Turbulence in non-ideal fluids

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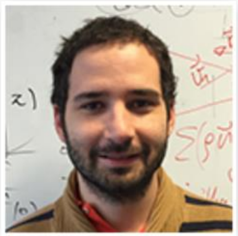

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FOM NWO PRACE SURF SARA cscs erc

Motivation

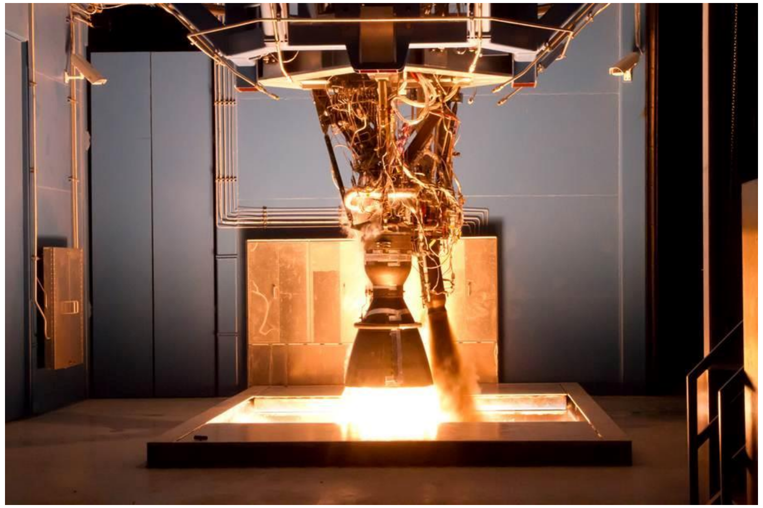
The continuous demand to increase

efficiency in **energy conversion** systems

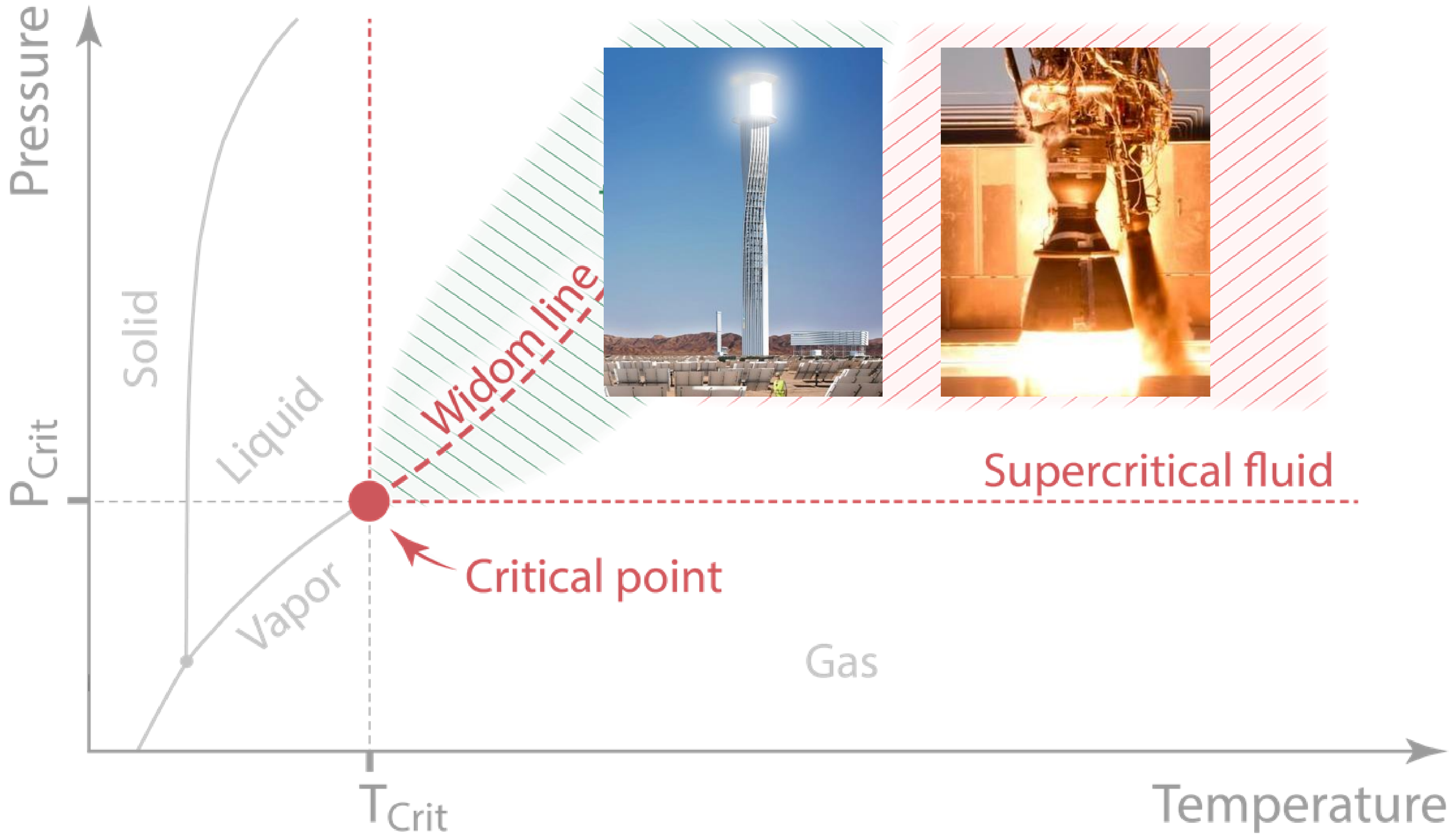
&

productivity in **chemical processes**

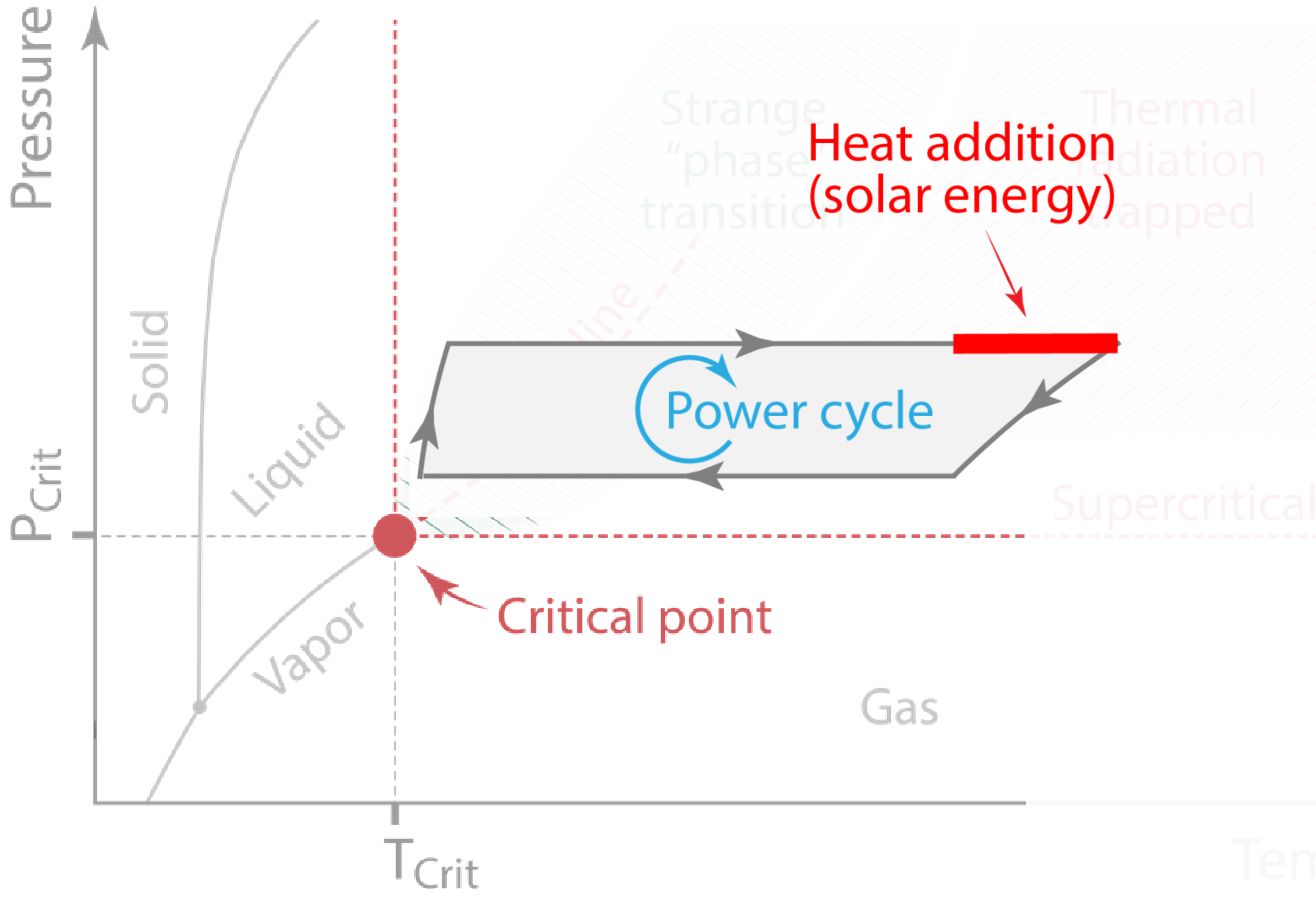
forces engineers to use fluids at increasingly higher **pressures** and **temperatures!**



