Advancements in Negative Refractive Index Metamaterials

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Abstract: The index of refraction is an important property of transparent materials, and has always been measured positive for every known material found in nature. However, a negative refractive index can be achieved by metamaterials, which are artificially created materials, consisting of so-called unit cells. In this review article, the applications of negative refractive index metamaterials of different research papers are discussed and compared. Furthermore, the theory behind negative refractive index metamaterials will also be discussed. From the comparison between the research papers, it was concluded that the metamaterial fabricated by Suzuki et al. (Opt. Express **26**, 8314, 2018) can be used in monochromatic optical systems for the low terahertz frequency range, the metamaterial fabricated by Islam et al. (Mater. Technol. **50**, 873, 2016) for applications in radio communication in the gigahertz frequency range, and the simulated metamaterial from Wi et al. (Opt. Commun. **412**, 85, 2018) in almost all applications in the low terahertz range, assuming that the fabricated metamaterial will match the simulated one.

1. Introduction

An important property of transparent every-day materials (media) is the index of refraction. The index of refraction is a measure of how light will bend on the interface between two different media, for example, the transition from air to glass. This electromagnetic interaction, between the light beam and the two media, is described by Snell's law, and is thus completely determined by the refractive indices of both media [1]. For all natural materials, discovered to this date, the refractive index of the material is always positive [2]. However, this did not stop the Soviet/Russian theoretical physicist Veselago, in 1968, to analyse what would happen if the refractive index was indeed negative [3]. Veselago constructed the theoretical framework for the electromagnetic interaction between positive and negative refractive materials, which from now on for theoretical reasons will be called right-handed (RH) and left-handed (LH) materials respectively. The properties that followed from letting the index of refraction be negative, were quite interesting and counter-intuitive, like for example, reverse wave propagation [3].

In over 30 years after the publication of Veselago's paper, still no natural LH materials were found [2]. However, then, in 1999, the theoretical physicist Pendry published a paper in which he describes a way to fabricate a new kind of material, with properties that cannot be achieved by natural materials [4]. He named these new artificially created materials, metamaterials. These metamaterials are unlike any material found in nature. The structure of these metamaterials consists of tiny periodically arranged unit cells. These unit cells are designed and fabricated to have a certain property that natural materials cannot achieve. The overall properties of the metamaterial are determined by the properties of the unit cells, thus since these unit cells can achieve unnatural properties, the metamaterial can achieve unnatural properties as well [5].

The most prominent property a metamaterial can achieve, that is not observed in nature, is a negative refractive index. By carefully designing the unit cells of the metamaterial, the metamaterial can achieve a negative refractive index for a certain frequency range [6]. In 2001, this was experimentally verified. A team of researchers from the University of California built a prism made out of metamaterials. From their measured data it can clearly be concluded that negative refraction was observed. This was the first experimental verification of negative refraction [7].

Modern day research on LH metamaterials focuses on creating LH metamaterials with a negative refractive index in a frequency range important for certain applications. The most important applications include: perfect flat lens imaging, super-resolution imaging, nanoscale antennae engineering, nano-photolithography, and laser physics [8].

The main question that will be discussed in this review article, is: What applications are suitable for different negative refractive index metamaterial designs? This discussion will be done by comparing three recent research papers in the field of LH metamaterials. To have a reliable and clear discussion, the method on finding the research papers used in this review article will be discussed first. After that, the theory behind negative refractive indices and metamaterials will be described. Then, the research articles used for the comparison will be individually analysed. Finally, a comparison between the different research papers as a discussion will be given, resulting in a conclusion to end this review.

2. Literature search method

The research articles used for the comparison were found using the TU Delft Library and Google Scholar. The search terms used were "Negative Index of Refraction Metamaterials" and "Negative Refractive Index Metamaterials". Both search results were filtered for peer reviewed articles that were published since 2016. These filters ensured a good search result of modern-day research articles. From these search results, three articles were chosen, namely:

1. "Negative refractive index metamaterial with high transmission, low reflection, and low loss in the terahertz waveband", by Suzuki et al. [9].
2. "A new wideband negative refractive index metamaterial for dual-band operation", by Islam et al. [10].
3. "Thermally tunable broadband terahertz metamaterials with negative refractive index", by Li et al. [11].

