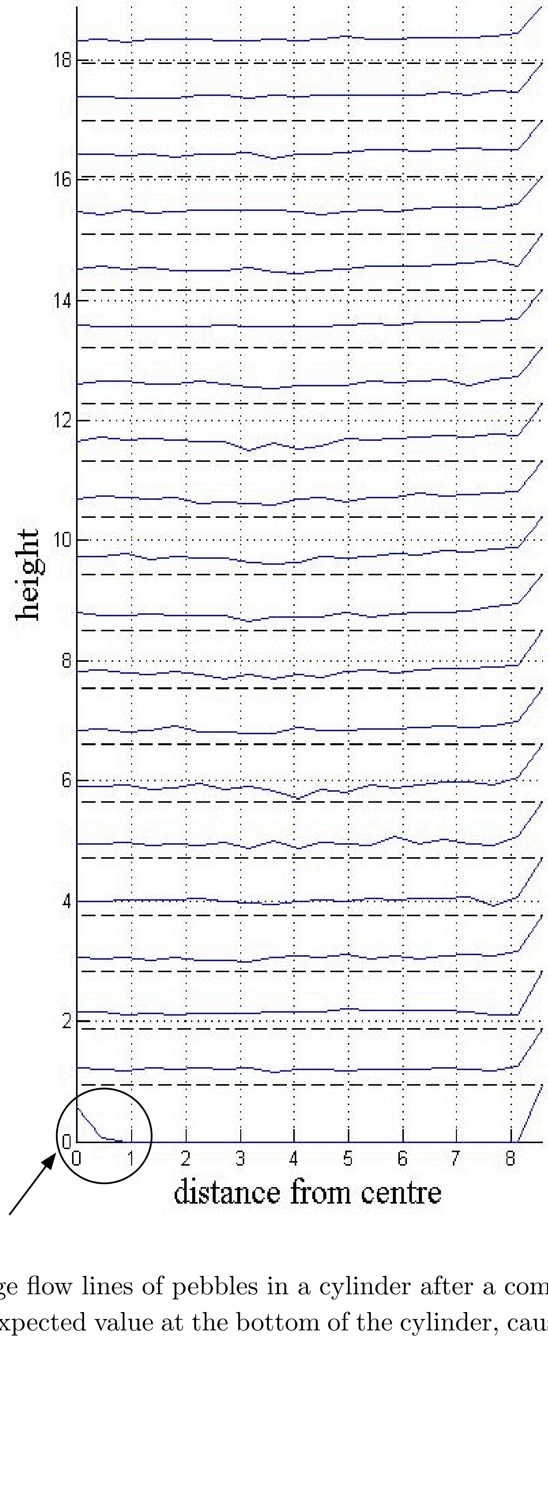


Figure 4.6: The average flow lines of pebbles in a cylinder after a complete pass through. The arrow highlights the unexpected value at the bottom of the cylinder, caused by a statistical error.

same horizontal positions as the pebbles removed. This is especially important for the pebbles added in the cylinder, because in that case, there are exactly enough pebbles to fill the diameter of the cylinder. In the case of the core with a funnel, the pebbles added are in a smaller diameter than the top of the reactor. The top layer of the reactor, has less or no pebbles near the outer reflector. The vertical displacement of each time step is averaged over the amount of pebbles in that area. To get the average flow lines, all the vertical displacements are summed and divided by the number of times the pebbles have been moved down. If, at some of the calculation steps, no pebbles originated from those rings, the final average vertical displacement will be influenced. The fact that this is only visible in the top layer of the reactor, shows that filling the reactor in this way, forms no pile of pebbles. The effect shown can be removed by adding the new pebbles in a different way. This can be done by adding the pebbles at random horizontal positions or at certain predetermined positions, that are more spread over the reactor. This last possibility will be discussed in more detail in chapter 5.

#### Comparison with experiments

To validate the results, they are compared with experiments performed by MIT [3]. The results of the experiments are shown figure 4.8. This graph shows a vertical cross section of a reactor, the colours represent the individual pebbles that are followed down in the experiment. The results from the computations only show a half cross section of a reactor. Figure 4.8 shows horizontal flow lines, until half the height of the reactor. After this, the flow lines are curved towards the centre of the reactor. The same behaviour is visible in figure 4.7. The flow lines are horizontal on average, but close to the funnel, they bend toward the centre of the reactor. This validates the results for the pebble-flow model with a funnel.

### 4.4.2 Radial void fraction

The radial void fraction, after the pebble-flow model has been applied to the cylinder and reactor described before, can be found in figures 4.9 and 4.10, respectively. Both figures do not show the radial void fraction that is known to belong to the static packing of a pebble bed reactor and can be seen in figure 3.5.

#### Cylinder

Figure 4.9 has a peculiar shape. There is a large bump in the middle of the graph. To get a better understanding of what causes the bump, the two-dimensional void fraction has been calculated, see figure 4.11. The figure shows there are four spots in the core where the void fraction is much larger than average. Because these spots all have about the same radial coordinate, it shows up at the radial void fraction plot. The higher void fraction is caused by large cavities in the pebble bed.

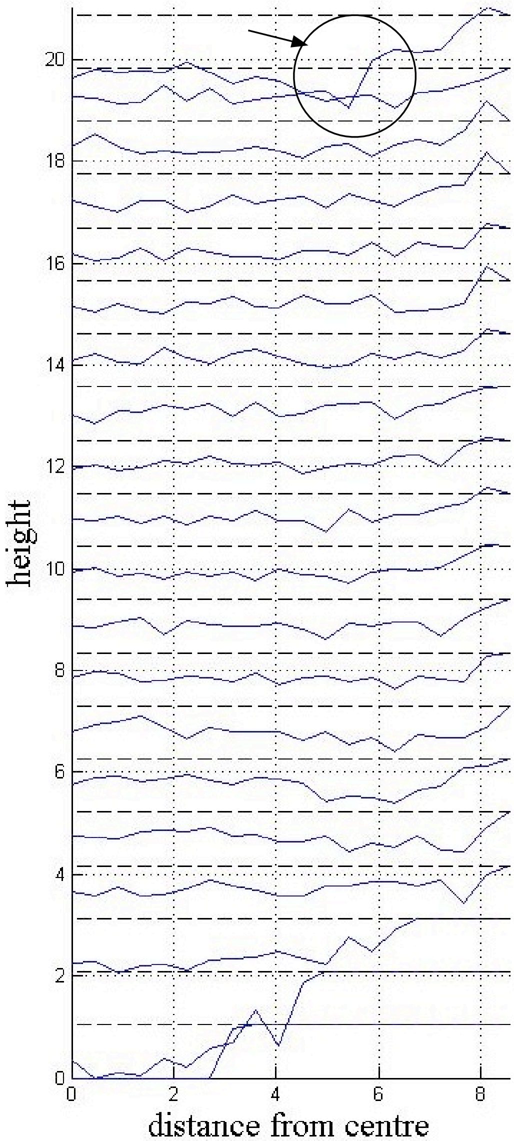


Figure 4.7: The average flow lines of pebbles in a reactor with funnel after 500 pebbles have been removed and added. The values are multiplied by ten to show more detail. The arrow highlights the influence of the way the pebbles are added to the reactor core.

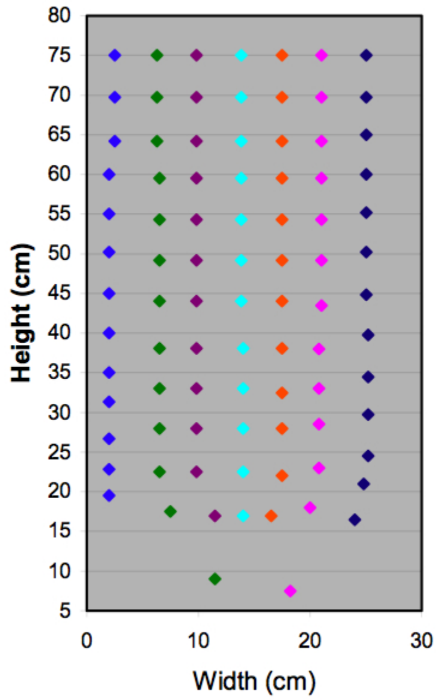


Figure 4.8: Experimental results [3]. Note that the results of the computational model in figure 4.7 show the distance from the centre and these experimental results show the full width of the reactor. Each colour indicates an individual pebble that has been tracked in the experiment.

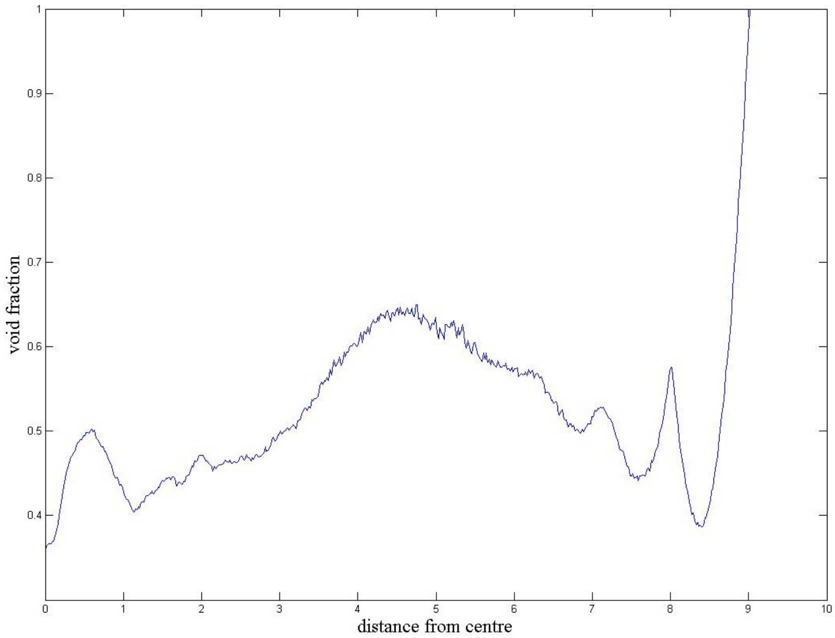


Figure 4.9: The radial void fraction of the cylinder as shown in figure 4.6.