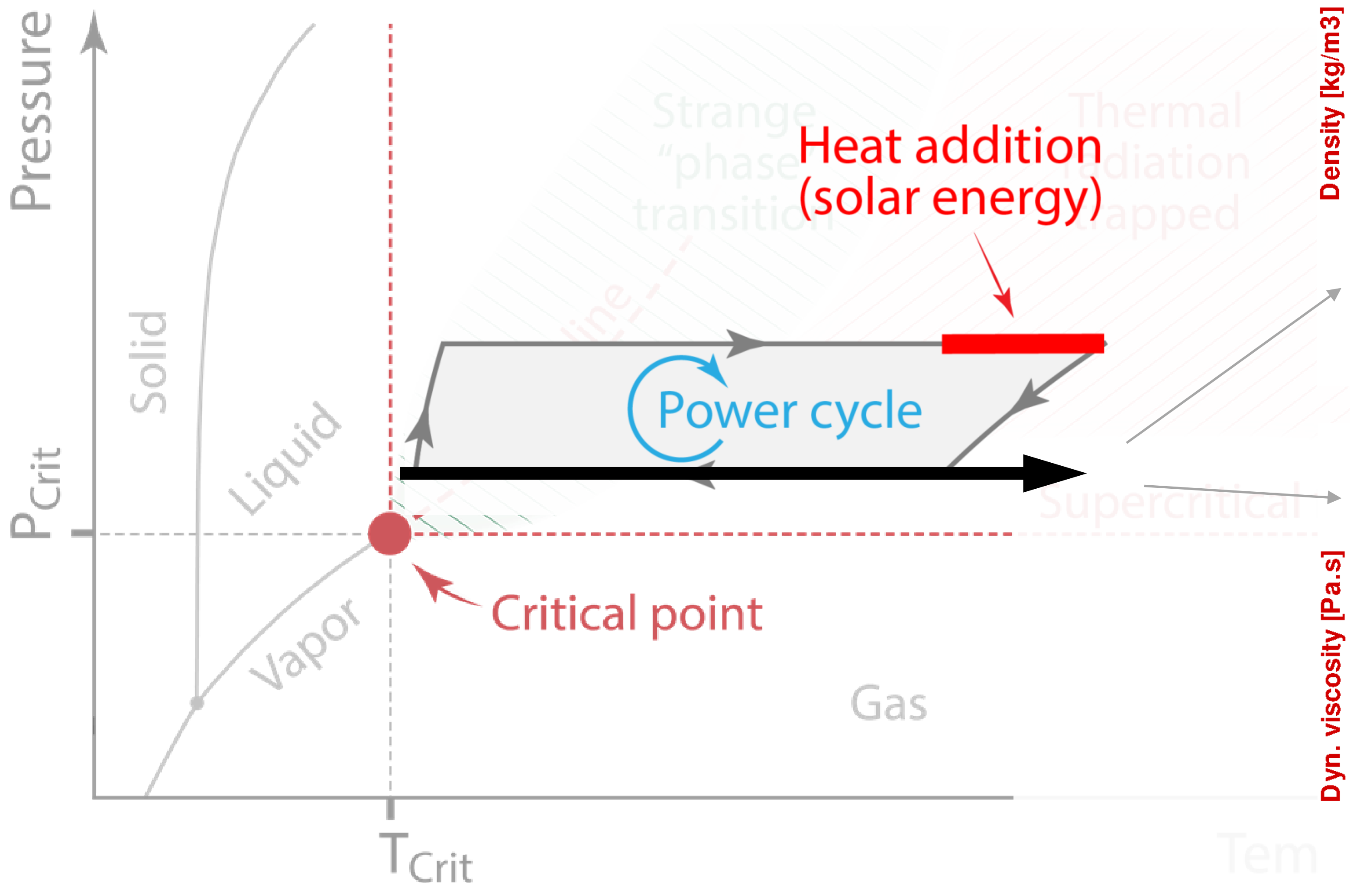
Phase diagram of an arbitrary substance

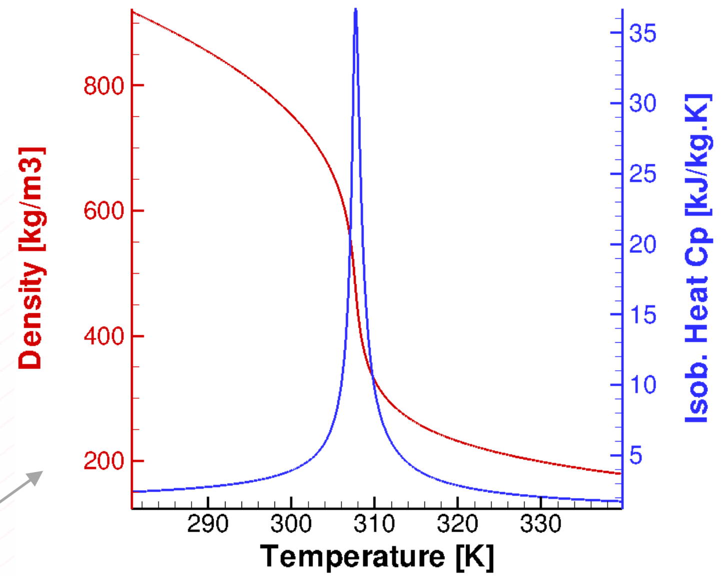


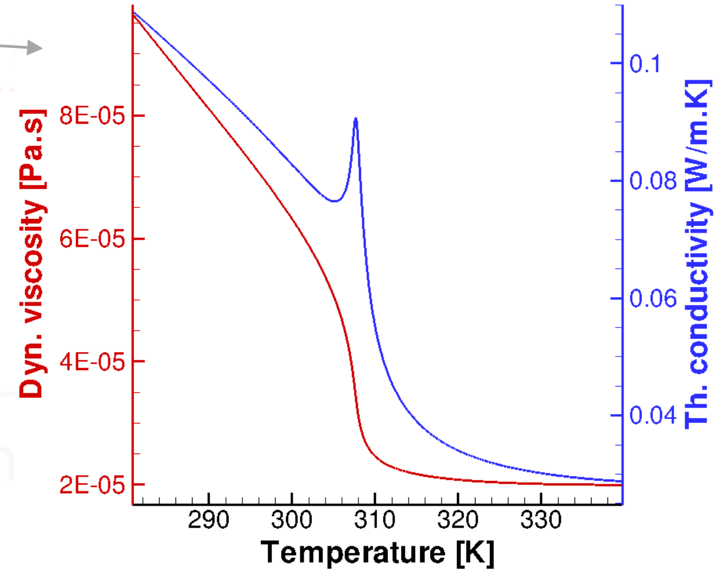
Supercritical power cycle



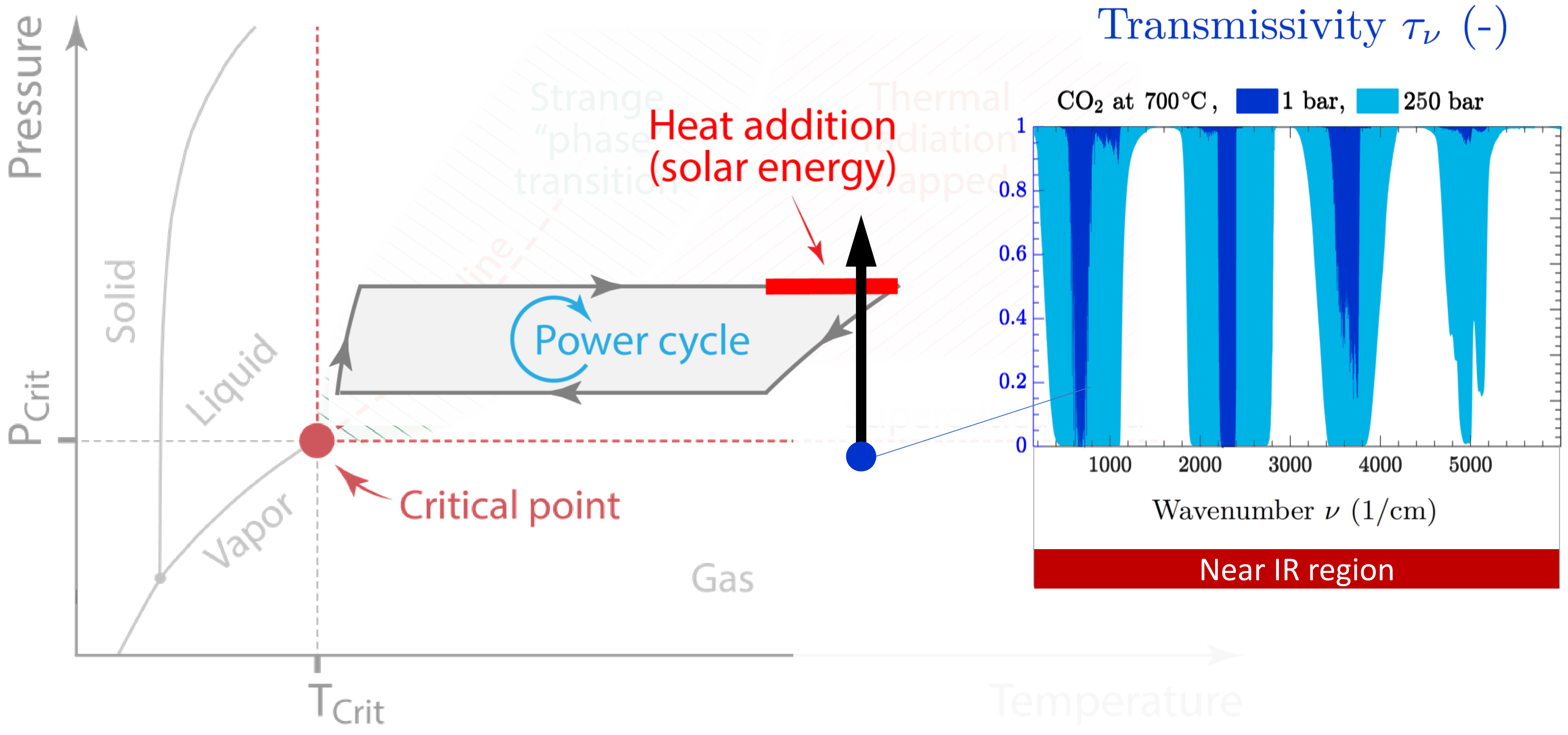
Properties of CO_2 @ 80 bar



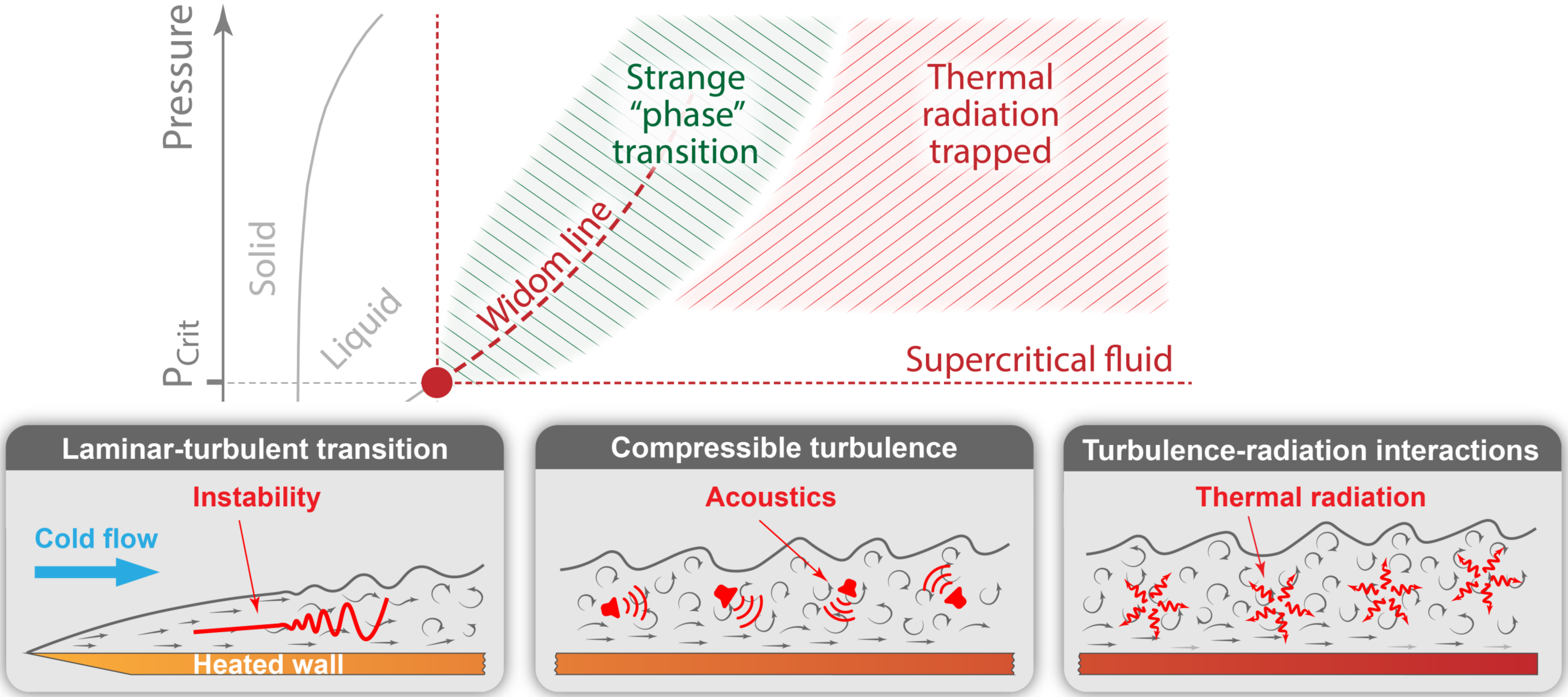




Transmissivity at high pressures

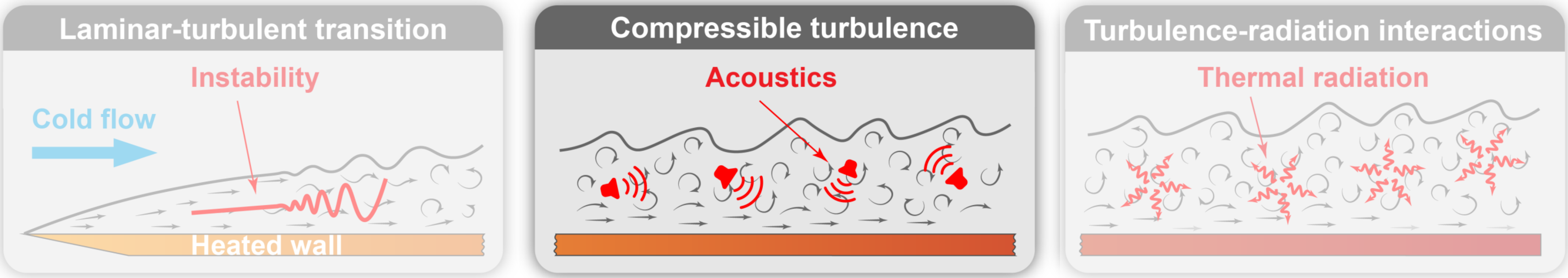


Challenge: turbulence in supercritical fluids



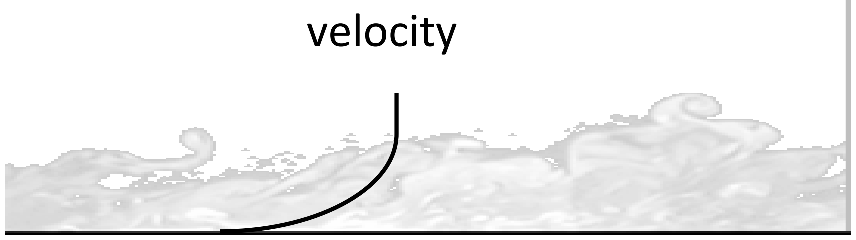
How does compressibility affect turbulence?