The reason behind picking these three articles in particular, is because these articles together describe a good portion of the modern research in the development of LH metamaterials, and also, because these articles really stood out to me. The first and second article stood out to me due to the keywords "high transmission" and "wideband" in the title of the articles. These keywords describe the important properties of LH metamaterials that need to be improved on for real world applications. The third article stood out to me because I had never heard of a thermally tunable LH metamaterial before, and the new applications it can offer.

The rest of the articles used for this review, either for the theory or additional information, were found using Google Scholar, the TU Delft Library, and using the sources from other articles.

3. Theory on negative refractive metamaterials

As stated earlier, the index of refraction is a measure of how light will bend on the interface between two different media. For example, consider the transition from a medium with refractive index n_1, to a medium with refractive index n_2, as shown in figure 1.

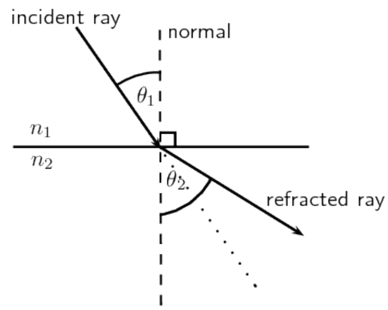

Fig. 1. A visual representation of the interface between two different media with refractive index n_1 and n_2 respectively. The angle between the incident ray and the normal of the interface is denoted with θ_1, and the angle between the refracted ray and the normal with θ_2. Adapted from [12], para. 2.

The scene set in figure 1, is completely described by Snell's law, which is given by:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \ , \tag{1}$$

where θ_1 and θ_2 respectively denote the angle of incidence and the angle of refraction [1]. From Eq. (1), it is thus clear that the relation between the incoming and outgoing angle of the light rays only depend on the refractive index of both media.

3.1. Negative Refractive Indices

While being a very useful quantity, the refractive index, is not the fundamental underlying quantity that describes the electromagnetic interaction between light and matter. The actual underlying quantities are the electric permittivity ϵ and the magnetic permeability μ, which describe how a medium responds to external electric and magnetic fields respectively [3]. These two fundamental properties of a medium determine the overall index of refraction. This relationship between the index of refraction and the underlying quantities ϵ and μ, reads as:

$$n^2(\omega) = \epsilon(\omega)\mu(\omega) \ , \tag{2}$$

where ω denotes the frequency dependence of n, ϵ and μ [6]. In general, ϵ and μ are complex valued, and can thus be written in complex notation as [13]:

$$\epsilon = \epsilon_r + i\epsilon_i = |\epsilon| \exp(i\phi_\epsilon) \ , \tag{3}$$

$$\mu = \mu_r + i\mu_i = |\mu| \exp(i\phi_\mu) \ . \tag{4}$$

Using Eq. (2, 3, 4), the resulting complex valued index of refraction n becomes [14]:

$$n \equiv \mathrm{Re}(n) + i\mathrm{Im}(n) = \sqrt{|\epsilon||\mu|} \exp\left(\frac{i}{2}(\phi_\epsilon + \phi_\mu)\right) \ . \tag{5}$$

The index of refraction thus consists of a real $\mathrm{Re}(n)$ and an imaginary part $\mathrm{Im}(n)$. The real and imaginary part both have different physical interpretations. The interpretation of the real part of the index of refraction is the same interpretation as stated before, how light will bend on the interface between two media, with the only difference being that in Snell's law (Eq. (1)) only $\mathrm{Re}(n)$ should be used [1]. The imaginary component of n describes the attenuation that light will encounter while traversing through a medium [1]. Due to these different physical interpretations, from now on, a negative refractive index will refer only to the sign of the real part of the index of refraction [14].