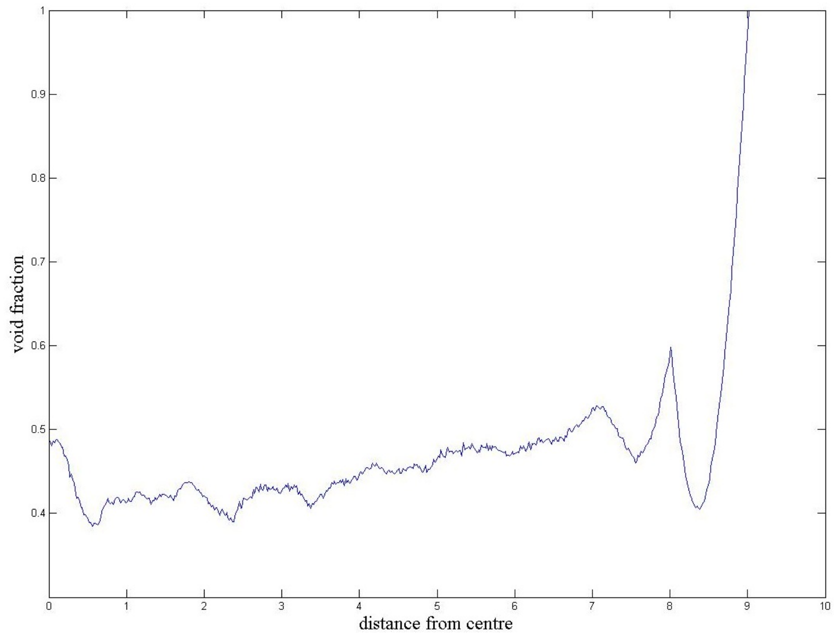


Figure 4.10: The radial void fraction of the cylinder as shown in figure 4.7.

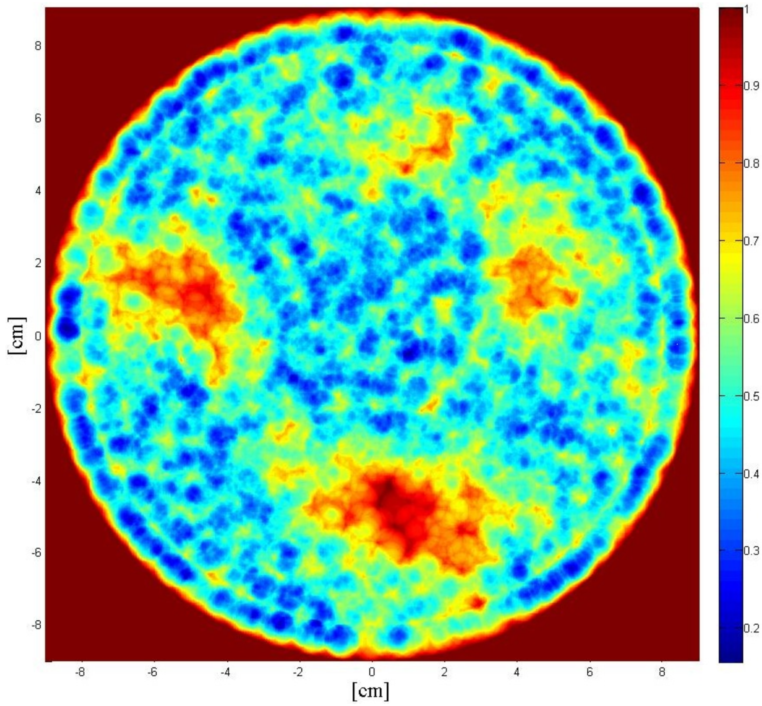


Figure 4.11: The two dimensional radial void fraction of the cylinder as shown in figure 4.6.

### Bridging

A similar effect is known to exist in pebble bed reactors at the exit of the funnel. If the exit diameter is too small, compared to the pebble diameter, bridging may occur. This means, the pebble flow is blocked because several pebbles form a stable curved structure. Empirically it has been found that this happens when the exit diameter is smaller than five pebble diameters. The ratio after which bridging is no longer expected is $\frac{50}{6}$ [22].

### Cavities

Because bridging is an effect caused by friction, bridging itself will not occur when the pebble-flow model is applied. This is also because the lowest pebbles are always removed in the model, they cannot block the movement. The pebbles however, move to the lowest possible stable position. Because a lot of pebbles are removed each batch, pebbles have to move down a significant distance. While moving down, the pebble will stay at the lowest stable position found. If there is a small pile of pebbles, the pebble will be placed at a low position on the outside of that pile. However, in this case this is probably not the lowest possible position because the pebble could also be placed in a stable position at the bottom of the pile of pebbles. There is a maximum distance the pebble can be moved down each calculation step. If, for example, pebble A reaches a stable position at this maximum distance after a calculation step, the next calculation step is to move pebble B. In that case, pebble A is considered to be at its lowest stable position. If another calculation step had been performed for pebble A, a lower stable position would be found if pebble A is on the edge of a cavity. Because of this, when a cavity happens to arise, it will be maintained and may even grow. The cavities shown in this random packing of a cylinder did not originate at the bottom of the reactor. Most cavities were not visible at the lower part of the cylinder.

Although the cavities are relatively large, they do not show up in the flow lines of the cylinder, in figure 4.6. This indicates that the cavities move. The flow lines are an average over multiple batches. The cavities do show up in the radial void fraction and the two-dimensional void fraction because those figures show a snap shot and not an average. Because of the way the cavities are formed and maintained, it is most likely the cavities move down. In a real reactor, cavities would move up, as described and approximated in the spot model [12].

There is no experimental data available to determine whether cavities occur in a pebble bed reactor or not. It seems unlikely that cavities, of the size found, could be maintained in a pebble bed reactor while the pebbles move. In the case of bridging, the pebbles rest on the funnel exit, which is stationary. In the case of cavities, the pebbles rest on other pebbles, that are also moving. This makes it unlikely for cavities to occur in a pebble bed.

Using batches with fewer pebbles can prevent the occurrence of cavities. This has been confirmed by taking a close look at the packing of figure 4.5, where 500 pebbles are removed and no new pebbles added. The situation where the pebbles are removed by layer, shows some small cavities. In the situation where the batch size is ten, no similar sized cavities are visible. Another confirmation that removing less pebbles between the calculation steps can prevent cavities, is that the cavities are not seen in the reactor with funnel.

It should be noted that cavities could also occur when the exit diameter of the funnel is large, compared to the pebble diameter. A large amount of pebbles can be removed in that situation as well. An optimum batch size should be sought when cavities occur. The situation where the batch size is ten pebbles in figure 4.5 also shows relative smooth flow lines. This shows that it is also possible to remove less pebbles than the complete layer, to get a reasonable result.

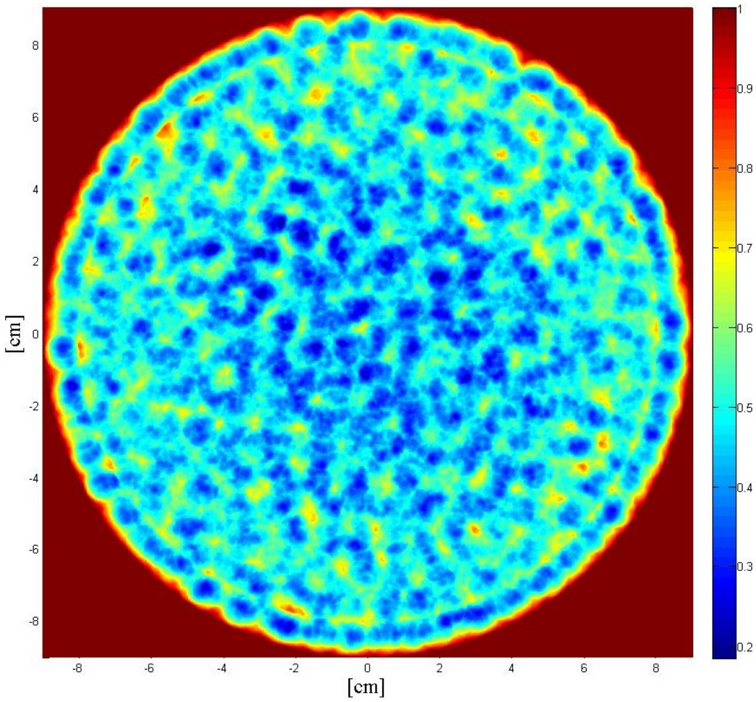


Figure 4.12: The two-dimensional radial void fraction of the reactor as shown in figure 4.7.

**Reactor with funnel**

The radial void fraction in figure 4.10 shows, besides the expected oscillations, a slope. The centre of the reactor, is denser than the outer region. To make sure this is not caused by a similar effect as cavities, the two-dimensional void fraction is also calculated for this situation, see figure 4.12. The figure shows that the reactor is indeed denser at the centre of the reactor. Also compared to the static packing of the reactor, as shown in figure 3.6. The outer region of the reactor is less dense. This is probably caused by both the effect of the funnel on the flow and because of the location where pebbles are added. Although the flow lines show that the pebbles added are spread homogeneously over the whole diameter of the reactor quickly, they are visible in the void fraction plot. This effect is enhanced by the removal of the pebbles at the centre of the reactor. Pebbles at the centre of the reactor always have the possibility to move down and in that way apparently generate a close packing. Pebbles near the outer reflector need to move sideways, when they get near the funnel, to be able to move down. This might cause the packing to be less dense in this region.

## 4.5 Alternative static packing

Besides using the pebble-flow model after the static packing, it is also possible to fill an empty reactor with the model. The pebbles will fall, one at a time, from the top of the reactor. To do this, a pebble is placed at a random position on top of the reactor and the flow is applied to the whole reactor, just one pebble in this case. The pebble will fall down and reach the bottom of the reactor. Then, a new pebble is placed on top of the reactor and the flow is applied again. This does not affect the first pebble because it is already at the bottom of the reactor, but the other pebble falls down as well. This has the advantage that it is more realistic, a pebble is first placed in a stationary position, before a new pebble is placed in the reactor. It is possible that on one side of the reactor there is already a small pile of pebbles, while there are still positions on the bottom of the reactor that can be filled by other pebbles. Because the flow is applied every time a new pebble is added, the pebbles already placed are also more dynamic. It is possible, that a pebble is placed at the lowest stable position from its original position, but the next step it might be able to slide down to a lower stable position that was too far away from the original position.