1. “Universal” scaling laws
2. Turbulence models for RANS (& LES)



Scaling of turbulent statistics

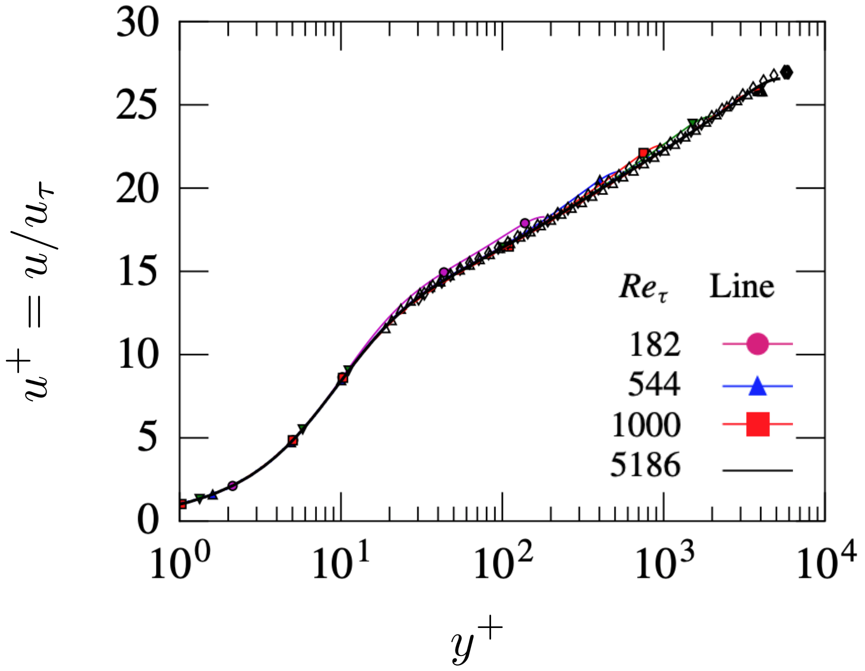
Conventional (incompressible) wall based scaling		Compressible semi-local scaling (Huang et al., 1995)
$u_\tau = \sqrt{\tau_w/\rho_w}$	Friction velocity	$u_\tau^\star = \sqrt{\tau_w/\overline{\rho}}$
$\delta_v = \mu_w/(\rho_w u_\tau)$	Viscous length scale	$\delta_v^\star = \overline{\mu}/(\overline{\rho} u_\tau^\star)$
$Re_\tau = h/\delta_v$	Friction Reynolds number	$Re_\tau^\star = h/\delta_v^\star = \sqrt{(\overline{\rho}/\rho_w)}/(\overline{\mu}/\mu_w) Re_\tau$
$y^+ = (y/h)\ Re_\tau$	Non-dim wall distance	$y^\star = (y/h)\ Re_\tau^\star$



Velocity scaling

Incompressible channel

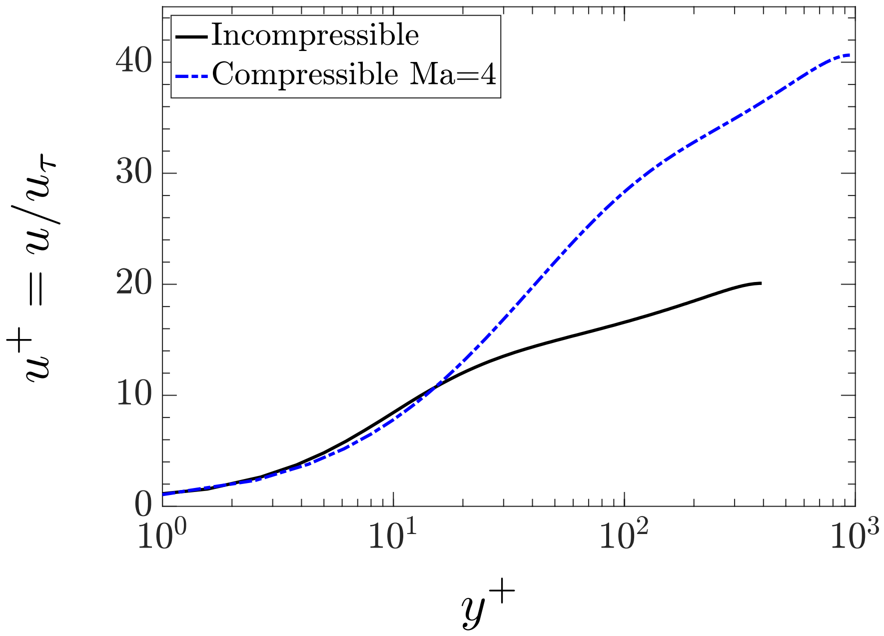
(Lee & Moser, JFM 2015)



Scaling based on wall units

Compressible channel

(Own unpublished work)



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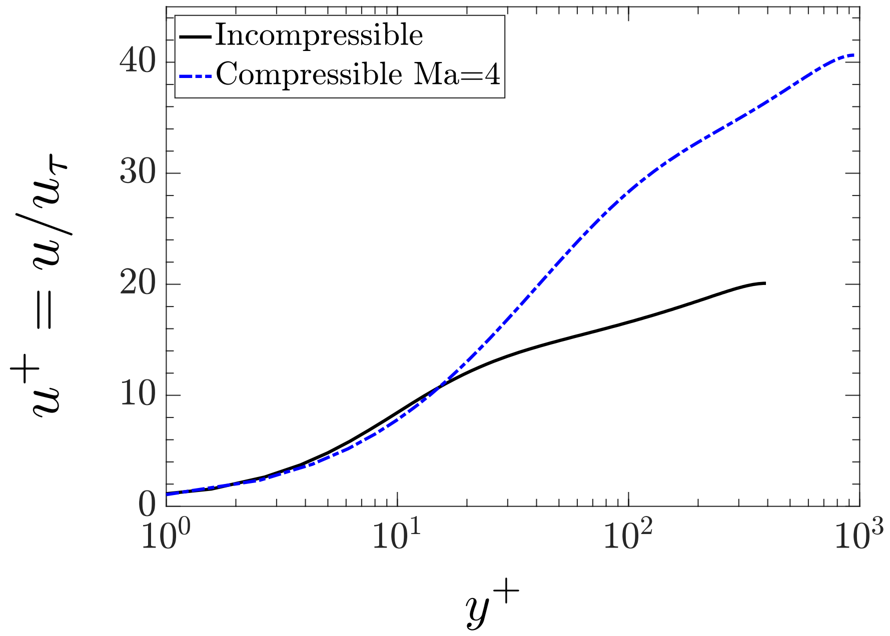
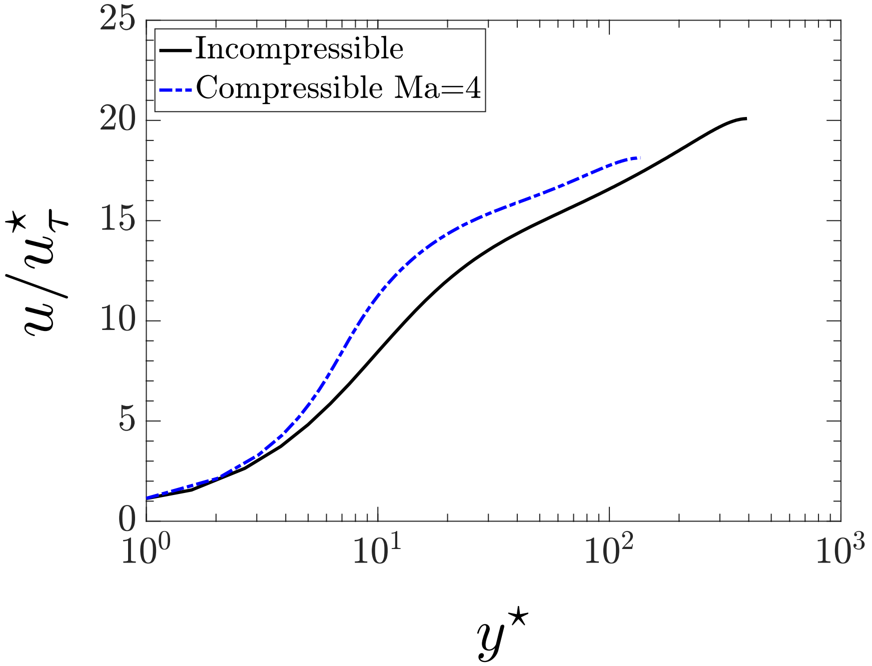


Scaling based on wall units

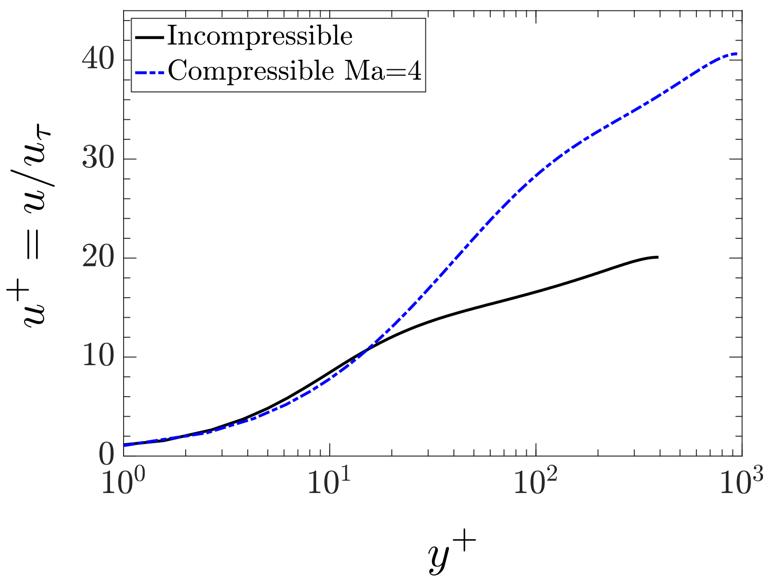


Scaling based on semi-local units

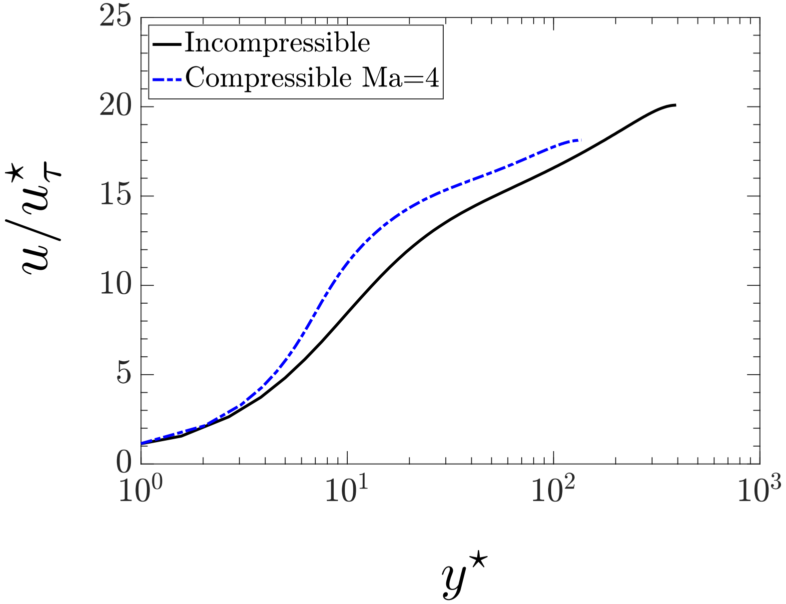
Velocity scaling

Compressible channel

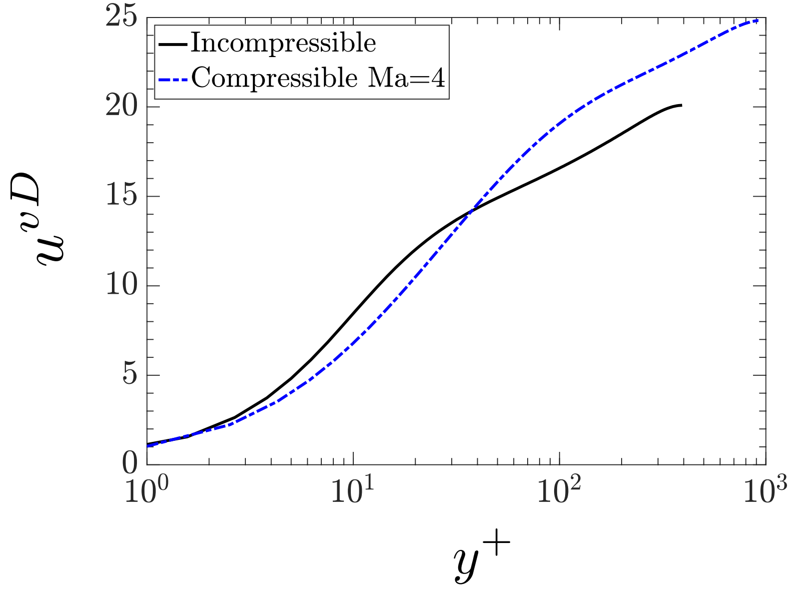
(Own unpublished work)



Scaling based on wall units



Scaling based on semi-local units



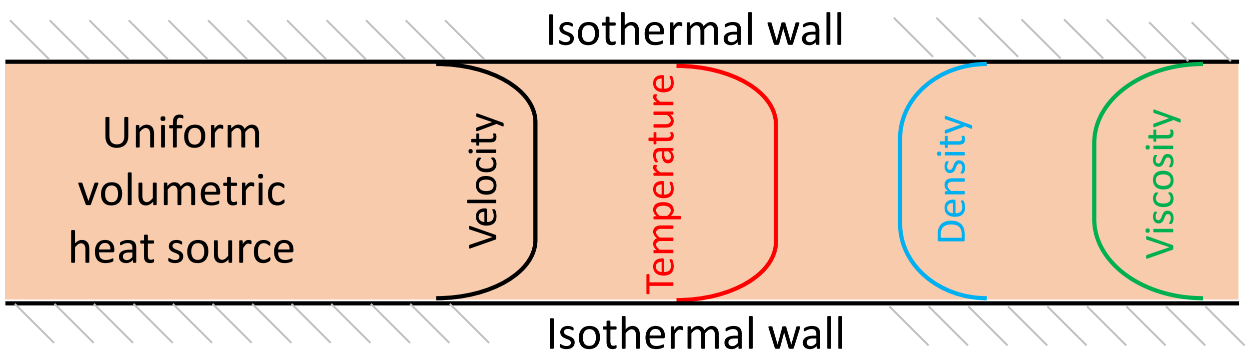
van Driest velocity scaling, (1951)

$$u^{vD} = \int_0^{u^+} \sqrt{\rho/\rho_w}\, du^+$$

Is it possible to derive a “universal” scaling for compressible flows?