Using Eq. (5), it can be shown that the refractive index must be negative if, and only if, the following condition holds [14]:

$$\epsilon_r|\mu| + \mu_r|\epsilon| < 0 \ . \tag{6}$$

From this condition it is clear, that when the real part of ϵ and μ are both negative, the resulting index of refraction must also be negative.

3.2. Resonators

Still, the question remains: how can one achieve both negative ϵ and μ? The answer lies in the physics of electric and magnetic resonators. These resonators are, simply put, tiny electronic circuits printed on a electrically non-conductive sheet (substrate) [15]. An example of a few resonator designs are shown in figure 2.

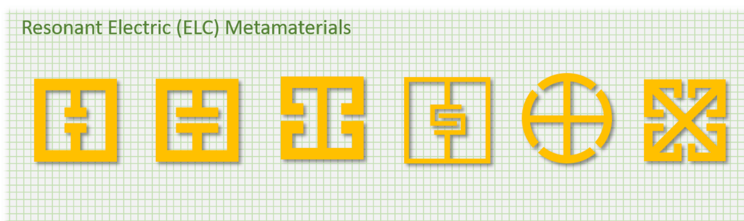

Fig. 2. A few examples of electromagnetic resonator circuits. Adapted from [16], para. 15.

The examples shown in figure 2 are ELC[1] resonators, which both function as an electric and as a magnetic resonator. All these different shapes respond differently to external electric and magnetic fields, and hence, also have different values for ϵ and μ. Thus, by changing the geometry of the resonator, the values of ϵ and μ can be altered as well. From electromagnetic theory, which goes beyond the scope of this article, it can be derived that by carefully designing the geometry of the resonator, the effective permittivity ϵ_{eff} and permeability μ_{eff} of the resonator can both be negative at the same time for a certain frequency range [15]. To evaluate if a resonator design will show simultaneous negative values of ϵ and μ, the resonator can be simulated by using an appropriate simulation software. For simple resonator shapes, the electromagnetic behaviour can also be derived analytically [15].

3.3. Constructing the metamaterial

Using the previously discussed resonators, a metamaterial can be constructed. The construction of the metamerial is done by periodically arranging the unit cells. To achieve a negative refractive index, ELC resonators can be used as unit cells [15]. An example of such a 2D configuration is shown in figure 3.

[1] electric-LC, with LC denoting LC circuits

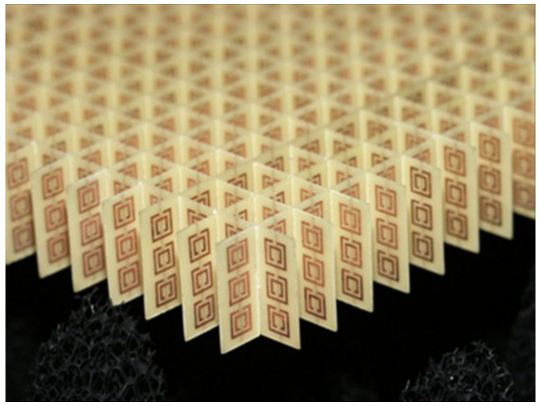

Fig. 3. A 2D metamaterial consisting of electromagnetic resonators. Adapted from [17], para. 1.

By designing the geometry of the resonators such that ϵ_{eff} and μ_{eff} are both negative for a certain frequency range, the metamaterial will have a negative refractive index for that frequency range. However, it is important, for theoretical reasons, that the spatial dimensions of the unit cells are smaller than the wavelength of light for which the metamaterial will be used [15].

4. Analysis of the research articles

4.1. High transmission THz metamaterial, by Suzuki et al.

In the first article, by Suzuki et al. [9], the authors designed, simulated and measured their metamaterial for applications in the terahertz frequency range, such as superlenses. For the unit cells of the metamaterial, the researchers chose for a shifted-cut-wire resonator design, which is shown in figure 4 [9].