The disadvantage of the method is that there is a large chance to create a compact and repetitive filling instead of a random one. This is because, when a pebble starts above one or multiple pebbles, it will be placed touching at least one of the other pebbles, which will create patterns.

Flow applied in this way is more sensitive to errors, because of the different starting positions and situations might occur that are not covered in the pebble-flow model. The large calculation time involved and these disadvantages did not allow for enough filling of the reactor to generate meaningful results.

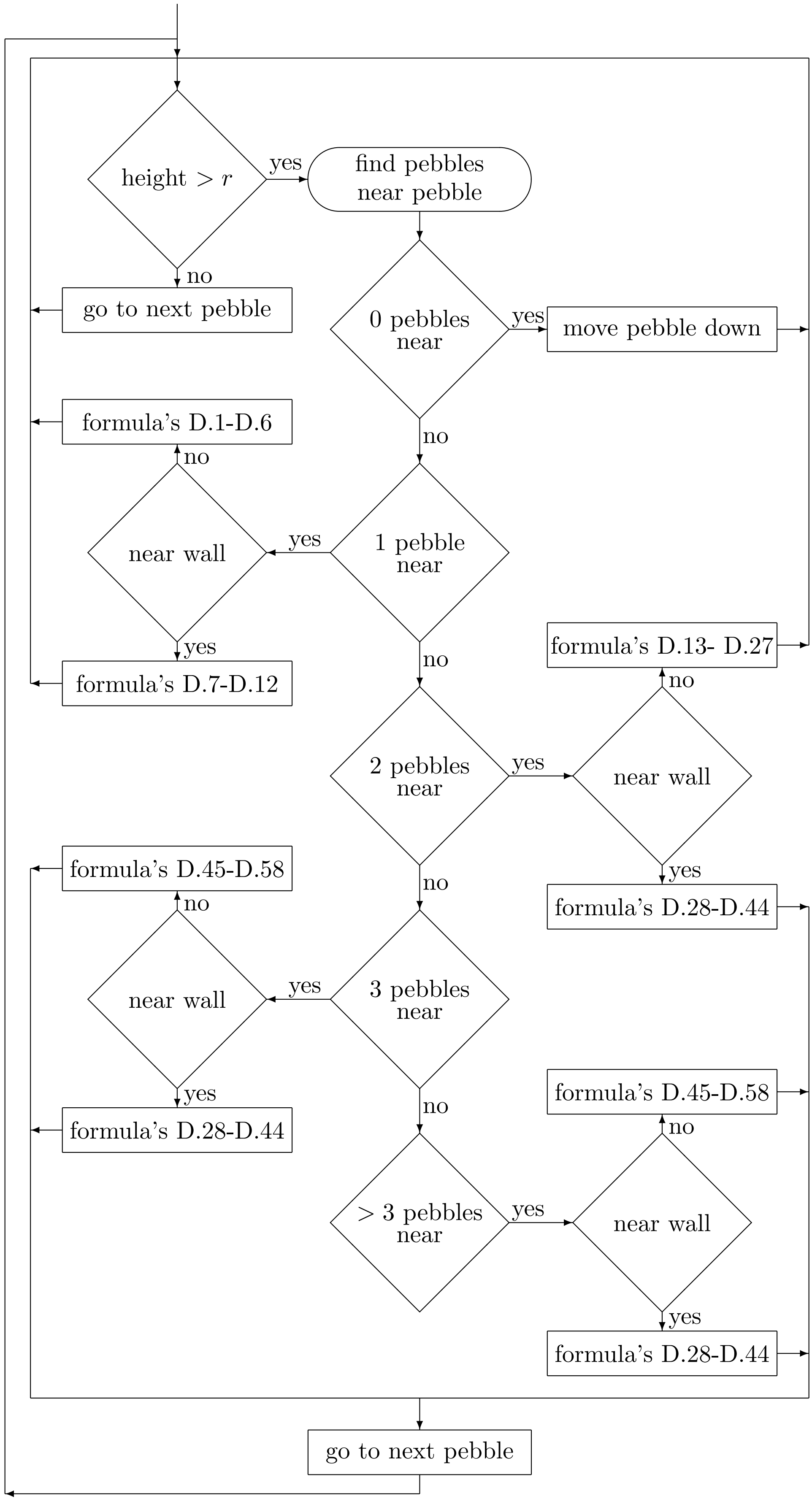


Figure 4.13: The flowchart for the dynamic packing.

# Chapter 5

# Radial displacement of pebbles

The radial displacement of pebbles is of importance for reactors that use radial zones with different material compositions. An example of a radial zone with a different material composition is an inner reflector made of moderator pebbles. An other example where the radial displacement of the pebbles is of importance, is fuel zoning, pebbles with different amounts of fissile material are added to different radial zones of the reactor. This might be pebbles with a different initial amount of TRISO particles, a burn-up dependent refuelling scheme or different sized pebbles in the radial zones. In a refuelling scheme the zone to which the pebbles are added, depends on the past number of passages through the reactor.

Knowing the radial displacement of the pebbles is also important for the possible formation of "hotspots", a cluster of fresh pebbles that produces more power than the average power generated. When there is a lot of mixing of the pebbles, there is a change that such hotspots arise.

At MIT experiments have been performed to test the radial displacement of pebbles, the results of these experiments will be used as a comparison[3].

## 5.1 Zones with equal width

To test the radial displacement, the reactor is divided into several radial zones. Each pebble gets a label with its initial zone. After the pebbles have been moved a predefined distance, each zone is checked for "foreign" pebbles. The percentage of pebbles from the different zones is calculated for each zone.

Dividing the reactor core in different radial zones can be done in different ways. One of the options is to have the same width for each zone, another is to have equal volume.

When the zones are created with equal width, the amount of inlets differs per zone. The central zone will have one inlet. This determines the amount of inlets in the other zones of the reactor core. The inlets have to be spread as evenly as possible, attributing to an even filling profile. This means the number of inlets in a zone should be proportional to the surface of that zone. The surfaces perpendicular to the flow of zone 1 till 4 are

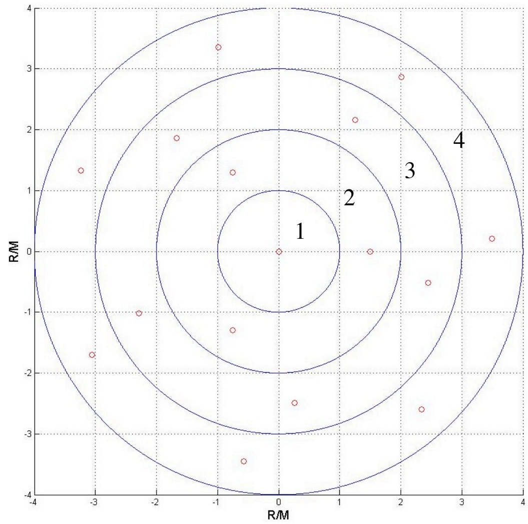


Figure 5.1: The arrangement of inlets in the case of four zones with equal width, the inlets are red and the zone boundaries are blue. The same arrangement can also be used when there are 2 or 3 zones, the outer zones should not be taken into account in those cases.

given by

$$\begin{array}{ll} \text{zone 1} & \pi\left(\frac{R}{M}\right)^2 \\ \text{zone 2} & \pi\left(\frac{2R}{M}\right)^2 - \pi\left(\frac{R}{M}\right)^2 = 3\pi\left(\frac{R}{M}\right)^2 \\ \text{zone 3} & \pi\left(\frac{3R}{M}\right)^2 - \pi\left(\frac{2R}{M}\right)^2 = 5\pi\left(\frac{R}{M}\right)^2 \\ \text{zone 4} & \pi\left(\frac{4R}{M}\right)^2 - \pi\left(\frac{3R}{M}\right)^2 = 7\pi\left(\frac{R}{M}\right)^2 \end{array}$$

where M is the number of zones and R the radius of the reactor. This means that zone 2, 3 and 4 need 3, 5 and 7 inlets respectively. They can be arranged as shown in figure 5.1. More zones can be added in the same way.

### 5.1.1 Results

This arrangement has been tested by removing pebbles at the bottom of the core and adding pebbles at the positions giving in figure 5.1. For each pebble that is removed, one pebble is added in a specified homogeneous order. This order is chosen such that the pebbles added, are divided as evenly as possible.

The diameter of the reactor is 18.03 pebble diameters, this means $\frac{R}{M} = 2.25$ pebble diameters. The reactor simulated is equipped with a funnel with an angle of $\frac{\pi}{6}$. The pebbles in the funnel are not considered for the radial displacement. This would influence

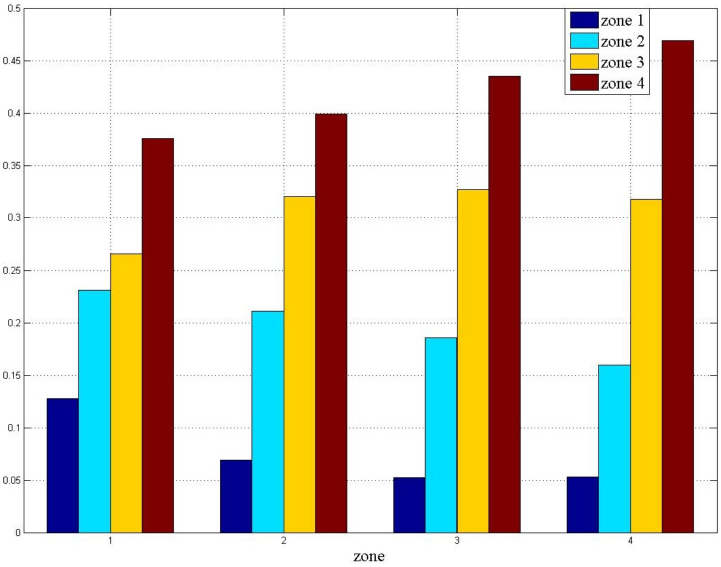


Figure 5.2: The composition of the different zones with equal width after 5000 pebbles have been removed and new pebbles have been added at the top of the reactor. The figure shows relatively where the pebbles in each zone originate.

the outcome because, depending on the diameter of the funnel exit, the outlet only consists of the inner zone(s). The funnel, however, is included in the reactor to evaluate the radial displacement due to its presence.

The result of a complete pass-through (5000 pebbles) is shown in figure 5.2. The histogram shows that only 12.5 % of the pebbles in zone one originate from zone one. 24 % of the pebbles in zone one originates from zone two, 26 % from zone three and 37.5 % from zone four.