Turbulent channel flow configuration

Simple setup



Governing equations

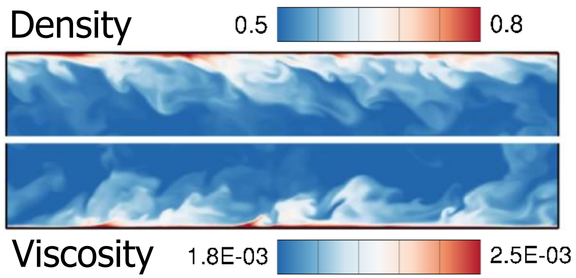
- Continuity $\partial_t \rho + \partial_{x_j}(\rho u_j) = 0,$
- Momentum $\partial_t(\rho u_i) + \partial_{x_j}(\rho u_i u_j) = -\partial_{x_i} p + \frac{1}{Re_\tau}\partial_{x_j}\left(2\mu S_{ij}\right),$
- Enthalpy $\partial_t(\rho H) + \partial_{x_j}(\rho u_j H) = \frac{1}{Re_\tau Pr_w}\partial_{x_j}(\lambda \partial_{x_j} T) + \frac{\phi}{Re_\tau Pr}$

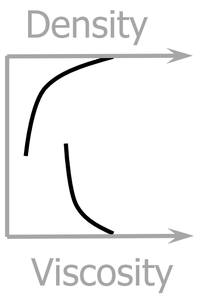
Numerical schemes

- Low-Mach number/anelastic approximation of Navier-Stokes equations (no acoustic waves)
- 6^{th} order compact finite difference (staggered) in wall normal direction
- Pseudo spectral method in periodic directions (skew-symmetric form of advective terms)

Volumetrically heated channel flows

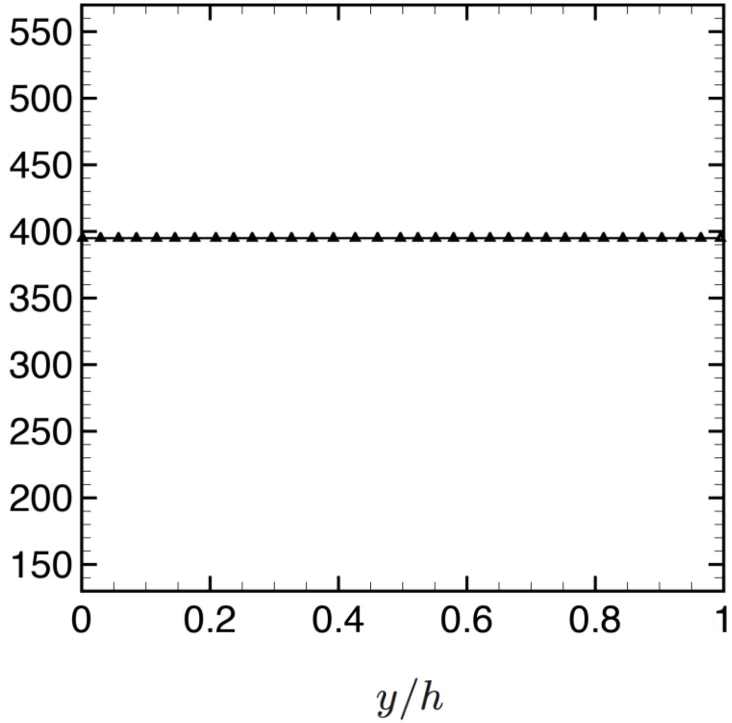
	Case	Density ρ/ρ_w	Viscosity μ/μ_w
▲	CP395	1	1
—	$\mathrm{CRe}_\tau^\star$	$(T/T_w)^{-1}$	$(T/T_w)^{-0.5}$





Semi-local Reynolds number

$$Re_\tau^\star = \sqrt{(\overline{\rho}/\rho_w)}/(\overline{\mu}/\mu_w)Re_\tau$$



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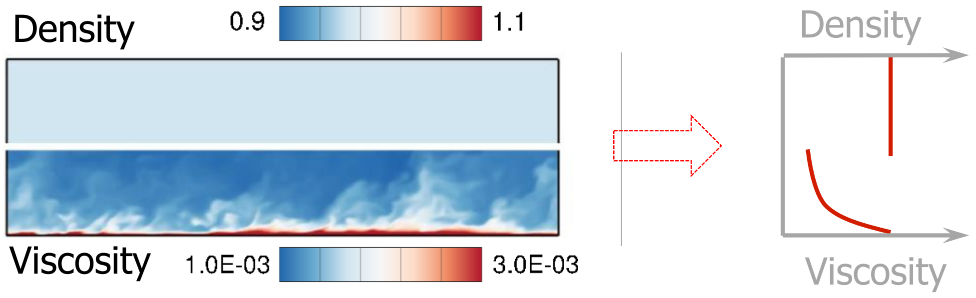
—	$\mathrm{CRe}_\tau^\star$	$(T/T_w)^{-1}$	$(T/T_w)^{-0.5}$
o	LL	1	$(T/T_w)^{-1}$
····	$\mathrm{SRe}_{\tau LL}^\star$	$(T/T_w)^{0.6}$	$(T/T_w)^{-0.75}$



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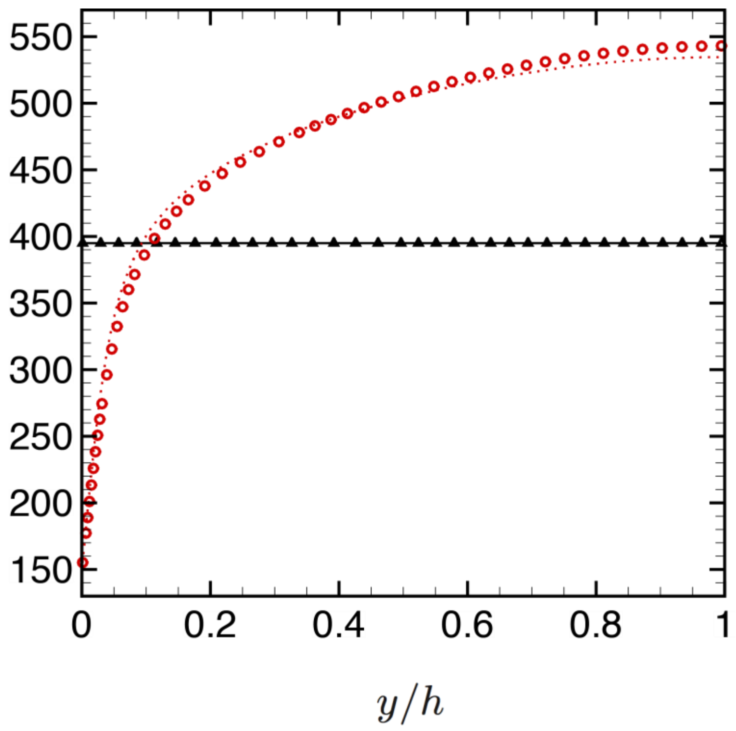

Volumetrically heated channel flows

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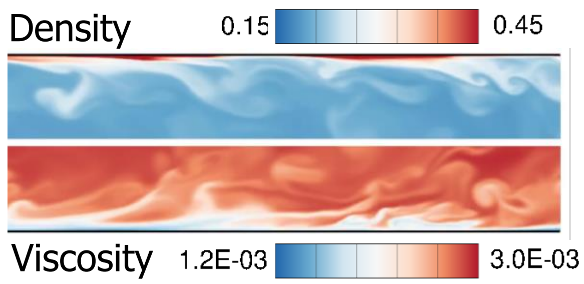
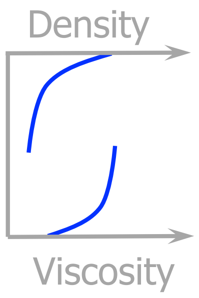
o	LL	1	$(T/T_w)^{-1}$
····	$SRe_{\tau LL}^\star$	$(T/T_w)^{0.6}$	$(T/T_w)^{-0.75}$
- -	GL	$(T/T_w)^{-1}$	$(T/T_w)^{0.7}$
▫	$SRe_{\tau GL}^\star$	1	$(T/T_w)^{1.2}$





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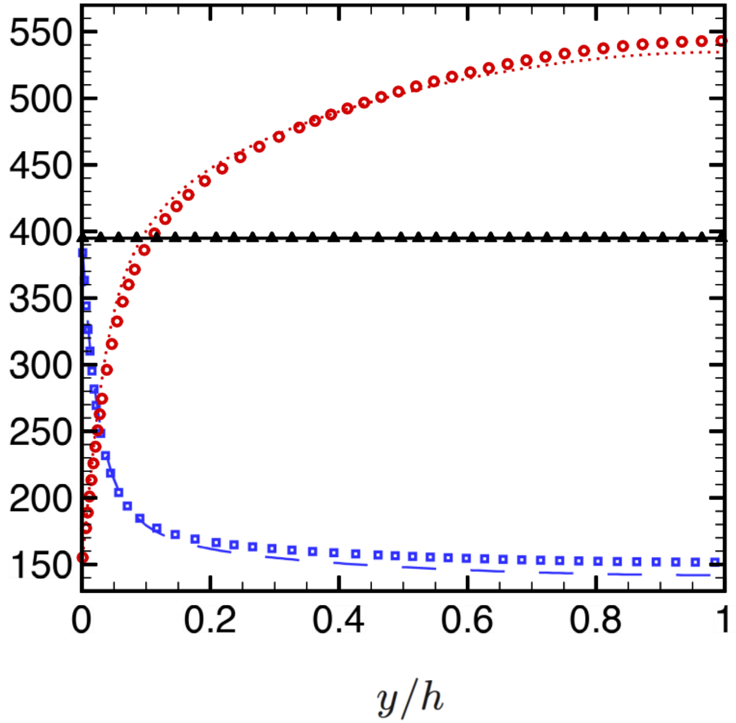
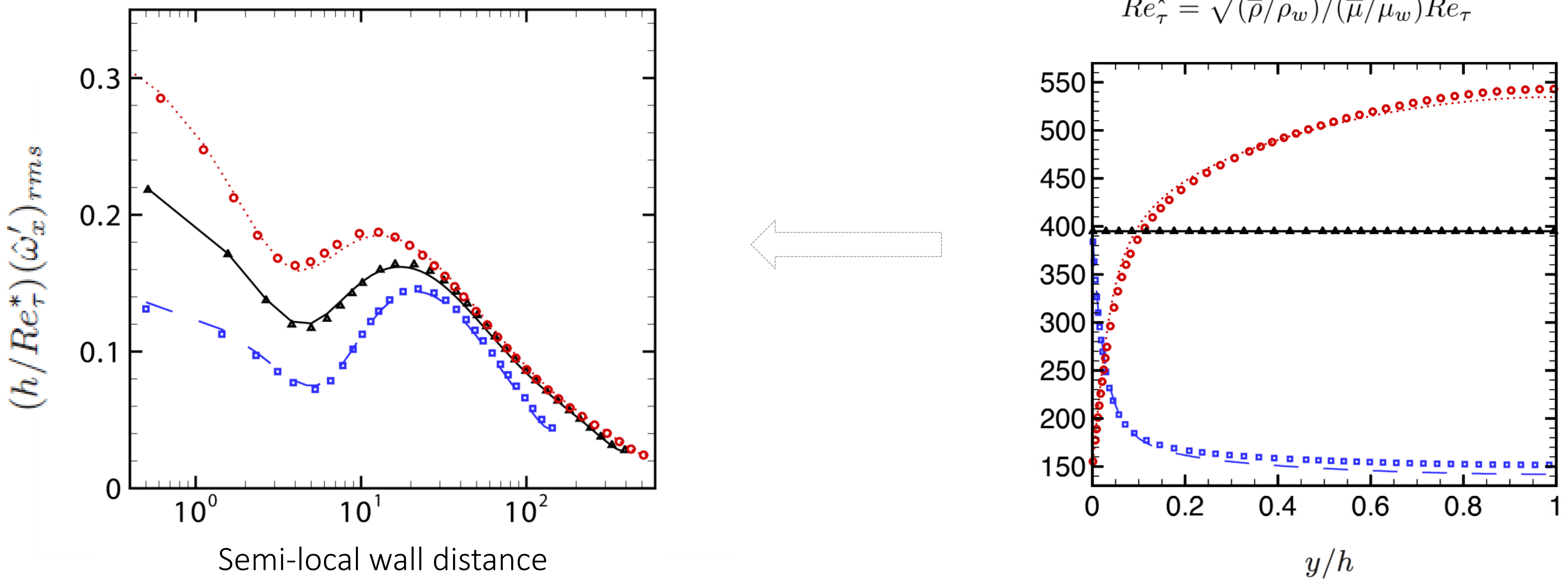


Turbulent statistics

Streamwise vorticity fluctuations

Semi-local Reynolds number

$$Re_\tau^\star = \sqrt{(\overline{\rho}/\rho_w)}/(\overline{\mu}/\mu_w)Re_\tau$$



Semi-local Reynolds number is governing parameter of turbulence statistics!