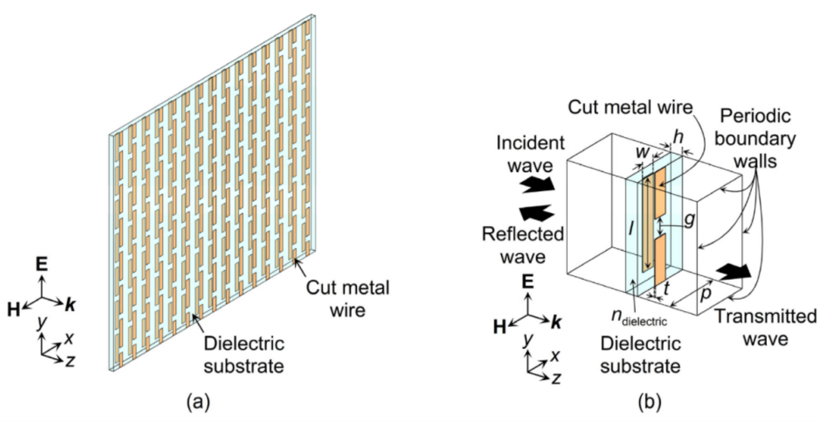

Fig. 4. (a) A 2D metamaterial design consisting of periodically arranged unit cells. (b) The unit cells are made out of cut metal wire, with the metal wire on the back of the dielectric substrate shifted vertically with respect to the cut metal wire on the front. Adapted from [9], p. 8318.

The unit cell shown in figure 4b consists of cut wire on the front and back of the substrate. The cut wire on the back of the substrate is shifted vertically some amount in respect to the cut wire on the front. This shifted-cut-wire design acts as the ELC resonator [9].

Not only did the researchers want to design a metamaterial with a negative refractive index, they also wanted a negative refractive index metamaterial with high transmission and low loss, meaning that most of the light should not be absorbed or reflected by the metamaterial. Using these design guidelines, the exact dimensions of the shifted-cut-wire pattern were determined using simulations. From the simulations, a metamaterial with refractive index $-5 + 0.22i$ at 0.42 THz and transmission coefficient of 83% was designed [10].

With the determined dimensions of the resonator pattern, the metamaterial was fabricated. The researchers measured the properties of their metamaterial, and the results were a refractive index of $-4.2 + 0.17i$ at 0.42 THz, and a transmission coefficient of 81.5%. The researchers also compared the measurements and simulation for the entire frequency range of their interest, which is shown in figure 5 [9].

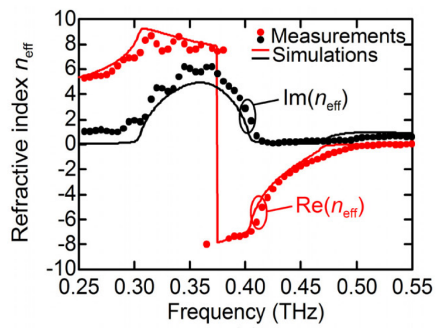

Fig. 5. A comparison between the measured and simulated refractive index of a metamaterial designed, simulated, fabricated and measured by Suzuki et al.. Adapted from [9], p. 8322

4.2. Wideband GHz metamaterial, by Islam et al.

In the article written by Islam et al. [10], the researchers designed, simulated, fabricated and measured their metamaterial for the gigahertz range, with applications in radio-communication. The main goal of this research project was to design a metamaterial with a negative refractive index for a wide range of frequencies in the gigahertz spectrum. For the unit cells, the researchers chose for a bare-H-shaped resonator design, which is shown in figure 6 [10].

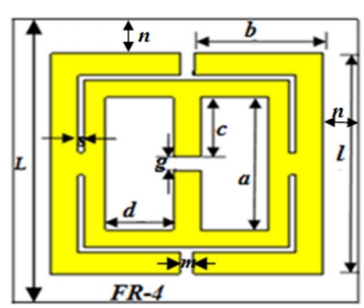

Fig. 6. Bare-H-shaped resonator design of a unit cell for a metamaterial using FR-4 substrate, with spatial dimensions $a, b, c, d, g, n, m, l, L$ and δ. Adapted from [10], p. 874.