In each zone, most pebbles originate from zone four. This is because zone four is the largest zone and in general most pebbles have originated from zone four. Only looking at the pebbles originating from zone one, the amount of pebbles is higher in zone one than in any other zone. The same is true for zone three. The differences with pebbles from zone two and four, however, are really small. Relatively most pebbles originating from zone two are found in zone one. This seems remarkable, but is due to the fact that zone one is the smallest zone.

## 5.2 Zones with equal volume

Arranging the zones in the reactor in a different way, with the same amount of pebbles in each zone might alter the results.

To test this, a new arrangement has been made as shown in figure 5.3. Here all the

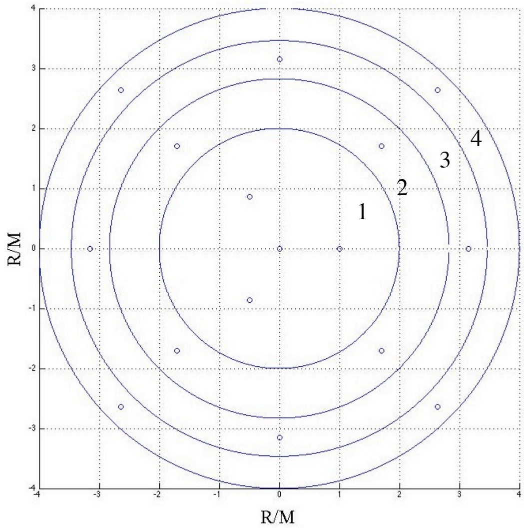


Figure 5.3: The arrangement of inlets in case of four zones with equal volume, the inlets are red and the zone boundaries are blue.

zones have the same surface $\frac{\pi R^2}{M}$. This results in zone boundaries at

$$\begin{aligned}
\text{zone 1} \qquad & R_1 = \frac{R}{\sqrt{M}} \\
\text{zone 2} \qquad & R_2 = R\sqrt{\frac{2}{M}} \\
\text{zone 3} \qquad & R_3 = R\sqrt{\frac{3}{M}} \\
\text{zone 4} \qquad & R_4 = R\sqrt{\frac{4}{M}}
\end{aligned}$$

and so on.

The new arrangement does not alter the pebble flow for the pebbles originating from the initial static packing of the reactor, but the new pebbles are added in different positions and the boundaries of the zones are in different positions.

### 5.2.1 Results

This configuration has been tested in the same way as the configuration with equal width. The same static packing of the reactor has been used as initial condition, thus $\frac{R}{M} = 2.25$ pebble diameters. The radius of zone 4, the zone with the smallest width, is $R - R\sqrt{\frac{3}{M}} = 1.2$ pebble diameters. The result of a complete pass-through can be seen in figure 5.4.

The results from this configuration are easier to interpret because each zone contains the same amount of pebbles. In each zone, the pebbles, which are found most in that

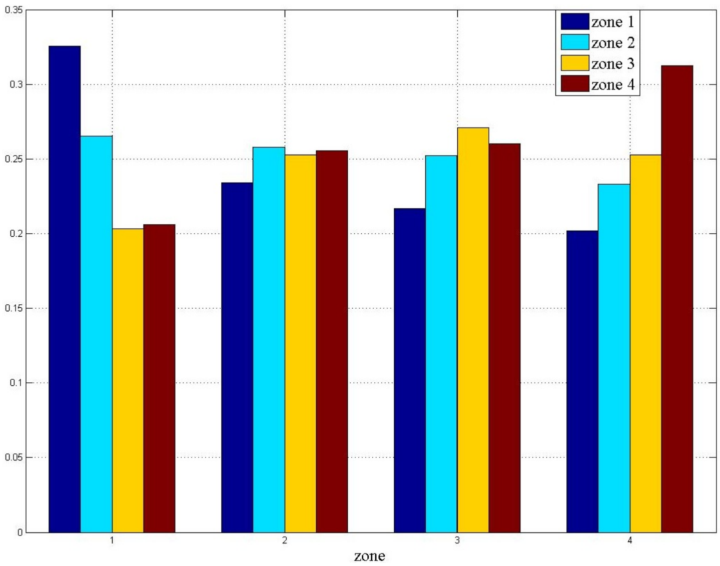


Figure 5.4: The composition of the different zones with equal volume after 5000 pebbles have been removed and new pebbles have been added at the top of the reactor. The figure shows relatively where the pebbles in each zone originate.

zone, are the pebbles that originate from that zone. The figure shows that, although there are still a lot of pebbles from the original zone in each zone, a fair amount of mixing has taken place in the core. Only a quarter of the pebbles will stay in (or return to) their original zone.

The zones with most pebbles still in their original zone, are zone one and zone four. This is because they both have only one adjoining zone and for that reason they have a smaller perimeter than zone two and zone three. In zone two and zone three there are more pebbles that start close to the zone barrier and those pebbles are more likely to cross to another zone.

### Conculsion

The two examples show that the arrangement of the zones matters for the amount of information that can be extracted. The configuration with zones with equal volume is easier to interpret than the configuration with zones with equal width.

When looking at the different zones, the width and perimeter of the zone should be noticed. If the zones are small and have a large perimeter, it is very likely that the pebbles will end up in the adjoining zones, while the pebble has only been moved a small distance. Having less or larger zones does not alter the amount of mixing in the core, although the results will be different than shown in this chapter.

## 5.3 Course of the radial displacement

To get more insight in the mixing of the pebbles, it is investigated how the mixing occurs during the different stages of the movement of the pebbles. To get this information, the final packing of the pebble bed with zones with equal volume has been split up in five parts, each containing 1000 pebbles. The first region contains the pebbles that have been added to the pebble bed last. The pebbles in the last region are in the funnel of the reactor, they have been in the reactor the longest. This way it can be made visible how quick the mixing of the pebbles occurs.

The results can be found in figure 5.5, where also the boundaries of the regions are shown. The first graph shows that the highest pebbles are not mixed yet. 55 % of the pebbles in zone one, originates from zone one. In the other zones a smaller part of the pebbles remains in their original zone, but in all zones it is more than 30 %.

In the other regions the pebbles are mixed. A maximum of 34 % of pebbles originating from zone one can be found in zone one, but this is in the second axial region of the core. In this region, zone one and zone four still have the highest percentage of pebbles originating from that zone, but in zone two and three, this no longer applies.

After the first two axial regions, the pebbles seem to be mixed randomly over the different zones. The percentage of pebbles in a zone has large fluctuations, for some zones, the difference between the axial regions can be higher than 5 %. An overall look would suggest the same amount of pebbles from, for example zone one, can be found in all the different zones.

The last region is not used when determining the overall mixing of the pebbles because of the funnel. The funnel exit is only in zone one, which makes the results from this graph misleading. In this region, there is not the same amount of pebbles in each zone. In zone one are 367 pebbles, while in zone four there are only 136 pebbles.

## 5.4 Comparison with experiments

MIT has performed experiments to determine the radial displacement of pebbles in a pebble bed reactor [3]. These experiments indicate that pebbles do not move in the radial direction of the reactor. The experiments, however, are performed on a half reactor, the experimental set-up is shown in figure 5.6. The radial displacement against the straight dividing wall is recorded by using coloured pebbles. It shows that pebbles touching the dividing wall, which is not curved like the reflector of a real reactor, will not move in a radial direction along the wall. This might explain the difference between the outcome of the experiments and the results of the pebble-flow model. The wall limits the movement of the pebbles and the results might not be valid for pebbles that are not in contact with a wall. To confirm the results from the experiments, a three-dimensional test has been performed at MIT as well. It is not clear what has been done in this test and what the results are. It is only known that the results support the tests with the half reactor. This would mean the results from the model are not comparable with reality.

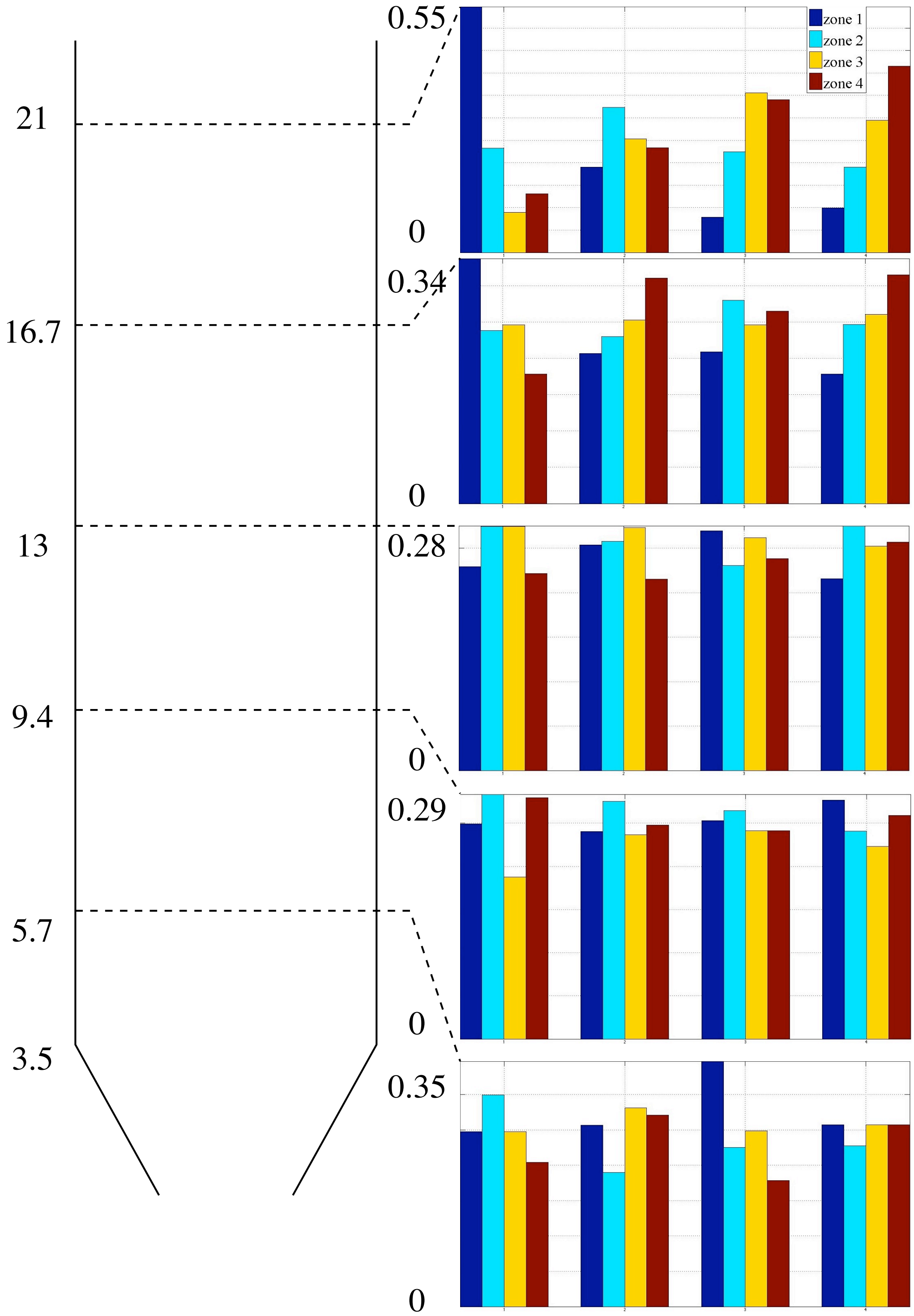


Figure 5.5: The radial displacement taken from different parts of the reactor, the height of the boundaries between the different regions is given in pebble diameters.