Extending semi-local scaling framework (Patel et al., PoF 2015)

- Idea: use semi-local scaling transformations for evolution equations

$$\widehat{\rho} = \rho/\overline{\rho}, \quad \widehat{\mu} = \mu/\overline{\mu}, \quad \widehat{u} = u/u_\tau^\star \qquad \text{with: } u_\tau^\star = \sqrt{\tau_w/\overline{\rho}}$$

- Evolution equation of fluctuating velocity components

Mean density gradient

$$\partial_{\hat{t}}(\hat{u}_i') + \partial_{\hat{x}_j}(\hat{u}_i'\hat{u}_j') + \hat{v}'\partial_{\hat{y}}(\overline{u}^{vd})\delta_{i1} + \bar{\hat{u}}_j\partial_{\hat{x}_j}(\hat{u}_i') \approx -\partial_{\hat{x}_i}\hat{p}' + \partial_{\hat{x}_j}(\overline{\hat{u}_i'\hat{u}_j'}) + \partial_{\hat{x}_j}\frac{1}{Re_\tau^*}\left(2\hat{S}_{ij}' - \hat{D}_{ij}\right)$$

Characteristic velocity

Effective viscosity

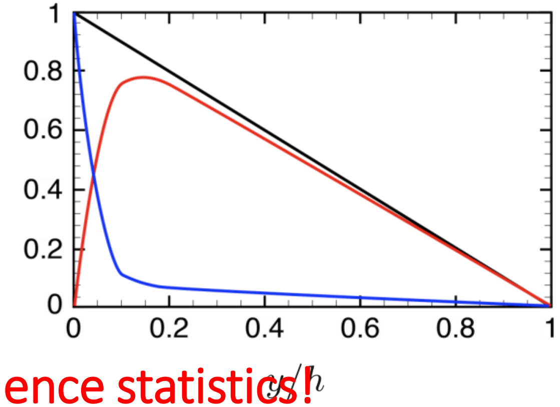
- Stress balance equation (fully dev. turbulent channel)

$$-\widetilde{\hat{u}''\hat{v}''} + \frac{h}{Re_\tau^\star}\frac{d\overline{u}^{\mathrm{vD}}}{d\hat{y}} \approx \frac{\tau}{\tau_w} = \left(1 - \frac{y}{h}\right)$$

Turbulent shear stress

Viscous stress

Total stress

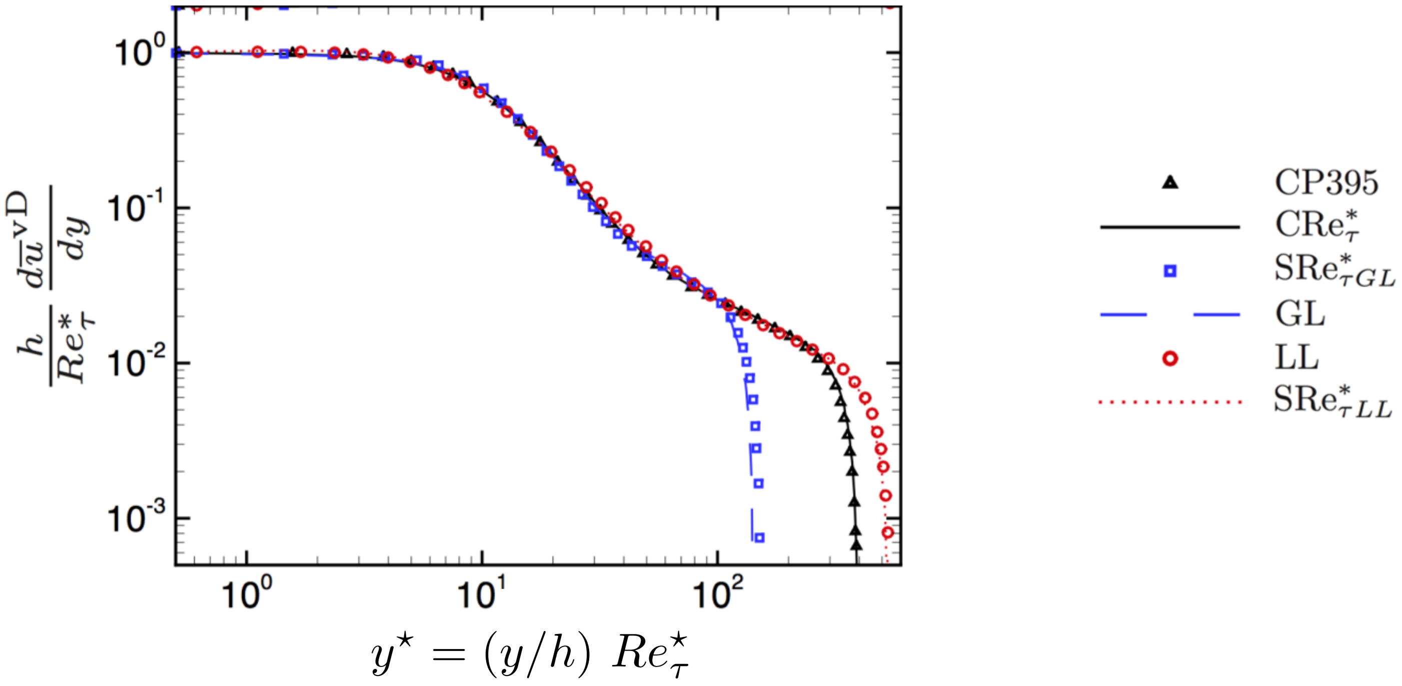


Semi-local Reynolds number is governing parameter of turbulence statistics!

Semi-locally scaled stress balance equation

Viscous stress

$-\widetilde{\hat{u}''\hat{v}''} +$



Semi-locally scaled stress balance equation

$$-\widetilde{\hat{u}''\hat{v}''} + \underbrace{\frac{h}{Re_\tau^\star}\frac{d\overline{u}^{\mathrm{vD}}}{d\hat{y}}}_{\text{Viscous stress}} \approx \frac{\tau}{\tau_w} = \left(1 - \frac{y}{h}\right)$$

Scaled viscous stress is basis for transformation:

$$\frac{h}{Re_\tau^\star}\left(\frac{dy^\star}{dy}\right)\frac{d\overline{u}^{\mathrm{vD}}}{dy^\star} = \Phi(y^\star)$$

with:

$$y^\star = yRe_\tau^\star/h$$

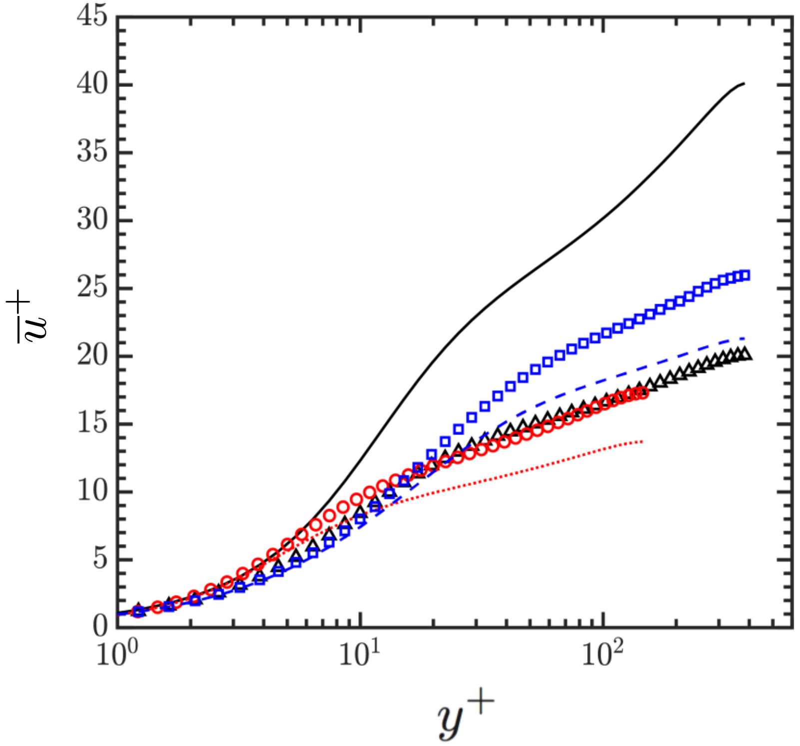
Universal velocity transformation:

$$\overline{u}^\star = \int_0^{\overline{u}^{\mathrm{vD}}}\left(1 + \frac{y}{Re_\tau^\star}\frac{dRe_\tau^\star}{dy}\right)d\overline{u}^{\mathrm{vD}}$$

Velocity scaling for channel flows

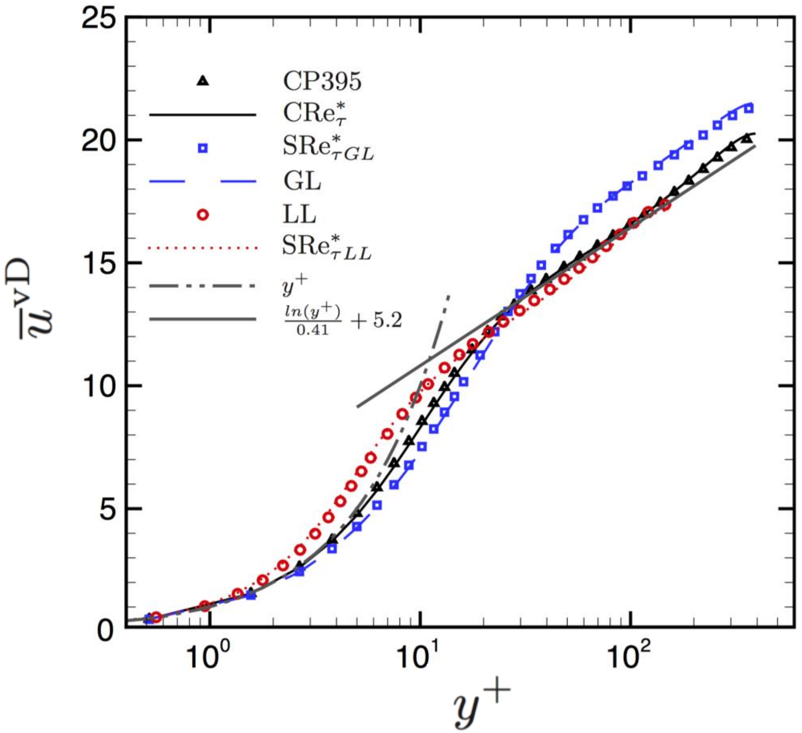
Scaling based on wall units

$$\overline{u}^+ = \overline{u}/u_\tau$$



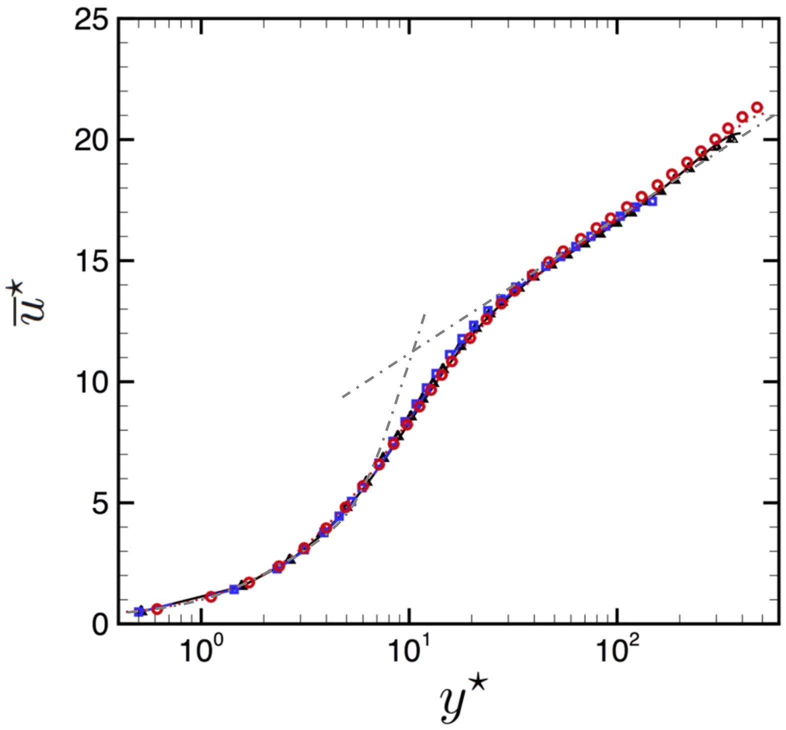
Van Driest transformation

$$\overline{u}^{\mathrm{vD}} = \int_0^{\overline{u}} \sqrt{\overline{\rho}/\rho_w}\, d(\overline{u}/u_\tau)$$



"Universal" transformation

$$\overline{u}^\star = \int_0^{\overline{u}^{\mathrm{vD}}} \left(1 + \frac{y}{Re^\star_\tau}\frac{dRe^\star_\tau}{dy}\right) d\overline{u}^{\mathrm{vD}}$$



How do conventional turbulence models perform?

How do conventional turbulence models perform?