Using simulations, the researchers determined the optimal dimensions of all the parameters of the resonator design. With these results, they fabricated two metamaterials. The difference between these metamaterials is in the substrates used, with one metamaterial using FR-4 as substrate and the other Rogers RT 3010. The resulting measured refractive index of both metamaterials is shown in figure 7 [10].

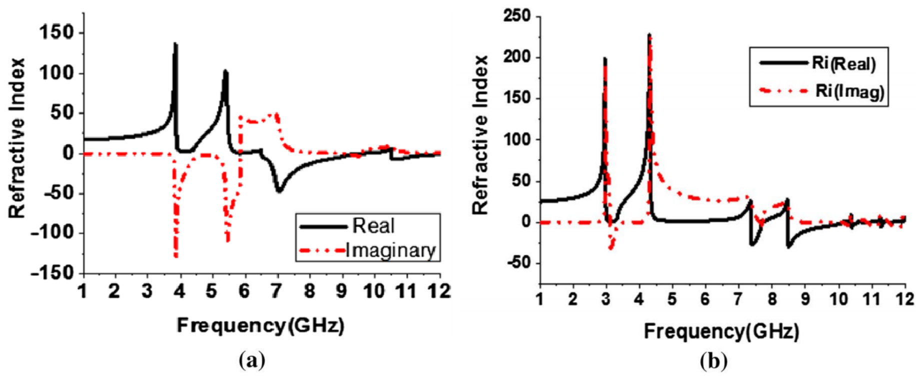

Fig. 7. Measurement results of the refractive index of two metamaterials designed, fabricated and measured by Islam et al.. The metamaterials differ in the substrate used, with one of the metamaterials using a standard FR-4 substrate (a) and the other one a Rogers RT 3010 substrate (b). Adapted from [10], pp. 876-877.

4.3. Thermally tunable THz metamaterial, by Li et al.

In the last article, written by Li et al. [11], the researchers simulated a metamaterial design in the terahertz range, with an interesting property, namely, a thermally tunable refractive index. By varying the temperature of the proposed metamaterial, the refractive index can be tuned to specific values. This property arises from the unit cells used for the metamaterial, which are shown in figure 8 [11].

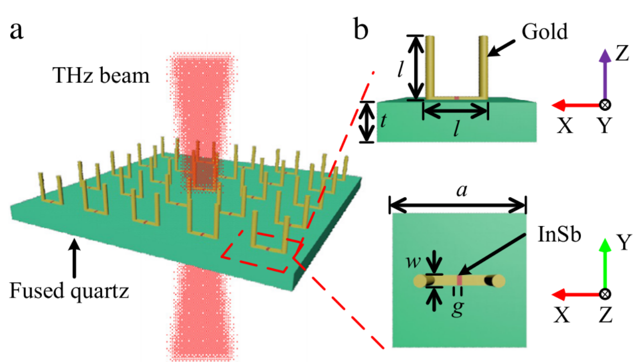

Fig. 8. (a) A metamaterial consisting of (b) U-shaped unit cells made out of gold. The two opposing gold L-shapes are connected via a piece of indium antimonide (InSb). Adapted from [11], p. 86.

As shown in figure 8(b), the unit cell consists out of two opposing gold L-shapes that are connected via a piece of indium antimonide (InSb). The conductivity of this piece of InSb is strongly temperature dependent, and hence, the overall ϵ_{eff} and μ_{eff} of the unit cell will also be temperature dependent [11].

The researchers validated their design using simulations. The results of the simulations are shown in figure 9 [11].

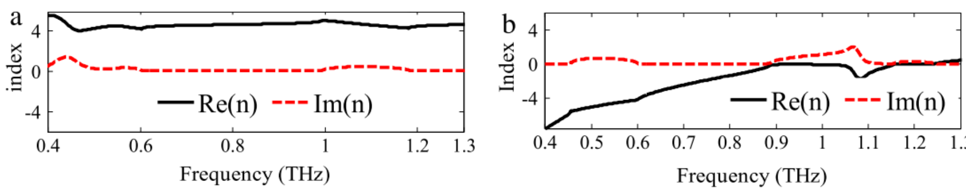

Fig. 9. Simulation results of the refractive index of a metamaterial simulated by Li et al. at temperatures (a) T = 250 K and (b) T = 400 K. Adapted from [11], p. 88.