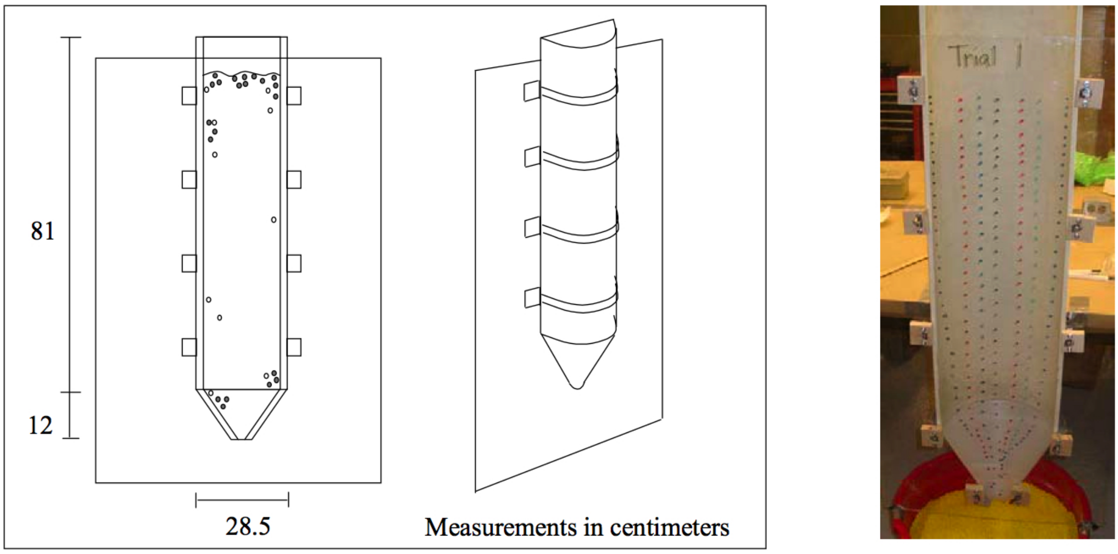


Figure 5.6: The experimental set-up used by MIT [3].

## 5.5 Vertical displacement

In chapter 4 it was suggested that the dip highlighted in figure 4.7 can be eliminated by adding the pebbles more homogeneously on top of the reactor. To confirm this, flow lines are generated for both cases discussed in this chapter. The flow lines do not show any significant differences because the flow of the examples is the same, only the positions of the new pebbles are different. For that reason only the flow lines of the example with equal volume is shown in figure 5.7. The difference between figure 5.7 and figure 4.7 is mainly seen in the smoothness of the flow lines. Because ten times more pebbles have been removed, the flow lines are averaged over ten times more pebble movements. Another important difference can be seen in the top flow line. As predicted, when the pebbles are added over the whole surface of the reactor, the top flow line shows similar behaviour as the other flow lines and is of the same width.

The last that can be noted from figure 5.7 are the displacements outside the funnel. Although small, two vertical movements are visible in parts of the reactor where no pebbles should be able to come, as highlighted in figure 5.7. This means some pebbles move through the funnel, in this case also made of pebbles. Because the distance the pebbles move outside the reactor is very small, it is likely the pebbles move back into the reactor the next calculation step.

Finally the void fraction is determined for this generated packing after flow, to compare with figure 4.10. Figure 5.8 shows the radial void fraction after a complete pass-through of the reactor, while the pebbles are added homogeneously over the whole surface of the reactor. The hypothesis was that the slope in figure 4.10 is caused by the way the

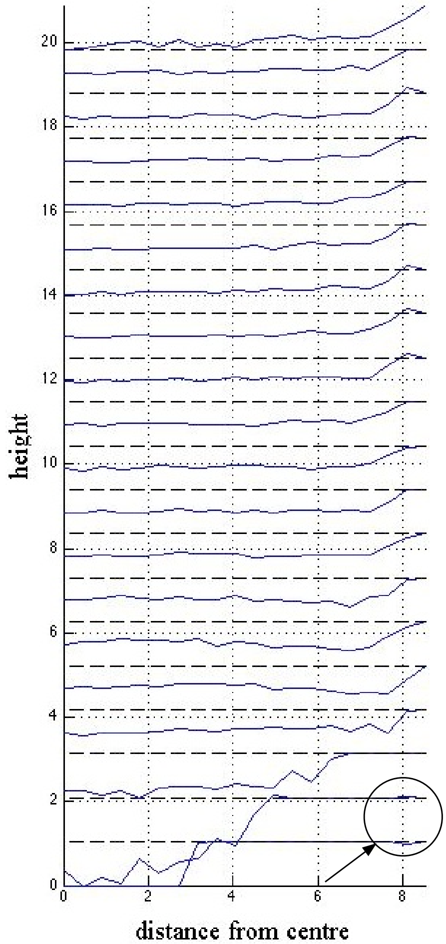


Figure 5.7: The average flow lines of a cylinder with funnel after 5000 pebbles have been removed and added. The flow lines are averaged over the movement steps and multiplied by ten to show more detail. The arrow highlights two displacements out side the reactor core, here pebbles have moved through the funnel.

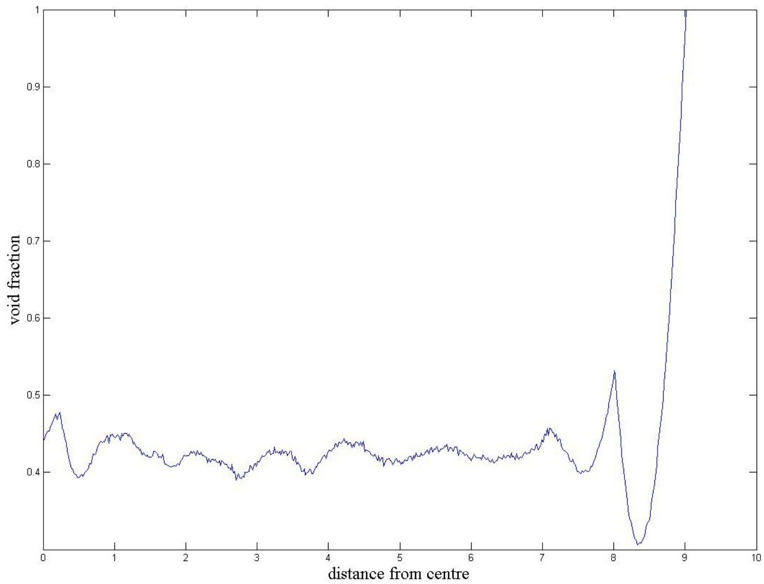


Figure 5.8: The radial void fraction after 5000 pebbles have been removed and new pebbles have been added at the top of the reactor.

pebbles are added to this reactor before the pebble-flow model is applied is confirmed, the figure does not show a slope in this case.

The shape of the radial void fraction after flow has been applied is comparable to the radial void fraction of a static packed pebble bed. To get a better insight into the composition of the pebble bed, the two-dimensional void fraction has also been determined. This can be found in figure 5.9 and can be compared to figure 3.6. It seems that the pebble bed is less dense after a complete pass through. However, this is not the conclusion from figure 5.8, the radial void fraction is similar to the void fraction of the static pebble bed, the overall void fraction also stays the same. The pebble bed shows more fluctuations after pebble flow has been applied.

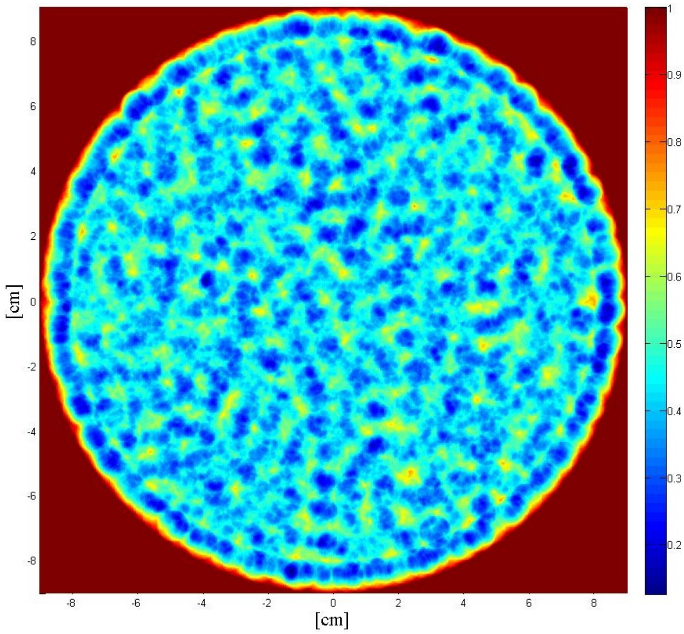


Figure 5.9: The two-dimensional void fraction after 5000 pebbles have been removed and new pebbles have been added at the top of the reactor.