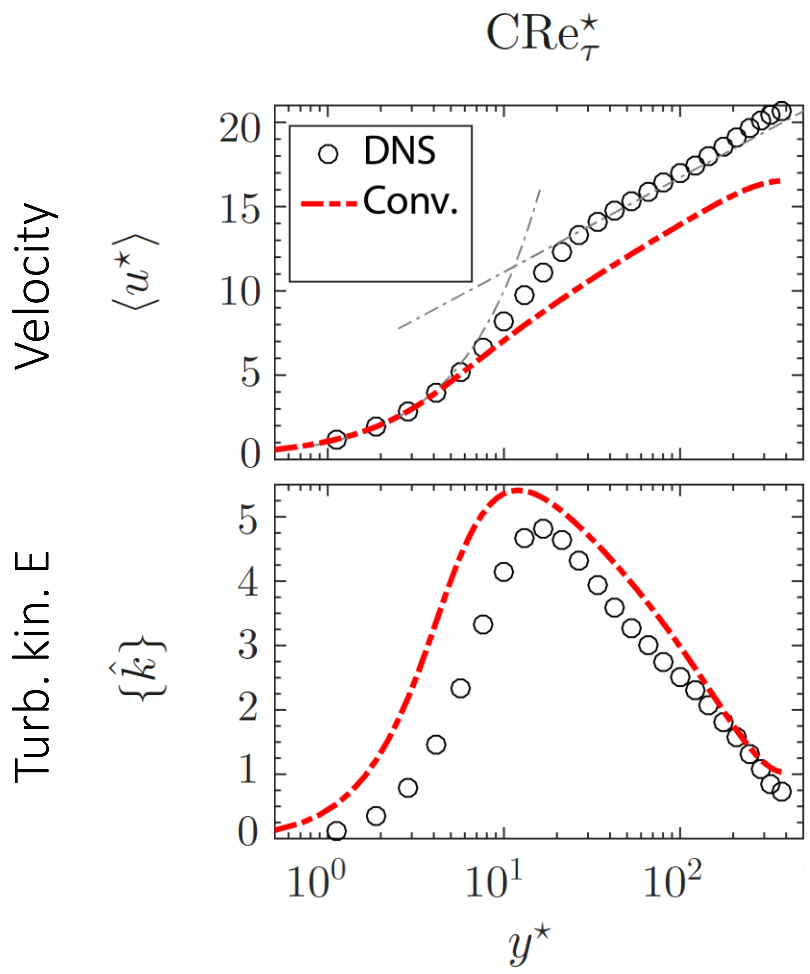
Reynolds/Favre averaged equations for fully developed turbulent channel

- Streamwise momentum $$\frac{\partial}{\partial y}\left[\left(\frac{\mu}{Re}+\mu_t\right)\frac{\partial u}{\partial y}\right]=-\rho f_x,$$
- Energy equation $$\frac{\partial}{\partial y}\left[\left(\frac{\lambda}{RePr_w Ec}+\frac{c_p\mu_t}{Pr_t}\right)\frac{\partial T}{\partial y}\right]=-\Phi,$$
- Turb kinetic energy $$-\frac{\partial}{\partial y}\left[(\overline{\mu}+\mu_t/\sigma_k)\frac{\partial k}{\partial y}\right]=P_k-\overline{\rho}\epsilon$$

  + supporting eqs. $$\vdots$$

- Eddy viscosity $$\mu_t = C_\mu f_\mu \overline{\rho}\, k^2/\epsilon$$

V2F turbulence model



Nusslet number error: 20.3%

Extending semi-local scaling framework

- Using semi-local scaling transformations to non-dimensionalize conservation equations

$$\widehat{\rho} = \rho/\overline{\rho}, \quad \widehat{\mu} = \mu\ /\overline{\mu}, \quad \widehat{u} = u/u_\tau^\star \qquad \text{with: } u_\tau^\star = \sqrt{\tau_w/\overline{\rho}}$$

- Momentum:

viscous terms governed by semi-local Reynolds number

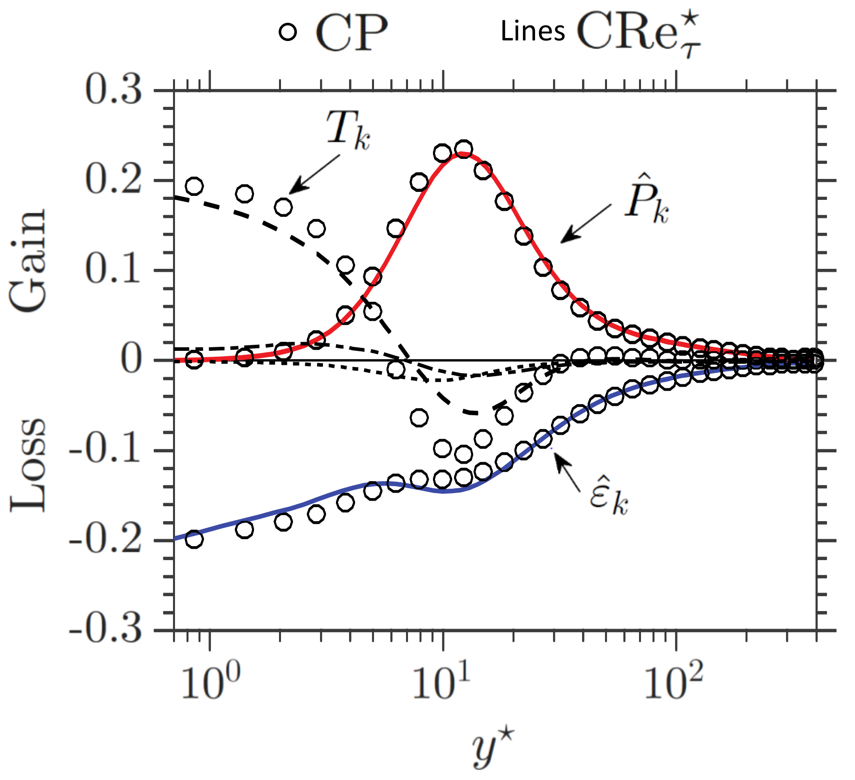
$$\hat{\rho}\frac{\partial \hat{u}_i}{\partial \hat{t}} + \hat{\rho}\hat{u}_j\frac{\partial \hat{u}_i}{\partial \hat{x}_j} - \hat{u}_i\hat{\rho}\hat{u}_j C_j = -\frac{\partial \hat{p}}{\partial \hat{x}_i} + \frac{\partial}{\partial \hat{x}_j}\left[\frac{2\hat{\mu}}{\mathrm{Re}_\tau^\star}\left(\hat{S}_{ij} - \hat{D}_{ij}\right)\right]$$

- Turbulent kinetic energy

$$\frac{\partial \overline{\hat{\rho}\hat{k}}}{\partial \hat{t}} + \frac{\partial \overline{\hat{\rho}\hat{k}}\widetilde{\hat{u}}_j}{\partial \hat{x}_j} = -P_k - \overline{\hat{\tau}_{ij}\frac{\partial \hat{u}_i''}{\partial \hat{x}_j}} + \frac{\partial}{\partial \hat{x}_j}\left(\overline{\hat{u}_i''\hat{\tau}_{ij}} - \overline{\hat{\rho}\hat{u}_j''\tfrac{1}{2}\hat{u}_i''\hat{u}_i''}\right)$$
$$+ \overline{\hat{u}_j''\frac{\partial \hat{p}}{\partial \hat{x}_j}} + \left(\overline{\hat{\rho}\hat{k}}\widetilde{\hat{u}}_j + \overline{\hat{\rho}\hat{u}_j''\tfrac{1}{2}\hat{u}_i''\hat{u}_i''}\right) C_j$$

Semi-locally scaled turbulent kinetic energy budget (Pecnik, JFM 2016)

$$0 = -P_k \underbrace{- \overline{\hat{\tau}_{ij} \frac{\partial \hat{u}_i''}{\partial \hat{x}_j}}}_{\hat{\varepsilon}_k} + \underbrace{\frac{\partial}{\partial \hat{x}_j}\left(\overline{\hat{u}_i'' \hat{\tau}_{ij}} - \overline{\hat{\rho} \hat{u}_j'' \tfrac{1}{2} \hat{u}_i'' \hat{u}_i''}\right)}_{\hat{T}_k} + \underbrace{\overline{\hat{u}_j'' \frac{\partial \hat{p}}{\partial \hat{x}_j}}}_{\hat{C}_k} + \underbrace{\left(\hat{\rho} \hat{k} \hat{u}_j + \overline{\hat{\rho} \hat{u}_j'' \tfrac{1}{2} \hat{u}_i'' \hat{u}_i''}\right) C_j}_{\hat{D}_k}$$



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$$0 = -P_k \underbrace{- \overline{\hat{\tau}_{ij} \frac{\partial \hat{u}_i''}{\partial \hat{x}_j}}}_{\hat{\varepsilon}_k} + \underbrace{\frac{\partial}{\partial \hat{x}_j} \left(\overline{\hat{u}_i'' \hat{\tau}_{ij}} - \overline{\hat{\rho} \hat{u}_j'' \tfrac{1}{2} \hat{u}_i'' \hat{u}_i''} \right)}_{\hat{T}_k} + \underbrace{- \overline{\hat{u}_j'' \frac{\partial \hat{p}}{\partial \hat{x}_j}}}_{\hat{C}_k} + \underbrace{\left(\hat{\rho} \hat{k} \hat{u}_j + \overline{\hat{\rho} \hat{u}_j'' \tfrac{1}{2} \hat{u}_i'' \hat{u}_i''} \right) C_j}_{\hat{D}_k}$$

Fully developed channel

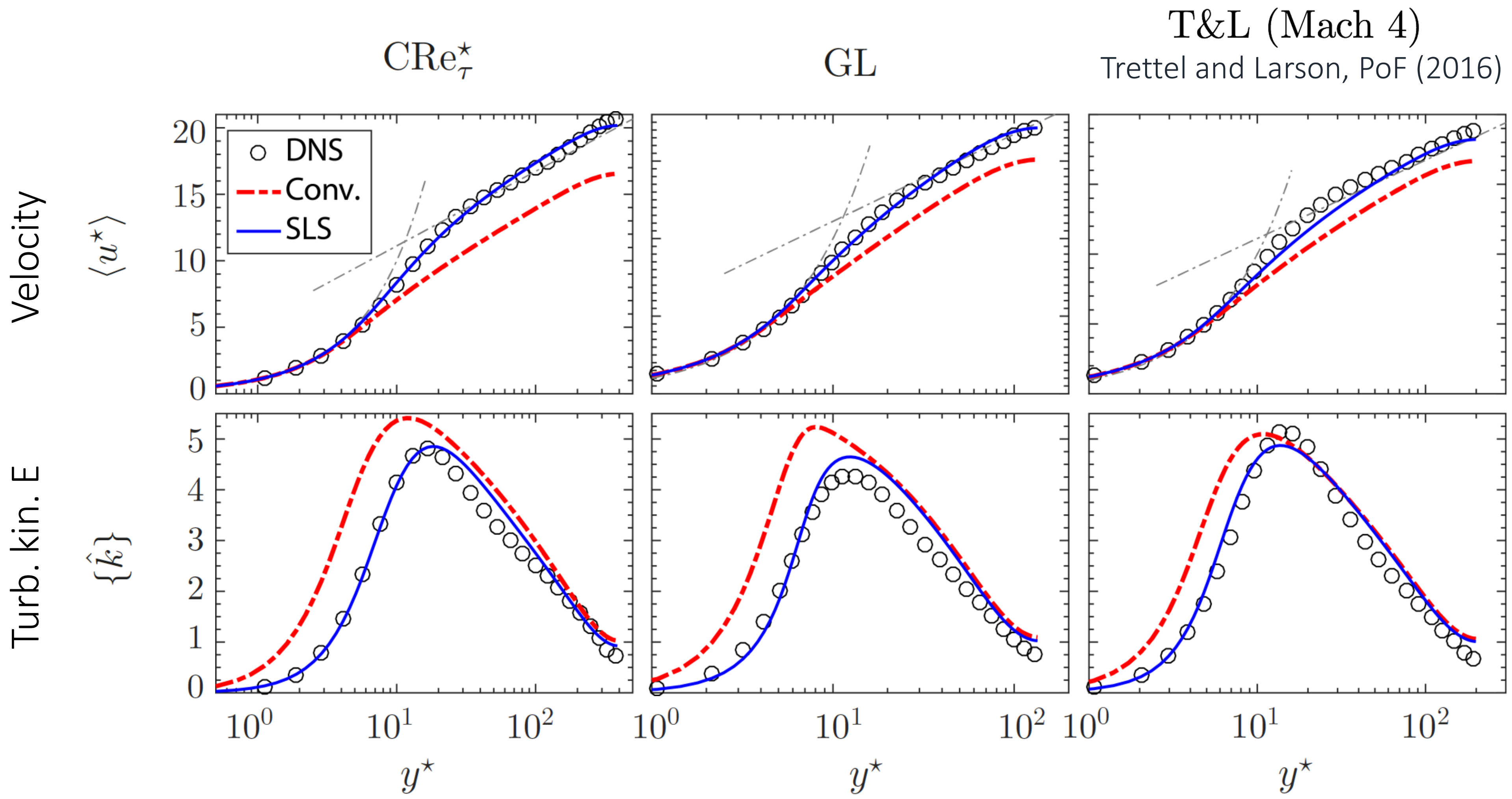
$$-\frac{\partial}{\partial y} \left[\left(\frac{1}{Re_\tau^\star} + \frac{\hat{\mu}_t}{\sigma_k} \right) \frac{\partial \hat{k}}{\partial y} \right] = \hat{\mu}_t \left(\frac{\partial u^{\mathrm{vD}}}{\partial y} \right)^2 - \hat{\varepsilon}$$

Transforming back to conventional scales

$$\frac{1}{\sqrt{\rho}} \frac{\partial}{\partial x_j} \left[\frac{1}{\sqrt{\rho}} \left(\frac{\mu}{Re_\tau} + \frac{\mu_t}{\sigma_k} \right) \frac{\partial \rho k}{\partial x_j} \right] = \mu_t \left(\frac{\partial u}{\partial y} \right)^2 - \rho \varepsilon$$