5. Discussion and comparison

The first and second article discussed in the previous section demonstrate a good portion of the modern-day research into metamaterials. In the first article, the frequency range of interest was the high-gigahertz / low-terahertz spectrum. One of the goals of metamaterial research is to eventually fabricate metamaterials that work in the frequency range of visible light (high-terahertz). To achieve this, it is important to first start at the lower frequencies, and work up to the higher frequencies, because the fabrication of metamaterial designs increases in difficulty the higher the application frequency [9].

The second article focuses on the low-gigahertz spectrum. This is an important spectrum, because this spectrum includes frequencies used for radio transmission. In the article, the authors wanted to design a metamaterial with a broad range of frequencies for which negative refraction occurs. This design goal directly translates to the application, because radio transmission uses a broad bandwidth of frequencies [10].

The last article was chosen for two reasons. Firstly, to demonstrate that metamaterials cannot only achieve a negative refractive index, but also that with the power of carefully designing the unit cells, some properties of the metamaterial can be made temperature dependent, or even dependent on another external variable for that matter [11]. Secondly, because this temperature controlled metamaterial can lead to new applications, such as thermally tunable optics.

To compare the different methods of designing a metamaterial with a negative refractive index, the following measures and categories will be used: Measurement (Bandwidth and Transmission), Design (Frequency and Manufacturing) and Use (Application and Further Research). The explanation of the measures and the comparison between the articles is given in table 1.

Table 1. A comparison between the designs of three metamaterials by Suzuki et al. [9], Islam et al. [10] and Li et al. [11]. The bandwidth is a measure for the frequency range that negative refraction is observed, and is calculated as a relative bandwidth. The transmission is a measure of the average transmission coefficient of the metamaterial. The frequency indicates for which frequency range the metamaterial will be used. Manufacturing is a measure of how well the resonator design can be fabricated, and is indicated with either '+' or '-'. The application and further research measures indicate the application and further research of the metamaterial.

		Article 1: Suzuki et al.	Article 2: Islam et al.	Article 3: Li et al.
Measurement	Bandwidth	20%	40%	70%[2]
	Transmission	83%	55%	85%[2]
Design	Frequency	THz	GHz	THz
	Manufacturing	+	+	-
Use	Application	Superlenses	Radio transmission	Tunable optics
	Further Research	Increasing bandwidth	Increasing transmission	Fabricating design

For the gigahertz range, the metamaterial by Islam et al. stands out. The bandwidth of the metamaterial covers the so-called C and X bands, which are frequently used in radio and satellite communication [10]. The transmittance of this metamaterial is also relatively good, but can be improved.

For the terahertz range, the simulated metamaterial by Li et al. not only is tunable, but also has a relatively great bandwidth and transmittance. However, this metamaterial was only simulated, so it may be that the fabricated version will not match the simulation. The metamaterial by Suzuki would be a great fit in optical systems that only use a specific frequency (monochromatic optical systems), due to the lower bandwidth.

6. Conclusion

To summarise, in this review article, three recent research papers [9–11] on negative refractive index metamaterials were discussed and compared. The metamaterials designed in these research papers have applications in the giga- and terahertz frequency band, ranging from superlenses to applications in radio transmission.

The comparison of the metamaterials of each of the papers gave good insight into what application is suitable for the different metamaterials, and what can be done for further research to improve on them. The metamaterial designed and fabricated by Suzuki et al., has applications in monochromatic optical systems in the terahertz spectrum, due to the high transmission and low bandwidth of the metamaterial. The metamaterial designed and fabricated by Islam et al. is suitable for applications in radio transmission, resulting from the great bandwidth and relatively good transmittance. Finally, the metamaterial designed and simulated by Li et al. shows great promises in all applications in the terahertz range, however, not with absolute certainty, because this metamaterial has not been fabricated and measured yet.

Future research on negative refractive metamaterials can be conducted to increase bandwidth, transmission and the application frequency.

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