# Chapter 6

# Conclusion and recommendations

There are multiple ways to simulate the pebble flow in a HTR. The simulation that has the best physical background is the Discrete Element Method, where all the forces are calculated for each pebble to determine the motion. The goal of this thesis is to create a code that would yield similar results to the DEM but uses less CPU time. To achieve this, a lot of assumptions have been made. The main one is that there are no forces on the pebbles, they just move to the lowest stable position. Unfortunately it was not possible to compare the results generated in this way with results generated with the DEM. The PAPA code, which uses the DEM, did not work properly and no data has been generated with this code. This leaves an important note that has to be considered in the conclusions given here. The conclusions are made assuming the flow generated with the pebble-flow model is accurate. Some results are confirmed using previously performed experiments, but some results could not be confirmed. There is experimental data that is used to confirm the static packing of the reactor. Currently experiments are performed at the TU Delft, but no results were yet available.

## 6.1 Static packing

For the static packing the method of placing pebbles at the lowest possible position per layer has been used. This method results in a similar shape for the radial void fraction as the experimental ones [21]. The overall void fraction is 0.45, which is higher than the theoretical void fraction of 0.3954. This is caused by the fact that pebbles are placed on grid points and do not necessarily have to touch one another, which in reality is of course impossible. When the pebble-flow model is applied, the pebbles do touch one another and the overall void fraction can be reduced to 0.44, which is still higher than the theoretical void fraction for this configuration. With a denser grid and more CPU time, a closer packing can be achieved of 0.42, this is without applying the pebble-flow model.

## 6.2 Dynamic packing

For the dynamic packing a new method has been developed. Pebbles are moved to the lowest available stable position near their original position. Because the pebble to funnel interaction is difficult to solve analytically, the funnel of the reactor is made by placing pebbles at the position where the funnel would be. It is possible for pebbles to go through this pebble-funnel. Since it does not happen very often and because the pebble is placed back into the reactor the next calculation step, this small error is accepted.

Different sized batches with pebbles removed are tested on a cylinder without a funnel. When the batch size contains an entire layer, the flat flow lines are flat, as expected. This is contrary to the case where the pebbles are removed one at a time. This latter situation gives a very irregular result. It has to be noted that using large batch sizes can also have undesired effects. After a complete pass through in the cylindrical set-up, cavities are found in the pebble bed. This influences the shape of the radial void fraction. These cavities are formed because of the large batch size and do not show up in the reactor with a funnel where the batch size is smaller. The cavities are most likely artificial and should for that reason be prevented. It is possible that a funnel with a large exit diameter could give the same problem. If that is the case, a smaller batch size should be used, which will not alter the flow lines of the reactor much. An optimum batch size should be sought in such cases.

When the model is tested on a reactor with a funnel, the shape of the flow lines is comparable to experiments performed on a similar reactor.

## 6.3 Radial displacement

The radial displacement has been determined for two methods that both divide the reactor in zones. The reactor has been divided in zones with equal width and in zones with equal volume. The situation where the reactor is divided in zones with equal volume is easier to work with, because there is an equal amount of pebbles in each zone. In the case where the reactor is divided in four zones more than 25 % of the pebbles is always left in the original zone after a complete pass-through of a core with 5000 pebbles. For the zones in the centre of the reactor and next to the outer reflector more than 30 % of the pebbles is left. If these results are accurate the pebbles mix through the whole reactor. It does not become a homogeneous mixture, but the pebbles are mixed. This could mean that the effect of fuel zoning on the power profile of the reactor would be minimal compared to adding the pebbles at one inlet in the centre of the reactor. These results differ from the behaviour of the pebbles seen in experiments; in the experiments the pebbles move in a straight vertical line downwards [3].

It has to be noted that the zones used to generate these results are relatively small and larger zones could give a clearer separation between the different zones. This does not, however, alter the pebble-flow and the amount of mixing in the core.

A complete pass-through of a core with 5000 pebbles has been calculated while the pebbles are added in the different zones of the reactor in both ways described above.

The radial void fraction shows a similar result as the radial void fraction of the static packing. The flow in a reactor does not alter the pebble density of the reactor.

## 6.4 Recommendations

As mentioned before, it was not possible to verify the assumptions made for the pebble-flow model. Before any further research is done with the pebble-flow model it should be verified whether the results are realistic and comparable to experiments and to the DEM. The radial displacement is the most important to confirm because the results differ from the experimental data found. Other results are confirmed by experimental data.

When the accuracy of the code is confirmed or alterations have been made, it is interesting to look into some effects with more detail. It should, for example, experimentally be determined whether cavities occur in a pebble bed or not. If cavities do not occur, it will be important to determine what the maximum amount of pebbles is that can be removed without the appearance of cavities. Another point of interest is the radial displacements of a reactor with a larger reactor diameter to pebble diameter ratio. Using larger zones, a larger percentage of pebbles should stay in their zone of origin.

When these points have been researched there are some other examples to be thought of that could be of interest;

- Results for a full scale HTR calculation
- The effect of different funnel angles
- The effect of different sized pebbles in one reactor
- The effect of a central column with moderator pebbles

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# Appendix A

# Abbreviations

| | |
|---|---|
| CPU | Central Processing Unit |
| DEM | Discrete Element Method |
| HTR | High Temperature Reactor |
| MIT | Massachusetts Institute of Technology |
| PAPA | Parallel Algorithm for Particle flow |
| TRISO | Tristructural-isotropic |
| VHTR | Very High Temperature Reactor |

# Appendix B

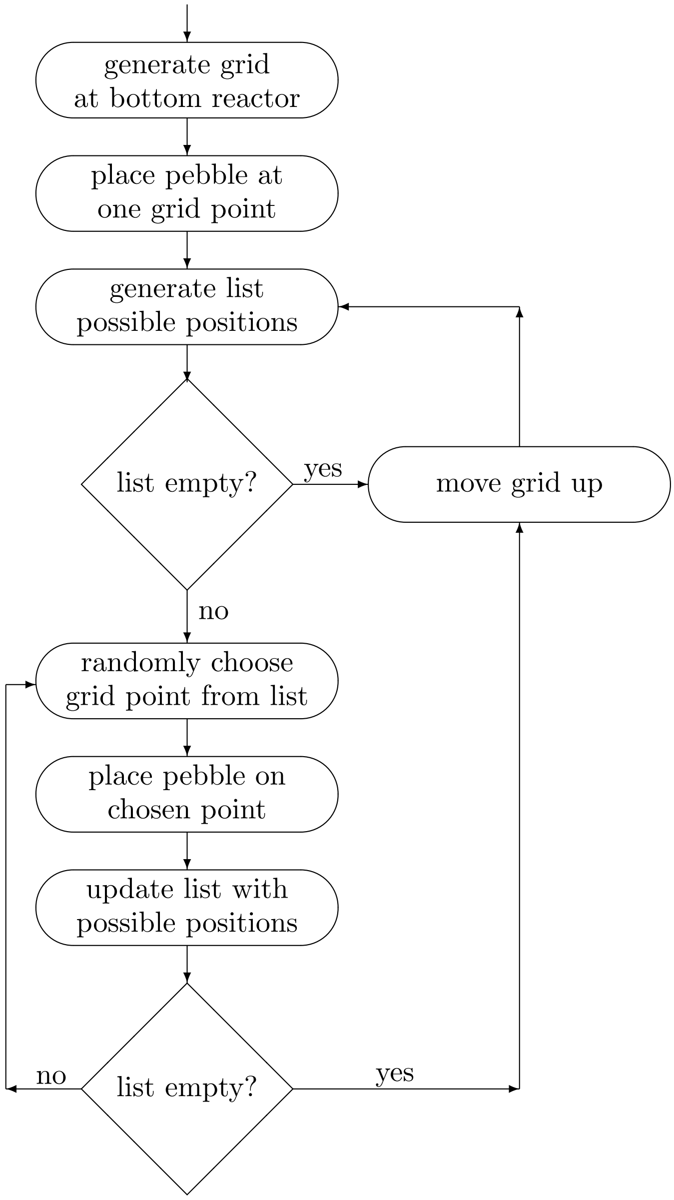
# Possible simulation methods

Table B.1: Possible simulation methods found in literature.

| | | |
|---|---|---|
| 1. | DEM simulation. | [4–12] |
| 2. | Start with a regular packing of pebbles with random initial speeds and apply the DEM until a stable situation is reached. | [8] |
| 3. | Spot model: a free volume moves through pebbles upwards in the reactor, this means small groups of pebbles move down in a cluster. | [12] |
| 4. | Generate random points, place pebble at lowest possible generated point. | [13] |
| 5. | Rainmodel: pebble falls down from a random position until it touches another pebble. It slides down until a stable position is reached. | [14, model a] |
| 6. | Pebble falls down on a random position until it touches another pebble and sticks at this position. When all pebbles are placed the reactor is *shaken* by a Monte Carlo technique. | [15] |
| 7. | Poring in the pebbles is simulated by generating a given number of pebbles in each time step in a small cylinder. | [16] |
| 8. | The pebble is placed at the lowest possible position. After each pebble is placed, the list with possible positions is up-dated. | [14, model c] [6, 17] |
| 9. | Same as method 8 but the list is only up-dated after all the possible positions are filled. The reactor is thus filled layer by layer. | [17] |
| 10. | Same as method 9 but there are restrictions on the placement of the pebbles, e.g. the pebbles are alternately placed touching the side of the reactor and in the middle of the reactor. | [17] |
| 11. | The total number of pebbles is placed randomly. Moving the pebbles in opposite directions until all the overlap is eliminated. | [18] |