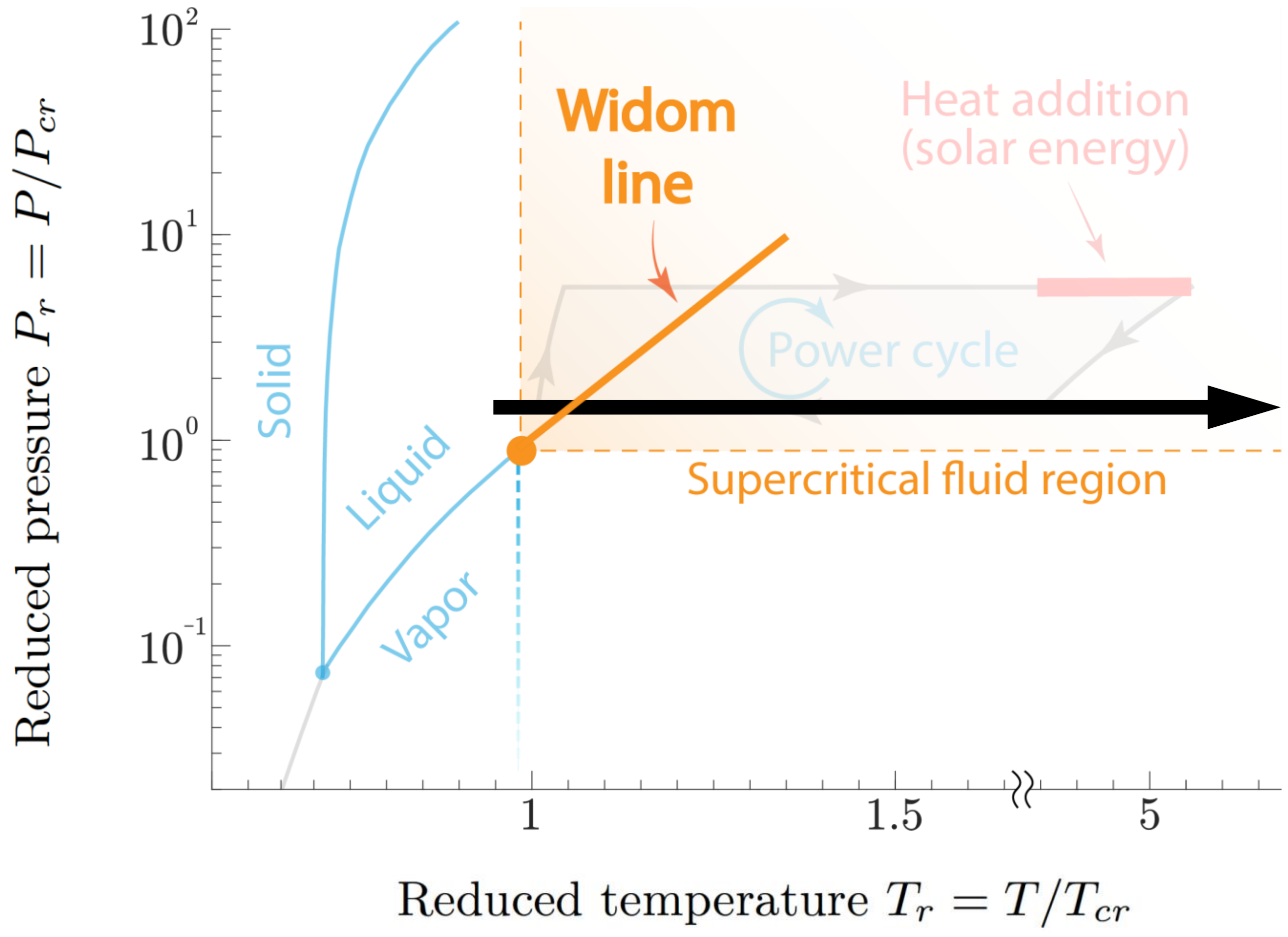
Diffusion of TKE acts upon energy per unit volume !

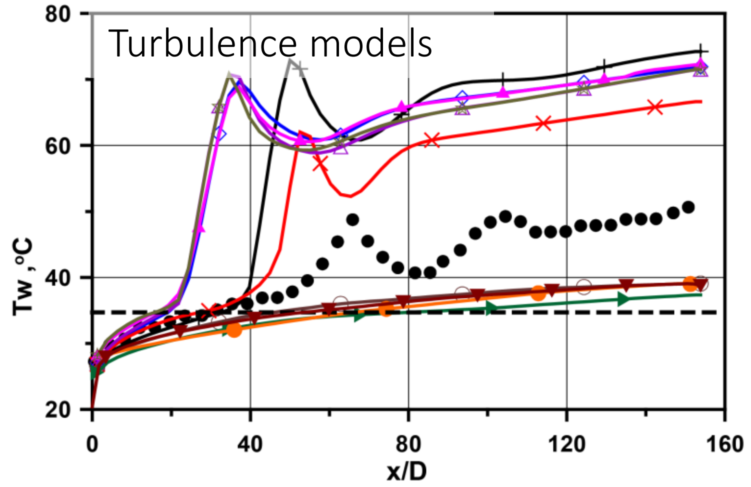
V2F turbulence model



Heat transfer to supercritical fluids

Turbulence models fail

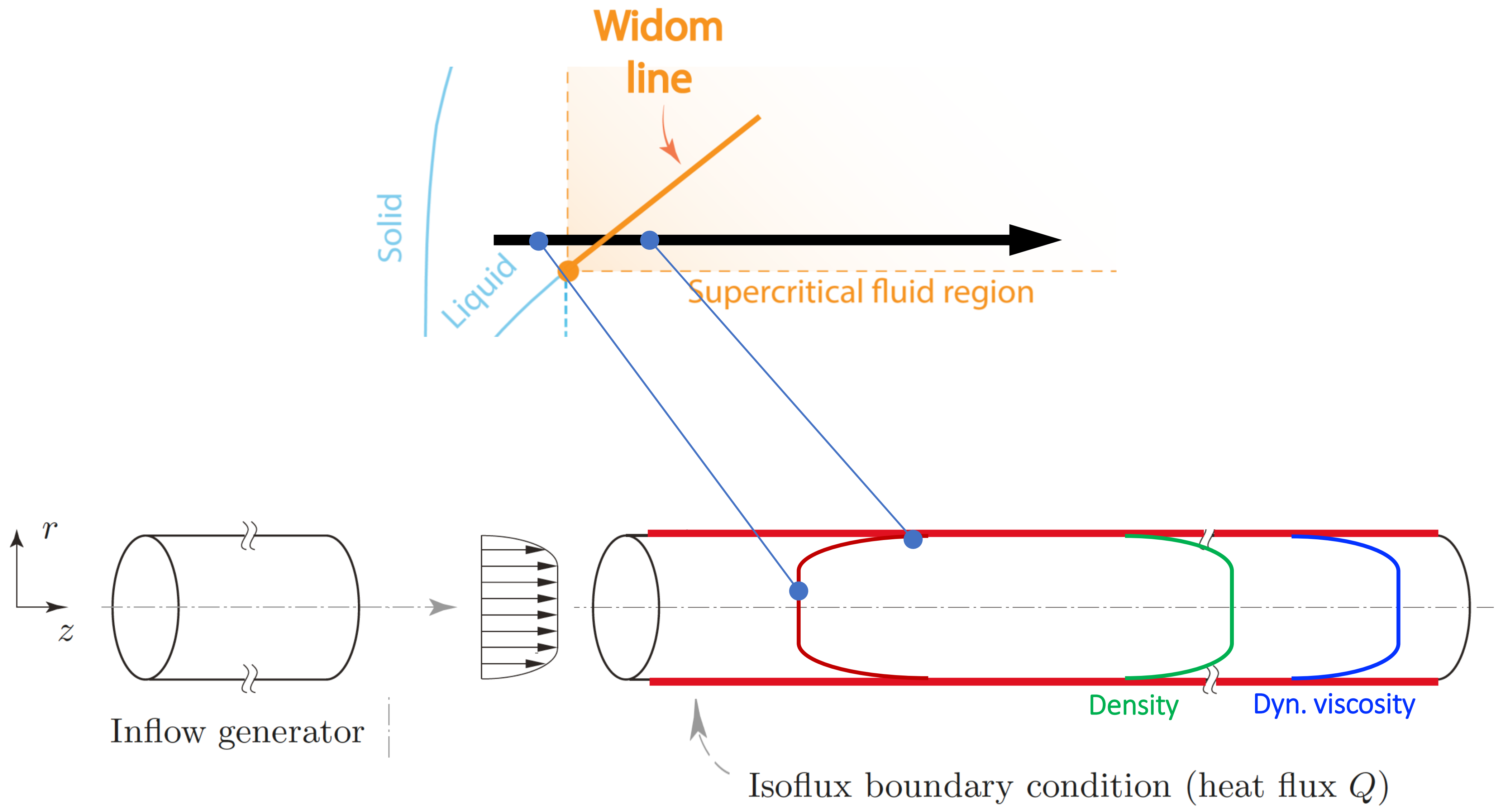




Turbulent heat transfer
(constant heat flux)

Sharabi, Ambrosini, Ann. Nuclear Energy (2009)

Numerical study of heat transfer using DNS



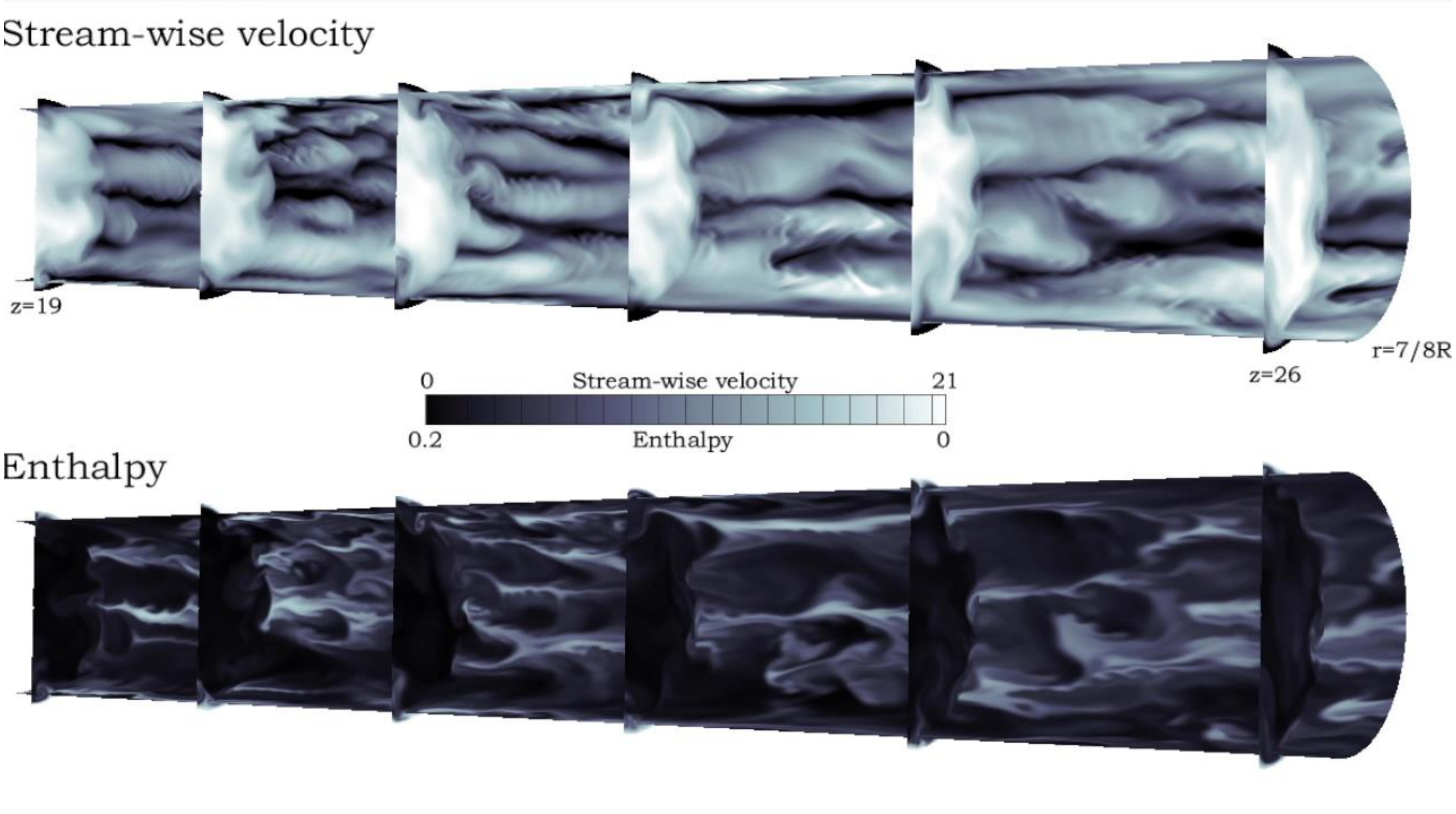
Considered cases

Case	Type	Direction / gravity	Richardson #
A	Forced	No gravity	0
B	Mixed	Upward flow ⇡	-10
C	Mixed	Upward flow ⇡	-270
D	Mixed	Downward flow ⇣	100

With:

- Reynolds number: $Re_{\tau,0} = \frac{\rho_0 u_{\tau,0} D}{\mu_0} = 360$
- Prandtl number: $Pr_0 = \frac{\mu c_{p,0}}{\lambda_0} = 3.19$
- Non-dimensional heat flux: $Q = \frac{q_w D}{\lambda_0 T_0} = 2.4$

Forced convection (case A)



Nemati et al., JFM (2016)

How do turbulence models perform?

