**AE4ASM003: Linear Modelling (incl. FEM)**
**Answers**

The answers to the three questions as part of the skill level test are given here in full.

**Answer 1**
This question is to check for continuity in the potential as described in the variational problem.

Given that,

$$\Pi = \frac{1}{2}\int_0^L EI\left(\frac{d^2w}{dx^2}\right)^2 .dx - \frac{P}{2}\int_0^L \left(\frac{dw}{dx}\right)^2 .dx + \frac{1}{2}kw_L^2$$

one can see the highest order derivative in the potential is $m = 2$. Therefore, the variational is a $C^{m-1}$ or $C^1$ problem. The statement that it is $C^0$ is hence, false.

**Answer 2**
This question is to check for basics in statics and sense of free body diagrams and correctness.

One can see that the truss system as a fixed support on the left vertex and a roller support on the right vertex. Therefore, three unknown reactions are present in the system. Two forces are acting on the system. As points of support on the left and right, now denoted as $A$ and $B$ in the figures, equilibrium equations can be setup in terms of force balance and moment equilibrium.

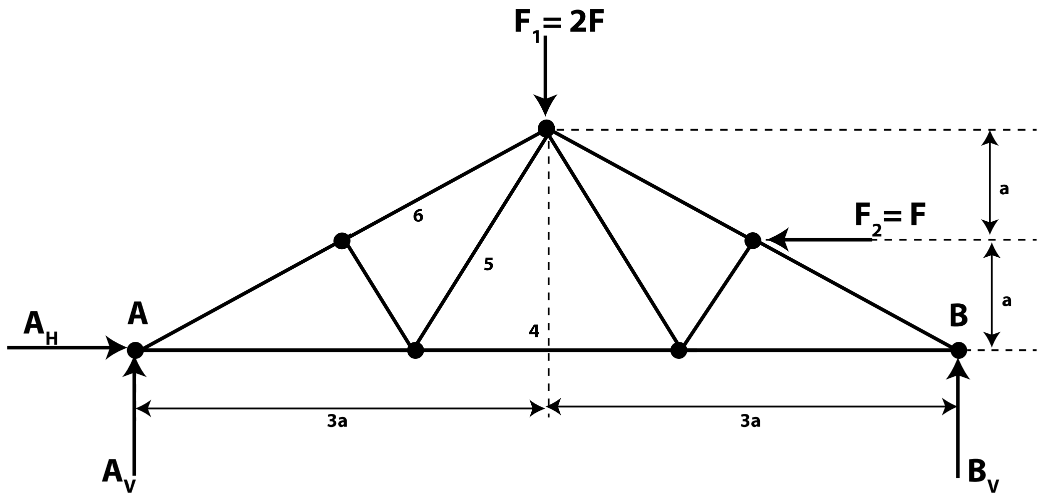


**Figure 1: Free body diagram 1**

Solving for force equilibrium,

$$\sum F_H = A_H - F_2 = 0 \quad (1)$$

$$\Rightarrow A_H = F_2 = F \quad (2)$$

and,

$$\sum F_V = B_V + A_V - F_1 = 0 \quad (3)$$

$$\Rightarrow B_V + A_V = F_1 = 2F \quad (4)$$

Taking moments about A,

$$\sum M_A = -F_1.3a + F_2.a + B_V.6a = 0 \tag{5}$$

$$\Rightarrow B_V = \frac{5}{6}F \tag{6}$$

Substituting (6) in (4), we get,

$$A_V = \frac{7}{6}F \tag{7}$$

Now, having computed all unknown reactions, we can splice the problem such that we can find out the force in element 4 as shown in the figure 2. The resultant free body diagram is given in figure 3.