# Appendix C

# Flowchart static packing

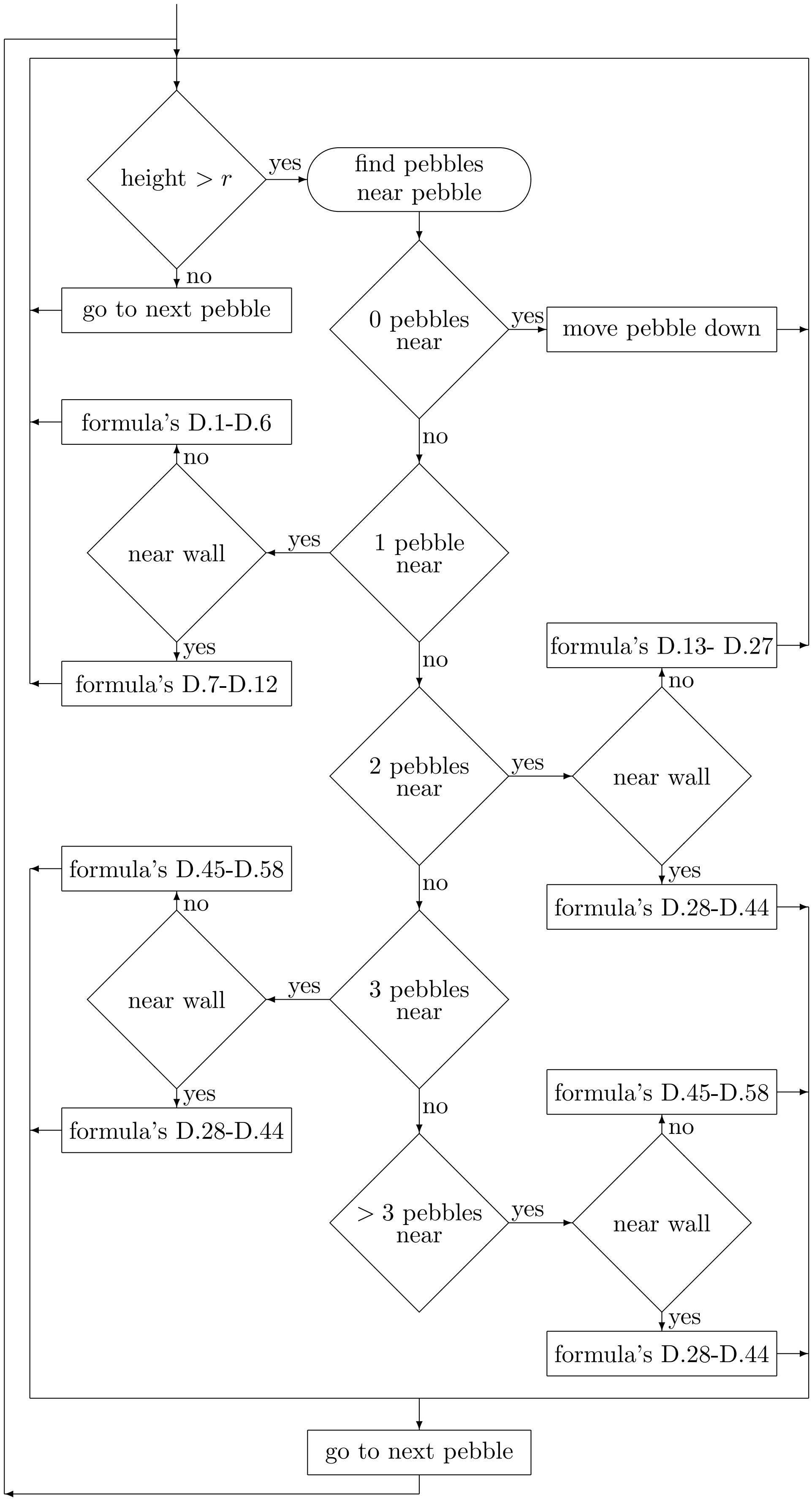


# Appendix D

# Formula's dynamic packing

The flowchart and the formulas for the dynamic packing are given next. In the formulas the following symbols are used;

$$
\begin{aligned}
r &= \text{the radius of the pebbles} \\
R &= \text{the effective radius of the reactor at the height of the pebble} \\
R_{min} &= \text{the radius of the reactor at the bottom of the funnel} \\
k &= \text{the angle of the funnel} \\
x_b &= \text{the x-position of pebble M} \\
y_b &= \text{the y-position of pebble M} \\
h_b &= \text{the z-position of pebble M} \\
r_{pb} &= \text{the radial position of pebble M} \\
x_e &= \text{the x-position of pebble M after calculation} \\
y_e &= \text{the y-position of pebble M after calculation} \\
h_e &= \text{the z-position of pebble M after calculation} \\
r_{pe} &= \text{the radial position of pebble M after calculation} \\
f_{1,j} &= \text{the x-position of the } j^{th} \text{ found pebble} \\
f_{2,j} &= \text{the y-position of the } j^{th} \text{ found pebble} \\
f_{3,j} &= \text{the z-position of the } j^{th} \text{ found pebble} \\
f_{4,j} &= \text{the radial position of the } j^{th} \text{ found pebble}
\end{aligned}
$$



height $> r$
yes
find pebbles near pebble
no
go to next pebble
0 pebbles near
yes
move pebble down
no
formula's D.1-D.6
no
near wall
yes
1 pebble near
yes
formula's D.7-D.12
no
formula's D.13- D.27
no
2 pebbles near
yes
near wall
yes
formula's D.28-D.44
no
formula's D.45-D.58
no
near wall
yes
3 pebbles near
yes
formula's D.28-D.44
no
formula's D.45-D.58
no
$>$ 3 pebbles near
yes
near wall
yes
formula's D.28-D.44
go to next pebble

When there is one pebble near pebble M, pebble M will be placed next to the pebble found.

$$\begin{aligned}
\mathrm{d}x &= x_b - f_{1,1} && \text{(D.1)}\\
\mathrm{d}y &= y_b - f_{2,1} && \text{(D.2)}\\
dist &= \sqrt{dx^2 + dy^2} && \text{(D.3)}\\
x_e &= f_{1,1} + \frac{2r\mathrm{d}x}{dist} && \text{(D.4)}\\
y_e &= f_{2,1} + \frac{2r\mathrm{d}y}{dist} && \text{(D.5)}\\
h_e &= f_{3,1} && \text{(D.6)}
\end{aligned}$$

But when pebble M is near the wall, this could cause pebble M to overlap with the wall, pebble M will then be placed against the wall.

$$\begin{aligned}
\phi &= \arctan\left(\frac{f_{2,1}}{f_{1,1}}\right) && \text{(D.7)}\\
\mathrm{d}\phi &= \arccos\left(\frac{R^2 + f_{4,1}^2 - 4r^2}{2Rf_{4,1}}\right) && \text{(D.8)}\\
r_{pe} &= R && \text{(D.9)}\\
x_e &= r_{pe}\cos(\mathrm{d}\phi + \phi) && \text{(D.10)}\\
y_e &= r_{pe}\sin(\mathrm{d}\phi + \phi) && \text{(D.11)}\\
h_e &= f_{3,1} && \text{(D.12)}
\end{aligned}$$

When there are two pebbles found near pebble M, pebble M will also be placed next to the two pebbles. This is done in the same way; pebble M is placed next to the higher of the two pebbles. Pebble M will then be moved down to the height of the lowest pebble.

$$\begin{aligned}
\mathrm{d}x_1 &= x_b - f_{1,1} && \text{(D.13)}\\
\mathrm{d}y_1 &= y_b - f_{2,1} && \text{(D.14)}\\
dist &= \sqrt{dx_1^2 + dy_1^2} && \text{(D.15)}\\
\mathrm{d}x &= \frac{2r\mathrm{d}x_1}{dist} && \text{(D.16)}\\
\mathrm{d}y &= \frac{2r\mathrm{d}y_1}{dist} && \text{(D.17)}\\
x_{e1} &= f_{1,1} + \frac{2r\mathrm{d}x}{dist} && \text{(D.18)}\\
y_{e1} &= f_{2,1} + \frac{2r\mathrm{d}y}{dist} && \text{(D.19)}\\
h_e &= f_{3,2} && \text{(D.20)}
\end{aligned}$$

If this position does not overlap with the lower pebble, this is the new position of pebble M. If it does overlap the new position will be as follows;

$$\begin{aligned}
dist_f &= (f_{1,1} - f_{1,2})^2 + (f_{2,1} - f_{2,2})^2 && \text{(D.21)}\\
dist_2 &= (x_{e1} - f_{1,2})^2 + (y_{e1} - f_{2,2})^2 && \text{(D.22)}\\
b &= \frac{dist_f + 4r^2 - dist_2}{2r} && \text{(D.23)}\\
c &= dist_f - 4r^2 && \text{(D.24)}\\
dist_3 &= \frac{b + \sqrt{b^2 - 4c}}{2r} && \text{(D.25)}\\
x_e &= f_{1,1} + \frac{dist_3 \mathrm{d}x_1}{dist} && \text{(D.26)}\\
y_e &= f_{2,1} + \frac{dist_3 \mathrm{d}y_1}{dist} && \text{(D.27)}
\end{aligned}$$

Unless pebble M is located near the side of the reactor and in between the side of the reactor and the pebbles found. In that case it is possible to place pebble M on the stable position touching the two pebbles and the outer reflector. The y-coordinate of the new position of pebble M can be calculated from the following quartic polynomial.

$$K_4 y^4 + K_3 y^3 + K_2 y^2 + K_1 y + K_0 = 0 \tag{D.28}$$

This equation gives four possible answers for each set of coefficients. The desired answer for $y$ needs to be selected from these possibilities. The coefficients can be calculated as follows.

$$\begin{aligned}
N_i &= \frac{f_{i,1} - f_{i,2}}{2} && \text{(D.29)}\\
M_i &= \frac{f_{i,1} + f_{i,2}}{2} && \text{(D.30)}\\
C_1 &= 4r^2 - \sum_{i=1}^{3} N_i^2 && \text{(D.31)}\\
C_2 &= \sum_{i=1}^{3} N_i M_i && \text{(D.32)}\\
K_0 &= -2C_1C_2^2N_3^2 - 4C_2N_3^3M_3^3 - 2C_1N_3^4M_3^2 - 2N_3^4M_1^2R^2\\
&\quad - 2N_3^4M_1^2r^2 - 4N_3^4rR^3 + 6N_3^4r^2R^2 + 2N_3^4M_2^2R^2\\
&\quad + 2N_1^2N_3^2R^4 + 2C_2^2N_3^2R^2 + 2N_3^4M_3^2R^2 - 2C_1N_3^4R_2\\
&\quad + 2N_3^4M_2^2r^2 + 2N_1^2N_3^2r^4 + 2C_2^2N_3^2r^2 - 2N_1^2N_3^2M_3^2R^2\\
&\quad - 2C_1N_3^4r^2 + 2N_3^4M_1^2M_2^2 + 2C_2^2N_3^2M_1^2 + 2N_3^4M_1^2M_3^2\\
&\quad - 2C_1N_3^4M_1^2 + 2C_2^2N_3^2M_2^2 + 2N_3^4M_2^2M_3^2 - 4N_1^4rR^3\\
&\quad + 6N_1^4r^2R^2 - 2C_2^2N_1^2R^2 - 4N_1^4r^3R - 4C_2N_3^3M_1^2M_3\\
&\quad - 8N_1^2N_3^2r^3R - 4C_2^2N_3^2rR - 4N_3^4M_3^2rR + 4C_1N_3^4rR
\end{aligned}$$