Reynolds/Favre averaged equations

- Momentum equations

$$\frac{\partial \bar{\rho}\tilde{u}_i\tilde{u}_j}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\left(\bar{\mu}+\mu_t\right)2\bar{S}^{c}_{ij}\right] + Ri_{0,z}\bar{\rho}$$

- Enthalpy equation

$$\frac{\partial \bar{\rho}\tilde{h}\tilde{u}_j}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\frac{\bar{\lambda}}{\bar{c}_p}+\frac{\mu_t}{Pr_t}\right)\frac{\partial \tilde{h}}{\partial x_j}\right]$$

- Turbulent kinetic energy equation

$$\frac{\partial \bar{\rho}\tilde{u}_j k}{\partial x_j} = P_k - \bar{\rho}\varepsilon + \frac{\partial}{\partial x_j}\left[\left(\bar{\mu}+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + B_k$$

Gradient diffusion hypothesis for buoyant production

$$B_k = Gr_{z,0}\beta c_T T_t\left(\frac{2}{3}\bar{\rho}k\delta_{ij} - 2\mu_t S^{c}_{ij}\right)\frac{\partial T}{\partial x_i}$$

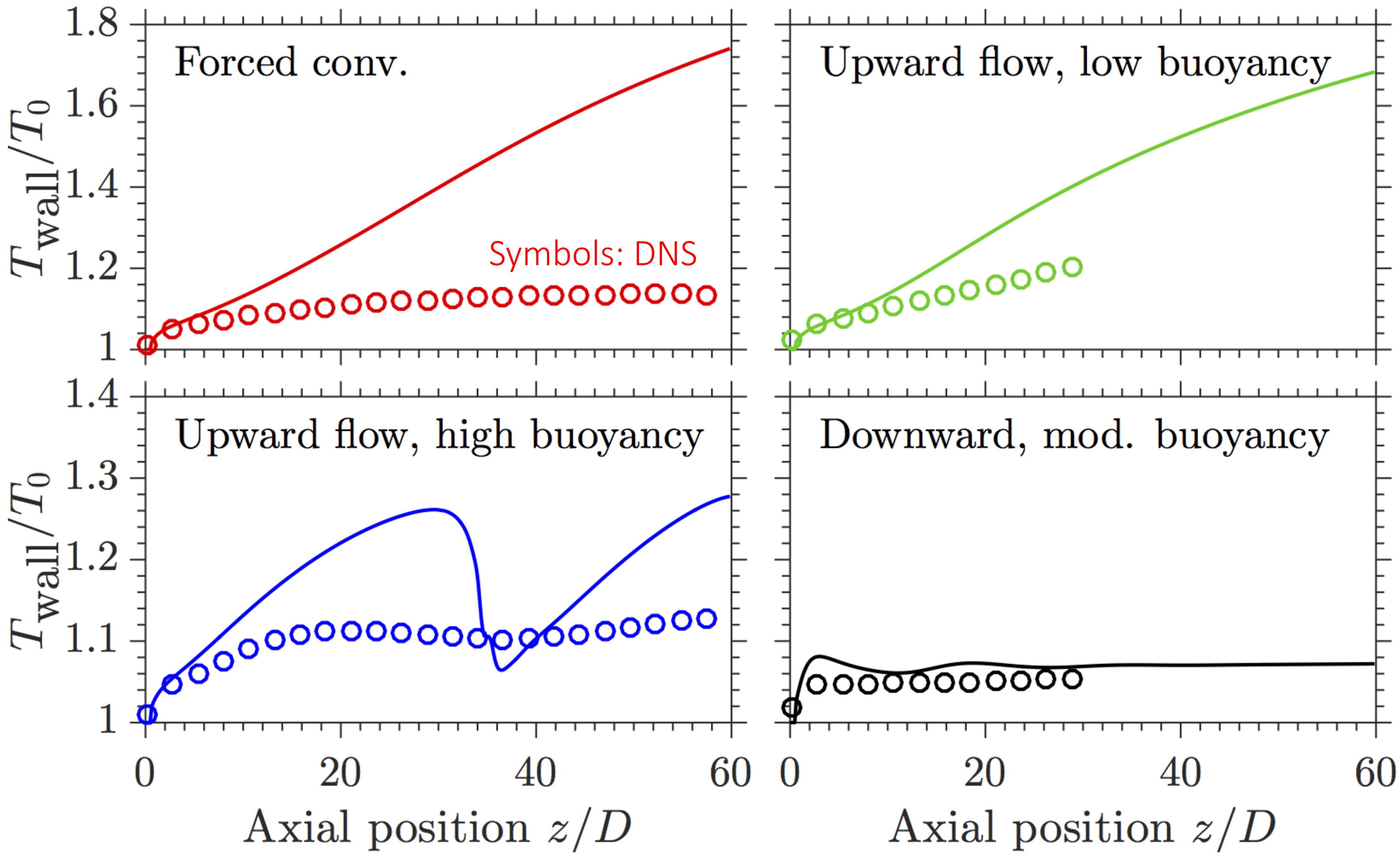
- Other turbulent model supporting equations, for example for V2F model (Durbin 1995)

$$\overline{v'^2}, \quad \varepsilon, \quad f$$

- Eddy viscosity

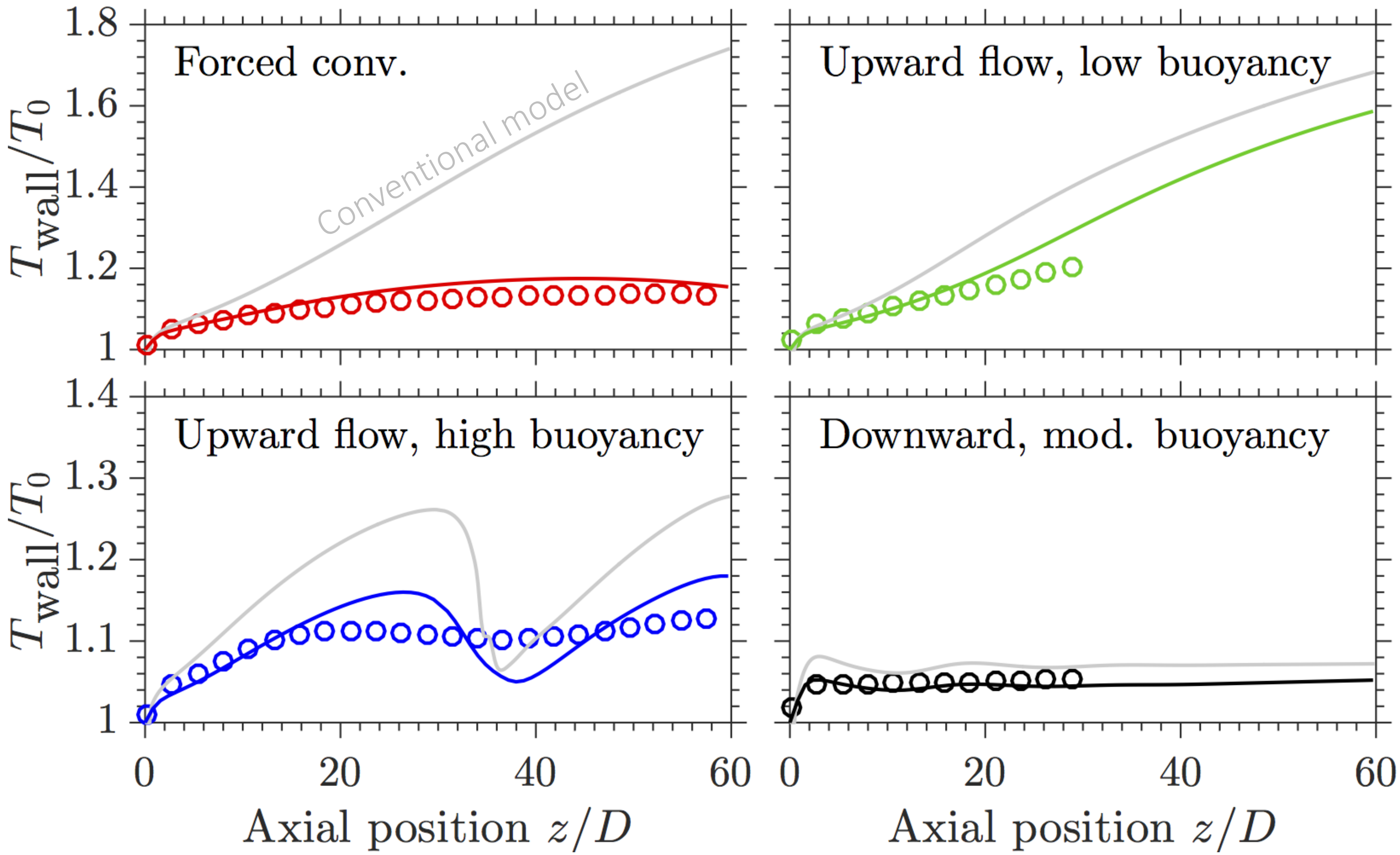
$$\mu_t = C_\mu \bar{\rho}\overline{v'^2}T_t$$

Model results V2F model for supercritical pipe flow



Model results – supercritical pipe flows

Symbols: DNS



https://github.com/Fluid-Dynamics-Of-Energy-Systems-Team/RANS_Channel

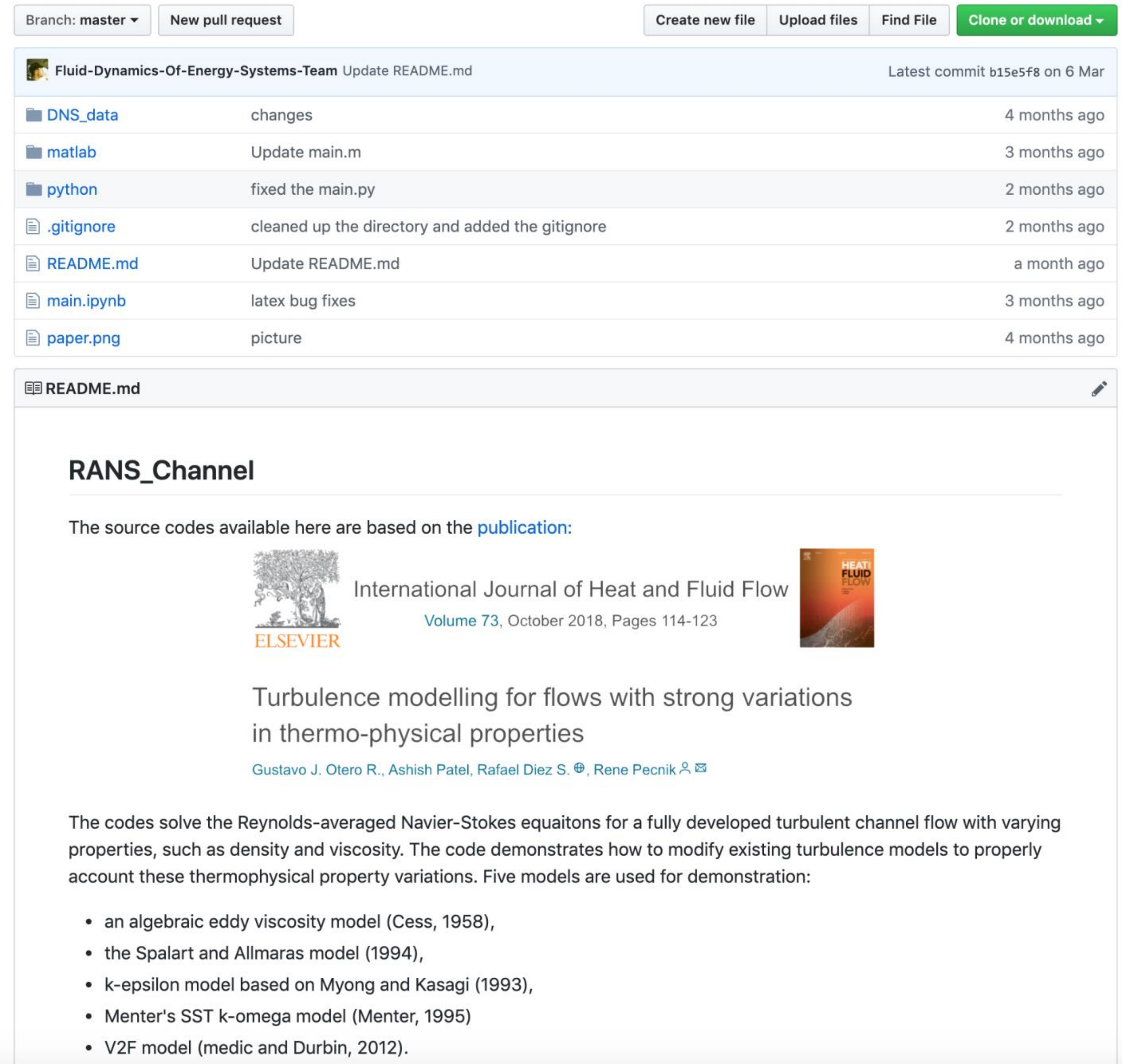
Conclusions

We proposed:

- Scaling for velocity
- Scaling for temperature
- Generic modification for turbulence models
- Improved results for supercritical flows

Future directions

- Target modeling buoyancy production term
- Test approach to experimental data at higher Reynolds numbers



Branch: master | New pull request | Create new file | Upload files | Find File | Clone or download

Fluid-Dynamics-Of-Energy-Systems-Team Update README.md — Latest commit b15e5f8 on 6 Mar

DNS_data	changes	4 months ago
matlab	Update main.m	3 months ago
python	fixed the main.py	2 months ago
.gitignore	cleaned up the directory and added the gitignore	2 months ago
README.md	Update README.md	a month ago
main.ipynb	latex bug fixes	3 months ago
paper.png	picture	4 months ago

README.md

RANS_Channel

The source codes available here are based on the publication:

ELSEVIER — International Journal of Heat and Fluid Flow, Volume 73, October 2018, Pages 114-123

Turbulence modelling for flows with strong variations in thermo-physical properties

Gustavo J. Otero R., Ashish Patel, Rafael Diez S., Rene Pecnik

The codes solve the Reynolds-averaged Navier-Stokes equaitons for a fully developed turbulent channel flow with varying properties, such as density and viscosity. The code demonstrates how to modify existing turbulence models to properly account these thermophysical property variations. Five models are used for demonstration:

- an algebraic eddy viscosity model (Cess, 1958),
- the Spalart and Allmaras model (1994),
- k-epsilon model based on Myong and Kasagi (1993),
- Menter's SST k-omega model (Menter, 1995)
- V2F model (medic and Durbin, 2012).