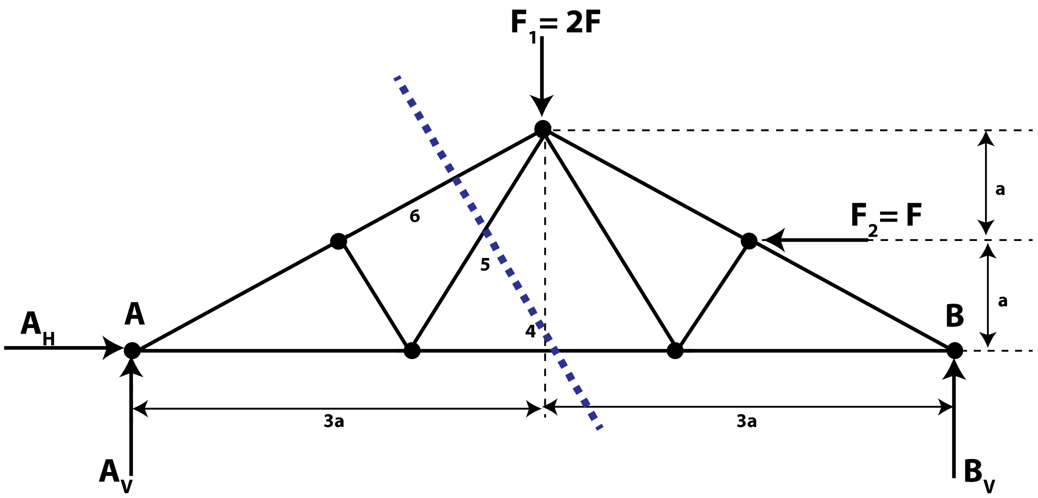


**Figure 2: Free body diagram with splice location**

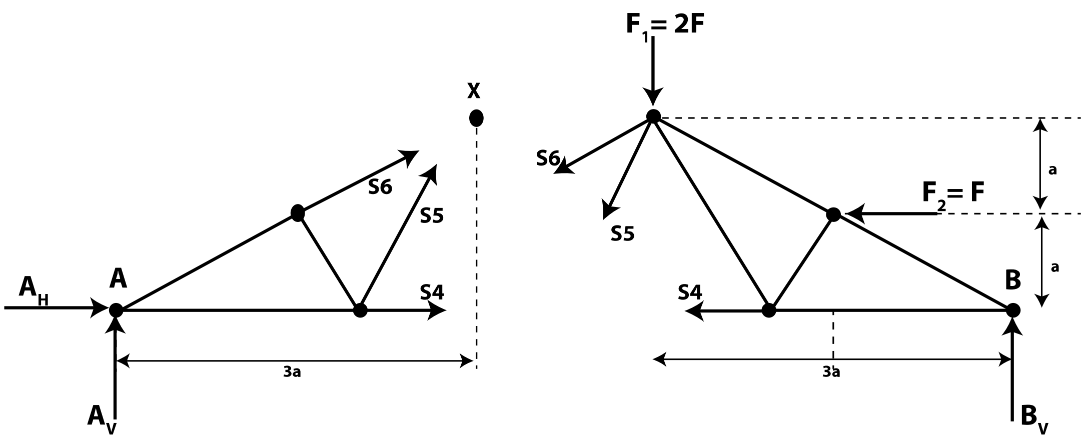


**Figure 3: Free body diagram at splice**

Now, taking moments about point X, for the left side of the splice, we get,

$$\sum M_X = -A_V.3a + A_H.2a + S_4.2a = 0 \tag{8}$$

$$\Rightarrow S_4 = \frac{3}{4}F \tag{9}$$

Similarly, taking moments about point X for the right side of the splice, we get,

$$\sum M_X = B_V.3a - F_2.a - \ S_4.2a = 0 \tag{10}$$

$$\Rightarrow S_4 = \frac{3}{4}F \tag{11}$$

$S_4$ is the truss force in member 4.

**Answer 3**

This question is to test the basics of using Newton’s second law for setting up equations of motion. First the free body diagrams have to be setup as shown in figure 4.

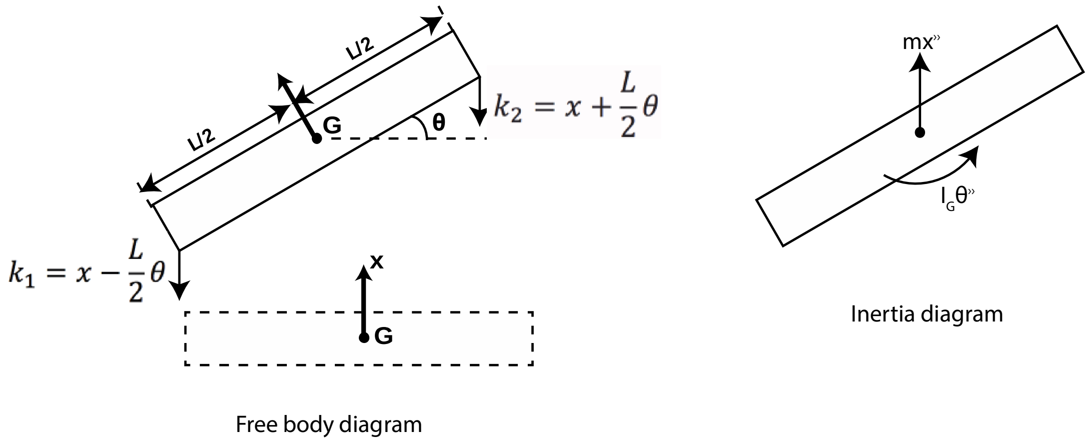


**Figure 4: Free body and inertia diagrams**

Using basic statics principle, the equilibrium of the weight of the rod and spring forces in opposite direction should cancel out. Using Newton’s second law,

$$\sum F_X = m(a_G)_X = mx" \tag{1}$$

$$\Rightarrow -k_1\left(x - \frac{L}{2}\theta\right) - k_2\left(x + \frac{L}{2}\theta\right) = mx" \tag{2}$$

Simplification of (2) gives,

$$mx" + (k_1 + k_2)x - (k_1 - k_2)\frac{L}{2}\theta = 0 \tag{3}$$

which is the translation motion equation.

Similarly, the rotational motion can be described as

$$\sum M_G = I_G\alpha = I_G\theta" \tag{4}$$

Substituting, we get,

$$I_G\theta \text{ - } (k_1\text{-}k_2)\frac{L}{2}x\text{+} + (k_1 + k_2)\left(\frac{L}{2}\right)^2 \theta = 0 \tag{5}$$

(3) and (5) are the equations of motion for the given system.