$$
\begin{aligned}
& - 4C_2N_3^3M_3r^2 + 2N_1^2N_3^2M_1^2R^2 + 2N_1^2N_3^2M_1^2r^2 - 2C_2^2N_1^2r^2 \\
& + 2N_1^2N_3^2M_2^2R^2 + 2N_1^2N_3^2M_2^2r^2 - 4C_2N_3^3M_2^2M_3 + 2N_3^4M_3r^2 \\
& - 2C_1N_1^2N_3^2R^2 + 4C_2^2N_1^2rR - 2N_1^2N_3^2M_3^2r^2 - 2C_1N_1^2N_3^2r^2 \\
& + 4C_1C_2N_3^3M_3 + 4N_3^4M_1^2rR - 4N_1^2N_3^2M_1^2rR - 8N_1^2N_3^2rR^3 \\
& + 4N_1^2N_3^2M_3^2rR - 4N_1^2N_3^2M_2^2rR - 8C_2N_1^2N_3M_3rR + 4C_1N_1^2N_3^2rR \\
& + 4C_2N_1^2N_3M_3r^2 + 12N_1^2N_3^2r^2R^2 - 4C_2N_3^3M_3R^2 - 4N_3^4M_2^2rR \\
& - 4N_3^4r^3R + 6C_2^2N_3^2M_3^2 - 2C_1N_3^4M_2^2 + C_2^4 + 4C_2N_1^2N_3M_3R^2 \\
& + N_1^4R^4 + N_1^4r^4 + N_3^4R^4 + N_3^4M_2^4 + N_3^4r^4 + N_3^4M_1^4 + N_3^4M_3^4 \\
& + C_1^2N_3^4 - 4C_2^3N_3M_3 + 8C_2N_3^3M_3rR \pm 8N_1N_3^3M_1M_3r^2 \\
& \mp 8C_2N_1N_3^2M_1R^2 \mp 8C_2N_1N_3^2M_1r^2 \pm 16C_2N_1N_3^2M_1rR \\
& \pm 8N_1N_3^3M_1M_3R^2 \mp 16N_1N_3^3M_1M_3rR \qquad \text{(D.33)}
\end{aligned}
$$

$$
\begin{aligned}
K_1 \quad = \quad & - 4N_3^4M_2R^2 + 4N_2N_3^3M_3^3 - 4N_3^4M_2M_3^2 - 4C_2^2N_3^2M_2 \\
& + 4C_1N_3^4M_2 - 4N_3^4M_1^2M_2 - 4N_3^4M_2r^2 - 4N_1^2N_2N_3M_3r^2 \\
& - 8C_2N_1^2N_2rR - 4N_1^2N_2N_3M_3R^2 + 8N_3^4M_2rR - 4C_2N_2N_3^2M_1^2 \\
& - 4N_1^2N_3^2M_2R^2 - 4N_1^2N_3^2M_2r^2 + 4C_2N_1^2N_2R^2 - 4C_2N_2N_3^2r^2 \\
& + 4N_2N_3^3M_3r^2 + 4N_2N_3^3M_1^2M_3 + 8C_2N_3^3M_2M_3 + 12C_2^2N_3N_2M_3 \\
& - 12C_2N_2N_3^2M_3^2 \pm 8N_1N_2N_3^2M_1r^2 \mp 16N_1N_2N_3^2M_1rR \\
& - 4C_2N_2N_3^2M_2^2 - 4N_3^4M_2^3 - 8N_2N_3^3M_3rR + 4N_2N_3^3M_3R^2 \\
& - 4C_2N_2N_3^2R^2 + 4N_2N_3^3M_2^2M_3 + 4C_1C_2N_2N_3^2 - 4C_1N_2N_3^3M_3 \\
& - 4C_2^3N_2 + 8N_1^2N_2N_3M_3rR + 4C_2N_1^2N_2r^2 + 8N_1N_2N_3^2M_1R^2 \\
& + 8C_2N_2N_3^2rR + 8N_1^2N_3^2M_2rR \qquad \text{(D.34)}
\end{aligned}
$$

$$
\begin{aligned}
K_2 \quad = \quad & - 2N_1^2N_2^2R^2 + 4N_1^4rR + 2N_2^2N_3^2R^2 - 2N1^2N_3^2R \\
& - 2N_1^2N_3^2M_2^2 + 2N_2^2N_3^2M_2^2 + 2C_1N_1^2N_3^2 - 2N_1^2N_3^2r^2 \\
& + 2N_2^2N_3^2M_1^2 + 2N_2^2N_3^2r^2 - 2N_1^2N_3^2M_1^2 - 2N_1^2N_2^2r^2 \\
& - 2C_1N_2^2N_3^2 + 6N_2^2N_3^2M_3^2 - 4N_2^2N_3^2rR \mp 8N_1N_3^3M_1M_3 \\
& + 4N_1^2N_3^2rR + 8C_2N_2N_3^2M_2 - 8N_2N_3^3M_2M_3 - 12C_2N_2^2N_3M_3 \\
& \pm 8C_2N_1N_3^2M_1 - 2N_1^4R^2 + 2C_2^2N_1^2 - 2N_1^4r^2 + 4N_1^2N_2^2rR \\
& + 4N_3^4M_2^2 + 4N_3^4M_1^2 + 2N_1^2N_3^2M_3^2 + 6C_2^2N_2^2 - 4C_2N_1^2N_3M_3 \qquad \text{(D.35)}
\end{aligned}
$$

$$
\begin{aligned}
K_3 \quad = \quad & 4N_1^2N_2N_3M_3 \mp 8N_1N_2N_3^2M_1 \mp 4C_2N_2^3 + 4N_2^3N_3M_3 \\
& + 4N_1^2N_3^2M_2 - 4N_2^2N_3^2M_2 - 4C_2N_1^2N_2 \qquad \text{(D.36)} \\
K_4 \quad = \quad & 2N_1^2N_2^2 + N_2^4 + N_1^4 \qquad \text{(D.37)}
\end{aligned}
$$

When $y_e$ is solved from this polynomial, $x_e$ and $h_e$ can be calculated from $y_e$.

$$
\begin{aligned}
x_e \quad &= \quad \pm\sqrt{(-R + r - y_e)(-R + r + y_e)} \qquad \text{(D.38)} \\
h_e \quad &= \quad -\frac{N_1x_e + N_2y_e - C_2}{N_3} \qquad \text{(D.39)}
\end{aligned}
$$

Because this causes problems when $N_3 = 0$, when all the pebbles found are at the same height, the new position of pebble M has to be calculated in an alternative way when that happens.

$$\vec{n} = \begin{bmatrix} (f_{2,2} - f_{2,1}) \\ (f_{1,1} - f_{1,2}) \\ 0 \end{bmatrix} \tag{D.40}$$

$$dis = -M_1 n_1 + M_2 n_2 \pm \sqrt{2M_1 n_1 M_2 N_2 + n_1^2((R-r)^2 - M_2^2) + n_2^2((R-r)^2 - M_1^2} \tag{D.41}$$

$$x_e = M_1 + dis n_1 \tag{D.42}$$

$$y_e = M_2 + dis n_2 \tag{D.43}$$

$$h_e = f_{1,3} + \sqrt{4r^2 - (f_{1,1} - x_e)^2 - (f_{1,2} - y_e)^2} \tag{D.44}$$

When three pebbles are found and they are not close to the side of the reactor, pebble M will reach a stable position on top of the three pebbles found. How this position can be calculated is shown next.

$$K_y = f_{2,3} - f_{2,1} + \frac{(f_{2,1} - f_{2,2})(f_{1,3} - f_{1,1})}{f_{1,2} - f_{1,1}} \tag{D.45}$$

$$K_z = f_{3,3} - f_{3,1} + \frac{(f_{3,1} - f_{3,2})(f_{1,3} - f_{1,1})}{f_{1,2} - f_{1,1}} \tag{D.46}$$

$$K = \sum_{i=1}^{3}(f_{i,1}^2 - f_{i,3}^2) + \sum_{l=1}^{3}(f_{l,2}^2 - f_{l,1}^2)\frac{f_{1,3} - f_{1,1}}{f_{1,2} - f_{1,1}} \tag{D.47}$$

$$h_t = 1 \tag{D.48}$$

$$y_t = \frac{K_z + \frac{K}{2}}{-K_y} \tag{D.49}$$

$$x_t = y_t(f_{2,1} - f_{2,2}) + f_{3,1} - f_{3,2} + \frac{\frac{1}{2}\sum_{i=1}^{3} f_{i,2}^2 - f_{i,1}^2}{f_{1,2} - f_{1,1}} \tag{D.50}$$

$$\vec{pos} = \begin{bmatrix} x_t \\ y_t \\ h_t \end{bmatrix} \tag{D.51}$$

$$\vec{n} = \begin{bmatrix} (f_{2,2} - f_{2,1})(f_{3,3} - f_{3,1}) - (f_{3,2} - f_{3,1})(f_{2,3} - f_{2,1}) \\ (f_{1,3} - f_{1,1})(f_{3,2} - f_{3,1}) - (f_{1,2} - f_{1,1})(f_{3,3} - f_{3,1}) \\ (f_{1,2} - f_{1,1})(f_{2,3} - f_{2,1}) - (f_{1,3} - f_{1,1})(f_{2,2} - f_{2,1}) \end{bmatrix} \tag{D.52}$$

$$b = -2\sum_{i=1}^{3}(n_i f_{i,1} - pos_i) \tag{D.53}$$

$$c = \sum_{i=1}^{3}(f_{i,1} - pos_i)^2 - 4r^2 \tag{D.54}$$

$$dis = \frac{-b \pm \sqrt{b^2 - 4c}}{2} \tag{D.55}$$

The sign in the function $dis$ is chosen in such a way that pebble M is on top of the three pebbles. This is done by a comparison of the possible values for $h_e$.

$$x_e = x_t + n_1 dis \tag{D.56}$$

$$y_e = y_t + n_2 dis \tag{D.57}$$

$$h_e = h_t + n_3 dis \tag{D.58}$$